Math 2001: Spring 2019 **EXAM 1**

RULES

- **No books**, **no notes**, and **no calculators**.
- **No bathroom breaks** until after you have completed and submitted the exam.
- **All phones must be completely silent** for the duration of the exam, so please *turn off your phone now!*
- Out of consideration for your classmates, do not make disturbing noises during the exam.

Cheating will not be tolerated. If there is any indication that a student may have given or received unauthorized aid on this test, the case will be referred to the Office of the Chair of the Mathematics Department. When you finish the exam, you must sign the following pledge:

"On my honor as a student I, _____, have neither given nor received unauthorized aid on this exam." (print name clearly)

Signature: _____ Date: 27 February 2019

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Points:	13	12	9	10	6	50
Score:						

(6pts) 1. (a) Let A and B be sets. Recall, $A \subseteq B$ is defined as $(\forall x)(x \in A \to x \in B)$. Write down a similar definition of $A \not\subseteq B$. That is, give a formula that defines $\not\subseteq$.

[**Hint.** Use symbols such as $\exists$, $\in$, $\notin$, $\wedge$, $\vee$, $\to$, etc.; do *not* simply write $\neg(A \subseteq B)$.]

(7pts) (b) Define the following sets:

$$C = \emptyset, \quad D = \{0\}, \quad E = \{1\}, \quad F = \{0, 1\}, \quad G = \{1, 2, 3\}, \quad H = \{0, 1, 2, 3\}.$$

and circle True or False, as appropriate.

i. $C \in D$ True False

ii. $C = D$ True False

iii. $D \subseteq E$ True False

iv. $D \in D$ True False

v. $F \cap G = E$ True False

vi. $F \cup G = H$ True False

vii. $G \cap H = G$ True False

Score for this page: _____ out of 13

2. Let the variables M, P, V, stand for the following statements:

   M = "my website makes **M**oney"

   P = "my website is behind a **P**aywall"

   V = "my website has **V**ideo content"

   Use these propositional variables and the logical connectives (i.e., $\wedge$, $\vee$, $\rightarrow$, $\neg$) to express statements (a), (b), and (c) below.

   Example. My website has Video and makes Money.

   Answer. $V \wedge M$

(2pts) (a) Either my website has Video or it makes no Money.

(2pts) (b) If my website is behind a Paywall, then it makes Money.

(2pts) (c) My website has no video, but it is behind a Paywall.

(6pts) (d) Show that the statements (a), (b), and (c) are contradictory by completing the following natural deduction proof skeleton. (Do **not** assume the statement in the Example.)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg V \wedge P}\,(\wedge \mathrm{E}_r)}{V}}{\bot}\qquad \dfrac{\dfrac{}{\neg V \wedge P}}{\neg M}}{\bot}\,(\rightarrow \mathrm{E})}{\bot}\,(\vee \mathrm{E})$$

Score for this page: _____ out of 12

(9pts) 3. Give a natural deduction proof of $(A \wedge B) \to ((A \to C) \to \neg(B \to \neg C))$.

[**Hint.** Start by assuming $A \wedge B$ and $A \to C$ and $B \to \neg C$; then derive a contradiction. Be sure your proof tree ends with $(A \wedge B) \to ((A \to C) \to \neg(B \to \neg C))$.]

Score for this page: _____ out of 9

4. Let P be the statement $A \vee \neg B \longrightarrow \neg(\neg A \wedge B)$;

(8pts) (a) Construct a truth table for P.

A	B	$\neg A$	$\neg B$				P

(2pts) (b) The statement P above is a (circle one)

tautology **contradiction** **neither**

Score for this page: _____ out of 10

(6pts) 5. Fill in the blanks in the following Lean proofs.

```
variables A B C : Prop
theorem exercise4 (h₁ : A ∨ B) (h₂ : A → C) (h₃ : B → D) : C ∨ D :=
or.elim _____________
   ( assume a : A,
     have c : C, from _____________,
     or.inl _____________
   )
   ( assume _____________,
     have _____________, from h₃ b,
     _____________
   )
```

Score for this page: _____ out of 6

– scratch –

– scratch –