HW 6

DUE: Friday, March 9, 9am
Section 2.9. 2, 6, 8, 18;
Section 3.1: 10, 28, 32, 38, 40;
Section 3.2: 2, (3).

Section 2.9

Exercises 2, 6, (8), 18

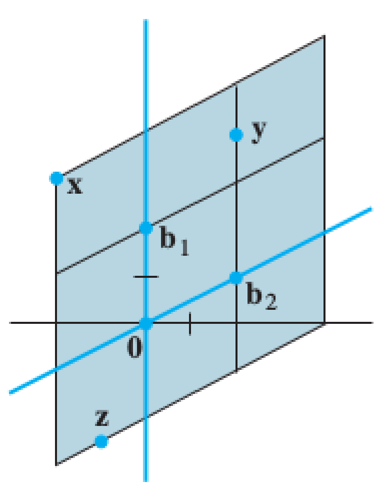
2.9.2. Find the vector $\mathbf{x}$ determined by the given coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ and the given basis $\mathcal{B}$. Illustrate your answer with a figure, as in the solution to Practice Problem 2 in the textbook.

$$\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}, \quad [\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}.$$

2.9.6. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, where $\mathbf{b}_1 = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix}$. The vector $\mathbf{x} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}$ belongs to $\text{Span}\{\mathbf{b}_1, \mathbf{b}_2\}$. Find the $\mathcal{B}$-coordinate representation of $\mathbf{x}$.

2.9.8. (recommended)

Let $\mathbf{b}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $\mathbf{z} = \begin{bmatrix} -1 \\ -2.5 \end{bmatrix}$, and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Use the figure to estimate $[\mathbf{x}]_{\mathcal{B}}, [\mathbf{y}]_{\mathcal{B}}$, and $[\mathbf{z}]_{\mathcal{B}}$. Confirm your estimates of $[\mathbf{y}]_{\mathcal{B}}$ and $[\mathbf{z}]_{\mathcal{B}}$ by using them and $\{\mathbf{b}_1, \mathbf{b}_2\}$ to compute $\mathbf{y}$ and $\mathbf{z}$.



2.9.18. Mark each statement True or False. Justify each answer. Here A is an $m \times n$ matrix.
a. If $\mathcal{B}$ is a basis for a subspace H, then each vector in H can be written in only one way as a linear combination of the vectors in $\mathcal{B}$.
b. If $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a basis for a subspace H of $\mathbb{R}^n$, then the correspondence $\mathbf{x} \mapsto [\mathbf{x}]_{\mathcal{B}}$ makes H look and act the same as $\mathbb{R}^p$.
c. The dimension of $\mathrm{Nul}\, A$ is the number of variables in the equation $A\mathbf{x} = \mathbf{0}$.
d. The dimension of the column space of A is $\mathrm{rank}\, A$.
e. If H is a p-dimensional subspace of $\mathbb{R}^n$, then a linearly independent set of p vectors in H is a basis for H.

Section 3.1

Exercises: 10, 28, 32, 38, 40

3.1.10. Compute the determinant of the following matrix using cofactor expansions. At each step, choose a row or column that involves the least amount of computation.

$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{vmatrix}$$

3.1.28. Compute the determinant of the elementary matrix,

$$\begin{bmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3.1.32. What is the determinant of an elementary scaling matrix with k on the diagonal?

3.1.38. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and let k be a scalar. Find a formula that relates $\det(kA)$ to k and $\det A$.

3.1.40. Let A be an $n \times n$ matrix. Mark each of the following statements True or False. Justify.

a. The cofactor expansion of $\det A$ down a column is equal to the cofactor expansion along a row.

b. The determinant of a triangular matrix is the sum of the entries on the main diagonal.

Section 3.2

Exercises: 2, (3)

3.2.2. The following equation illustrates a property of determinants. State the property.

$$\begin{vmatrix} 1 & 2 & 2 \\ 0 & 3 & -4 \\ 3 & 7 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 0 & 3 & -4 \\ 0 & 1 & -2 \end{vmatrix}$$

3.2.3. (recommended, not required) The following equation illustrates a property of determinants. State the property.

$$\begin{vmatrix} 3 & -6 & 9 \\ 3 & 5 & -5 \\ 1 & 3 & 3 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 & 3 \\ 3 & 5 & -5 \\ 1 & 3 & 3 \end{vmatrix}$$
