HW 12

DUE Wednesday, May 2

Note: This assignment may be submitted anytime before Friday, May 4, 3pm. If you don't have it ready to turn in during Wednesday's class, please drop your completed homework off in my office (Math Room 202) sometime before 3pm on Friday, May 4. (Slip it under the door if no one is in the office when you arrive.)

List of Exericses

Section 6.3: 6, 8, 12, (13), 16
Section 6.4: 4, 8, 12, 16, 18,
Section 6.5: 4, 8, 10, 18.

Section 6.3

Exercises: 6, 8, 12, (13), 16

6.3.6. Let $\mathbf{y} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Verify that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set, and then find the orthogonal projection of $\mathbf{y}$ onto $\mathrm{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

6.3.8. Let $\mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$. Let W be the subspace spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$, and write $\mathbf{y}$ as the sum of a vector in W and a vector orthogonal to W.

6.3.12. Let $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$. Find the closest point to $\mathbf{y}$ in the subspace W spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.

6.3.13. (recommended)
Let $\mathbf{z} = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$. Find the best approximation to $\mathbf{z}$ by a vector of the form $c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

6.3.16. Let $\mathbf{y}$, $\mathbf{v}_1$, $\mathbf{v}_2$ be as in Exercise 12. Find the distance from $\mathbf{y}$ to the subspace W spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.

Section 6.4

Exercises 4, 8, 12, 16, 18

6.4.4. Let W be a subspace with basis given by the vectors $\begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix}$. Use the Gram-Schmidt process to produce an orthogonal basis for W.

6.4.8. Find an orthonormal basis for the subspace W in Exercise 4.

6.4.12. Find an orthogonal basis for the column space of the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}$.

6.4.16. Find a QR factorization for the matrix in Exercise 12.

6.4.18. Mark each of the following statements True or False. Justify your answers. Assume all vectors and subspaces are in $\mathbb{R}^n$.

a. If $W = \text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$, where $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is a linearly independent set, and if $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set in W, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for W.

b. If $\mathbf{x}$ does not belong to the subspace W, then $\mathbf{x} - \text{proj}_W\, \mathbf{x}$ is not zero.

c. In a QR factorization, say $A = QR$ (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A.

Section 6.5

Exercises 4, 8, 10, 18

6.5.4. Let $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$. Find a least-squares solution to $A\mathbf{x} = \mathbf{b}$ in two ways:

(a) by constructing the normal equations for $\hat{\mathbf{x}}$ and

(b) by solving for $\hat{\mathbf{x}}$.

6.5.8. Compute the error associated with the least-squares solution found in Exercise 4.

6.5.10. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$. Find

(a) the orthogonal projection of $\mathbf{b}$ onto $\text{Col}\, A$ and

(b) a least-squares solution to $A\mathbf{x} = \mathbf{b}$.

6.5.18. Mark each statement True or False. Justify each answer. Assume A is an $m \times n$ matrix and $\mathbf{b}$ is in $\mathbb{R}^m$.

a. If $\mathbf{b}$ is in the column space of A, then every solution to $A\mathbf{x} = \mathbf{b}$ is a least-squares solution.

b. The least-squares solution to $A\mathbf{x} = \mathbf{b}$ is the point in the column space of A closest to $\mathbf{b}$.

c. A least-squares solution to $A\mathbf{x} = \mathbf{b}$ is a list of weights that, when applied to the columns of A, produces the orthogonal projection of $\mathbf{b}$ onto $\text{Col}\, A$.

d. If $\hat{\mathbf{x}}$ is the least-squares solution to $A\mathbf{x} = \mathbf{b}$, then $\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}$

e. The normal equations always provide a reliable method for computing least-squares solutions.

f. If A has a QR factorization, say $A = QR$, then the best way to find the least-squares solution to $A\mathbf{x} = \mathbf{b}$ is to compute $\hat{\mathbf{x}} = R^{-1} Q^T \mathbf{b}$.