HW 2

Due Friday, February 2, 9am
Section 1.3. 2, 6, 8, 10, 14, (19), (21), 24, 26;
Section 1.4. 2, 10, (13), 14, 18, 20, 22, (27), (29), (31), 32.
(Numbers in parentheses are recommended exercises; please do not submit solutions to these exercises.)

Section 1.3

Exercises 2, 6, 8, 10, 14, (19), (21), 24, 26

1.3.2. Compute $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-2\mathbf{v}$, where $\mathbf{u}=\begin{bmatrix}3\\2\end{bmatrix}$ and $\mathbf{v}=\begin{bmatrix}2\\-1\end{bmatrix}$.

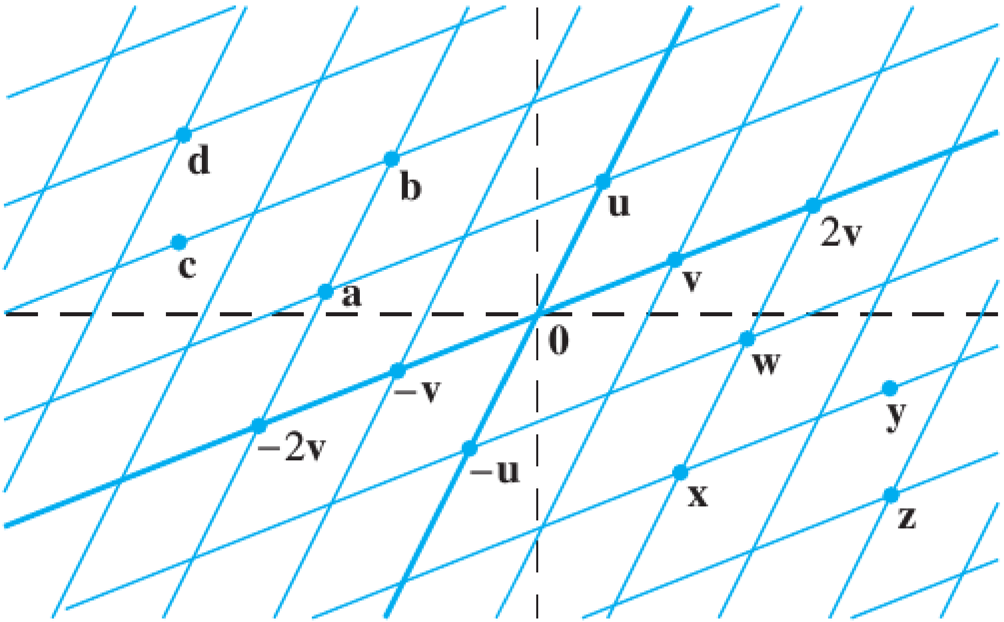
1.3.6. Write a system of equations that is equivalent to the vector equation,

$$x_1\begin{bmatrix}-2\\3\end{bmatrix}+x_2\begin{bmatrix}8\\5\end{bmatrix}+x_3\begin{bmatrix}1\\-6\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

1.3.8. Answer parts (a) and (b).

(a) Using the accompanying figure, write each of the vectors $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

(b) Is every vector in $\mathbb{R}^2$ a linear combination of $\mathbf{u}$ and $\mathbf{v}$?



1.3.10. Write a vector equation that is equivalent to the following system of equations:

$$\begin{aligned} 4x_1 + x_2 + 3x_3 &= 9 \\ x_1 - 7x_2 - 2x_3 &= 2 \\ 8x_1 + 6x_2 - 5x_3 &= 15 \end{aligned}$$

1.3.14. Determine if $\mathbf{b}$ is a linear combination of the vectors formed from the columns of the matrix A, where

$$A = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}.$$

(1.3.19) Give a geometric description of Span$\{\mathbf{v}_1, \mathbf{v}_2\}$ for the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}.$$

(1.3.21) Let $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. Show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ for all h and k.

1.3.24. Mark each statement True or False. Justify each answer.

a. Any list of five real numbers is a vector in $\mathbb{R}^5$.

b. The vector $\mathbf{u}$ results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector $\mathbf{v}$.

c. The weights $c_1, \ldots, c_p$ in a linear combination $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p$ cannot all be zero.

d. When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains the line passing through $\mathbf{u}$ and the origin.

e. Asking whether the linear system corresponding to an augmented matrix $[\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{b}]$ has a solution amounts to asking whether $\mathbf{b}$ is in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.

1.3.26.

Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$, let $\mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$, and let W be the set of all linear combinations of the columns of A. Answer (a) and (b).

(a) Is $\mathbf{b}$ in W?

(b) Show that the third column of A is in W.

Section 1.4

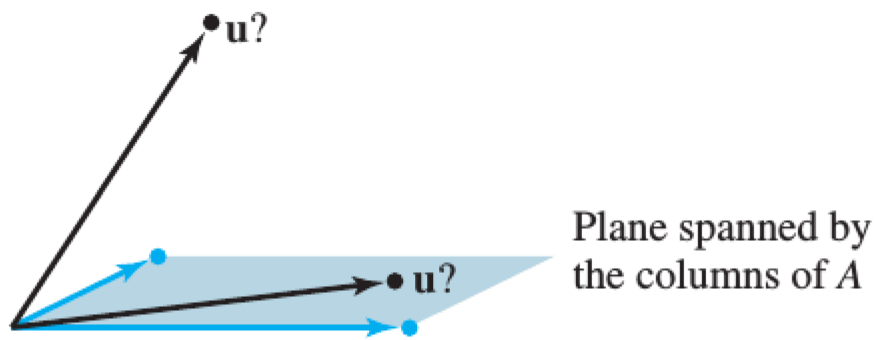
Exercises 2, 10, (13), 14, 18, 20, 22, (27), (29), (31), 32

1.4.2. Compute the product $\begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \end{bmatrix}$. If the product is undefined, explain why.

1.4.10. Write the following system first as a vector equation and then as a matrix equation:

$$\begin{aligned} 8x_1 - x_2 &= 4 \\ 5x_1 + 4x_2 &= 1 \\ x_1 - 3x_2 &= 2 \end{aligned}$$

(1.4.13) Let $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is $\mathbf{u}$ in the plane spanned by the columns of A? (See the figure below.) Why or why not?



Where is **u**?

1.4.14. Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the plane spanned by the columns of A? (See the figure above.) Why or why not?

Exercises 18 and 20 refer to the matrix B below. Make appropriate calculations that justify your answers and mention an appropriate theorem.

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix}$$

1.4.18. Do the columns of B span $\mathbb{R}^4$? Does the equation $B\mathbf{x} = \mathbf{y}$ have a solution for each $\mathbf{y}$ in $\mathbb{R}^4$?

1.4.20. Can every vector in $\mathbb{R}^4$ be written as a linear combination of the columns of the matrix B above? Do the columns of B span $\mathbb{R}^3$?

1.4.22. Let $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}$.

Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^3$? Why or why not?

(1.4.27) Let $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$, and $\mathbf{v}$ be vectors in $\mathbb{R}^5$, and let x_1, x_2, and x_3 denote scalars. Write the vector equation

$$x_1\mathbf{q}_1 + x_2\mathbf{q}_2 + x_3\mathbf{q}_3 = \mathbf{v}$$

as a matrix equation. Identify any symbols you choose to use.

(1.4.29) Construct a 3×3 matrix, not in echelon form, whose columns span $\mathbb{R}^3$. Show that the matrix you construct has the desired property.

(1.4.31) Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^3$. Generalize your argument to the case of an arbitrary A with more rows than columns.

1.4.32. Could a set of three vectors in $\mathbb{R}_4$ span all of $\mathbb{R}_4$? Explain. What about n vectors in $\mathbb{R}_m$ when n is less than m?