# HW 10

**Due: Friday, April 20, 9am.**

## Exercises

**Section 5.3:** 8, 16, 22
**Section 5.4:** 2, 12, (21), (25), 26, (27)
**Section 6.1:** 6, 10, 14, 20

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## Section 5.3

**Exercises: 8, 16, 22**

Diagonalize the matrices in Exercises 8 and 16, if possible. The eigenvalues for Exercise 16 are $1$ and $2$.

**5.3.8.** $\begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$

**5.3.16.** $\begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$

**5.3.22.** Let $A$, $B$, $P$, and $D$ be $n \times n$ matrices. Mark each statement True or False. Justify each answer. (Study Theorems 5 and 6 and the examples in this section carefully before you try this exercise.)

**a.** $A$ is diagonalizable if $A$ has $n$-eigenvectors.

**b.** If $A$ is diagonalizable, then $A$ has $n$ distinct eigenvalues.

**c.** If $AP = PD$, with $D$ diagonal, then the nonzero columns of $P$ must be eigenvectors of $A$.

**d.** If $A$ is invertible, then $A$ is diagonalizable.

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## Section 5.4

**Exercises: 2, 12, (21), (25), 26, (27)**

**5.4.2.** Let $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be bases for vector spaces $V$ and $W$, respectively. Let $T : V \to W$ be a linear transformation satisfying

$$T(\mathbf{d}_1) = 2\mathbf{b}_1 - 3\mathbf{b}_2 \quad \text{and} \quad T(\mathbf{d}_2) = -4\mathbf{b}_1 + 5\mathbf{b}_2.$$

Find the matrix for $T$ relative to $\mathcal{D}$ and $\mathcal{B}$.

**5.4.12.** Find the $\mathcal{B}$-matrix for the transformation $\mathbf{x} \mapsto A\mathbf{x}$, when $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$, where

$$A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

**5.4.21.** (recommended)
Prove the following statement for square matrices, $A$, $B$, $C$. If $B$ is similar to $A$ and $C$ is similar to $A$, then $B$ is similar to $C$.

**5.4.25.** (recommended)
The *trace* of a square matrix $A$ is the sum of the diagonal entries in $A$ and is denoted by $\mathtt{tr}\, A$. It can be verified that $\mathtt{tr}(FG) = \mathtt{tr}(GF)$ for any two $n \times n$ matrices $F$ and $G$. Show that if $A$ and $B$ are similar, then $\mathrm{tr}\, A = \mathrm{tr}\, B$.

**5.4.26.** It can be shown that the *trace* (see Exercise 5.4.25 for definition) of a matrix $A$ equals the sum of the eigenvalues of $A$. Verify this statement for the case when $A$ is diagonalizable.

**5.4.27.** (recommended)
Let $V$ be $\mathbb{R}^n$ with a basis $\mathcal{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$, let $W$ be $\mathbb{R}^n$ with a the standard basis, denoted $\mathcal{E}$. Consider the identity transformation $I : V \to W$, where $I(\mathbf{x}) = \mathbf{x}$. Find the matrix for $I$ relative to $\mathcal{B}$ and $\mathcal{E}$. What was this matrix called in Section 4.4?

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## Section 6.1

**Exercises: 6, 10, 14, 20**

**6.1.6.** Let $\mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$. Compute $\left(\frac{\mathbf{x}\cdot\mathbf{w}}{\mathbf{x}\cdot\mathbf{x}}\right)\mathbf{x}$

**6.1.10.** Find a unit vector in the direction of the vector $\begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$

**6.1.14.** Find the distance between $\mathbf{u} = \begin{bmatrix} 0 \\ -5 \\ 2 \end{bmatrix}$ and $\mathbf{z} = \begin{bmatrix} -4 \\ -1 \\ 8 \end{bmatrix}$

**6.1.20.** Mark each statement True or False. Justify each answer. All vectors are assumed to be in $\mathbb{R}^n$.

**a.** $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$

**b.** For any scalar $c$, $\|c\mathbf{v}\| = c\|\mathbf{v}\|$.

**c.** If $\mathbf{x}$ is orthogonal to every vector in a subspace $W$, then $\mathbf{x}$ is in $W^\perp$.

**d.** If $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \|\mathbf{u} + \mathbf{v}\|^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

**e.** For an $m \times n$ matrix $A$, vectors in the null space of $A$ are orthogonal to vectors in the row space of $A$.