HW 10

Due: Monday, April 23, 9am.

Exercises

Section 5.1: 6, 16, 22abde, 24;
Section 5.2: 8, 12, 16, 22bcd;
Section 5.3: 8, 16, 22.

Section 5.1

Exercises: 6, 16, 22abde, 24

5.1.6. Is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ and eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, find the corresponding eigenvalue.

5.1.16. Let $A = \begin{bmatrix} 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Find a basis for the eigenspace corresponding to the eigenvalue $\lambda = 4$.

5.1.22. Let A be an $n \times n$ matrix. Mark each statement True or False.

a. If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ, then $\mathbf{x}$ is an eigenvector of A.

b. If $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues.

d. The eigenvalues of a matrix are the numbers on its main diagonal.

e. An eigenspace of A is the null space of a certain matrix.

5.1.24. Construct an example of a 2×2 matrix with only one distinct eigenvalue.

Section 5.2

Exercises: 8, 12, 16, 22bcd

5.2.8. Find the characteristic polynomial and the eigenvalues of the matrix $\begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}$.

5.2.12. Find the characteristic polynomial of the matrix $\begin{bmatrix} -1 & 0 & 1 \\ -3 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

5.2.16. List the eigenvalues, repeated according to their multiplicity, for the matrix $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 8 & -4 & 0 & 0 \\ 0 & 7 & 1 & 0 \\ 1 & -5 & 2 & 1 \end{bmatrix}$.

5.2.22. Suppose A and B are $n \times n$ matrices. Mark each statement True or False.

b. $\det A^T = (-1) \det A$.

c. The multiplicity of a root r of the characteristic equation of A is called the algebraic multiplicity of r.

d. A row replacement operation on A does not change the set of eigenvalues.

Section 5.3

Exercises: 8, 16, 22

Diagonalize the matrices in Exercises 8 and 16, if possible.
Hint: The eigenvalues for Exercise 16 are 1 and 2.

5.3.8. $\begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$

5.3.16. $\begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$

5.3.22. Let A, B, P, and D be $n \times n$ matrices. Mark each statement True or False. Justify each answer. (Study Theorems 5 and 6 and the examples in this section carefully before you try this exercise.)

a. A is diagonalizable if A has n-eigenvectors.

b. If A is diagonalizable, then A has n distinct eigenvalues.

c. If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A.

d. If A is invertible, then A is diagonalizable.