HW 5

Due Friday, February 23, 9am
Section 2.3. 6, 12, 18, 22, 34;
Section 2.8. 6, 10, 18, 22, 24;

Section 2.3

Exercises 6, 12, 18, 22, 34

2.3.6. Determine whether the matrix $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 0 \end{bmatrix}$ is invertible. Use as few calculations as possible. Justify your answers.

2.3.12. In this exercise, the matrices are all $n \times n$. Each part of the exercise is an implication of the form "If 'statement 1', then 'statement 2'." Mark an implication as True if the truth of 'statement 2' always follows whenever 'statement 1' happens to be true. An implication is False if there is an instance in which 'statement 2' is false but 'statement 1' is true. Justify each answer.

a. If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$.
b. If the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$.
c. If the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b} \in \mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$.
d. If the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, then A has n pivot positions.
e. If there is a $\mathbf{b}$ in $\mathbb{R}^n$ such that the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is not one-to-one.

2.3.18. If C is a 6×6 matrix and the equation $C\mathbf{x} = \mathbf{v}$ is consistent for every $\mathbf{v}$ in $\mathbb{R}^6$, is it possible that, for some $\mathbf{v}$, the equation $C\mathbf{x} = \mathbf{v}$ has more than one solution? Why or why not?

2.3.22. If the equation $H\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbb{R}^n$, what can you say about the equation $H\mathbf{x} = \mathbf{0}$? Why?

2.3.34. Suppose T is the linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ given by $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$. Show that T is invertible and find a formula for T^{-1}.

Section 2.8

Exercises 6, 10, 18, 22, 24

2.8.6. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ -7 \\ 9 \\ 7 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -8 \\ 6 \\ 5 \end{bmatrix}$, and $\mathbf{u} = \begin{bmatrix} -4 \\ 10 \\ -7 \\ -5 \end{bmatrix}$. Determine if $\mathbf{u}$ is in the subspace of $\mathbb{R}^4$ generated by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

2.8.10. Let $\mathbf{v}_1 = \begin{bmatrix} -3 \\ 0 \\ 6 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -6 \\ 3 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$, and $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$. Determine if $\mathbf{u}$ is in $\operatorname{Nul} A$.

2.8.18. Determine whether the set $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$ is a basis for $\mathbb{R}^2$ or $\mathbb{R}^3$. Justify your answer.

2.8.22. Mark each statement True or False. Justify each answer.
a. A subset H of $\mathbb{R}^n$ is a subspace if the zero vector is in H.
b. Given vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$, the set of all linear combinations of these vectors is a subspace of $\mathbb{R}^n$.
c. The null space of an $m \times n$ matrix is a subspace of $\mathbb{R}^n$.
d. The column space of a matrix A is the set of solutions of $A\mathbf{x} = \mathbf{b}$.
e. If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A.

2.8.24. Consider the following echelon form of the matrix A:

$$A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Col A and a basis for $\operatorname{Nul} A$.