HW 11

DUE Friday, April 27, 9am

List of Exercises

Section 6.1: 16, (27), 28, (30)
Section 6.2: 10, 12, (13), 16, 24, (33)
Section 6.3: 6, 8, 12, (13), 16

Section 6.1

Exercises: 16, (27), 28, (30)

6.1.16. Determine whether $\mathbf{u} = \begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$ are orthogonal vectors.

6.1.27. (recommended)
Suppose a vector $\mathbf{y}$ is orthogonal to vectors $\mathbf{u}$ and $\mathbf{v}$. Show that $\mathbf{y}$ is orthogonal to the vector $\mathbf{u} + \mathbf{v}$.

6.1.28. Suppose $\mathbf{y}$ is orthogonal to $\mathbf{u}$ and $\mathbf{v}$. Show that $\mathbf{y}$ is orthogonal to every $\mathbf{w}$ in $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$. [*Hint:* An arbitrary $\mathbf{w}$ in $\mathrm{Span}\{\mathbf{u}, \mathbf{v}\}$ has the form $\mathbf{w} = c_1\mathbf{u} + c_2\mathbf{v}$; show that $\mathbf{y}$ is orthogonal to every such a vector.]

6.1.30. (recommended)
Let W be a subspace of $\mathbb{R}^n$, and let $W^\perp$ be the set of all vectors orthogonal to W. Show that $W^\perp$ is a subspace of $\mathbb{R}^n$ using the following steps.

a. Take $\mathbf{z}$ in $W^\perp$, and let $\mathbf{u}$ represent any element of W. Then $\mathbf{z} \cdot \mathbf{u} = 0$. Take any scalar c and show that $c\mathbf{z}$ is orthogonal to $\mathbf{u}$. (Since $\mathbf{u}$ was an arbitrary element of W, this will show that $c\mathbf{z}$ is in $W^\perp$.)

b. Take $\mathbf{z}_1$ and $\mathbf{z}_2$ in $W^\perp$, and let $\mathbf{u}$ be any element of W. Show that $\mathbf{z}_1 + \mathbf{z}_2$ is orthogonal to $\mathbf{u}$. What can you conclude about $\mathbf{z}_1 + \mathbf{z}_2$? Why?

c. Finish the proof that $W^\perp$ is a subspace of $\mathbb{R}^n$.

Section 6.2

Exercises: 10, 12, (13), 16, 24, (33)

6.2.10. Let $\mathbf{u}_1 = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$. Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$. Then express $\mathbf{x}$ as a linear combination of the $\mathbf{u}$'s.

6.2.12. Compute the orthogonal projection of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ onto the line through $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

6.2.13. (recommended)
Let $\mathbf{y} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$. Write $\mathbf{y}$ as the sum of two orthogonal vectors, one in $\text{Span}\{\mathbf{u}\}$ and the other orthogonal to $\mathbf{u}$.

6.2.16. Let $\mathbf{y} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Compute the distance from $\mathbf{y}$ to the line passing through $\mathbf{u}$ and the origin.

6.2.24. Mark each statement True or False. Justify each answer. All vectors are assumed to belong to $\mathbb{R}^n$.

a. Not every orthogonal set in $\mathbb{R}^n$ is linearly independent.

b. If a set $S = \{\mathbf{u}_1, \ldots, \mathbf{u}_p\}$ has the property that $\mathbf{u}_i \cdot \mathbf{u}_j = 0$ whenever $i \neq j$, then S is an orthonormal set.

c. If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $\mathbf{x} \mapsto A\mathbf{x}$ preserves lengths.

d. The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{v}$ whenever $c \neq 0$.

e. An orthogonal matrix is invertible.

6.2.33. (recommended)
Suppose $\mathbf{u}$ is a nonzero vector in $\mathbb{R}^n$, and let $L = \text{Span}\{\mathbf{u}\}$. Show that the mapping $\mathbf{x} \mapsto \text{proj}_L\, \mathbf{x}$ is a linear transformation.

Section 6.3

Exercises: 6, 8, 12, (13), 16

6.3.6. Let $\mathbf{y} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Verify that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set, and then find the orthogonal projection of $\mathbf{y}$ onto $\text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$.

6.3.8. Let $\mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$, $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$. Let W be the subspace spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$, and write $\mathbf{y}$ as the sum of a vector in W and a vector orthogonal to W.

6.3.12. Let $\mathbf{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$. Find the closest point to $\mathbf{y}$ in the subspace W spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.

6.3.13. (recommended)
Let $\mathbf{z} = \begin{bmatrix} 3 \\ -7 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$. Find the best approximation to $\mathbf{z}$ by a vector of the form

$c_1\mathbf{v}_1 + c_2\mathbf{v}_2$.

6.3.16. Let $\mathbf{y}$, $\mathbf{v}_1$, $\mathbf{v}_2$ be as in Exercise 12. Find the distance from $\mathbf{y}$ to the subspace W spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$.