HW 4

DUE: Friday, February 16, 9am

Section 1.9. 2, (8), 16, (18), (20), 24, (29), 30, (32), 38, (40);
Section 2.1. 2, 4, 6, 16, 18, (22);
Section 2.2. (3), 4, 6, (15), 18, 32.

Section 1.9

Exercises 2, (8), 16, (18), (20), 24, (29), 30, (32), 38, (40)

In Exercises 1–10, assume that T is a linear transformation. Find the standard matrix of T.

1.9.2. $T: \mathbb{R}^3 \to \mathbb{R}^2$, $T(\mathbf{e}_1) = (1,3)$, $T(\mathbf{e}_2) = (4,-7)$, and $T(\mathbf{e}_3) = (-5,4)$, where $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are the columns of the 3×3 identity matrix.

1.9.8. (recommended) $T: \mathbb{R}^2 \to \mathbb{R}^2$ first reflects points through the horizontal x_1-axis and then reflects points through the line $x_2 = x_1$.

1.9.16. Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables.

$$\begin{bmatrix} ? & ? \\ ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ -2x_1 + x_2 \\ x_1 \end{bmatrix}$$

In Exercises 17–20, show that T is a linear transformation by finding a matrix that implements the mapping.

1.9.18. (recommended) $T(x_1, x_2) = (2x_2 - 3x_1, x_1 - 4x_2, 0, x_2)$

1.9.20. (recommended) $T(x_1, x_2, x_3, x_4) = 2x_1 + 3x_3 - 4x_4 \quad (T: \mathbb{R}^4 \to \mathbb{R})$

1.9.24. Mark each statement True or False. Justify each answer.

a. Not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.
b. The columns of the standard matrix for a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images of the columns of the $n \times n$ identity matrix.
c. The standard matrix of a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that reflects points through the horizontal axis, the vertical axis, or the origin has the form $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, where a and d are 1 or -1.
d. A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ is one-to-one if each vector in $\mathbb{R}^n$ maps onto a unique vector in $\mathbb{R}^m$.
e. If A is a 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbb{R}^2$ onto $\mathbb{R}^3$.

1.9.29. (recommended) Suppose $T: \mathbb{R}^3 \to \mathbb{R}^4$ is one-to-one. Describe the possible echelon forms of the standard matrix for T. Use the notation of Example 1 in Section 1.2.

1.9.30. Suppose $T: \mathbb{R}^4 \to \mathbb{R}^3$ is onto. Describe the possible echelon forms of the standard matrix for T. Use the notation of Example 1 in Section 1.2.

1.9.32. (recommended) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, with A its standard matrix. Complete the following statement to make it true: "T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if A has ___ pivot columns." Find some theorems that explain why the statement is true.

In Exercises 38 and 40, let T be the linear transformation whose standard matrix is given. In Exercises 38, decide if T is a one-to-one mapping. In Exercises 40, decide if T maps $\mathbb{R}^5$ onto $\mathbb{R}^5$. Justify your answers.

1.9.38.

$$\begin{bmatrix} 7 & 5 & 4 & -9 \\ 10 & 6 & 16 & -4 \\ 12 & 8 & 12 & 7 \\ -8 & -6 & -2 & 5 \end{bmatrix}$$

1.9.40. (recommended)

$$\begin{bmatrix} 9 & 13 & 5 & 6 & -1 \\ 14 & 15 & -7 & -6 & 4 \\ -8 & -9 & 12 & -5 & -9 \\ -5 & -6 & -8 & 9 & 8 \\ 13 & 14 & 15 & 2 & 11 \end{bmatrix}$$

Section 2.1

Exercises 2, 4, 6, 16, 18, (22)

2.1.2. Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, $E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$. Compute each expression if it is defined:
a. $A + 2B$
b. $3C - E$
c. CB
d. EB
If an expression is undefined, explain why.

2.1.4. Compute $A - 5I_3$ and $(5I_3)A$, when $A = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix}$.

2.1.6. Let $A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$. Compute the product AB in two ways: (a) by the definition, where $A\mathbf{b}_1$ and $A\mathbf{b}_2$ are computed separately, and (b) by the row-column rule for computing AB.

2.1.16. Let A, B, and C be arbitrary matrices for which the indicated sums and products are defined. Mark each statement True or False. Justify each answer.

a. If A and B are 3×3 and $B = [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$, then $AB = [A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3]$.

b. The second row of AB is the second row of A multiplied on the right by B.

c. $(AB)C = (AC)B$

d. $(AB)^\top = A^\top B^\top$

e. The transpose of a sum of matrices equals the sum of their transposes.

2.1.18. Suppose the first two columns, b_1 and b_2, of B are equal. What can you say about the columns of AB (assume AB is defined)? Why?

2.1.22. (recommended) Show that if the columns of B are linearly dependent, then so are the columns of AB.

Section 2.2

Exercises (3), 4, 6, (15), 18, 32

Find the inverses of the matrices in Exercises 1–4.

2.2.3. (recommended) $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}$

2.2.4. $\begin{bmatrix} 3 & -4 \\ 7 & -8 \end{bmatrix}$

2.2.6. Use the inverse found in Exercise 3 to solve the system

$$\begin{aligned} 8x_1 &+ 5x_2 = -9 \\ -7x_1 &- 5x_2 = 11 \end{aligned}$$

2.2.15. (recommended) Suppose A, B, and C are invertible $n \times n$ matrices. Show that ABC is also invertible by producing a matrix D such that $(ABC)D = I$ and $D(ABC) = I$.

2.2.18. Suppose P is invertible and $A = PBP^{-1}$. Solve for B in terms of A.

2.2.32. Find the inverses of the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$ if it exists. Use the algorithm introduced in this section.