HW 3

Due Friday, February 9, 9am

Section 1.5. 6, (14), 16, (22), 24, 26, 34;
Section 1.7. 2, 16, 18, (20), 22, (28), (30), 34, (36), (38);
Section 1.8. 6, 10, 12, 17, 22.

Section 1.5

Exercises 6, (14), 16, 22, 24, 26, 34, 38

1.5.6. Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 0 \\ x_1 + 4x_2 - 8x_3 &= 0 \\ -3x_1 - 7x_2 + 9x_3 &= 0 \end{aligned}$$

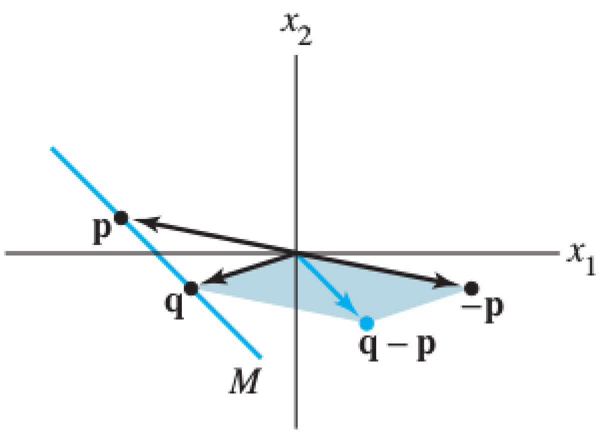
1.5.16. Describe the solutions of the following system in parametric vector form, and provide a geometric comparison with the solution set in Exercise 6.

$$\begin{aligned} x_1 + 3x_2 - 5x_3 &= 4 \\ x_1 + 4x_2 - 8x_3 &= 7 \\ -3x_1 - 7x_2 + 9x_3 &= -6 \end{aligned}$$

1.5.22. (Recommended, not required.) Find a parametric equation of the line M through the vectors

$$\mathbf{p} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{q} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$$

[Hint: M is parallel to the vector $\mathbf{qp}$. See the figure below.]



The line through **p** and **q**.

1.5.24. Mark each statement True or False. Justify each answer on the basis of a careful reading of the text.

a. If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero.

b. The equation $\mathbf{x} = x_2\mathbf{u} + x_3\mathbf{v}$, with x_2 and x_3 free (and neither $\mathbf{u}$ nor $\mathbf{v}$ a multiple of the other), describes a plane through the origin.

c. The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

d. The effect of adding $\mathbf{p}$ to a vector is to move the vector in a direction parallel to $\mathbf{p}$.

1.5.26. Suppose $A\mathbf{x} = \mathbf{b}$ has a solution. Explain why the solution is unique precisely when $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

1.5.34. Given $A = \begin{bmatrix} 4 & -6 \\ -8 & 12 \\ 6 & -9 \end{bmatrix}$, find one nontrivial solution of $A\mathbf{x} = \mathbf{0}$ by inspection.

[Hint: you should not have to reduce the matrix using elementary row operations; you can just “see” an answer. This is what “by inspection” means.]

Section 1.7

Exercises 2, 16, 18, (20), 22, (28), (30), 34, (36), (38)

1.7.2. Determine if the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -8 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

1.7.16. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix}$$

1.7.18. Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

1.7.20. (Recommended, not required.) Determine by inspection whether the vectors are linearly independent. Justify your answer.

$$\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1.7.22. Mark each statement True or False. Justify each answer on the basis of a careful reading of the text.

a. Two vectors are linearly dependent if and only if they lie on a line through the origin.

b. If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.

c. If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in $\mathrm{Span}\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent.

d. If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector.

1.7.28. (Recommended, not required.) How many pivot columns must a 5×7 matrix have if its columns span $\mathbb{R}^5$? Why?

1.7.30. (Recommended, not required.)

a. Fill in the blank in the following statement: “If A is an $m \times n$ matrix, then the columns of A are linearly independent if and only if A has ___ pivot columns.”"

b. Explain why the statement in **a** is true.

Each statement in Exercises 34–38 is either true (in all cases) or false (for at least one example). If false, construct a specific example to show that the statement is not always true. (Such an example is called a counterexample to the statement.) If a statement is true, give a justification. (One specific example cannot explain why a statement is always true.)

1.7.34. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are in $\mathbb{R}^4$ and $\mathbf{v}_3 = \mathbf{0}$ then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

1.7.36. (Recommended, not required.) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are in $\mathbb{R}^4$ and $\mathbf{v}_3$ is *not* a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

1.7.38. (Recommended, not required.) If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are linearly independent vectors in $\mathbb{R}^4$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly independent.

Section 1.8

Exercises 6, 10, 12, 17, 22

1.8.6. Let

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 3 & -4 & 5 \\ 0 & 1 & 1 \\ -3 & 5 & -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 9 \\ 3 \\ -6 \end{bmatrix}$$

Let T be defined by $T(\mathbf{x}) = A\mathbf{x}$. Find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

1.8.10. Let $A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$. Find all $\mathbf{x}$ in $\mathbb{R}^4$ that are mapped into the zero vector by the transformation $\mathbf{x} \mapsto A\mathbf{x}$.

1.8.12. Let $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$, and let A be the matrix in Exercise 10. Is $\mathbf{b}$ in the range of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$? Why or why not?

1.8.17. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ into $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and maps $\mathbf{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$. Use the fact that T is linear to find the images under T of $3\mathbf{u}$, $2\mathbf{v}$, and $3\mathbf{u} + 2\mathbf{v}$.

1.8.22. Mark each statement True or False. Justify each answer.

a. Every matrix transformation is a linear transformation.

b. The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A.

c. If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ in the range of T?”

d. A linear transformation preserves the operations of vector addition and scalar multiplication.

e. The superposition principle is a physical description of a linear transformation.
