Optimization and Related Rates Examples

1. After acing your calculus exam, you decide to apply what you have learned. You move to Colorado and start a marijuana farm. You have $8000 to spend on an enclosure for a rectangular garden. Along one side of the garden is a brick wall (which you don't have to pay for). Two sides of the garden will be perpendicular to the wall and made of wood fencing which costs $20 per foot. The side parallel to the wall will be made of chain link fence which costs $10 per foot. If you have $8000 to spend on the enclosure, what dimensions will maximize the area of the garden?

   (Hint: draw a picture; write down an area function, which you want to maximize; write down a cost function, which will equal 8000.)

length of perpendicular side: _____ ft

length of parallel side: _____ ft

total area: _____ ft^2

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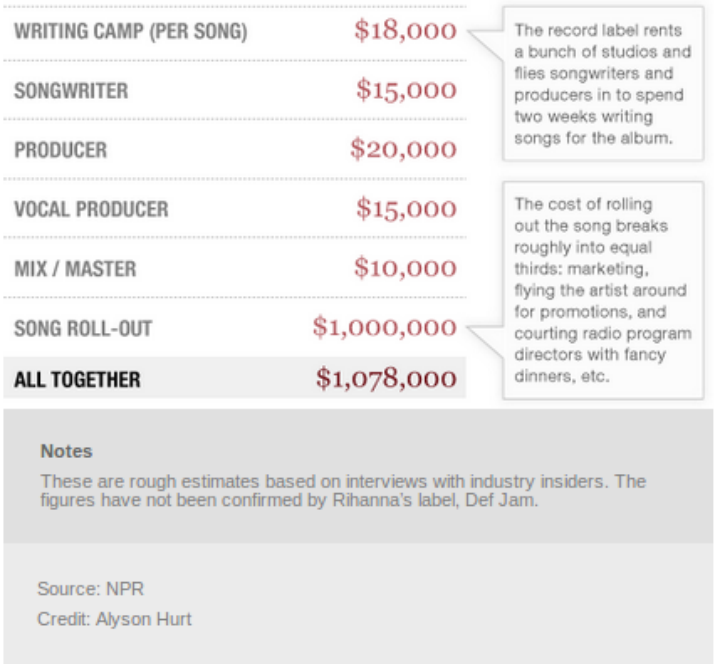
2. According to a recent study, Def Jam Records spent \$1,078,000 producing Rihanna's single "Man Down."
Generally speaking, the demand for song downloads goes down as the price per download goes up. Suppose the quantity q of downloads demanded is given by the following function of price:

$$q = \frac{12000000}{p^2 + 9}. \qquad (1)$$

The revenue generated from selling q downloads at price p (dollars) is price times quantity: $R(p) = qp$.



What price should Def Jam charge for each download in order to maximize revenue? (*Hint:* Use Eq. (1) to express revenue as a function of price only, then differentiate.)

3. Each side of a square is increasing at a rate of 2 cm/s. At what rate is the area of the square increasing when the area of the square is 49 cm^2? (Include appropriate units.)

Answer: $A' =$ _____

4. After acing your calculus exam, you are hired at the Near Earth Object Observatory atop a volcano on the island of Maui. Your job is to support the early detection system for meteors that might impact earth. On your first day you discover a meteor, in the shape of a perfect sphere, fast approaching earth. As it travels through the earth's atmosphere and burns up, its surface area decreases at a rate of 24π m^2/second.

   (a) At what rate is the radius decreasing when the radius is 3 meters?
   (Hint: Surface area of a sphere is $S = 4\pi r^2$; apply the chain rule.)

   (b) In solving Part i, you hopefully found $r'(t)$, the rate of change of the radius. Now write down an integral expression that gives the total change in the radius as time goes from a seconds to b seconds.

   (We won't have the tools to answer this part until we cover Chapter 7.)

   (c) Suppose at time $t = 1$ the meteor is observed to have a radius of 32 meters, and suppose it will reach the earth after $e^{10} \approx 22026$ seconds, what will be its radius upon impact?

   (We won't have the tools to answer this part until we cover Chapter 7.)