Optimization and Related Rates Examples

1. After acing your calculus exam, you decide to apply what you have learned. You move to Colorado and start a marijuana farm. You have \$8000 to spend on an enclosure for a rectangular garden. Along one side of the garden is a brick wall (which you don't have to pay for). Two sides of the garden will be perpendicular to the wall and made of wood fencing which costs \$20 per foot. The side parallel to the wall will be made of chain link fence which costs \$10 per foot. If you have \$8000 to spend on the enclosure, what dimensions will maximize the area of the garden?

(Hint: draw a picture; write down an area function, which you want to maximize; write down a cost function, which will equal 8000.)

length of perpendicular side:	ft
length of parallel side:	ft
total area:	ft^2

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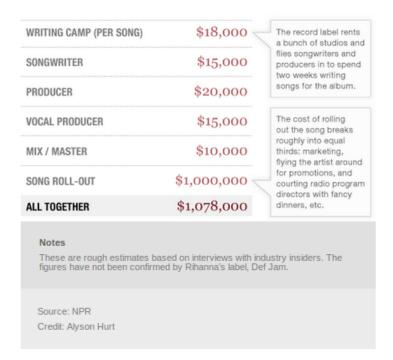
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2. According to a recent study, Def Jam Records spent \$1,078,000 producing Rihanna's single "Man Down."

Generally speaking, the demand for song downloads goes down as the price per download goes up. Suppose the quantity q of downloads demanded is given by the following function of price:

$$q = \frac{12000000}{p^2 + 9}. (1)$$

The revenue generated from selling q downloads at price p (dollars) is price times quantity: R(p) = qp.



What price should Def Jam charge for each download in order to maximize revenue? (*Hint:* Use Eq. (1) to express revenue as a function of price only, then differentiate.)

3.	Each side of a square is increasing at a rate of 2 cm/s. At what rate is the area of the square increasing when the area of the square is 49 cm ² ? (Include appropriate units.)
	Answer: $A' =$

- 4. After acing your calculus exam, you are hired at the Near Earth Object Observatory atop a volcano on the island of Maui. Your job is to support the early detection system for meteors that might impact earth. On your first day you discover a meteor, in the shape of a perfect sphere, fast approaching earth. As it travels through the earth's atmosphere and burns up, its surface area decreases at a rate of 24π m²/second.
 - (a) At what rate is the radius decreasing when the radius is 3 meters? (Hint: Surface area of a sphere is $S = 4\pi r^2$; apply the chain rule.)

(b) In solving Part i, you hopefully found r'(t), the rate of change of the radius. Now write down an integral expression that gives the total change in the radius as time goes from a seconds to b seconds.

(c) Suppose at time t=1 the meteor is observed to have a radius of 32 meters, and suppose it will reach the earth after $e^{10} \approx 22026$ seconds, what will be its radius upon impact?