Math 374 – Equivalence Rules and Inference Rules
(Updated February 1, 2013)

TABLE 1. Equivalence Rules of Propositional Logic

Expression	Equivalent to	Name (abbreviation)
$P \vee Q$ $P \wedge Q$	$Q \vee P$ $Q \wedge P$	Commutative (comm)
$(P \vee Q) \vee R$ $(P \wedge Q) \wedge R$	$P \vee (Q \vee R)$ $P \wedge (Q \wedge R)$	Associative (ass)
$P \vee (Q \wedge R)$ $P \wedge (Q \vee R)$	$(P \vee Q) \wedge (P \vee R)$ $(P \wedge Q) \vee (P \wedge R)$	Distributive (dist)
$(P \wedge Q)'$ $(P \vee Q)'$	$P' \vee Q'$ $P' \wedge Q'$	De Morgan's laws (DeMorgan)
$P \to Q$	$P' \vee Q$	Implication (imp)
P	$(P')'$	Double negation (dn)
$P \leftrightarrow Q$	$(P \to Q) \wedge (Q \to P)$	Equivalence (equ)
$(P \wedge Q) \to R$	$P \to (Q \to R)$	Deduction (ded)
$P \to Q$	$Q' \to P'$	Contraposition (cont)

TABLE 2. Inference Rules of Propositional Logic

From	Can Derive	Name (abbreviation)
P, $P \to Q$	Q	Modus Ponens (mp)
$P \to Q$, Q'	P'	Modus Tollens (mt)
P, Q	$P \wedge Q$	Conjunction (conj)
$P \wedge Q$ $P \wedge Q$	P Q	Simplification (sim)
P	$P \vee Q$	Addition (add)
$P \to Q$, $Q \to R$	$P \to R$	Hypothetical syllogism (hs)
$P \vee Q$, P'	Q	Disjunctive syllogism (ds)
P	$P \wedge P$	Self-reference (self)
$P \vee P$	P	Self-reference (self)
P', P	Q (for any Q)	Inconsistency (inc)

TABLE 3. Inference Rules of Predicate Logic

From	Can Derive	Name (abbreviation)	Restrictions
$(\forall x)P(x)$	$P(t)$ where t is a variable or constant.	Universal Instantiation (ui)	If t is a variable, it must not be quantified inside $P(x)$.
$(\exists x)P(x)$	$P(a)$ where a is a constant.	Existential Instantiation (ei)	Must be the first use of the constant a
$P(x)$	$(\forall x)P(x)$	Universal Generalization (ug)	$P(x)$ hasn't been deduced from hypotheses where x is free, nor by using ei on a wff with x free.
$P(x)$ or $P(a)$	$(\exists x)P(x)$	Existential Generalization (eg)	To get $(\exists x)P(x)$ from $P(a)$, x can't already appear in $P(a)$.

Remarks: We proved in class that Deduction (ded) is an equivalence rule. (See Exercise 49a, p.34.) That is, we showed that the following is a tautology:

$$[(P \wedge Q) \to R] \quad \longleftrightarrow \quad [P \to (Q \to R)].$$

So the wff's on either side of $\longleftrightarrow$ are equivalent. We denote this by

$$(P \wedge Q) \to R \quad \Longleftrightarrow \quad P \to (Q \to R).$$

(Note that the book calls ded "exportation" (exp) and lists it as a inference rule in Table 1.14, p.33, rather than an equivalence rule. Pay no mind to this, call it ded, not exp, and use it as an equivalence rule, which is more powerful since you can go in either direction.)