MATH 321 **Homework 7 (due 12/07)** **NAME:**

(3pts) 1. Prove the following identity for all natural numbers $n \geq 1$.

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n} \tag{1}$$

(3pts) 2. (V 6.1.4) Find a formula for $1 + 3 + 5 + \cdots + (2n - 1)$, for $n \geq 1$, and prove that your formula is correct. (*Hint:* First try some particular values of n and look for a pattern.)

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(3pts) 3. Use "strong" induction (i.e., the 2nd Principle of Induction) to prove that any amount of postage greater than or equal to 12 cents can be obtained using only 4-cent and 5-cent stamps.

(3pts) 4. In any group of k people, $k \geq 1$, each person is to shake hands with every other person. Find a formula for the number of handshakes, and prove the formula by induction.

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(3pts) 5. (6.2.3) Suppose R is a total order on a set A. Prove that every finite, nonempty set $B \subseteq A$ has an R-smallest element.

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6. The sequence $1, 1, 2, 3, 5, 8, 13, \ldots$ of *Fibonacci numbers* is defined recursively as follows:

$$F(0) = 1, \quad F(1) = 1,$$
$$F(n) = F(n-1) + F(n-2), \quad n \geq 2.$$

(2pts) (a) Prove the following property of the Fibonacci numbers directly (i.e., without using induction): $(\forall n \geq 6)\ (F(n) = 5F(n-4) + 3F(n-5))$.

(2pts) (b) Prove the following property of the Fibonacci numbers using the 2nd Principle of Induction: $(\forall n \geq 1)\ (F(n) < 2^n)$.

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(3pts) 7. Solve the recurrence relation. That is, give a closed form (i.e., non-recursive) expression defining $F(n)$.

(i) $F(1) = 2$.

(ii) $F(n) = 2F(n-1) + 2^n$ for $n \geq 2$.

(3pts) 8. (6.2.15) What's wrong with the following proof that if $A \subseteq \mathbb{N}$ and $0 \in A$ then $A = \mathbb{N}$?

Proof. We will prove by induction that $(\forall n \in \mathbb{N})(n \in A)$.

Base case: If $n = 0$, then $n \in A$ by assumption.

Induction step: Let $n \in \mathbb{N}$ be arbitrary, and suppose that $n \in A$. Since n was arbitrary, it follows that every natural number is an element of A, and therefore in particular $n + 1 \in A$. $\square$

Question:	1	2	3	4	5	6	7	8	Total
Points:	3	3	3	3	3	4	3	3	25
Score:									