Equivalence Rules and Inference Rules
(Updated September 13, 2016)

Table 1. Equivalence Rules of Propositional Logic

Expression	Equivalent to	Name (abbreviation)
$P \vee Q$ $P \wedge Q$	$Q \vee P$ $Q \wedge P$	Commutative (comm)
$(P \vee Q) \vee R$ $(P \wedge Q) \wedge R$	$P \vee (Q \vee R)$ $P \wedge (Q \wedge R)$	Associative (ass)
$P \vee (Q \wedge R)$ $P \wedge (Q \vee R)$	$(P \vee Q) \wedge (P \vee R)$ $(P \wedge Q) \vee (P \wedge R)$	Distributive (dist)
$\neg(P \wedge Q)$ $\neg(P \vee Q)$	$\neg P \vee \neg Q$ $\neg P \wedge \neg Q$	De Morgan's laws (DeMorgan)
$P \to Q$	$\neg P \vee Q$	Implication (imp)
P	$\neg\neg P$	Double negation (dn)
$P \leftrightarrow Q$	$(P \to Q) \wedge (Q \to P)$	Equivalence (equ)
$(P \wedge Q) \to R$	$P \to (Q \to R)$	Deduction (ded)
$P \to Q$	$\neg Q \to \neg P$	Contraposition (cont)

Table 2. Inference Rules of Propositional Logic

From	Can Derive	Name (abbreviation)
P, $P \to Q$	Q	Modus Ponens (mp)
$P \to Q$, $\neg Q$	$\neg P$	Modus Tollens (mt)
P, Q	$P \wedge Q$	Conjunction (conj)
$P \wedge Q$ $P \wedge Q$	P Q	Simplification (sim)
P	$P \vee Q$	Addition (add)
$P \to Q$, $Q \to R$	$P \to R$	Hypothetical syllogism (hs)
$P \vee Q$, $\neg P$	Q	Disjunctive syllogism (ds)
P	$P \wedge P$	Self-reference (self)
$P \vee P$	P	Self-reference (self)
$\neg P$, P	Q (for any Q)	Inconsistency (inc)

Table 3. Inference Rules of Predicate Logic

From	Can Derive	Name (abbreviation)	Restrictions
$(\forall x)P(x)$	$P(t)$ where t is a variable or constant.	Universal Instantiation (ui)	If t is a variable, it must not be quantified inside $P(x)$.
$(\exists x)P(x)$	$P(a)$ where a is a constant.	Existential Instantiation (ei)	Must be the first use of the constant a
$P(x)$	$(\forall x)P(x)$	Universal Generalization (ug)	$P(x)$ hasn't been deduced from hypotheses where x is free, nor by using ei on a wff with x free.
$P(x)$ or $P(a)$	$(\exists x)P(x)$	Existential Generalization (eg)	To get $(\exists x)P(x)$ from $P(a)$, x can't already appear in $P(a)$.

Remarks: We can prove that Deduction (ded) is an equivalence rule. That is, we showed that the following is a tautology:

$$[(P \wedge Q) \to R] \quad \longleftrightarrow \quad [P \to (Q \to R)].$$

So the wff's on either side of $\longleftrightarrow$ are equivalent. We denote this by

$$(P \wedge Q) \to R \quad \Longleftrightarrow \quad P \to (Q \to R).$$

(Some authors call ded "exportation" (exp) and consider it an inference rule.)