MATH 321 **Homework 3 (due 9/28)** **NAME:**

(3pts) 1. What is the truth value of each wff when the domain is the set $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$?

Example: $(\forall x)(\forall y)(x < y \lor y < x)$

Answer: FALSE.

Reason: $(\forall x)(\forall y)$ includes values for which $x = y$. Alternatively, note that the negation of this statement is $(\exists x)(\exists y)(x = y)$, which is TRUE.

(a) $(\forall x)[x < 0 \to (\exists y)(y > 0 \land x + y = 0)]$

(b) $(\exists x)(\exists y)(x^2 = y)$

(c) $(\forall x)(x^2 > 0)$

(a) *Answer:*

Reason:

(b) *Answer:*

Reason:

(c) *Answer:*

Reason:

(3pts) 2. Say which of the following are equivalent to the statement,
"Cats are smarter than dogs."

(a) Some cats are smarter than some dogs.

(b) There is a cat that is smarter than all dogs.

(c) All cats are smarter than all dogs.

(d) Only cats are smarter than dogs.

(e) All cats are smarter than any dog.

Just write down the correct letter(s); no explanation needed.

Answer:

(4pts) 3. Give English translations of the following wffs, where

$L(x,y)$ is "x loves y", $H(x)$ is "x is handsome", $P(x)$ is "x is pretty",
$M(x)$ is "x is a man", $W(x)$ is "x is a woman"
j is "John", k is "Kathy"

(a) $(\forall x)(W(x) \to (\forall y)[L(x,y) \to M(y) \wedge H(y)])$

(b) $(\exists x)[M(x) \wedge H(x) \wedge L(x,k)]$

(c) $(\exists x)(W(x) \wedge P(x) \wedge (\forall y)[L(x,y) \to H(y) \wedge M(y)])$

(d) $(\forall x)[W(x) \wedge P(x) \to L(j,x)]$

Answer:

(a)

(b)

(c)

(d)

Score for this page: ____________ out of 4

(4pts) 4. Give interpretations to prove that each of the following wffs is not valid.

Example: $(\forall x)(\exists y)P(x,y) \to (\exists x)(\forall y)P(x,y)$

Answer: Recall, to prove an implication is not valid, find an interpretation for which the antecedent is true and the consequent is false. Let the universe of discourse be the set of integers, $\mathbb{Z}$. Let $P(x,y)$ denote $x < y$. Then the antecedent $(\forall x)(\exists y)P(x,y)$ is true since for each (fixed) $x \in \mathbb{Z}$ we can find a $y \in \mathbb{Z}$ such that $x < y$. (For example, pick $y = x + 1$.) However, the consequent $(\exists x)(\forall y)P(x,y)$ is false since it is impossible to find a single x for which $x < y$ holds for all $y \in \mathbb{Z}$.

(a) $(\forall x)[P(x) \to Q(x)] \to [(\exists x)P(x) \to (\forall x)Q(x)]$

(b) $(\forall x)[\neg A(x)] \leftrightarrow \neg[(\forall x)A(x)]$

Answer:

(a)

(b)

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(4pts) 5. Describe each of the following sets by listing its elements. (If the set is infinite, list a few of its members followed by an ellipsis, ...)

Example: $\{x \in \mathbb{N} \mid x^2 - 5x + 6 = 0\}$

Answer: The equation $x^2 - 5x + 6 = 0$ holds if and only if $(x-2)(x-3) = 0$ if and only if $x = 2$ or $x = 3$. Therefore, $\{x \in \mathbb{N} \mid x^2 - 5x + 6 = 0\} = \{2, 3\}$.

(a) $\{x \in \mathbb{N} \mid (\exists q)(q \in \{2,3\} \wedge x = 2q)\}$.

(b) $\{x \in \mathbb{N} \mid (\exists y)(\exists z)(y \in \{0,1\} \wedge z \in \{3,4\} \wedge y < x < z)\}$.

(c) $\{x \mid x \in \mathbb{R} \wedge x^2 = 7\}$

(d) $\{x \in \mathbb{N} \mid (\forall y)(y \text{ even} \rightarrow x \neq y)\}$.

Answer:

(a)

(b)

(c)

(d)

(5pts) 6. What is the cardinality of each of the following sets?

(a) $A = \{a, \{a, \{a\}\}\}$.

(b) $B = \{\{a\}, \{\{a\}\}\}$.

(c) $C = \{\emptyset\}$.

(d) $D = \{a, \{\emptyset\}, \emptyset\}$.

(e) $E = \{a, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}$.

Answer:

(a)

(b)

(c)

(d)

(e)

Score for this page: ____________ out of 5

(4pts) 7. Let

$$A = \{a, \{a\}, \{\{a\}\}\}; \qquad B = \{a\}; \qquad C = \{\emptyset, \{a, \{a\}\}\}.$$

Say which statements are TRUE and which are FALSE? For those that are FALSE, where do they fail?

a. $B \subseteq A$	d. $\emptyset \subseteq C$	g. $\{a, \{a\}\} \subseteq A$
b. $B \in A$	e. $\emptyset \in C$	h. $B \subseteq C$
c. $C \subseteq A$	f. $\{a, \{a\}\} \in A$	i. $\{\{a\}\} \subseteq A$

Answer:

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

Score for this page: ____________ out of 4

(3pts) 8. Write TRUE if the statement is true for *all* sets A, B, and C. Write FALSE otherwise. If a statement is false, give a counterexample.

(a) If $A \subset B$ and $B \subseteq C$, then $A \subset C$.

(b) If $A \neq B$ and $B \neq C$, then $A \neq C$.

(c) If $A \in B$ and $B \not\subseteq C$, then $A \notin C$.

Answer:

(a)

(b)

(c)

Question	Points	Score
1	3	
2	3	
3	4	
4	4	
5	4	
6	5	
7	4	
8	3	
Total:	30	