1. Suppose that

$$A = \{x : x \in \mathbb{N} \text{ and } x \text{ is even}\},$$
$$B = \{x : x \in \mathbb{N} \text{ and } x \text{ is prime}\},$$
$$C = \{x : x \in \mathbb{N} \text{ and } x \text{ is a multiple of } 5\}.$$

Describe each of the following sets.

(a) $A \cup B$

(b) $A \cap (B \cup C)$

2. If $A = \{a, b, c\}$, $B = \{1, 2, 3\}$, and $C = \emptyset$, list all of the elements in each of the following sets.

(a) $B \times A$

(b) $A \times C$

3. Prove that if A and B are nonempty sets, then the following are equivalent:

(i) $A \times B = B \times A$

(ii) $A \times B \subseteq B \times A$

(iii) $A = B$

4. Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

5. (a) Define a function $f : \mathbb{N} \to \mathbb{N}$ that is one-to-one but not onto.

(b) Define a function $f : \mathbb{N} \to \mathbb{N}$ that is onto but not one-to-one.

6. Let $f : A \to B$ and $g : B \to C$ be maps.
   (a) If f and g are both one-to-one functions, show that $g \circ f$ is one-to-one.
   (b) If $g \circ f$ is onto, show that g is onto.
   (c) If $g \circ f$ is one-to-one, show that f is one-to-one.
   (d) If $g \circ f$ is one-to-one and f is onto, show that g is one-to-one.
   (e) If $g \circ f$ is onto and g is one-to-one, show that f is onto.

7. Let $f : X \to Y$ be a map with $A_1, A_2 \subset X$ and $B_1, B_2 \subset Y$.
   (a) Prove $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.
   (b) Prove $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$. Give an example in which equality fails.
   (c) Prove $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$, where

   $$f^{-1}(B) = \{x \in X : f(x) \in B\}.$$

   (d) Prove $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.
   (e) Prove $f^{-1}(Y \setminus B_1) = X \setminus f^{-1}(B_1)$.

8. Determine whether or not the following relations are equivalence relations on the given set. If the relation is an equivalence relation, describe the partition given by it. If the relation is not an equivalence relation, state why it fails to be one.
   (a) $x \sim y$ in $\mathbb{R}$ if $x \geq y$
   (b) $m \sim n$ in $\mathbb{Z}$ if $mn > 0$
   (c) $x \sim y$ in $\mathbb{R}$ if $|x - y| \leq 4$
   (d) $m \sim n$ in $\mathbb{Z}$ if $m \equiv n \pmod 6$

9. Find the error in the following argument by providing a counterexample. "The reflexive property is redundant in the axioms for an equivalence relation. If $x \sim y$, then $y \sim x$ by the symmetric property. Using the transitive property, we can deduce that $x \sim x$."

10. Let $n \in \mathbb{N}$. Use the division algorithm to prove that every integer is congruent mod n to precisely one of the integers $0, 1, \ldots, n - 1$. Conclude that if r is an integer, then there is exactly one s in $\mathbb{Z}$ such that $0 \leq s < n$ and $[r] = [s]$. Hence, the integers are indeed partitioned by congruence mod n.

11. Find all $x \in \mathbb{Z}$ satisfying the following equation: $9x \equiv 3 \pmod 5$.