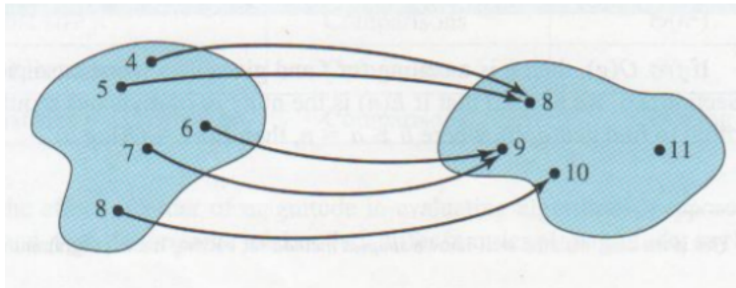
MATH 321 **Homework 6 (due 11/16)** **NAME:**

(3pts) 1. The accompanying figure represents a function with domain $\{4, 5, 6, 7, 8\}$.



(a) The codomain is ____________

(b) The range is ____________

(c) The image of 5 is ____________

(d) The image of 8 is ____________

(e) The preimage of 9 is ____________

(f) This function is onto. True False

(g) This function is one-to-one. True False

(3pts) 2. Circle True or False, as appropriate.

(a) A function is onto if and only if every element in the domain has an image. True False

(b) A function is onto if and only if every element in the codomain has an image. True False

(c) A function is onto if and only if every element in the codomain has a preimage. True False

(d) A function is onto if and only if every element in the codomain has a unique preimage. True False

(e) A function is onto if and only if (the range) $\cap$ (the codomain) $= \emptyset$. True False

(f) A function is one-to-one if and only if every element in the codomain has a unique preimage. True False

(g) A function is one-to-one if and only if distinct elements in the domain map to distinct elements in the codomain. True False

Score for this page: ____________ out of 6

(3pts) 3. Let $S = \{0, 2, 4, 6\}$ and $T = \{1, 3, 5, 7\}$. Determine whether each of the following sets of ordered pairs is a function with domain S and codomain T. If so, it is one-to-one? Is it onto?

(a) $\{(0, 2), (2, 4), (4, 6), (6, 0)\}$ is

○ a function ○ one-to-one ○ onto ○ none of these

(b) $\{(6, 3), (2, 1), (0, 3), (4, 5)\}$ is

○ a function ○ one-to-one ○ onto ○ none of these

(c) $\{(2, 3), (4, 7), (0, 1), (6, 5)\}$ is

○ a function ○ one-to-one ○ onto ○ none of these

(d) $\{(2, 1), (4, 5), (6, 3)\}$ is

○ a function ○ one-to-one ○ onto ○ none of these

(e) $\{(6, 1), (0, 3), (4, 1), (0, 7), (2, 5)\}$ is

○ a function ○ one-to-one ○ onto ○ none of these

(3pts) 4. Let $S = \{a, b, c, d\}$ and $T = \{x, y, z\}$.

(a) If possible, give an example of a function from S to T that is onto.

(b) If possible, give an example of a function from S to T that is one-to-one.

(c) The number of functions from S to T is ____________.

Score for this page: ____________ out of 6

5. (V 5.1.9) Suppose $f\colon A \to C$ and $g\colon B \to C$ are functions.
(See V 5.1.7 for the meaning of notation used in this exercise.)

(2pts) (a) Prove that if A and B are disjoint, then $f \cup g\colon A \cup B \to C$.

(1pts) (b) More generally, prove that $f \cup g\colon A \cup B \to C$ iff $f \restriction (A \cap B) = g \restriction (A \cap B)$.

Score for this page: __________ out of 3

6. (V 5.1.17)

(1pts) (a) Suppose $g\colon A \to B$ and let $\operatorname{Ker} g = \{(x, y) \in A \times A \mid g(x) = g(y)\}$. Show that $\operatorname{Ker} g$ is an equivalence relation on A.

(2pts) (b) Suppose R is an equivalence relation on A and let $g\colon A \to A/R$ be the function defined by the formula $g(x) = [x]_R$. Show that $R = \{(x, y) \in A \times A \mid g(x) = g(y)\}$.

Score for this page: ____________ out of 3

(2pts) 7. (V 5.2.16) Suppose R is an equivalence relation on A and suppose $f\colon A \to B$ is *compatible* with R. Ex 5.1.18 gives the definition of compatible, and also asks you to prove that there is a unique function $h\colon A/R \to B$ such that for all $x \in A$, $h([x]_R) = f(x)$. Now prove that h is one-to-one iff $(\forall x \in A)(\forall y \in A)(f(x) = f(y) \longrightarrow (x, y) \in R)$.

Score for this page: ____________ out of 2

8. (cf. V 5.3.13) Suppose $f\colon A \to B$ is onto. Let $K = \mathrm{Ker}(f) = \{(x, y) \in A \times A \mid f(x) = f(y)\}$. By Ex 5.1.17 K is an equivalence relation on A.

(2pts) (a) Consider the relation $h \subseteq A/K \times B$ defined by $h = \{([x]_K, f(x)) \mid x \in A\}$. Prove that h is actually a *function* from A/K to B, and satisfies $h([x]_K) = f(x)$.

(1pts) (b) Prove that h is one-to-one and onto. (Hint: See Ex 5.2.16)

(1pts) (c) It follows from part (b) that h has an inverse function, $h^{-1}\colon B \to A/K$. Prove that for all $b \in B$, $h^{-1}(b) = \{x \in A \mid f(x) = b\}$.

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Suggested Problems

The following exercises are also recommended: 5.1.7, 5.1.18, 5.2.8, 5.2.17, 5.2.18, 5.3.9.

Page	Points	Score
1	6	
2	6	
3	3	
4	3	
5	2	
6	4	
Total:	24	