Math 654 Homework #1 (due on 9/9/2016)

Your name:

Problem 4. Prove that every transitive set models extensionality. Is every model of extensionality transitive?

Problem 5. If α is an ordinal, prove that $S(\alpha)$ is an ordinal, $\alpha < S(\alpha)$ and $(\forall \gamma \in ON)\big(\gamma < S(\alpha) \leftrightarrow \gamma \leq \alpha\big)$. (Exercise I.8.11.)

Problem 6. Suppose $\alpha_0, \alpha_1, \ldots$ are ordinals and $\alpha_i < \alpha_{i+1}$ for all i. Prove that $X = \bigcup\{\alpha_i : i \in \omega\}$ is an ordinal, is the least upper bound of $\alpha_0, \alpha_1, \ldots$ and is a limit ordinal.

Definition 1.

(1) 2^ω is the set of functions with domain ω and range $2 = \{0, 1\}$. We often identify the members of 2^ω with infinite sequences of 0's and 1's. (This is called Cantor space.)
(2) 2^k is the set of functions with domain $k \in \omega$ and range $2 = \{0, 1\}$. We often identify 2^k with the set of strings of 0's and 1's with length exactly k.
(3) $2^{<\omega} = \bigcup\big\{2^k : k \in \omega\big\}$. We often identify $2^{<\omega}$ with the set of finite strings of 0's and 1's.

Problem 7. Prove that there is no well-ordering of 2^ω which has type ω. Also, prove there is a well-ordering of $2^{<\omega}$ which has order type ω.