Statistics 243: *class notes*

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We want to find an alternative to computing $X^t X$.

Let X be an $n \times p$ full rank matrix. Then there exists an orthonormal matrix Q, and an upper triangular matrix R, such that $X = QR$.

Normal equations

$$\begin{aligned} X^t X \hat{\beta} &= X^t y \\ (QR)^t (QR) \hat{\beta} &= (QR)^t y \\ R^t R \hat{\beta} &= R^t Q^t y \end{aligned}$$

Note that R is full rank, so

$$R\hat{\beta} = Q^t y$$

a system of p equations, where the rhs, $Q^t y$ involves np multiplications. We can solve this triangular system with back substitution.

Now let's find the QR decomposition of $n \times p$ matrix X.

Modified Gram Schmitt:

for $j = 1$ to p {
define $r_{jj} = \sqrt{\sum_i x_{ij}^2}$, the norm of x_j
check if $abs(r_{jj}) <$ some small number (say, $\sqrt{macheps}$)

for $i = 1$ to n {
    $x_{ij} = x_{ij}/r_{jj}$
} end for i
for $k = j + 1$ to p {
    $r_{jk} = \sum_i x_{ij} x_{ik}$
    for $i = 1$ to n {
        $x_{ik} = x_{ik} - x_{ij} r_{jk}$
    } end for k
} end for j

Now we want to compute $Var(\hat{\beta})$

$$\begin{aligned} Var(\hat{\beta}) &= \sigma^2 (X^t X)^{-1} \\ &= \sigma^2 (R^{-1})^t R^{-1} \end{aligned}$$