Measure Theory

Bill Last Updated:

27 January, 2020

Contents

Chapter 1

Measure Spaces

1.1 σ-Fields

Let $\mathcal{F}$ be a collection of subsets of a set Ω. Then, F is called a sigma field (or, sigma algebra; written as σ-field or σ-algebra) if and only if it satisfies the following properties:

(i) The empty set $\emptyset \in \mathcal{F}$.

(ii) If $A \in \mathcal{F}$, then the complement $A^c \in \mathcal{F}$.

(iii) If $A_1, A_2, ...$ is a sof elements of $\mathcal{F}$, then their union $\cup_{i=1}^{\infty} A_i \in \mathcal{F}$.

A pair $(\Omega, \mathcal{F})$ consisting of a set Ω and a $\sigma - field$ of subsets $\mathcal{F}$ is called a measurable space. The elements of $\mathcal{F}$ are called measurable sets or events.

Remarks

(1) The set Ω is called sample space in probability, but in general measure theory it is called the underlying set or underlying space.

(2) Since $\emptyset^c = \Omega$, it follows (i) and (ii) $\Omega \in \mathcal{F}$.

(3) Given any set Ω, the trivial σ-field is $\mathcal{F} = \{\emptyset, \Omega\}$. One can easily verify that this is a σ-field, and is in fact the smallest σ-field on Ω.

(4) Given any set Ω, the power set:

$$\mathcal{P}(\Omega) = \{A : A \in \Omega\}$$

Consisting of all subsets of Ω is also a σ-field on Ω