# Final Project

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## 1

The original problem

$$\begin{aligned} & min_{w,\gamma,y,z} \quad (\frac{1}{m}e^T y + \frac{1}{k}e^T z) + \frac{\mu}{2}w^T w \\ & s.t. \quad Mw - e\gamma + y \geq e, -Bw + e\gamma + z \geq e, y \geq 0, z \geq 0 \end{aligned}$$

can be formulated as a quadratic programming

$$\begin{aligned} & min \quad \frac{1}{2}x^T Q x + p^T x \\ & s.t. \quad Ax \leq b;\ y,z \geq 0;\ w,\gamma \text{ free} \end{aligned}$$

where

$$x = \begin{bmatrix} w_{n\times 1} \\ \gamma \\ y_{m\times 1} \\ z_{k\times 1} \end{bmatrix}, \quad Q = \begin{bmatrix} \mu & & & & \\ & \ddots & & & \\ & & \mu & & \\ & & & 0 & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix}, \quad p = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1/m \\ \vdots \\ 1/m \\ 1/k \\ \vdots \\ 1/k \end{bmatrix}, \quad A = \begin{bmatrix} -M & e_{m\times 1} & -I_m & 0_{m\times k} \\ B & -e_{k\times 1} & 0_{k\times m} & -I_k \end{bmatrix}, \quad b = \begin{bmatrix} -e_{(m+k)\times 1} \end{bmatrix}$$

Let's first define a function `fitModel` as below

```
function [w gamma obj] = fitModel(mu, Mtrain, Btrain)
% syntax: [w gamma] = fitModel(mu, Mtrain, Btrain)
% given the tuning param, training data, fit the model and output coeffs and
% the objective value for the QP.

% formulate the QP
n = size(Btrain,2);
m = size(Mtrain,1);
k = size(Btrain,1);
Q = [mu*eye(n) zeros(n,1+m+k); zeros(1+m+k,n+1+m+k)];
p = [zeros(n+1,1); (1/m)*ones(m,1); (1/k)*ones(k,1)];
A = [-Mtrain ones(m,1) -eye(m) zeros(m,k);
     Btrain -ones(k,1) zeros(k,m) -eye(k)];
b = -ones(m+k,1);
lb = [-inf((n+1),1); zeros(m+k,1)];
ub = inf(n+1+m+k,1);
```

```
% solve the QP
[x obj] = cplexqp(Q,p,A,b,[],[],lb,ub);
w = x(1:n);
gamma = x(n+1);
end
```

Let the tuning parameter $\mu = 0.0001$. The following is the code and output shows the solved $w$ and $\gamma$.

```
[train, tune, test] = getdata('wdbc.data', 30);
Btrain = train(find(train(:,1) == 66), 2:31);
Mtrain = train(find(train(:,1) == 77), 2:31);
mu = 0.0001;
% fit the model
[w gamma obj] = fitModel(mu, Mtrain, Btrain)

w =

   -4.1056
   -0.1242
   -3.3016
   -1.6735
    2.4501
   -4.8508
   -0.3210
    3.7928
   -1.2820
    0.2479
    4.1126
   -0.8100
   -0.3688
    3.8027
    0.5084
   -0.0471
   -0.1307
    4.2478
    0.7133
   -7.1841
    6.5976
    5.0381
    4.1452
    8.0556
   -1.6099
    0.1599
   -0.3038
    0.6853
    0.6642
    6.4163


gamma =

   -3.5688
```

## 2.

To pick the best trade off parameter, we should count the number of missclassified cases in the tuning set. Meanwhile, we should break the tie by considering the error of missclassification. The error for the missclassified cases in the tuning set could be defined as

$$\frac{1}{M}\sum_{i\in\mathcal{M}}|wx^i-\gamma|I(wx^i-\gamma\leq 0)+\frac{1}{B}\sum_{i\in\mathcal{B}}|wx^i-\gamma|I(wx^i-\gamma>0)$$

where $I(\cdot)$ is a indicator function.

Now we can define a function `evaluate` as below, for picking up the best $\mu$.

```
function [numMissed errors] = evaluate(mues, Mtrain, Btrain, Mtest, Btest)
% syntax: missed = getMissed(mu, Mtrain, Btrain, Mtest, Btest)
% given a vector of tuning parameter, training sets for positive and
% negative examples, test sets for positive and negative examples
% (features only, no class labels), fit the svm model and evaluate
% the model accuracy on test data.

numMissed = [];
errors = [];
for idx = 1:numel(mues)
  mu = mues(idx);
  [w gamma obj] = fitModel(mu, Mtrain, Btrain);
  % evaluate accuracy on the test data
  predictM = Mtest * w - gamma;
  predictB = Btest * w - gamma;
  correctM = (predictM > 0);
  wrongM = (predictM <= 0);
  correctB = (predictB <= 0);
  wrongB = (predictB > 0);
  numMissed = [numMissed sum(wrongM)+sum(wrongB)];
  m = size(predictM,1);
  k = size(predictB,1);
  err = (1/m)*sum(abs(predictM(find(wrongM == 1))));
  err = err + (1/k)*sum(abs(predictB(find(wrongB == 1))));
  errors = [errors err];
end
end
```

Using the function, we can investigate the number of missclassified cases in tuning set for each $\mu$'s.

```
Btune = tune(find(tune(:,1) == 66), 2:31);
Mtune = tune(find(tune(:,1) == 77), 2:31);
% evaluate the model when mu = 0.0001
[missed obj] = evaluate(mu, Mtrain, Btrain, Mtune, Btune)

missed =

     3


obj =
```

```
    0.1670

% evaluate the model for different mu's
mues = 5e-5:5e-5:5e-4;
[missed errors] = evaluate(mues, Mtrain, Btrain, Mtune, Btune)

missed =

     3     3     2     2     2     2     2     2     2     2


errors =

  Columns 1 through 8

    0.2763    0.1670    0.1313    0.1054    0.0925    0.0867    0.0820    0.0673

  Columns 9 through 10

    0.0534    0.0468
```

The number of missclassified points are all 2 for $\mu = 1.5e-4, \cdots, 5e-4$. We can break the tie by the error on the tuning set. When $\mu = 5e-4$ the error is the smallest, which is 0.0468. So, the best $\mu = 5e-4$.

# 3.

```
mu = 5e-4;
minMissed = size(Mtune,1);
for i = 1:29
  for j = (i+1):30
    Msubtrain = Mtrain(:,[i j]);
    Bsubtrain = Btrain(:,[i j]);
    Msubtune = Mtune(:,[i j]);
    Bsubtune = Btune(:,[i j]);
    numMissed = evaluate(mu,Msubtrain,Bsubtrain,Msubtune,Bsubtune);
    if numMissed < minMissed
      minMissed = numMissed;
    end
    fprintf('atts %2d %2d: misclass %3d\n',i,j, numMissed);
  end
end
minMissed

atts  1  2: misclass  14
atts  1  3: misclass  11
atts  1  4: misclass  12
atts  1  5: misclass  10
atts  1  6: misclass   8
atts  1  7: misclass   8
atts  1  8: misclass   8
atts  1  9: misclass   9
atts  1 10: misclass  11
atts  1 11: misclass   9
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atts  2  3: misclass  13
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atts  2  7: misclass  18
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atts  4  5: misclass   9
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atts 23 27: misclass   9
atts 23 28: misclass   7
atts 23 29: misclass   8
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atts 24 25: misclass   3
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atts 27 28: misclass   9
atts 27 29: misclass  20
atts 27 30: misclass  15
atts 28 29: misclass   9
atts 28 30: misclass   8
atts 29 30: misclass  30

minMissed =

     3
```

# 4.

From the last part, we saw that the minimum number of missclassified cases in tuning set is 3. We can use the same definition of error as in part 2 to break the tie.

```
mu = 5e-4;
for i = 1:29
  for j = (i+1):30
    Msubtrain = Mtrain(:,[i j]);
    Bsubtrain = Btrain(:,[i j]);
    Msubtune = Mtune(:,[i j]);
    Bsubtune = Btune(:,[i j]);
    [numMissed err] = evaluate(mu,Msubtrain,Bsubtrain,Msubtune,Bsubtune);
    if numMissed == 3
      fprintf('atts %2d %2d: misclass %3d\t error %f\n',i,j,numMissed,err);
    end
  end
end


atts  8 14: misclass   3     error 0.052644
atts 24 25: misclass   3     error 0.028699
```

So, we can see that attribute pair (24,25) gives the least missclassified cases and the best tuning set error.

Using feature 24 and 25, we can fit the model and test on the testing set. We can see that there are 6 missclassified points.

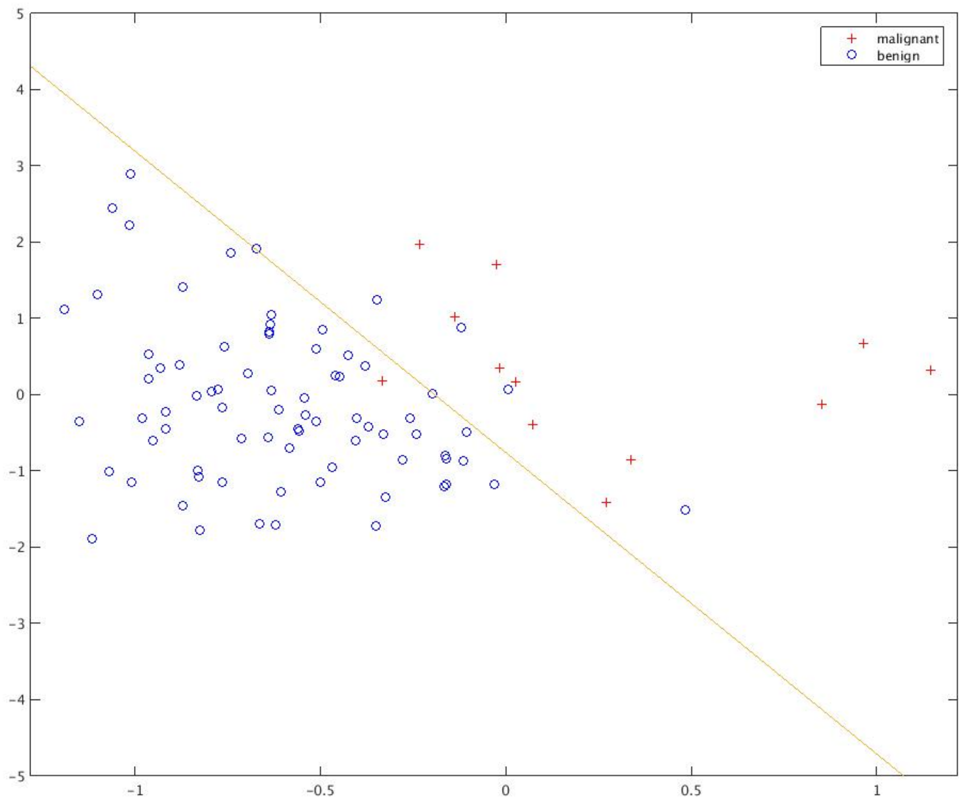
```
Btest = test(find(test(:,1) == 66), 2:31);
Mtest = test(find(test(:,1) == 77), 2:31);
mu = 5e-4;
idxs = [24 25];
Mtrain2 = Mtrain(:,idxs);
Btrain2 = Btrain(:,idxs);
Mtest2 = Mtest(:,idxs);
Btest2 = Btest(:,idxs);
numMissed = evaluate(mu,Mtrain2,Btrain2,Mtest2,Btest2)

numMissed =

     6
```

Let's now plot the testing data points, along with the separation plane.

```
% plot
[w,gamma] = fitModel(mu,Mtrain2,Btrain2);
w1 = w(1);
w2 = w(2);
xs = linspace(-2,4,1000);
ys = (-w1/w2)*xs + gamma/w2;
```

```
plot(Mtest2(:,1),Mtest2(:,2),'+r'), hold on;
plot(Btest2(:,1),Btest2(:,2),'ob'), hold on;
plot(xs,ys)
ylim([-5 5])
legend('malignant','benign')
```

The outcoming plot is shown below. It can be seen that, after some magnifying, six data points are incorrectly classified: 5 benigns are missclassified as malignant and one malignant is missclssified as benign.




separation line on feature 24 and 25, with testing data points