Problem Set 2

Submission deadline: February 16, 2022

Submission type: Report (soft or hard copy) and source code (soft copy)

Tsunami

Open ocean propagation of tsunami waves is approximated by the long-wavelength, shallow-water equation:

$$\partial_t^2 P = \boldsymbol{\nabla} \cdot (v^2 \, \boldsymbol{\nabla} P) \quad , \tag{1}$$

$$= \partial_x v^2 \, \partial_x P + \partial_y v^2 \, \partial_y P + v^2 (\partial_x^2 P + \partial_y^2 P) \quad . \tag{2}$$

where $P = P(x, y, t)$ is the height of tsunami waves above sea level, $v = \sqrt{g \, H(x, y)}$ the wave speed, g the acceleration due to gravity, and H the ocean depth.

When the term v^2 is not changing rapidly, an approximate form of the governing equation that will accurately predict the speed, dispersion, and focusing of the waves is:

$$\partial_t^2 P = v^2 \, \nabla^2 P \quad . \tag{3}$$

Equation (3) is called *homogeneous*, since it assumes that v^2 is constant, i.e., a homogeneous medium. Extra terms in equation (2) are called *inhomogeneous* terms.

Here a C-code, **class_tsunami.c**, is provided to you, which can solve the homogeneous equation to the 2nd-order in space and time.

(a) Modify the code to make it a 4th-order scheme in space while keeping the 2nd-order scheme in time.

(b) Derive formula for the inhomogeneous terms, both in 2nd-order and 4th-order schemes. You could play with them to appreciate their importance for various models of v.

Additional information:

1. List of files

   (a) **class_tsunami.c**: Main source file.

   (b) **bathymetry.in**: Bathymetry input file [Default].

(c) **bathymetryBE.in**: If you get maximum value of velocity 0 during program run, use this file instead of **bathymetry.in** replacing **char vmodel[] ="bathymetry.in"** with **char vmodel[] ="bathymetryBE.in"** in the main source file.

(d) **slices.out**: Output file.

(e) **class_movies.m**: Matlab script to plot output file.

2. The C-code can be compiled with, for example,

   gcc -o class_tsunami class_tsunami.c -lm

3. The code is executed by simply typing:

   ./class_tsunami

4. The output is in the file **slices.out**, which can be viewed in Matlab with the script **class_movie.m**.