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Working on understanding Majorana polarization. So we want to find two Majoranas γ^+ and γ^- (left/right Majorana) and form a non-local fermion

$$f = \frac{1}{2}(\gamma^+ + i\gamma^-), \tag{1}$$

such that $f^\dagger |o\rangle = |e\rangle$. Then we get $|e\rangle = \gamma^+ |o\rangle$ and $|e\rangle = -i\gamma^- |o\rangle$. We can expand these Majoranas in the Hermitian site-Majoranas $\gamma_j^+ = d_j^\dagger + d_j$ and $\gamma_j^- = i(d_j^\dagger - d_j)$

$$\begin{aligned} \gamma^+ &= \sum_{j,s} a_j^s \gamma_j^s \\ \gamma^- &= \sum_{j,s} b_j^s \gamma_j^s. \end{aligned} \tag{2}$$

We then calculate e.g.,

$$\begin{aligned} \langle o|\gamma_j^+|e\rangle = \langle o|\gamma_j^+\gamma^+|o\rangle = \langle o|\gamma_j^+ \sum_{k,s} a_k^s \gamma_k^s|o\rangle = \langle o|a_j^+ + \sum_{(k,s)\neq(j,+)} a_k^s \gamma_j^+\gamma_k^s|o\rangle \\ = a_j^+ + \sum_{(k,s)\neq(j,+)} a_k^s \langle o|\gamma_j^+\gamma_k^s|o\rangle \end{aligned} \tag{3}$$

but $\langle o|\gamma_j^+\gamma_k^s|o\rangle^* = \langle o|\gamma_k^s\gamma_j^+|o\rangle = -(2)\langle o|\gamma_j^+\gamma_k^s|o\rangle$ (by using anti-commutation). Hence the second term is purely imaginary (however, the anti-commutation has a factor 2?) and we can write (analogously for the other coefficients)

$$\begin{aligned} a_j^+ &= \mathrm{Re}\{\langle o|\gamma_j^+|e\rangle\} \\ a_j^- &= -\mathrm{Im}\{\langle o|\gamma_j^-|e\rangle\} \\ b_j^+ &= -\mathrm{Im}\{\langle o|\gamma_j^+|e\rangle\} \\ b_j^- &= \mathrm{Re}\{\langle o|\gamma_j^-|e\rangle\} \end{aligned} \tag{4}$$

since the a, b coefficients are real (γ^+ and γ^- are Hermitian). If the Hamiltonian is real (?), $a_j^- = b_j^+ = 0$ and we can skip taking the real part.