1 19 January 2023

Working on understanding Majorana polarization. So we want to find two Hermitian Majoranas γ^+ and γ^- (left/right Majorana) and form a non-local fermion

$$f = \frac{1}{2}(\gamma^+ + i\gamma^-), \tag{1}$$

such that $f^\dagger |o\rangle = |e\rangle$. Then we get $|e\rangle = \gamma^+ |o\rangle$ and $|e\rangle = -i\gamma^- |o\rangle$. We can expand these Majoranas in the Hermitian site-Majoranas $\gamma_j^+ = d_j^\dagger + d_j$ and $\gamma_j^- = i(d_j^\dagger - d_j)$

$$\begin{aligned} \gamma^+ &= \sum_{j,s} a_j^s \gamma_j^s \\ \gamma^- &= \sum_{j,s} b_j^s \gamma_j^s. \end{aligned} \tag{2}$$

We then calculate e.g.,

$$\begin{aligned} \langle o|\gamma_j^+|e\rangle &= \langle o|\gamma_j^+ \gamma^+|o\rangle = \langle o|\gamma_j^+ \sum_{k,s} a_k^s \gamma_k^s|o\rangle = \langle o|a_j^+ + \sum_{(k,s)\neq(j,+)} a_k^s \gamma_j^+ \gamma_k^s|o\rangle \\ &= a_j^+ + \sum_{(k,s)\neq(j,+)} a_k^s \langle o|\gamma_j^+ \gamma_k^s|o\rangle \end{aligned} \tag{3}$$

but $\langle o|\gamma_j^+ \gamma_k^s|o\rangle^* = \langle o|\gamma_k^s \gamma_j^+|o\rangle = -\langle o|\gamma_j^+ \gamma_k^s|o\rangle$ (by using anti-commutation). Hence the second term is purely imaginary and we can write (analogously for the other coefficients)

$$\begin{aligned} a_j^+ &= \mathrm{Re}\{\langle o|\gamma_j^+|e\rangle\} \\ a_j^- &= -\mathrm{Im}\{\langle o|\gamma_j^-|e\rangle\} \\ b_j^+ &= -\mathrm{Im}\{\langle o|\gamma_j^+|e\rangle\} \\ b_j^- &= \mathrm{Re}\{\langle o|\gamma_j^-|e\rangle\} \end{aligned} \tag{4}$$

since the a, b coefficients are real (γ^+ and γ^- are Hermitian). If the Hamiltonian is real (?), $a_j^- = b_j^+ = 0$ and we can skip taking the real part.

1. What have we assumed when doing this? Have we assumed that we have good left/right majoranas and then derived their expansion?

2 20 January 2023

The Majorana polarization is defined in e.g., Aksenov 2020 as roughly (though not taking into account taking the real part as above)

$$\mathrm{MP} = \frac{\sum_j' {a_j^+}^2 - {b_j^-}^2}{\sum_j' {a_j^+}^2 + {b_j^-}^2} \tag{5}$$

where the prime means to sum over half of the chain. This seems to depend on how much of γ^- has leaked into the left side.

1. What is the meaning of the MP? Relation to original paper?
2. Is it reasonably defined? What if we have an imaginary part? Spin?
3. What if one Majoranas leaks to the opposite half but the other does not?
4. Could you do a normal scalar product to calculate overlap?
5. If no overlap, the denominator is not necessary since the sum of the coefficients squared is unity (see below).

Proved that $\sum_k {a_k^+}^2 = 1$ by induction, anti-commutation relations and that the majoranas square to one, assuming that we only get the + coefficients. Also tried out the MP on the Poor mans geometry.