January 19th 2023

Working on understanding Majorana polarization. So we want to find two Hermitian Majoranas γ^+ and γ^- (left/right Majorana) and form a non-local fermion

$$f = \frac{1}{2}(\gamma^+ + i\gamma^-), \tag{1}$$

such that $f^\dagger |o\rangle = |e\rangle$. Then we get $|e\rangle = \gamma^+ |o\rangle$ and $|e\rangle = -i\gamma^- |o\rangle$. We can expand these Majoranas in the Hermitian site-Majoranas $\gamma_j^+ = d_j^\dagger + d_j$ and $\gamma_j^- = i(d_j^\dagger - d_j)$

$$\begin{aligned} \gamma^+ &= \sum_{j,s} a_j^s \gamma_j^s \\ \gamma^- &= \sum_{j,s} b_j^s \gamma_j^s. \end{aligned} \tag{2}$$

We then calculate e.g.,

$$\begin{aligned} \langle o|\gamma_j^+|e\rangle = \langle o|\gamma_j^+\gamma^+|o\rangle = \langle o|\gamma_j^+ \sum_{k,s} a_k^s \gamma_k^s|o\rangle = \langle o|a_j^+ + \sum_{(k,s)\neq(j,+)} a_k^s \gamma_j^+ \gamma_k^s|o\rangle \\ = a_j^+ + \sum_{(k,s)\neq(j,+)} a_k^s \langle o|\gamma_j^+\gamma_k^s|o\rangle \end{aligned} \tag{3}$$

but $\langle o|\gamma_j^+\gamma_k^s|o\rangle^* = \langle o|\gamma_k^s\gamma_j^+|o\rangle = -\langle o|\gamma_j^+\gamma_k^s|o\rangle$ (by using anti-commutation). Hence the second term is purely imaginary and we can write (analogously for the other coefficients)

$$\begin{aligned} a_j^+ &= \mathrm{Re}\{\langle o|\gamma_j^+|e\rangle\} \\ a_j^- &= \mathrm{Re}\{\langle o|\gamma_j^-|e\rangle\} \\ b_j^+ &= -\mathrm{Im}\{\langle o|\gamma_j^+|e\rangle\} \\ b_j^- &= -\mathrm{Im}\{\langle o|\gamma_j^-|e\rangle\} \end{aligned} \tag{4}$$

since the a, b coefficients are real (γ^+ and γ^- are Hermitian). If the Hamiltonian is real (?), $a_j^- = b_j^+ = 0$ and we can skip taking the real part.

1. What have we assumed when doing this? Have we assumed that we have good left/right majoranas and then derived their expansion?

January 20th 2023

The Majorana polarization is defined in e.g., Aksenov 2020 as roughly (though not taking into account taking the real part as above)

$$\mathrm{MP} = \frac{\sum_j' {a_j^+}^2 - {b_j^-}^2}{\sum_j' {a_j^+}^2 + {b_j^-}^2} \tag{5}$$

where the prime means to sum over half of the chain. This seems to depend on how much of γ^- has leaked into the left side.

1. What is the meaning of the MP? Relation to original paper?
2. Is it reasonably defined? What if we have an imaginary part? Spin?
3. What if one Majoranas leaks to the opposite half but the other does not?
4. Could you do a normal scalar product to calculate overlap?
5. If no overlap, the denominator is not necessary since the sum of the coefficients squared is unity (see below).

Proved that $\sum_k {a_k^+}^2 = 1$ by induction, anti-commutation relations and that the majoranas square to one, assuming that we only get the + coefficients. Also tried out the MP on the Poor mans geometry.

January 23rd, 2023

1. Confused about Fourier transform of second quantization operators. Turns out it can be seen as a basis change between position and momentum space bases. See p. 16 to 18 in Flensberg Many body.
2. Started trying to derive bulk energies of Kitaev chain with periodic boundary condition (Alicea).
3. Discussed MP with Viktor, seems like the normalization is not good. One special case with overlap only on right side, still unity MP. To include spin one can subtract the spin part, such that it measures how lonely one of the spin species is.
4. Then I started looking through the derivation in Akhmerov to try and understand how the Kitaev chain emerges from local coupling.

January 24th, 2023

Working on the derivation in Akhmerov, struggling with Maple. Might have a problem with one of my rules, giving the wrong sign in the new operators.

Januray 25th, 2023

Found problem with sign, just took other branch of Δ. Struggling with getting the correct phases of the a and b operators in Maple, but I think I got it. Expressed the old c operators in the new a and b. Inserted it in Hamiltonian. The dot hamiltonian of course reduces to only $a^\dagger a$ and $b^\dagger b$ so only spin-orbit/rotation left. Found out what $e^{i\boldsymbol{\lambda}\boldsymbol{\sigma}}$ actually means, it is the matrix representation of the

rotation operator $D(n, 2\lambda)$ which rotates the spin 2λ around the n axis. Also, very convenient to use Pauli matrix exponent formula. Very messy algebra however, try to get to Akhmerov. Don't understand how to/the reason why you can project onto the a operators.

Janurary 26th

Looked at job openings before lunch. After lunch I managed to get to the expression for the tunneling in Akhmerov. Still confused about how you can remove all b operators and what the conditions of doing it are. Thinking about the conditions on the parameters to get to the topological phase.

January 27th

Rewrote the derivation of Akhmerov more neatly. Discussed with Viktor:

- We can neglect the b operators at $V \gg w$ since the b-states are far off in energy and do not affect the ground state (which is what we are interested in). Perhaps one can think about it in terms of line width?
- The limit $V \gg w$ also implies $\Delta_{ind} \propto \Gamma_{SC}, \mu_n \gg w$. So in essence we need large Zeeman splitting, SC-dot coupling and chemical potential to obtain Kitaev chain with local SC-coupling.
- Also interesting that the particles in the emerging Kitaev chain are these a operators (Bogoliubov-like), not electrons.
- Next up is to
    1. Write up the Akhmerov Hamiltonian with constant SC-phase, spin-orbit rotation and spin-orbit direction, and maybe do a gauge transformation to only get one phase.
    2. Implement the model as well as the Kitaev chain in code.
    3. Implement a MP function for each model.
    4. Compare MP and energy gap in Kitaev and local coupling model.
- I don't understand how to do the scanning of MP and gap. I need to scan λ probably such that it takes linear steps in Δ?
- For the MP I now need to sum over three indices.

January 30th

Wrote down and implemented Akhmerov model with constant spin-orbit direction. Implemented the Kitaev model. Made heatmaps of MP and energy gap

for Kitaev model. SC lab and lecture. Tomorrow, write down SC summary and hand in exercises. Do the lab plotting. Maybe structure my code, move calculations to src and plotting to scripts.

January 31st

Done with SC summaries and lab data analysis.

February 1st

Handed in hand-in. Structuring code. Confused about Revise when editing multiple files. I think I need to create a package/module in src! Meeting with Martin:

- Discussing Akhmerov
- I need to do 1D scan in my model with constant w and ϵ, varying λ such that Δ changes but t constant.
- Look at adding Coulomb interaction, see how it looks like with a and b operators.
- MP, try both with a and b operators and with original c operators (add spin up and down, they are the same majorana!), should be same.
- A little confused about V_z being positive/negative.

February 2nd

Worked on figuring out workflow. Created package/module in src and structured everything, works now with Revise. Started with some scans of MP and gap for the local pairing model. Need to figure out how to set variables and scan λ to reach sweet spot $t = \Delta$. Should try implementing MP with a, b operators.

February 3rd

Did scans of λ, keeping everything else constant and reached sweet spot at $\lambda = 2\alpha$. Tried working on scanning λ, changing Kitaev Δ but keeping t constant, a bit tricky. Also worked on the setup of the Hamiltonian. I believe it's best to keep all parameters in a dictionary. Contemplating whether to keep the option for having varying parameters across the chain. Wrote something to be able to vary different stuff in a 1d scan, don't know if it's great. Should write a general Majorana polarization function which automatically does the Kitaev/spinful calculation depending on input.

February 13th

Changed the hamiltonians to taking the parameters in a dictionary instead. Kept the option to vary parameters across chain, even expanded it to Coulomb and Zeeman also. Fixed the function to vary stuff in a 1d scan. Now it can be used for both Kitaev and local pairing. Fixed the Majorana polarization function such that it can do both spinless and spinful. Did some plots scanning λ, confused about 2 things

- Seems to be more sweet spots than expected
- Odd number of dots seems to drastically change how much the energy gap varies

Should do a proper calculation of sweet spots and think about how to do the local pairing vs kitaev plot.

February 14th

Now I have a working comparison between local pairing and kitaev, the former approaching the latter with increasing V_z. Also added all Majorana coefficients to the MP calculation functions. Next maybe to investigate finite U?

February 15th

Meeting with Viktor:

- A bit unsure about how to change V_z in local vs Kitaev plots, scale it with w or t?
- Probably a good idea to calculate how Coulomb interaction affects how to hit $\epsilon = 0$.
- Idea: do the above, then for each U, find the optimal sweet spot.
- Viktor explained how he has worked with interpretation of MP and the complications with many-body Majoranas
- Majorana: degenerate, robust against perturbations, spatially separate majoranas?
- A new measure of robustness? $||\delta\rho_R||_F^2$

Did plots with finite U.

February 16th

Worked on the algebra for the renormalization due to interaction. A bit confusing and tedious, but I think that one misses sweet spot in energy by $U \sin \alpha$ if one chooses Akhmerov parameters, so no supression due to Zeeman. However, the projection does not work for low Zeeman either. Looked at Majorana braiding, cool. Should look at the new measure of robustness tomorrow.

February 17th

Meeting about Quantum seminars and then device meeting. Discussion about summer student on machine learning. Meeting with Martin, discussion about Coulomb. Seems like I can expand my equation for μ in U to get some expression for how to change μ to get to sweet spot. Should try reach sweet spot by solving it numerically also, alternatively, scanning in μ direction also.

February 20th

Been checking the algebra for U term. Seems ok. Solved the equation numerically, however does not reach sweet spot anyways. Might have to do with the rest of the scan (restrictions on w and λ). Wrote something to do 2d scans instead.

February 21–22nd

21st, Copenhagen. 22nd, did 2d scans with effective Kitaev Delta and mu, found bug. Now I see sweet spots. Consider moving hamiltonian to QuantumDots package instead with homogeneous chain. Seems like something is wrong with my renormalization due to Coulomb, my guess for the sweet spot is wrong. Annoying. Looked at reduced density matrix. Trying to understand again. Tomorrow, use reduced density matrix function from quantum dots package. Plot. Maybe move Hamiltonian. Write tests (should've done before). Look at outline for thesis.

February 23–24th

Implementing the $\delta\rho$ measure. Started looking at the outline of thesis. Created document. Still something wrong with guess for sweet spot. Viktor found the mistake! I cannot remove $bb^\dagger = 1$, only $b^\dagger b = 0$! Fixed the problem. Wrote a function to find sweet spot from scan. Ideas to do next week:

- Journal club Dvir?
- Plot measures at sweet spot with varying V.

- Write tests
- Performance issues? Tidy up. Maybe move hams to QuantumDots.
- What plots do I want?
- Look more into the tuning in Akhmerov?
- 2D scans with a different x-axis, more experimentally feasible?

February 27th

Removed the type from Matrix and set it to hermitian, sped things up. Could use parity thing to speed up more. Maybe change the hamiltonian such that it only takes scalars (maybe not μ and Φ)? Changed the δE axis, now I divide by the topological gap also. Added more contour lines, 5% of the gap. Added the possibility of properly optimizing towards the sweet spot. It works, but slow. Ideas:

- Plot the optimal sweet spot over V_z.
- Tests
- Tidy up. Maybe restrict to scalars
- Prep meeting

February 28th

Worked a lot on tidying up code. Now I can choose to use scalars or vectors how I want. Might be a bit slow though. Also worked on plotting sweet spot values for varying zeeman. Works but slow. Need to find a way to store values so I don't have to calculate every time I change something in the plot. Questions:

- What sites do I use for $\delta\rho$?

To do:

- Prep meeting
- Store calculations
- Plot the optimal sweet spot over V_z for different N and adding U.
- Tests