NANYANG TECHNOLOGICAL UNIVERSITY

SC2001/CE2101/CZ2101: Algorithm Design and Analysis

Introduction to Sorting

Instructor: Assoc. Prof. ZHANG Hanwang

Courtesy of Dr. Ke Yiping, Kelly's slides

Learning Objectives

At the end of this lecture, students should be able to:

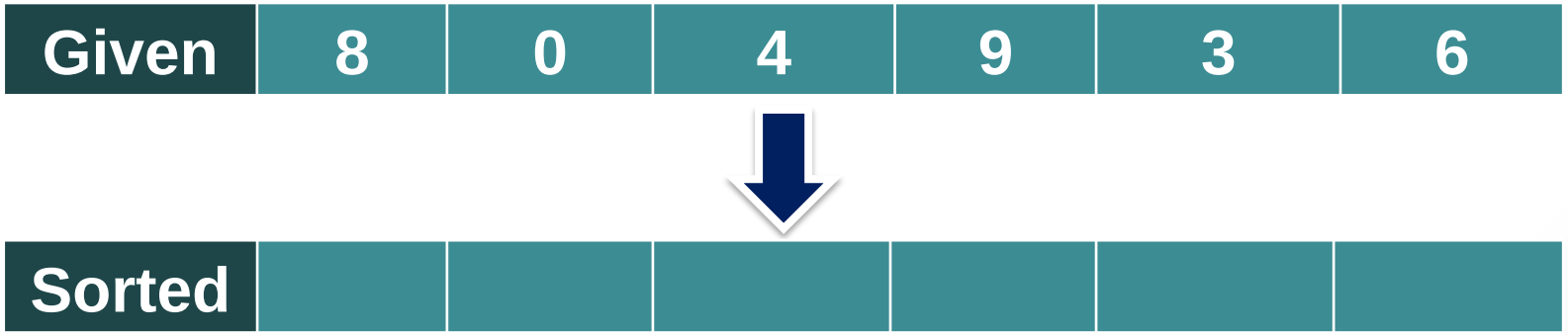
- Define what sorting is
- Explain why we learn sorting
- Analyze the objective and evaluation of sorting algorithms

What is Sorting?

Definition (sorting in ascending order):

- Given a set of records $\mathbf{r_1}, \mathbf{r_2}, \ldots, \mathbf{r_n}$ with key values $k_1, k_2, \ldots, k_n$, arrange records in order s such that records $\mathbf{r_{s1}}, \mathbf{r_{s2}}, \ldots, \mathbf{r_{sn}}$ have keys with property $k_{s1} \leq k_{s2} \leq \ldots \leq k_{sn}$.

Example:



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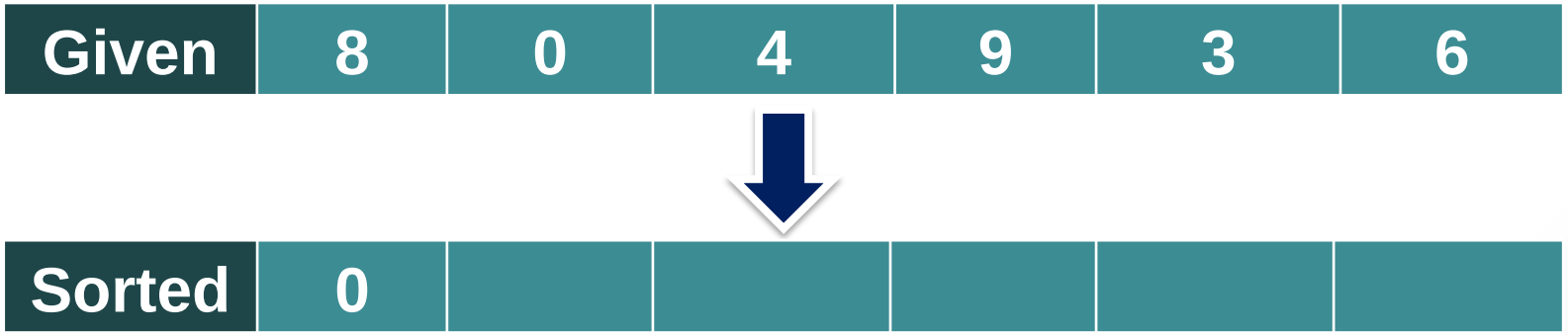

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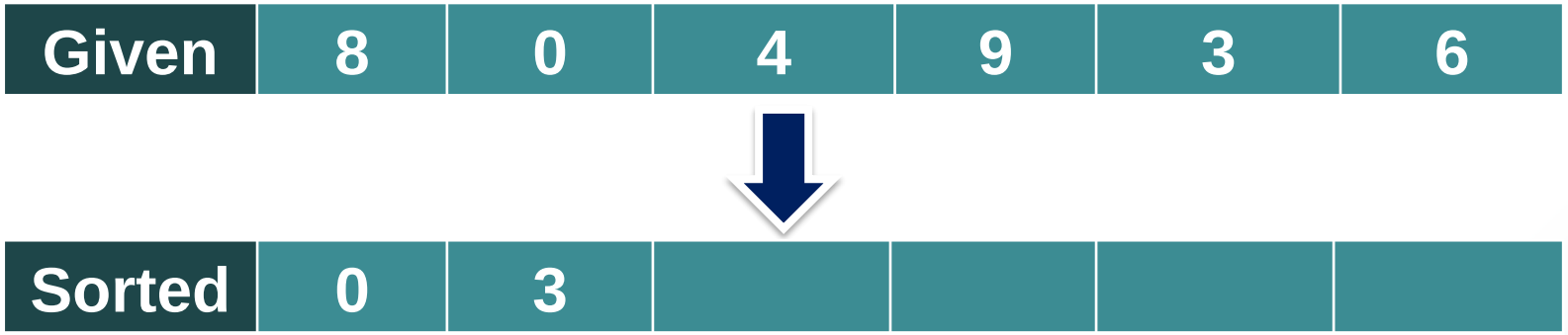

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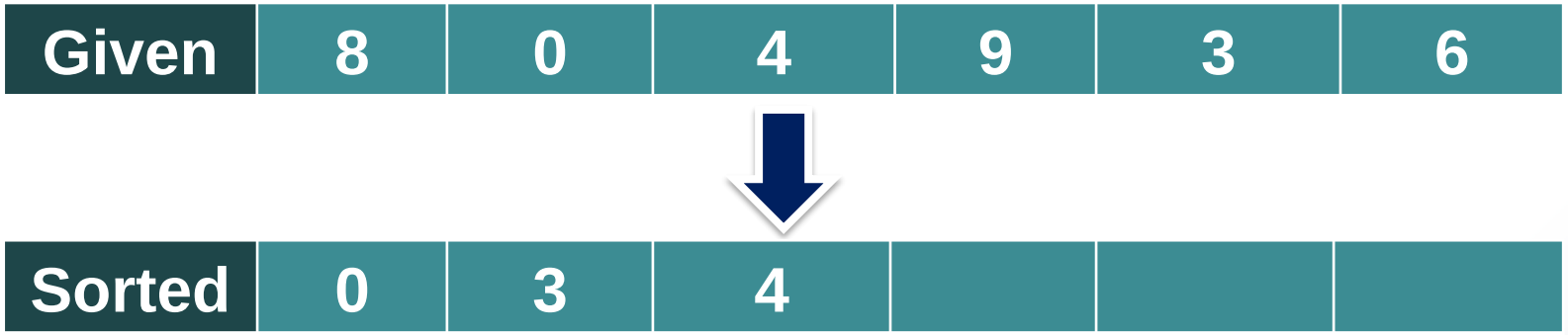

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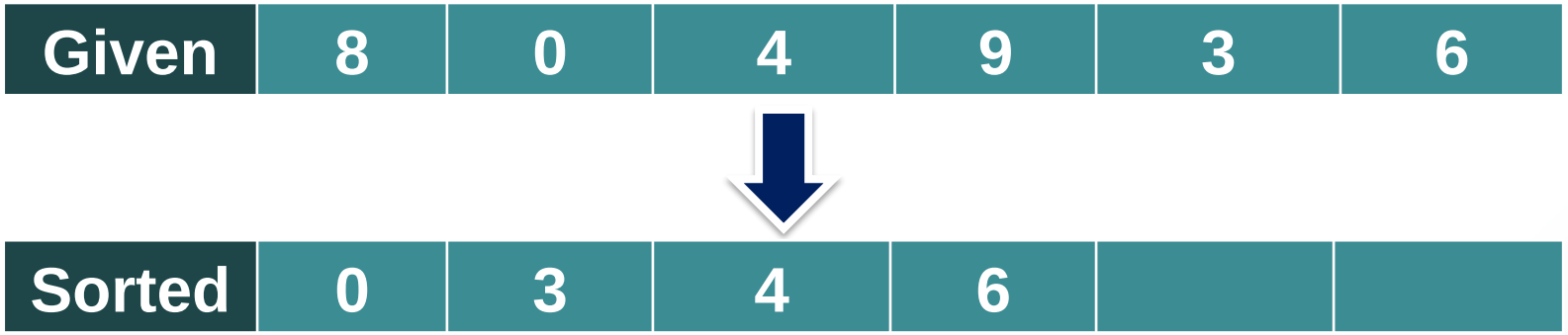

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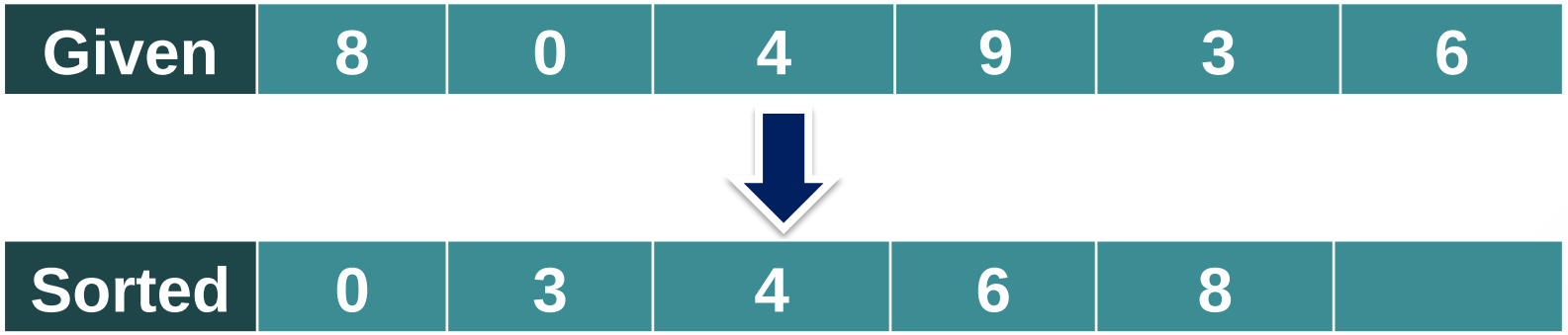

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Example:

Given	8	0	4	9	3	6

Sorted	0	3	4	6	8	9

Why do we learn sorting?

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- Things must be kept in some order if we want to find them quickly.

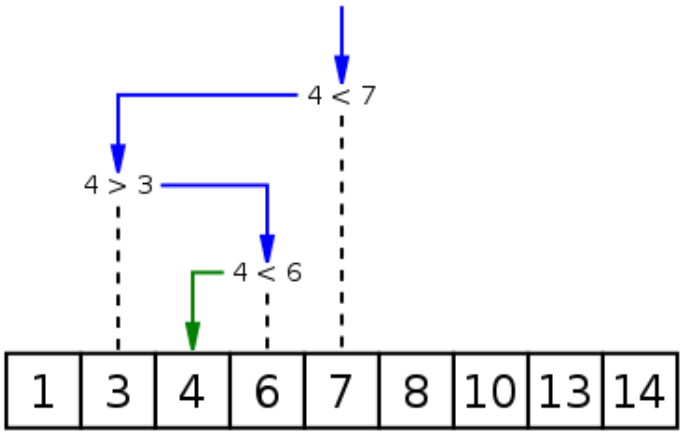
Dictionary

Reference: Row Of Books Clipart. (n.d.). Retrieved May 16, 2016, from http://cliparts.co/clipart/39651.

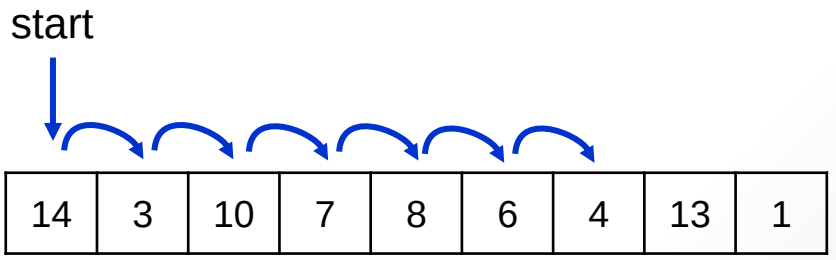
Why do we learn sorting?

- Things must be kept in some order if we want to find them quickly.
- How to arrange things in order? Sorting algorithms.
- Sorting is a basic building block for many algorithms.

Binary Search



Sequential Search



Reference: T. (2015, April 19). Binary search in a sorted array. Retrieved May 18, 2016, from https://commons.wikimedia.org/wiki/File:Binary_search_into_array.png#/media/File:Binary_search_into_array.png.

Why do we learn sorting?

- Things must be kept in some order if we want to find them quickly.

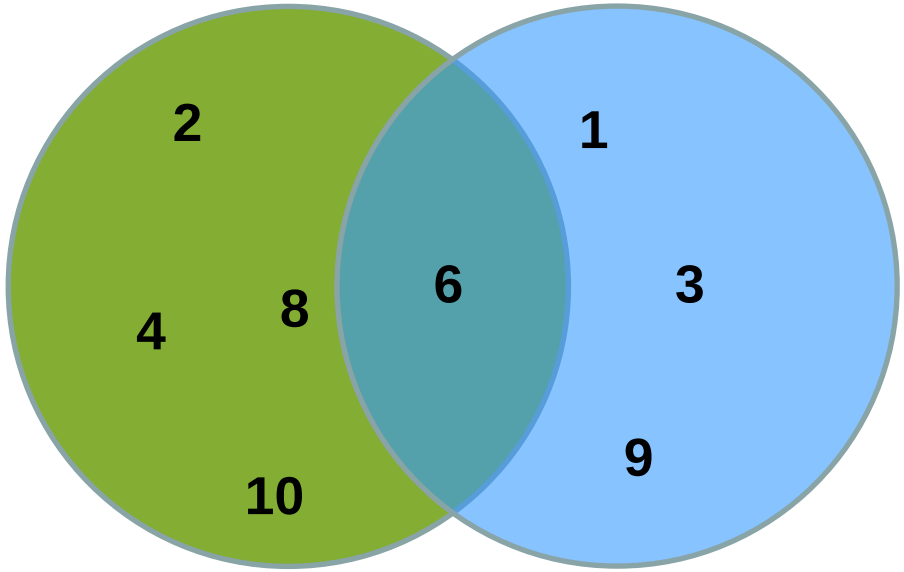
- How to arrange things in order? Sorting algorithms.
- Sorting is a basic building block for many algorithms.

- Most thoroughly studied problem in Computer Science.
- To learn ideas in Algorithm Design derived from techniques in sorting.

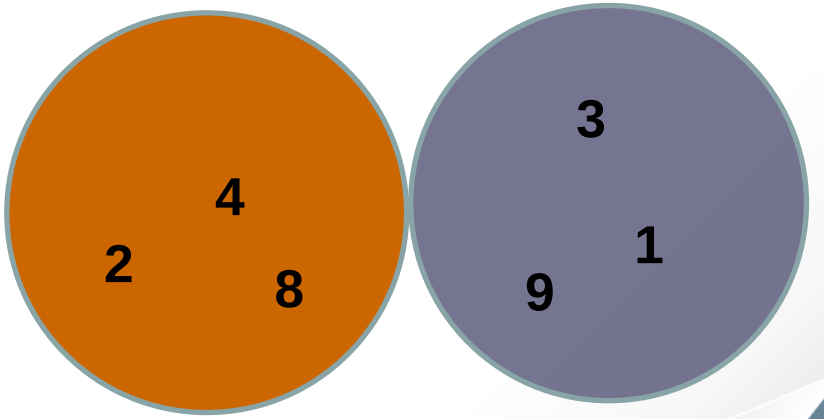
Example: Disjoint Sets

- **Problem:**

  Determine whether two sets (both of size *n*) are disjoint.



Disjoint



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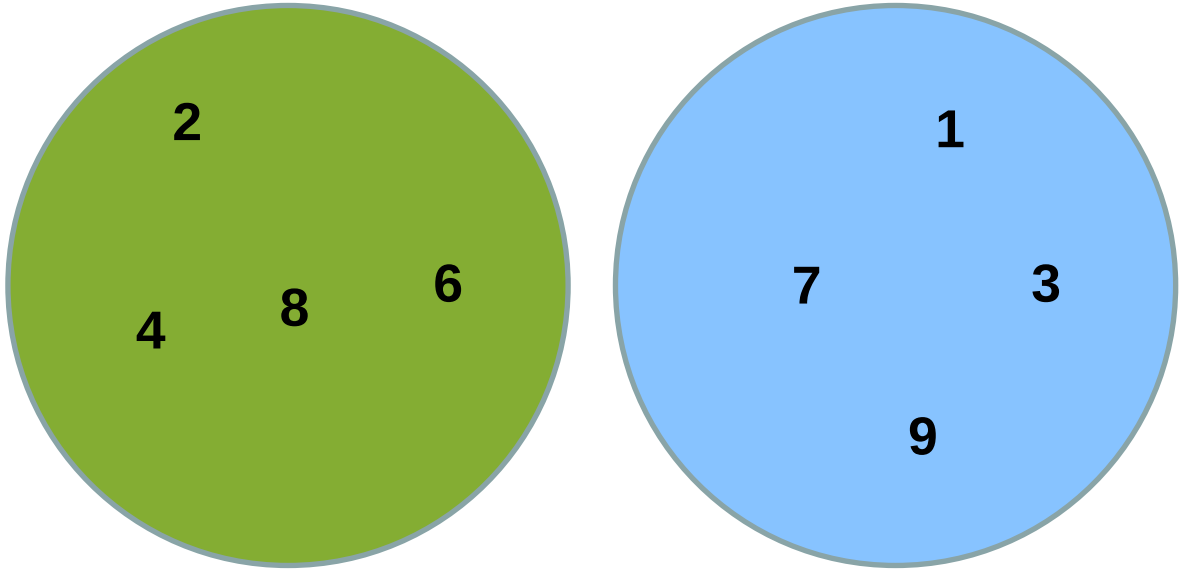
  Determine whether two sets (both of size *n*) are disjoint.

  - **Solution 1:** Compare each element of the 1st set with each element of the 2nd set. That is, n^2 comparisons.



Example: Disjoint Sets

- **Problem:**

  Determine whether two sets (both of size n) are disjoint.

  - **Solution 1:** Compare each element of the 1st set with each element of the 2nd set. That is, n^2 comparisons.
  - **Solution 2:**

    Step 1: We first **sort the first set into ascending order**. This takes $O(n \lg n)$ effort using Mergesort or Heapsort.

    Step 2: For each element in the 2nd set, we **use Binary Search** to find it in the 1st set. This takes $O(n \lg n)$ time.

Comparison of Performance

Solution 1: $O(n^2)$

Solution 2: $O(n\lg n)$

Savings:

n	=	64	128	256	512
n^2	=	4,096	16,384	65,536	262,144
$n\lg n$	=	384	896	2,048	4,608

Comparison of Performance

The data items to be sorted:

- Given a (very large) list of records.
- Each record has the following form: key; rest info of record:

```
class ALIST {
    KeyType     key;
    DataType    data;
};
```

- Key domain is an ordered set.
- **Objective:** To arrange records in 'ascending' or 'descending' order.

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Comparison of Performance

- Sorting can be classified into **internal sorting** and **external sorting.**
  - We focus on internal sorting only,

    i.e., all records are in (high speed) main memory during sorting.
- **Sorting involves two basic actions:**
  1) key comparisons between two records
  2) swapping records around
- **Goal:** Use minimum working space and do as few key comparisons as possible.

Summary

- Sorting is to arrange a set of records so that their key values are in ascending or descending order.

- It is important to learn sorting, because:
  - Sorting has important applications
  - Ideas of sorting can be used for other algorithms

- Objective is to design sorting algorithms with:
  - Minimum usage of memory
  - Minimum number of key comparisons or swaps