SC2001/CX2101 Algorithm Design and Analysis

Tutorial 4: Dynamic Programming

College of Computing and Data Science *Nanyang Technological University*

This tutorial helps you develop skills in the learning outcome of the course: "Able to design algorithms using suitable strategies (dynamic programming, etc) to solve a problem, able to analyse the efficiencies of different algorithms for problems like optimal sequencing for matrix multiplication, the longest common subsequence, etc".

Week 10 (Q1 – Q3)

1. Find the length of the longest common subsequence and a longest common subsequence of CAGAG and ACTGG by the dynamic programming algorithm in the lecture notes.

2. The H-number H(n) is defined as follows:

   H(0) =1, and for n>0:
   H(n) = H(n-1) + H(n-3) + H(n-5)+ ….+H(0) when n is odd
   H(n) = H(n-2) + H(n-4) + H(n-6)+ ….+H(0) when n is even.

   a) Give a recursive algorithm to compute H(n) for an arbitrary n as suggested by the recurrence equation given for H(n). Draw the tree that represents the recursive calls made when H(8) is computed.
   b) Draw the subproblem graph for H(8) and H(9).
   c) Write an iterative algorithm using the dynamic programming approach (bottom-up). What are the time and space required?

3. The binomial coefficients can be defined by the recurrence equation:

   $C(n, k) = C(n-1, k-1) + C(n-1, k)$ for $n > 0$ and $k > 0$
   $C(n, 0) = 1$ for $n >= 0$
   $C(0, k) = 0$ for $k > 0$

   C(n, k) is also called "n choose k". This is the number of ways to choose k distinct objects from a set of n objects.

   (a) Give a recursive algorithm as suggested by the recurrence equation given for C(n, k).
   (b) Draw the subproblem graph for C(5, 3).
   (c) Write a recursive algorithm using the dynamic programming approach (top-down) stating the data structure used for the dictionary. What is the space and time complexity respectively?
   (d) Write an iterative algorithm using the dynamic programming approach (bottom-up). What is the space and time complexity respectively?