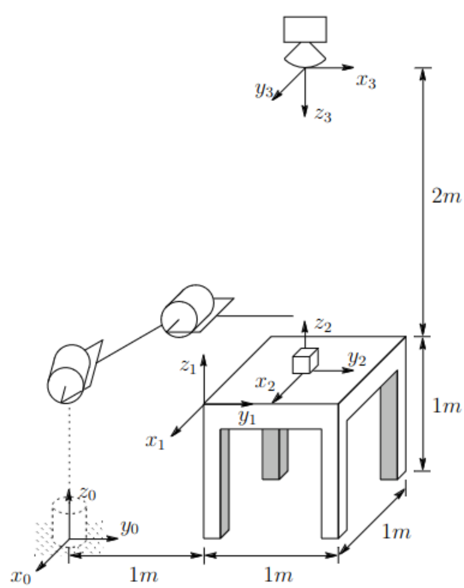
Fall 2024—16-642 Manipulation, Estimation and Control
Problem Set 4

Due: 25 November 2024

GUIDELINES:

- You must *neatly* write up your solutions and submit all required material electronically via Canvas by **11:59 pm of the posted due date.**
- Include any MATLAB scripts that were used in finding the solutions. Also include any plots generated with MATLAB as a part of the question.
- You are encouraged to work with other students in the class, however you must turn in your own *unique* solution.
- Late Policy: You are given **in total 72 hours** of grace period for **ALL** homework across the semester, which you can use to give yourself extra time without penalty. Late work handed in when you have run out of grace will not be accepted.

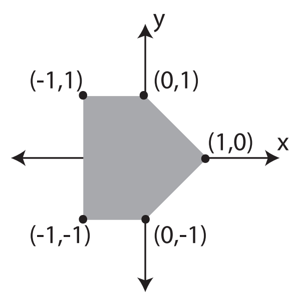
1. Consider the diagram on the next page. A robot is set up 1 meter from a table. The table top is 1 meter high and 1 meter square. A frame $O_1x_1y_1z_1$ is fixed to the edge of the table as shown. A cube measuring 20 cm on a side is placed in the center of the table with frame $O_2x_2y_2z_2$ established at the center of the cube as shown. A camera is situated directly above the center of the block 2m above the table top with frame $O_3x_3y_3z_3$ attached as shown.



(a) (5 points) Find the homogeneous transformations relating each of these frames from the base frame $O_0x_0y_0z_0$. Find the homogeneous transformation relating the frame $O_2x_2y_2z_2$ from the camera frame $O_3x_3y_3z_3$.

(b) (10 points) The block is rotated $\pi/2$ radians about z_2 and then both it and frame $O_2x_2y_2z_2$ are moved along the x_2y_2-plane so that the center of the cube has coordinates [-1.0, 0.5, 0.1]^T relative to frame $O_1x_1y_1z_1$. The camera is then tilted along one of the axes in the camera frame $O_3x_3y_3z_3$ so that z_3 is again pointed directly at the center of the cube. Compute the new homogeneous transformation relating the block frame $O_2x_2y_2z_2$ from the camera frame $O_3x_3y_3z_3$, and the new transformation relating the block frame from the robot base frame $O_0x_0y_0z_0$.

2. (20 points) **Fun With SE(2):** Consider the planar rigid body defined below. It is possible to "move" this body by transforming all of the corner points according to a 2D homogeneous transformation matrix and plotting the result.



Consider the two transformation matrices:

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.866 & 0.500 & 0 \\ -0.500 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Move the rigid body according to the motions described below and plot the results. (It's probably best to write some sort of program to do this – we recommend using MATLAB, but you can use any language you want. Please include the source file of your program with your solution.)

(a) A, relative to the fixed frame, followed by B, relative to the current frame.

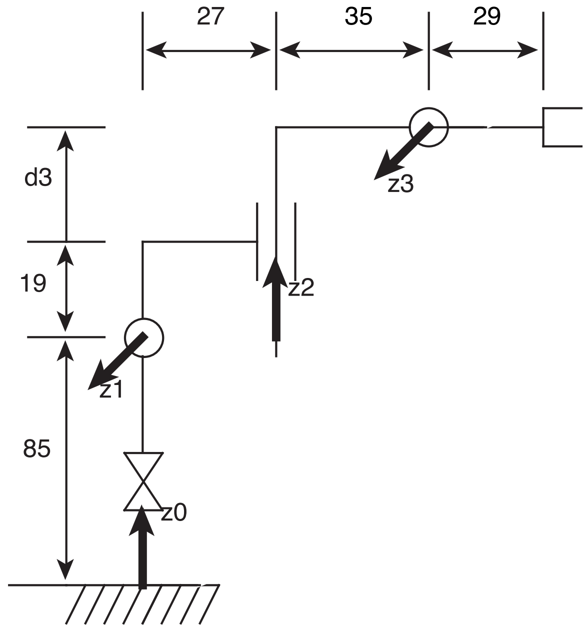
(b) A, relative to the fixed frame, followed by B, relative to the fixed frame.

(c) B, relative to the fixed frame.

(d) B, relative to the fixed frame, followed by A, relative to the fixed frame.

(e) B, relative to the fixed frame, followed by A, relative to the current frame.

3. (20 points) For the manipulator drawn below, draw the location of the DH frames and create a table of DH parameters. The positive direction of each joint is depicted by the z axis associated for that joint, which has conveniently been included for you. For each frame, explain whether the frame was uniquely defined by the DH convention. If it was not, describe the choices you made in defining it. (and don't forget to include the last frame!)

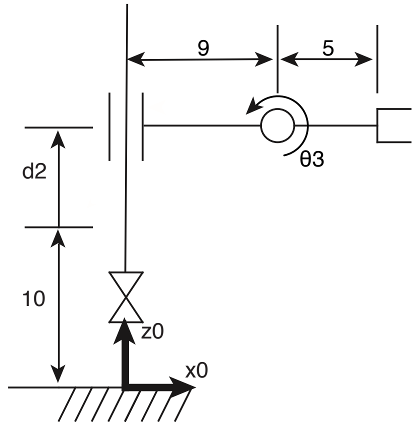


4. (15 points) Given the DH parameters in the table below, draw the manipulator that they describe. Assume that frame 0 is oriented with the z-axis pointing up, x-axis pointing right, and y-axis pointing into the page. θ_1, θ_2, θ_3 and ℓ_2 are joint variables. Provide a clearly labelled figure, with the axes and the link types depicted. No detailed explanation necessary.

i	θ_i	d_i	a_i	α_i
1	θ_1	5	7	0
2	0	ℓ_2	4	-90^o
3	θ_3	-2	3	90^o
4	θ_4	-3	0	0

hint: First use the parameters to determine where all of the frames should be, then determine where the joints should go, then draw in the links connecting them.

5. Consider the manipulator drawn below in a configuration where $q_1 = q_3 = 0$ at $t = 0$, and the task space is assumed to be the position only of the end effector (i.e., $\mathbb{X} = \mathbb{R}^3$). The positive direction of the first joint is given by the z_0 axis.



(a) (10 points) Find the analytical jacobian $J_a(q)$ at $t = 0$ by representing the end-effector position explicitly in terms of joint coordinates and differentiation

(b) (10 points) Find the geometric jacobian $J(q)$ at $t = 0$ by solving $v_n^T = J_n \dot{q}$ for each link and building the matrix. Make sure to explain your answers

(c) (10 points) Are there any singular configurations? If so list them. You can use whatever method you want to find them, but make sure to explain your answer.