16-665

HW 2

Boxiang Fu

boxiangf

10/02/2024

Q1.1.

Given parameters

cohesion c

angle of internal friction φ

Pressure-sinkage parameters k_c and k_φ

exponent of sinkage n

rigid tire of diameter d and width b

gross vehicle weight W_G

Q1.1.1. Let $n=0.5$, the compaction resistence per wheel is given as

$$R^{wheel} = \frac{(3W_{wheel}/\sqrt{d})^{\frac{(2n+2)}{(2n+1)}}}{(3-n)^{\frac{(2n+2)}{(2n+1)}}(n+1)(k_c+bk_\varphi)^{\frac{1}{(2n+1)}}}$$

So
$$\frac{R^{total}_{4\text{-wheeled}}}{R^{total}_{6\text{-wheeled}}} = \frac{4R^{wheel}_{4\text{-wheeled}}}{6R^{wheel}_{6\text{-wheeled}}}$$

$$= \frac{\dfrac{4\cdot(3\cdot\frac{1}{4}W_G/\sqrt{d})^{\frac{(2\times0.5+2)}{(2\times0.5+1)}}}{\cancel{(3-n)^{\frac{(2n+2)}{(2n+1)}}(n+1)(k_c+bk_\varphi)^{\frac{1}{(2n+1)}}}}}{\dfrac{6\cdot(3\cdot\frac{1}{6}W_G/\sqrt{d})^{\frac{(2\times0.5+2)}{(2\times0.5+1)}}}{\cancel{(3-n)^{\frac{(2n+2)}{(2n+1)}}(n+1)(k_c+bk_\varphi)^{\frac{1}{(2n+1)}}}}}$$

$$= \frac{4}{6} \times \frac{\left(\frac{3}{4} W_G \cancel{/\sqrt{d}}\right)^{\frac{3}{2}}}{\left(\frac{1}{2} W_G \cancel{/\sqrt{d}}\right)^{3/2}}$$

$$= \frac{2}{3} \times \left(\frac{3}{4} \times 2\right)^{3/2}$$

$$= \left(\frac{3}{2}\right)^{-1} \left(\frac{3}{2}\right)^{3/2}$$

$$= \left(\frac{3}{2}\right)^{1/2}$$

$$= \frac{\sqrt{6}}{2}$$

$$\simeq 1.2247$$

So the 4-wheeled vehicle has 1.2247 times more total compaction resistance than the 6-wheeled vehicle

Therefore, the 6-wheeled concept is better to minimize compaction resistance compared to the 4-wheeled concept.

Q1.1.2.

Let $A_{6\text{-wheeled}}$ denote the contact area of each wheel on the 6-wheeled vehicle.

Let H dnote soil thrust

Given $A_{4\text{-wheeled}} = 1.2 A_{6\text{-wheeled}}$ and the soil thrust equation for no slip

$H = cA + W\tan\varphi$, we have

$$\frac{H^{total}_{4\text{-wheeled}}}{H^{total}_{6\text{-wheeled}}} = \frac{4H^{wheel}_{4\text{-wheeled}}}{6H^{wheel}_{6\text{-wheeled}}}$$

$$= \frac{4\left(cA_{4\text{-wheeled}} + \frac{1}{4}W_G \tan\varphi\right)}{6\left(cA_{6\text{-wheeled}} + \frac{1}{6}W_G \tan\varphi\right)}$$

$$= \frac{4\left(1.2cA_{6\text{-wheeled}} + \frac{1}{4}W_G \tan\varphi\right)}{6\left(cA_{6\text{-wheeled}} + \frac{1}{6}W_G \tan\varphi\right)}$$

$$= \frac{4.8cA_{6\text{-wheeled}} + W_G \tan\varphi}{6cA_{6\text{-wheeled}} + W_G \tan\varphi}$$

We note that for physical values of c and $A_{6\text{-wheeled}}$, the ratio $\frac{H^{total}_{4\text{-wheeled}}}{H^{total}_{6\text{-wheeled}}} < 1$

So $H^{total}_{4\text{-wheeled}} < H^{total}_{6\text{-wheeled}}$

So the 6-wheeled concept is better at maximizing soil thrust assuming no slip compared to the 4-wheeled concept.

Q1.1.3. Assuming the only motion resistance is compaction, the drawbar pull for a single wheel of the 6-wheeled vehicle is:

$$DP^{wheel}_{6\text{-wheeled}} = H^{wheel}_{6\text{-wheeled}} - R^{wheel}_{6\text{-wheeled}}$$

$$= cA_{6\text{-wheeled}} + \frac{1}{6} W_G \tan\varphi$$

$$- \frac{\left(3 \cdot \frac{1}{6} W_G / \sqrt{d}\right)^{\frac{(2n+2)}{(2n+1)}}}{(3-n)^{\frac{(2n+2)}{(2n+1)}} (n+1)(k_c + bk_\varphi)^{\frac{1}{(2n+1)}}}$$

Assuming $n=0.5$ from Q1.1.1, we obtain

$$DP^{wheel}_{6\text{-wheeled}} = cA_{6\text{-wheeled}} + \frac{1}{6} W_G \tan\varphi - \frac{\left(W_G / 2\sqrt{d}\right)^{3/2}}{(2.5)^{3/2}(1.5)(k_c + bk_\varphi)^{1/2}}$$

Q1.1.4. From Class 2 Slide 17, the cohesion c and angle of internal friction φ of packed clay and loose sand are approx.

Clay

$c = 69 \times 10^3$ Pa

$\varphi = 20°$

Sand

$c = 1000$ Pa

$\varphi = 30°$

For a vehicle with weight W distributed on each wheel and contact area A for each wheel, we obtain the max. soil thrust for each wheel as

$$H^{wheel}_{clay} = 69000 A_{clay} + W_{clay} \tan 20°$$
$$= 69000 A_{clay} + 0.364 W_{clay}$$

$$H^{wheel}_{sand} = 1000 A_{sand} + W_{sand} \tan 30°$$
$$= 1000 A_{sand} + 0.577 W_{sand}$$

From the above equations, we see that holding weight W constant, a 0.01 increase/decrease in contact area A will increase/decrease H^{wheel}_{clay} by 690 and H^{wheel}_{sand} by 10.
So the increase/decrease for clay is 69 times more than sand.
Similarly, increasing/decreasing W by 1kg holding A constant will increase/decrease H^{wheel}_{clay} by 0.364 and H^{wheel}_{sand} by 0.577.
So the increase/decrease for sand is more than clay.

However, W and A are usually interrelated and are seldom independent. It is usually the case that W and A are positively correlated. So increasing W or A will almost always cause H^{wheel}_{clay} to increase more than H^{wheel}_{sand} (and viceversa) due to the fact that the soil cohesion in clay is 69 times that of sand. In other words, it is almost always the case that more soil thrust is avaliable in clay as compared to sand. The only case where this does not happen is if W is exceedingly large while A is exceedingly small, which is likely to be physically infeasible as heavy weights usually require high wheel contact areas. However, it is not impossible, but further contact pressure analysis is required to determine the relationship between A and W before concrete conclusions can be drawn.

Q1.2.1.

If choosing between two vehicles with a MMP=40 and another with MMP=100, I would choose the design with MMP=40.

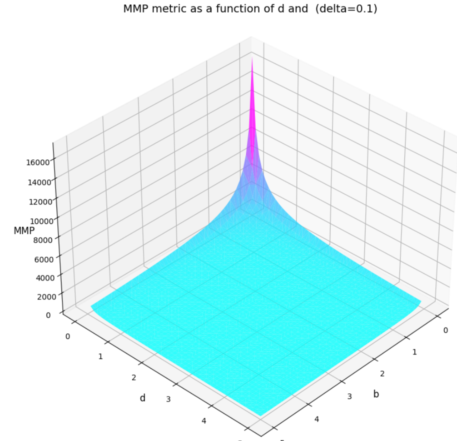
This is because the MMP (Mean maximum pressure) is defined as the mean value of the maxima occuring under all the wheels. A lower MMP means less pressure is exerted on the ground. So the vehicle is able to operate on soil with lower load-bearing capacity and is less likely for the vehicle to become stuck or damage the road due to the pressure exerted. In other words, the MMP=40 vehicle is more mobile compared to the MMP=100 one.

Q1.2.2.

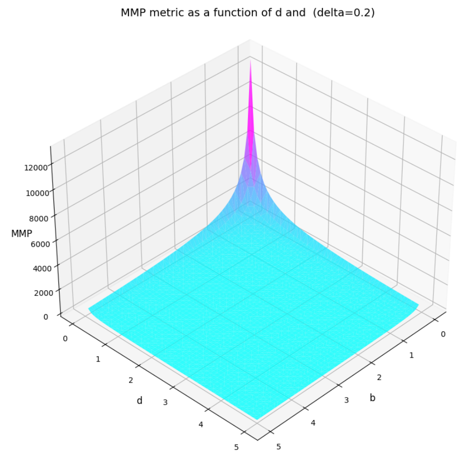
For $MMP = \dfrac{6.895 W_G}{n a b^{0.8} d^{0.8} \delta^{0.4}}$

With $W_G = 100\,kN$, $a=2$, $n=2$

Plot with $\delta = 0.1m$:



Plot with $\delta=0.2m$:



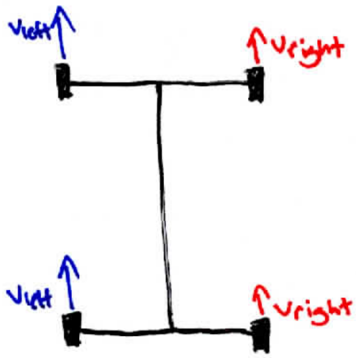
We see that b and d are symmetrical in its effects on MMP. Increasing/decreasing b or d decreases/increases MMP, holding the other constant. The increase/decrease in MMP is higher for values close to 0 (note that physically $d>0$, $b>0$) since b and d are on the reciprocal of MMP (in other words, small increases in b or d while the unloaded tire width and diameter are small will contribute greatly in lowering MMP). MMP asymptotes towards 0 as b and d gets larger.

When we increase $\delta=0.1m$ to $\delta=0.2m$, we see that the results are broadly similar. b and d are symmetrical, and increasing/decreasing b or d decreases/increases MMP, with the effect more noticeable at small values of b and d. However, for each fixed value of b and d, the MMP value is always lower for $\delta=0.2m$ than $\delta=0.1m$.

This is because the constant of multiplication for $\delta = 0.1m$ is $\frac{6.895 W_G}{na(0.1)^{0.4}} \approx 17.32 \frac{W_G}{na}$ while for $\delta = 0.2m$ is $\frac{6.895 W_G}{na(0.2)^{0.4}} \approx 13.13 \frac{W_G}{na}$. So for a fixed b and d, we always multiply by a lesser number to get MMP when using $\delta = 0.2m$ compared to $\delta = 0.1m$. Thus, the MMP plot for $\delta = 0.2m$ would always lie below the plot for $\delta = 0.1m$ for all physical values of b and d. Physically speaking, the greater tire deflection separates the force to the ground over a greater area, thereby reducing pressure (MMP).

Q2.1.1. v stands for velocity

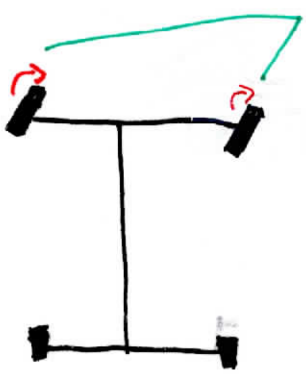
a.



Skid steering

Note: No portion of the vehicle can move, arrows show velocities of the wheels

b.



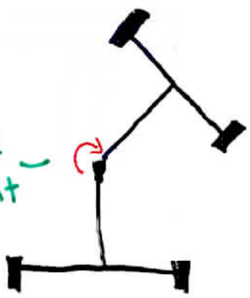
Can rotate left or right

Ackermann steering

Note: Showing front-axle Ackermann steering, also possible (not drawn) is back-axle or double-axle Ackermann steering

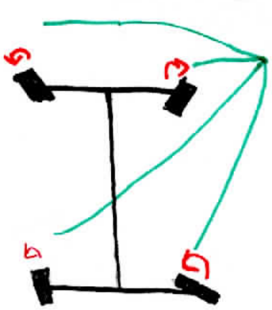
c.

can rotate left or right



Articulated steering

d.



can rotate left or right

Explicit/Independent steering

Q2.1.2.

a. Skid steering:

+: Simple design with few moving parts, does not need a dedicated steering mechanism (uses velocity differential instead).

−: Requires extra torque and power at the wheel output to overcome lateral resisting forces (usually 3x more than needed for only longitudinal motion).

b. Ackermann steering:

+: Increased stability during high-speed corners as each wheel follows a distinct turning radius, reducing the likelihood of skidding.

−: Not as maneuverable in tight spaces as turning radius is large.

c. Articulated steering:

+: Excellent maneuverability in confined spaces as turning radius is small due to the pivot in the middle.

−: Single failure point at the pivot, prone to structural failure or breaking under heavy load.

d. Explicit/independent steering:

+: Path following/navigation error is very low due to high controllability and ability to make tight maneuvers.

−: Higher mechanical complexity and cost, as each wheel needs an independent motor.

Q2.1.3.

F_x is the traction/thrust force that sustains the longitudinal motion of the vehicle. It is caused by friction between the wheel and the ground. For skid steering, due to the difference in velocity of the left and right side wheels, the force on the left wheels (F_{xo}) is different than the force on the right wheels (F_{xi}).

F_y is the forces resisting the turning motion of the vehicle that must be overcome. It is caused by skidding and scrubbing friction in the lateral direction.

R_r is the aggregation of all motion resistance forces against the driving direction. Its components include: compaction and bulldozing, gravitational resistance on slopes, obstacle climbing resistance, etc.

In other words, R_r and F_y represent the longitudinal and lateral resisting forces to motion. F_x represents the force necessary to overcome these resisting forces and make the vehicle accelerate (when the resultant force and moment is positive) or turn steadily at constant velocity (when the resultant force and moment is zero).

To estimate R_f, we seperate each component of R_f into segments such as compaction, bulldozing, gravitational, obstacles, etc. For hard surfaces, usually only friction is present and $R_r = f_r W$, where f_r is friction and W is weight of the vehicle. For granular soils, usually only $R_{compaction}$ and $R_{bulldozing}$ is relevant. We obtain the 2 using the following equations:

$$R_{compaction} = \frac{(3F_z/\sqrt{d}w)^{\frac{(2n+2)}{(2n+1)}}}{(3-n)^{\frac{(2n+2)}{(2n+1)}}(n+1)(k_c+bk_\phi)^{\frac{1}{(2n+1)}}}$$

(Refer to Class 3, Slide 5 for parameters)

$$R_{bulldozing} = \left(\frac{b\sin(\alpha+\phi)}{2\sin\alpha\cos\phi}\right)(2cK_c z + \gamma K_\gamma z^2) + \frac{\pi\gamma l_r^2(90-\phi)}{540} + \frac{\pi c l_r^2}{180} + c l_r^2 \tan(45+\frac{\phi}{2})$$

(Refer to Class 3, Slide 9 for parameters)

Finally, we add appropriate $R_{gravitational}$, $R_{aerodynamic}$, $R_{obstacles}$, etc as needed.

To estimate F_y, we first need to know if the turning radius is small or large. If it is large, then F_y is most likely elastic sideslip and can be estimated by $F_y = C\alpha$, where α is the slip angle and C is the tire cornering stiffness. If the turning radius is small, then F_y is most likely lateral slippage and scrubbing, and can be estimated by $F_y = \sqrt{(\mu W)^2 - F_x^2}$, where μW is the frictional limit (μ is the coefficient of friction and W is weight) and F_x is the traction/thrust force.

Q2.2. For a high-speed, all-terrain vehicle, we want to utilize a semi-active suspension over a passive one since:

1. We can adjust the damping characteristics of the vehicle to suit the terrain. I.e., we can switch to a softer damping for rough off-road terrain and switch to a stiffer setting for smoother terrain. This cannot be done for passive damping as the stiffness is fixed, thereby decreasing the vehicle's controllability.
2. We can improve the ride quality by dynamically adjusting damping to minimize road vibrations and increase stability. Having a single passive suspension means that the damper cannot cope with all terrain types. Sudden changes in terrain type could lead to excess vibrations and cause the vehicle to become unstable and potentially roll over.
3. We can quickly adjust the suspension to increase traction and grip. This is especially important for high-speed scenarios as losing grip could mean the vehicle becomes uncontrollable and potentially crash. Passive suspensions are usually too slow to react. We need a semi-active one to keep the wheels on the ground so grip is maintained.

Additionally, an active suspension may be more suitable over a semi-active one in scenarios where the drawbar pull is low. To prevent the vehicle from becoming stuck, real-time predictive algorithms are used to maximize thrust and reduce resistance. A semi-active system cannot predict ahead, and may result in the soil failing due to shear and trapping the vehicle. This should be avoided at all costs for vehicles operating in soil where the drawbar pull is near zero.

Appendices:

A.1. Code for Q1.2.2

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.tri import Triangulation

W = 100
a = 2
n = 2
delta1 = 0.1
delta2 = 0.2

b = np.linspace(0, 5, num=51)
d = np.linspace(0, 5, num=51)

B, D = np.meshgrid(b, d)

mmp = (6.895 * W)/(n * a * pow(B,0.8) * pow(D,0.8) * pow(delta2,0.4))

tri = Triangulation(B.ravel(), D.ravel())

fig = plt.figure(figsize=(5, 5))
ax = fig.add_subplot(111, projection='3d')

ax.plot_trisurf(tri, mmp.ravel(), cmap='cool', edgecolor='none', alpha=0.8)

ax.set_title('MMP metric as a function of d and  (delta=0.2)', fontsize=14)
ax.set_xlabel('b', fontsize=12)
ax.set_ylabel('d', fontsize=12)
ax.set_zlabel('MMP', fontsize=12)

plt.show()

```