Q1.1. For the image projection of the same point X, the correspondence satisfies

$$x^T F x' = 0$$

where $x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is the homogenous coords. of the projection of X onto the left image,

$x' = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is the homogenous coords. of the projection of X onto the right image,

F is the fundamental matrix

So $x^T F x' = 0$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_{13} \\ f_{23} \\ f_{33} \end{pmatrix} = 0$$

$$\Rightarrow f_{33} = 0$$

$\therefore f_{33} = 0$ when both image projections of X are at pixel $(0,0)$

Q1.2.

Let X denote a 3D point in world frame.
Referring to Figure 2, the point X in camera frames are:

$X_1 = R_1 X + t_1$ (1.1) (camera 1 frame)

$X_2 = R_2 X + t_2$ (1.2) (camera 2 frame)

From (1.1), we have

$R_1 X = X_1 - t_1$

$X = R_1^{-1}(X_1 - t_1)$

$X = R_1^T(X_1 - t_1)$ (1.3) ($R_1^{-1} = R_1^T$ since orthogonality)

Into (1.2):

$X_2 = R_2(R_1^T(X_1 - t_1)) + t_2$

$\therefore X_2 = R_2 R_1^T X_1 - R_2 R_1^T t_1 + t_2$ (1.4)

By definition of relative rotation and translation from camera 1 to camera 2, we have

$R_{rel} = R_2 R_1^T$

$\therefore R_{rel} = R_{i+1} R_i^T$

$t_{rel} = t_2 - R_2 R_1^T t_1$

$\therefore t_{rel} = t_{i+1} - R_{i+1} R_i^T t_i$

Defining $[t_{rel}]_\times$ as the matrix representation of the cross product with t_{rel}, and denoting $(K^{-1})^T = K^{-T}$, we have:

$\therefore E = [t_{rel}]_\times R_{rel}$

$F = K^{-T} E K^{-1}$

$\therefore F = K^{-T} [t_{rel}]_\times R_{rel} K^{-1}$