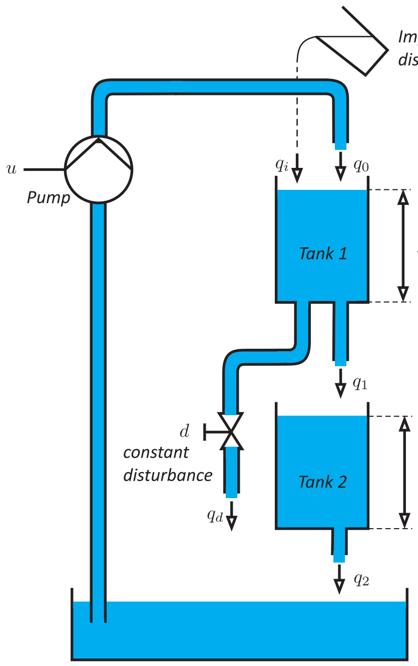
Double tank system

Manual and modelling guide

1 Introduction

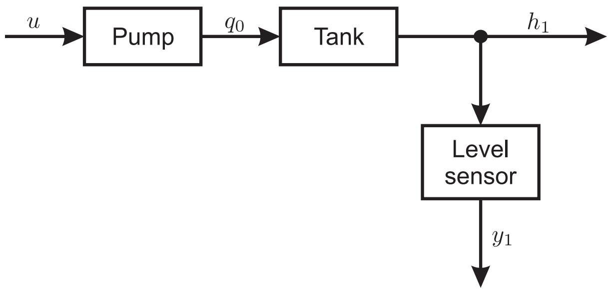
This manual describes in short how the double tank system works and can be modelled. This manual gives you a quick start in case you have not been working with such a setup before.



Figur 1: Schematics of the tank process

Tank processes are very common in industry and a usual control problem is to keep the level in a tank constant while there are flow variations and disturbances. In the lab room of the control engineering group there are miniatures of two tanks in series with the option to add disturbances. In this assignment you should derive a process model that describes the behavior between input signals, which is voltage that is fed to the pump and the level in the upper tank.

The schematics in Fig. 1 contains many details which are superficial for the aim of this assignment. Thus, we should draw a simplified block diagram which only contains the essential parts. The following block diagram of the tank process shows u as the voltage fed to the pump, q_0 as the flow into to the tank, h_1 as the level in the tank and y_1 represents the output voltage of the level sensor



2 First principle model

We will now model the tank process based on physical relationsships. First of all, we can review the schematics and block diagram and start with the upper tank *Tank 1*:

1. For a tank, the change of fluid volume is the difference between the flow in and out of the tank. In other words
$$\frac{dV}{dt} = q_{in} - q_{ut}$$
We assume the tank has a constant cross-section area which yields $V = Ah$, where A is the cross-section area of the tank and h is the level in the tank. Inflow q_{in} is equal to the outflow of the pump, see the block diagram at the beginning of section 2, denoted q_0. The outflow from the tank can be approximated by Bernoullis equation which can be simplified to $q_{ut} = a\sqrt{2g(h_1 + h_o)}$, where a cross-section area of the exit hole and h_o is the additional head height that is added by the outlet. For the tank processes h_o is a fixed constant and it is $3.2cm$.

2. Compared to the dynamics of the tank the response of the pump is relatively fast. This means that the model for the pump can be assumed to be a static function of the voltage u. In other words
$$q = f(u)$$

3. Similarly, we can assume that the sensor measuring the level is fast so that the model for the sensor can be assumed to be a static function as well. It becomes $y = g(h_1)$

The dynamic model can now be summarized as

$$\dot{h}_1 = \frac{f(u)}{A} - \frac{a}{A}\sqrt{2g(h_1 + h_o)} \tag{1}$$
$$y = g(h_1) \tag{2}$$

where A is known, but $f(u)$ and $g(h_1)$ are unknown. A similar model can be derived for the lower tank *Tank 2*. For this end we assume that the parameters a and A are the same for both tanks.

3 Linearization of the first principle model

Linear process models are determined for an operating point and they are usually valid for a region close to the operating point. The size of the region depends on the system that you work with. The operating point information is needed for the linear process model that is derived from first principles.

A general operating point for the tank process is given by nominal levels h_{10} in tank 1 and h_{20} in tank 2. These nominal levels are the result of a nominal voltage u_0 to the pump.

The equations for the linear process model for a general operating point from the first principle model equations can be derived in accordance with the first lecture (Introduction)

The resulting linear process model can now be stated in terms of a state space system which has u as input and h_1, h_2 as output.

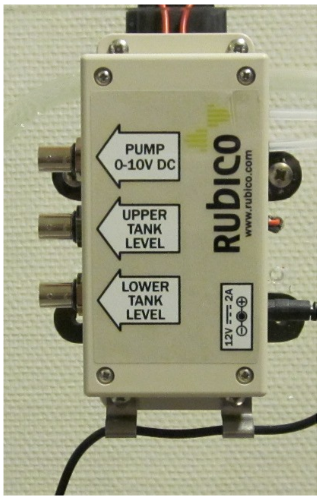
4 Determining the parameters, functions and operating conditions of the models

In order to determine the parameters of the model and register them in an efficient way, we will use Matlab in parallel with the experiments.

4.1 Check of the equipment

Take one of the tank processes which is in the shelf of the A1521 lab room. Fill the plastic bucket up to at least half its height with water and place the tank process in the bucket.



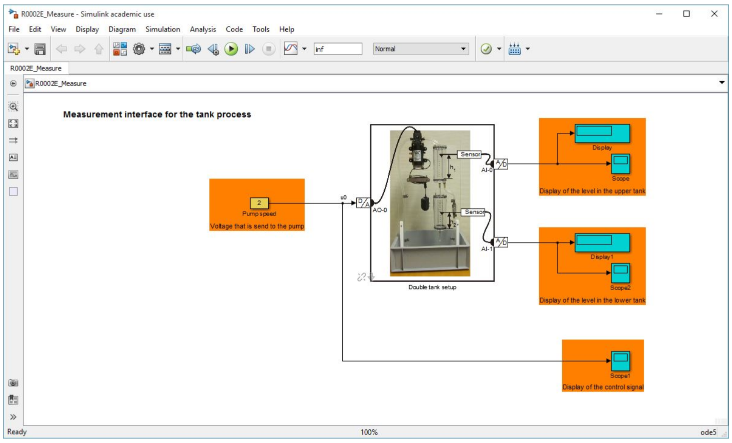


The lab assignment A and B of the course will be conducted with the use of Matlab and Simulink. So, we start Matlab from the START menu. When Matlab is started we change to the directory where the `R7014E_lib_R001.zip` file was extract.

Start by entering the following in the Matlab command window:

```
≫R7014E_Measure
```

After a short while simulink is started and the following block diagram is displayed:



In this block diagram we can see the double tank setup and how it is connected. Now we have to connect the computer in the same way with the real setup. The input to the `Double tank setup` block will send the signal value to the D/A card and will generate a voltage there. The outputs of the block will produce a voltage that represents the A/D converter value at the chosen port.

Now we connect the junction box to the computer, see Fig. 2. On the `Double tank setup` block you have an indication on the left hand side, which AO connector you should connect with the pump.

Similarly, connect the sensor signals for the level of the upper tank and the lower tank with the according AI connectors on the junction box. When this is done, you are ready to control the double tank process and read the level in the tanks.

1. Now you need to check that the pump works. Press the `Run` button and after a short moment the pump should start at a slow speed. In the default setting, 2 Volts are supplied to the pump. Double click on the Pump speed block and adjust the voltage to a higher value. Be aware that the upper tank may overflow if too much water is supplied.

2. Next step is to check the sensor. It provides a nominal value between 0 and 10 Volt. During the pump operation the two number display `Display` and `Display` should present values that represent the level.

3. Also check that a change in the level of the tank is followed by a change in the voltage values that are displayed in the blocks.

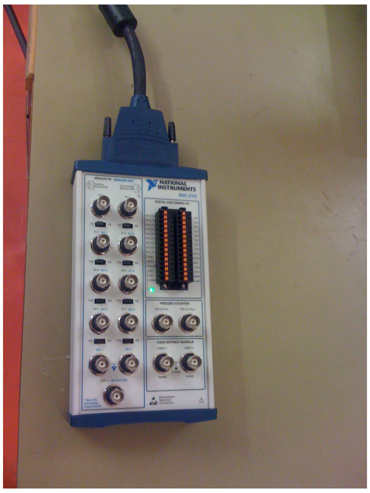
If everything works fine, you are all set to continue. If not, have a look at the trouble shooting at the end of the manual or talk to the course team and get their help!

4.2 Preparation of a parameter file

Matlab is already started and you are in the same directory as the file. For the subsequent experiments you can either use Simulink to adjust the pump speed and measure the level in the tank, or you can use the power source and a multimeter. This is up to you.

When the parameters for model of the tank process is derived, it is good to be able to

- Easily change the parameter values



Figur 2: Junction box for analog in and out.

- Comment and add descriptions to the parameters
- Load them into Matlab's workspace memory

A very simplistic way to achieve this, is by creating a so called M-file. This is a small Matlab script that executes all Matlab commands in the script which can be commented and easily changed, since it is a text file. First we create the M-file

```
≫edit mytank.m
```

If the file `mytank.m` does not exist, you will be asked if the file should be created. In order to indicate what this file is used for, you can add a comment in the beginning of the file

```
% Parameters and settings for the tank process
```

4.3 Cross section area A

First you can choose a unit for A and then measure it.

$A =$

We can now add the parameter to the M-file. For example if the value would be $10cm^2$, we could add the following line to the M-file

```
A=10; % Cross section area of the tanks in [cm^2]
```

Using this line the value 10 is assigned to the variable `A` and there is a comment on how to interpret the variable and its value. The symbol `;` prevents Matlab from displaying the value on the command line when the script `mytank.m` is executed. Press now `save` in the editor and enter

```
≫mytank
```

on the command line. This executes your script and will create the variable `A` in the workspace. You can see that in the window to the right of the command line. Let's continue!

4.4 The sensor

The characteristics of the sensors can be determined by filling the tank to different levels and reading the voltage for the different levels. The readings that represent samples of the function $g(h)$ can be entered in the following table.

h_1 [V]										
h_1 [cm]										

Now we want to add the table to the M-file. We will add a vector for each row and results in the following code

```
% Measurement points for the pump function
gV=[ value1, value2, ... , valueN ]; % given in [V]
gcm=[ value1, value2, ... , valueN ]; % given in [cm]
```

Usually, it is safe to assume that the function g is valid for both sensor. If you want to remove this uncertainty, you can also determine the function for the lower tank.

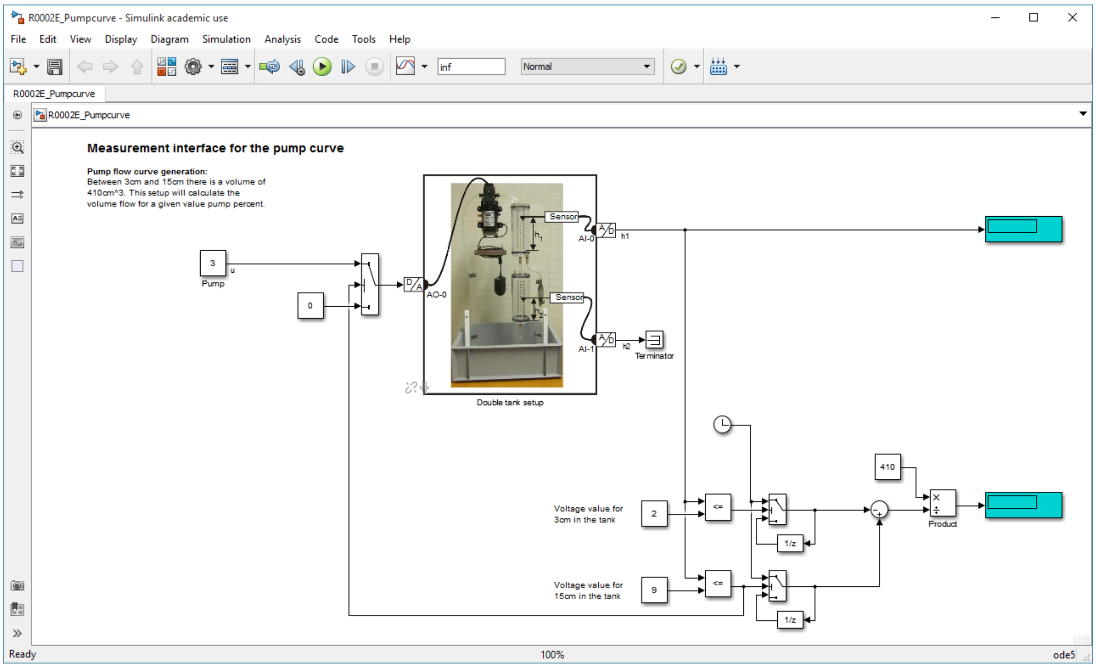
4.5 The pump

It is straight forward to determine the $f(u)$.

1. Start with $u = 0$ and increase the power to the pump slowly until a flow starts to emerge from the pipe.
2. Measure the flow from the pump for a number of value for u between 0 and 10 Volt.

The flow can be measured by starting with an empty tank, close the outlet with the yellow plug, keep the power to the pump constant and measure the time t until the tank is at nearly full. Having the level h_1 at the stop time gives the average flow as $q = \frac{Ah_1}{t}$. Alternatively, you can directly read the volume in the tank on the scale at the side. The measurement pairs (u, q_0) are points of the function $q = f(u)$. Register the pairs in the following table

For your convenience and to reduce the measurement error, the course team has provided you with a block diagram to perform the individual measurements:



In that block diagram you still need to adjust the voltage values for the level of 3 cm and the level of 15 cm. This block diagram measures the elapsed time to fill the tank between two levels.

u										
q_0										

In order to have a correct scaling for the function you need to know the unit for the flow.

Determine the function $f(u)$. You can use the function `plot` to display a graph. You can get help on the function by entering `doc plot` on the command line.

There are now two options to work with the function f(u). Either use a piece-wise linear representation or perform an approximation using a function representation.

We add a similar code as above to add the table to the M-file

```
% Measurement points for the pump function
fu=[ value1, value2, ... , valueN ]; % given in [V]
fq=[ value1, value2, ... , valueN ]; % given in [...]
```

4.6 Cross section area of the outlet a

Finally, the only remaining unknown is the cross-section area of the outlet of the tank, a. In order to determine this constant we can make use of (1). If the level is constant in the tank, we have a stationary state where $\dot{h}_1 = 0$ which means

$$0 = f(u) - a\sqrt{2g(h_1 + h_o)} \tag{3}$$

By measuring the level in the tank for some constant voltage values to the pump, we can derive an approximate value for the area a.

$a =$

Again we will also add this parameter to the M-file. If `a` would be $1cm^2$, then we would add the line
```
a=1; % Cross section area of the hole in the tank in [cm^2]
```

Now, we have all model parameters for one of the two tanks. We can assume that the second tank has exactly the same construction and dimensions and therefore parameters have the same parameter values.

We can now save the Matlab script and execute it by entering

```
≫mytank
```

All our parameters are now available on Matlab's workspace!

4.7 Operating conditions

Now that all parameters for the tank are known, the operating point need to be determined. In lab assignment B & C, the objective is to derive a control system for the double tank process that should operate at a nominal level h_{20} of the lower tank of $6cm$. We therefore need to find the parameters that define the operating point, namely voltage to the pump u_0 and nominal level h_{10} in the upper tank.

Perform an experiment, where you adjust the voltage to the pump to u_0 so that you reach approximately the target value in both tank. Keep the final set of values u_0, h_{10} and h_{20} as operating point.

It is important to note that deviations in the operating point between calculation and experiment could be caused by:

- Simplifications in the modeling.
- Approximations in the representation of certain elements like the pump and the sensors.
- Drift of sensors.
- Modeling errors.

The operating conditions can also be added to your Matlab script.

5 Complete process model

Now we have a process model for the double tank process. Since we will use the complete process model in the next assignments it is important to collect all the equations for double tank process including the parameters and operating point values.

This process model can also be implemented in Simulink and all the work can be conducted off-line without the process.

6 Alternative linear model from experiment

You can also derive a linear model for the process directly from experiments, where you assume the following serial model structure:

$$G_1(s) = \frac{H_1(s)}{U(s)} = \frac{K}{(Ts+1)}$$
$$G_2(s) = \frac{H_2(s)}{H_1(s)} = \frac{1}{(Ts+1)}$$

Perform a step response experiment around an operating point and then you will be able to determine K from the transfer from $u \to h_2$ and T from the transfer from $u \to h_1$. Thereafter you have to represent the transfer function as a state space realization, preferrably with the levels h_1 and h_2 as states.

7 Expected values for the parameters

Cross section area tank	$A \approx 33$ cm^2
Cross section area outlets	$a \approx 0.16$ cm^2
Time constant	$T \approx 35$ s $\pm$ 6 s
Gain	$K \approx 5$ cm/V $\pm$ 1 cm/V

8 Trouble shooting

Sample rate problem: When you start a simulation or a real-time run, and you get an error message relating to the sample rates and transitions between sample rates, then there is a problem in the model.
Generally, signals that are at different rates can not be combined with each other in mathematical operations. You have to analyze your model if this is the case. The error tells you where the problem occurred in the block diagram.
You can visualize the rate of the different signal by activating the option `Sample Time Display` in the `Format` menu. Choose `All`, then the signals will be displayed in colors according to their rate and an reference to the sample rate is given.
You can resolve the problem by adding `Rate transition` blocks, which change the signal rate.

Data type problem: When you make use of logical operations, then you will get signals of type `Boolean`. These signal can not be combined with the standard data type `Double` in mathematical operations. You need to manually add data type conversion blocks which will convert the `Boolean` to a `Double`.

Signal flat: In some rare cases, a channel in the A/D-card might be broken. IN this event a measurement signal might be flat and is not reflecting the actual level in the tank.
Double click on the `Double tank setup`. Choose a different channel for the signal that is flat. Press OK. Then re-connect the sensor with the new channel on the junction box. This should resolve the problem. Please inform the laboratory assistants.