Differential Calculus

Project Leibniz

November 26, 2019

0.1 Introduction

0.1.1 What is Project Leibniz?

Project Leibniz is a collection of real-world motivated maths problems aimed at secondary students to illustrate the power of calculus.

Calculus has a reputation as being abstract, impenetrable and poorly understood. Many secondary students struggle with this topic because it is not taught well. The aim of this project is to allow students to improve their calculus skills by practicing with real-world problems to inspire their curiosity and help them retain the knowledge.

Chapter 1

Level 1 Problems

Chapter 2

Level 2 Problems

2.1 Introduction

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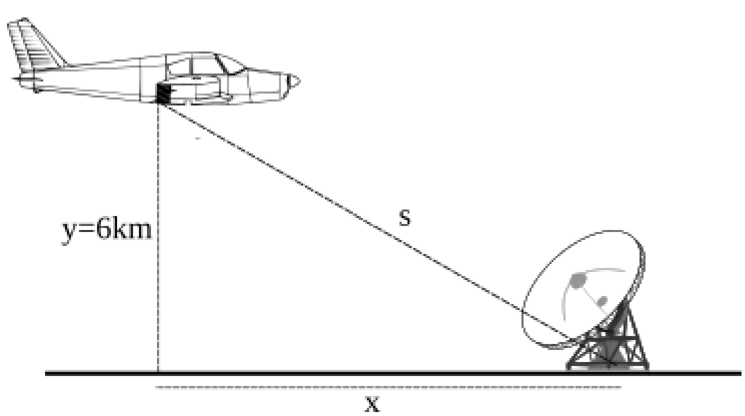
2.2 Problems

1. A radar station can be used to deduce the speed of an airplane. The radar station measures the distance s to the airplane in km. The airplane is cruising at a height of 6km above the ground. The radar station finds that s is decreasing at a rate of 400km per hour when $s = 10$km.



   (a) When $s = 10$km, write down $\frac{ds}{dt}$

(b) The altitude of the plane is not changing. Write down $\frac{dy}{dt}$.

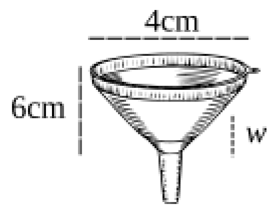
(c) Use Pythagoras' Theorem to write down the relationship between the values of x, y and s

(d) Hence find x when $s = 10$

(e) The displacement of the airplane from the tower is denoted x, write down how we would denote the speed of the airplane.

(f) Hence, using rates of change, find the horizontal speed of the airplane in km per hour.

2. A conical funnel has a diameter of $4cm$ and a height of $6cm$. Water flows out of the bottom of the funnel at a rate of $2cm^3/s$.



(a) The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$, find the capacity of the funnel (the maximum volume of water it can hold).

(b) When the cone contains water the shape of the water makes a cone even when the funnel is not full. This smaller cone of water is related to the full volume of the funnel because the ratio between the radius of the cross-section and the height of the cone is constant. Let w represent the height of water in the funnel and ρ represent the radius of the cross-section at that height then we have the equation

$$\frac{\rho}{w} = \text{constant}$$

i. Find the value of this constant for this cone by using the situation when then cone is full of water.

ii. Write down a formula for the radius of the cross-section of the cone ρ when the water level is w cm

iii. Hence, write an expression for the volume of water in the cup in terms of w

(c) Hence, find an expression for the rate of change of volume V in terms of the rate of change of w

(d) The volume of water that flows out of the funnel every second is $2cm^3$. Write down $\frac{dV}{dt}$.

(e) Hence find the rate of change of w - how fast is the water level dropping - when the funnel is filled with water to $w = 3cm$.

Chapter 3

Level 3 Problems

Chapter 4

Level 4 Problems