Success Story 1 — Detecting Illicit Mesh Networks

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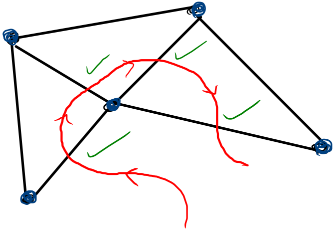
Challenges in the electromagnetic environment 1: Challenge 2

Arose at first CEME as:

"Challenge 2: Optimising a search route to discover networks in a landscape of constraints"



- N agents communicating in a network of general (possibly unknown) size and topology
- Movable detector capable of detecting when a beam is "crossed"
- How does one construct a strategy to determine locations of nodes?
- How can one determine number of nodes/network topology?

Work continued for a 10 day project developing the approach and increasing realism of the example for a proof-of-principle project.

Bayesian mathematical framework

1. Parameterise the problem via a parameter vector $\Theta = (N, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N, \mathbf{T})$, where
   - N is the (unknown) number of nodes,
   - $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N$ are the locations of the nodes,
   - $\mathbf{T}$ is the network topology (e.g. list of pairwise integers indicating network connectivity).
2. Determine the likelihood $P(D|\Theta, \mathbf{y})$ of observing data D from the beam-crossing dector at location $\mathbf{y}$ if you knew precisely the network configuration Θ.
3. Define your current state of knowledge via a prior probability distribution $P(\Theta)$.
4. Given this knowledge, choose spatial location of detector $\mathbf{y}$ to optimise the chance of detection.
5. Collect data D and update knowledge using Bayes theorem: $P(\Theta|D) = \frac{P(D|\theta)}{P(D)} \times P(\theta)$
6. Repeat steps 4 & 5 with new data with the posterior distribution $P(\theta|D)$ as a new prior knowledge.

Numerical Bayesian distributions can be computed and manipulated reliably using nested sampling algorithms, e.g. `PolyChord` [1506.00171]

Posterior updating ($N = 3$)

Toy example (conducted in CEME1 workshop):

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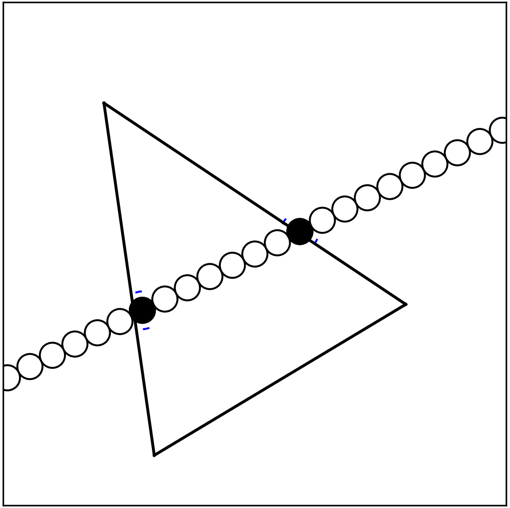

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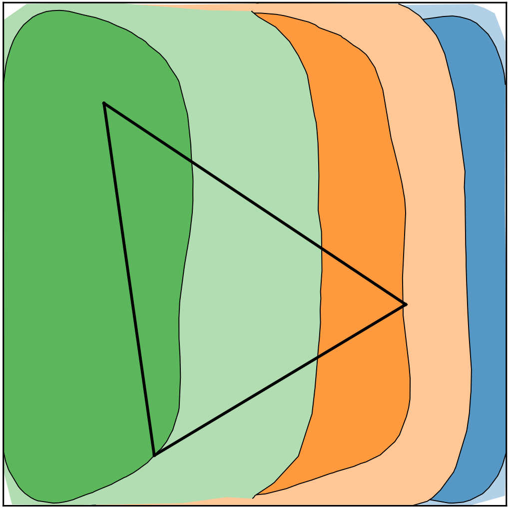

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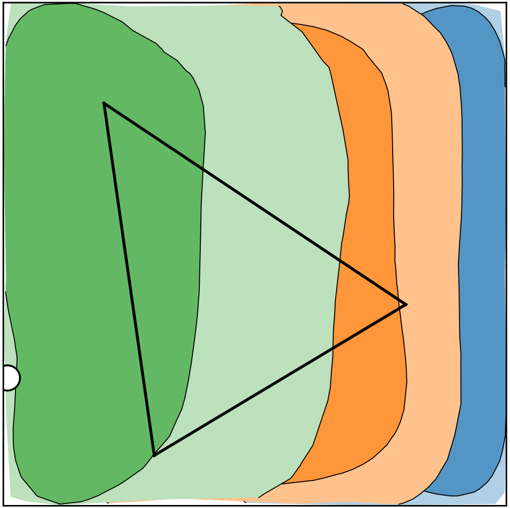

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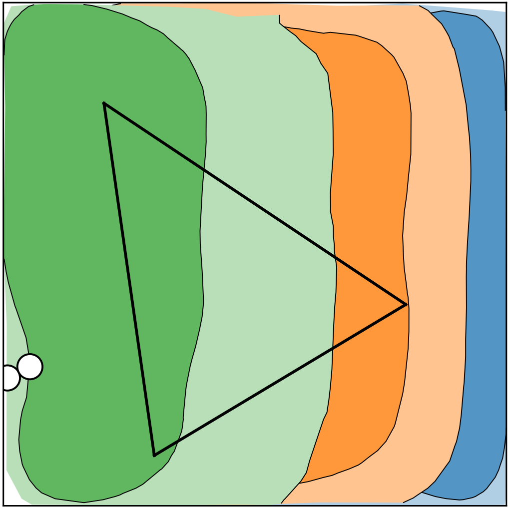

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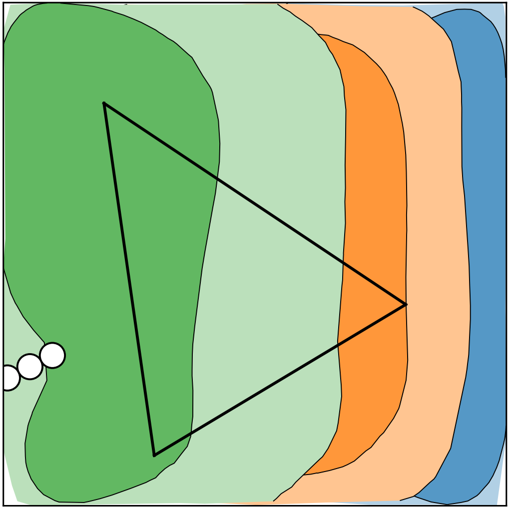

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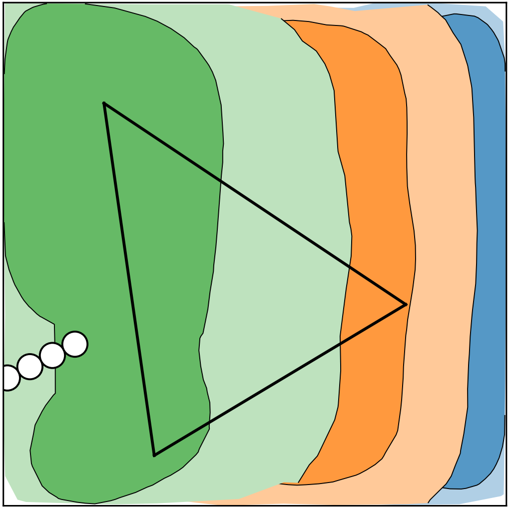

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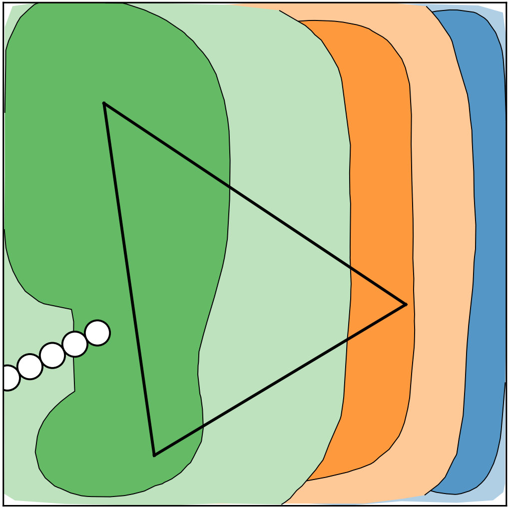

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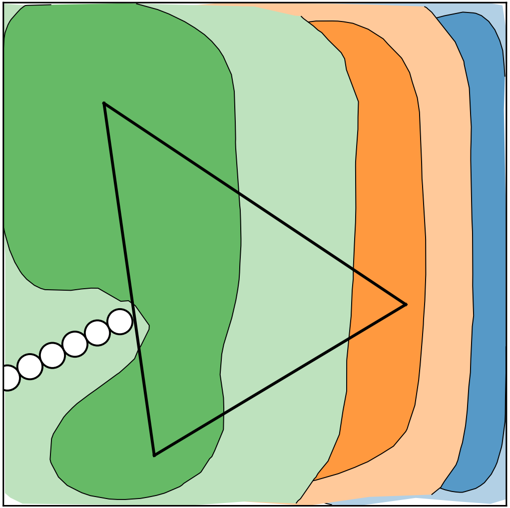

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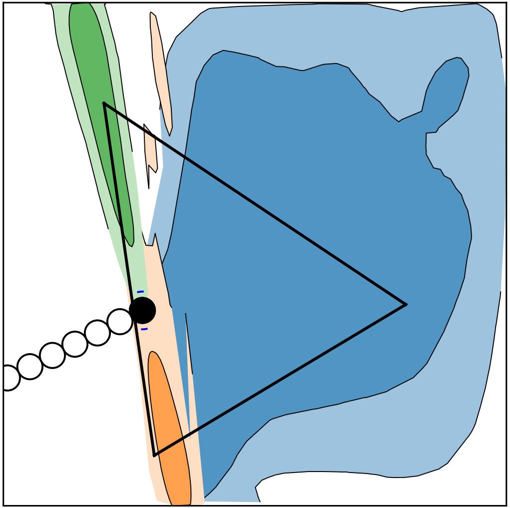

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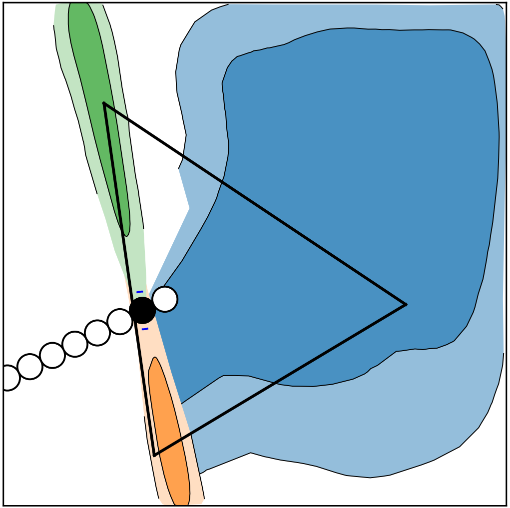

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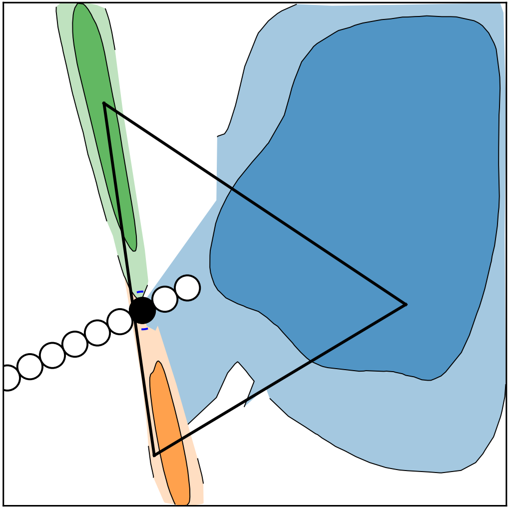

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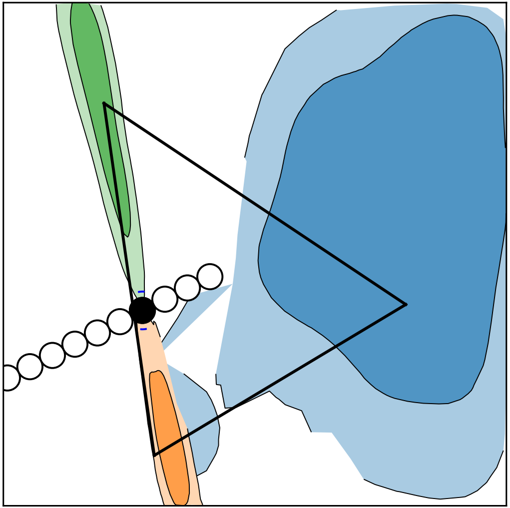

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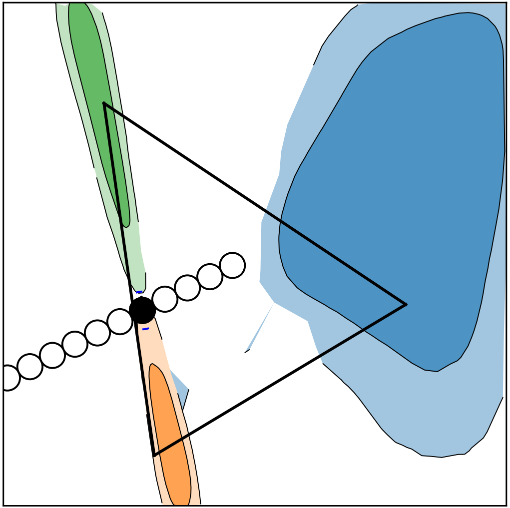

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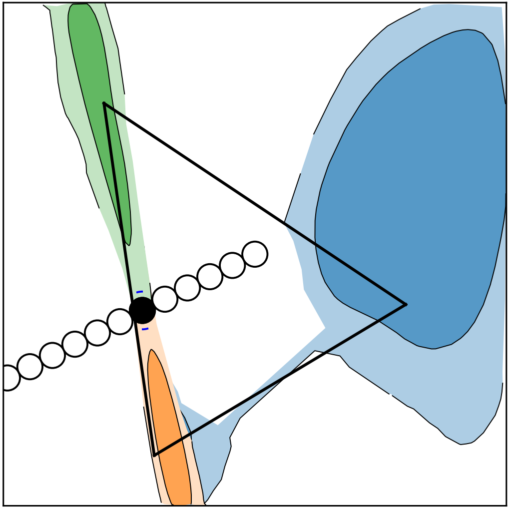

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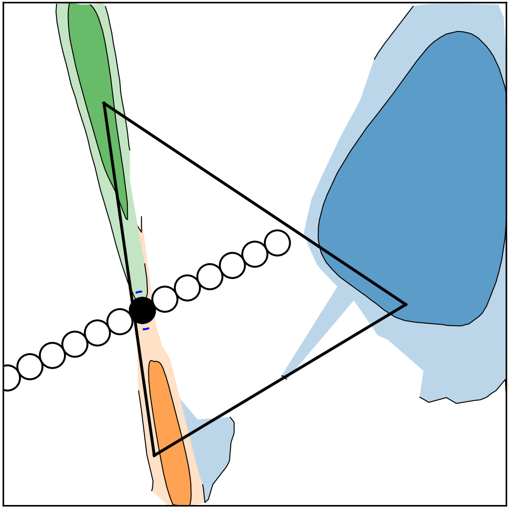

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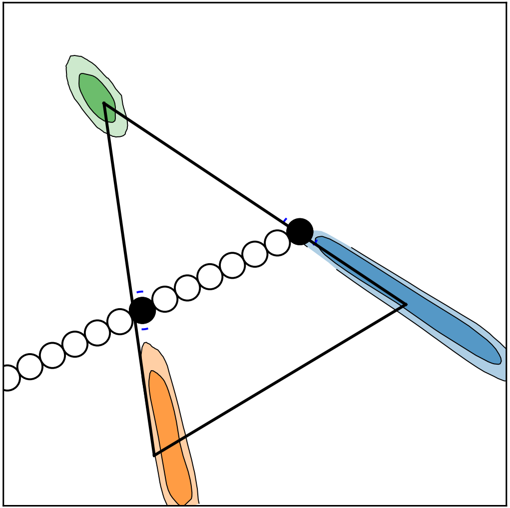

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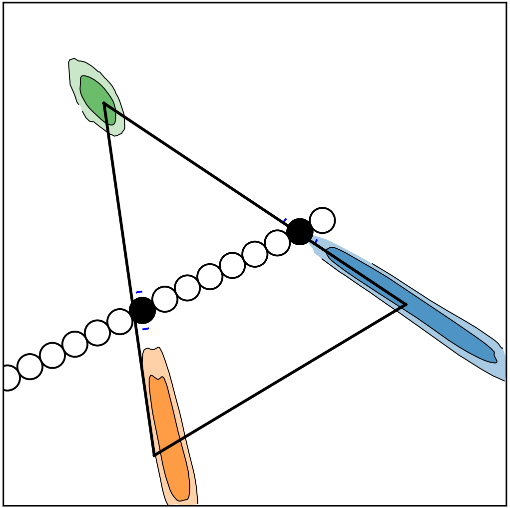

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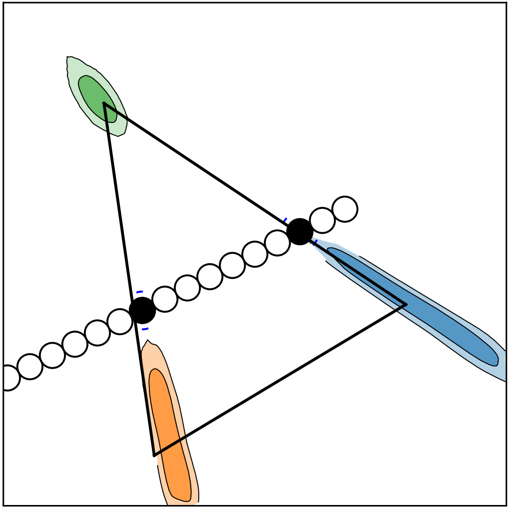

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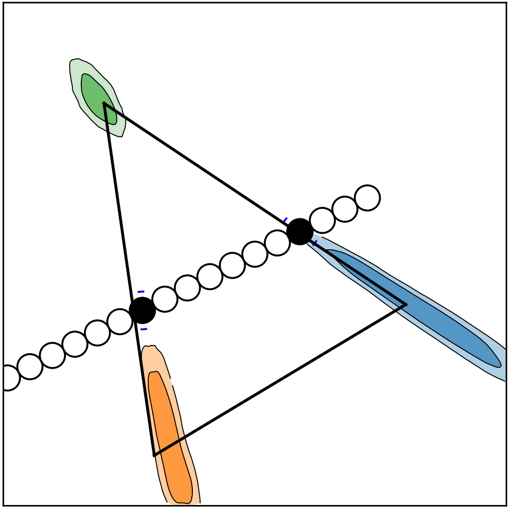

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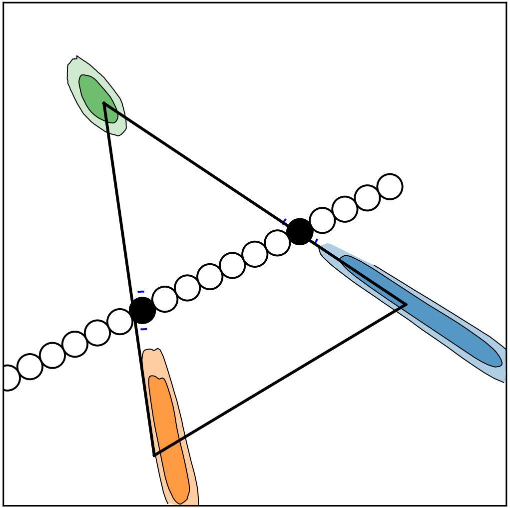

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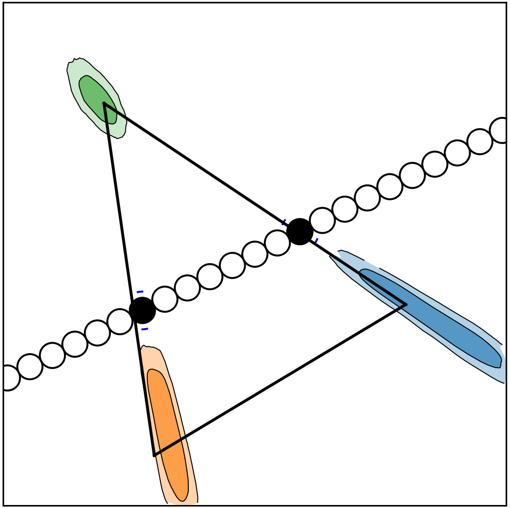
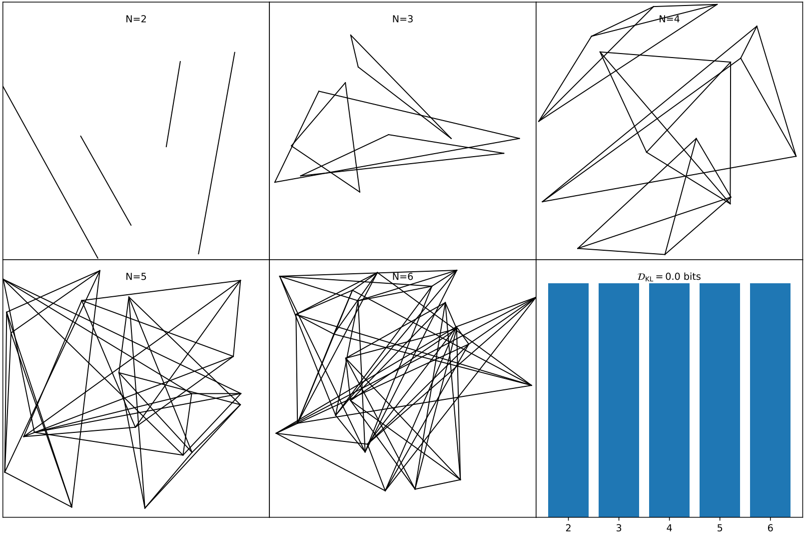


Posterior updating (varying N)

- Extend example so that N can also vary.
- Marginal posterior on $P(N|D)$ (bottom right).
- Instead of plotting posterior *contours*, we plot *samples* from the full posterior distribution.
- In general this is the least compressed representation of the posterior.



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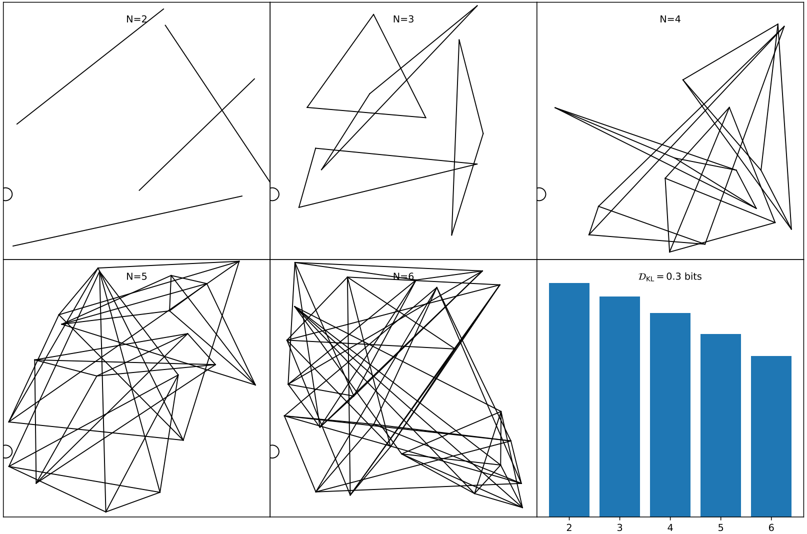

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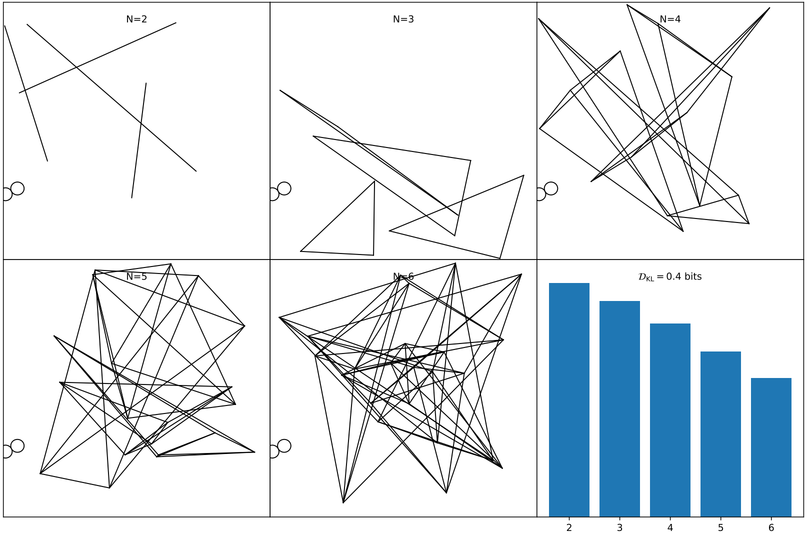

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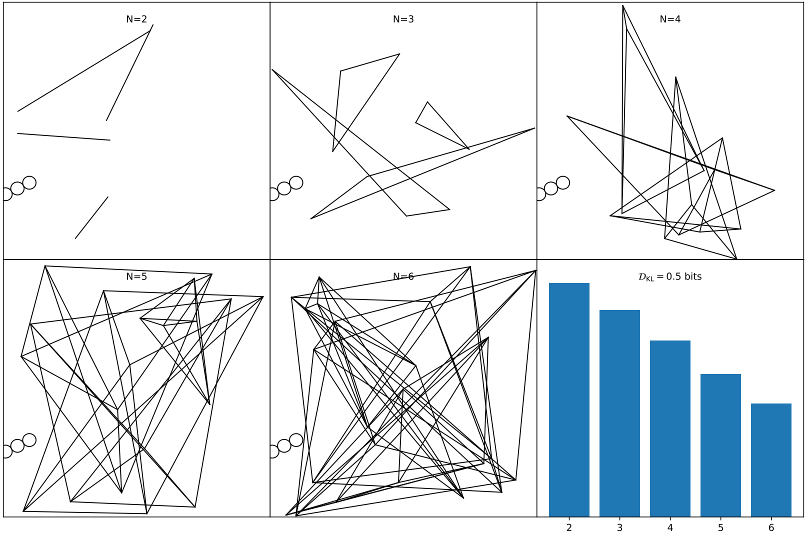

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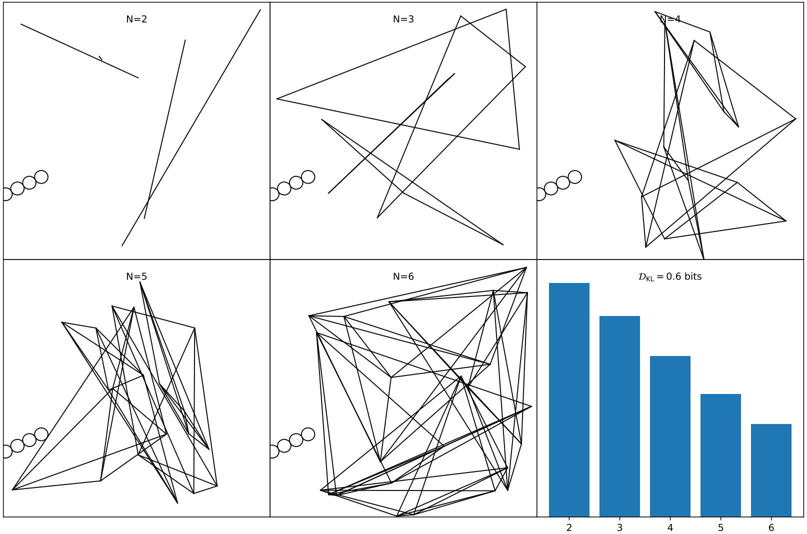

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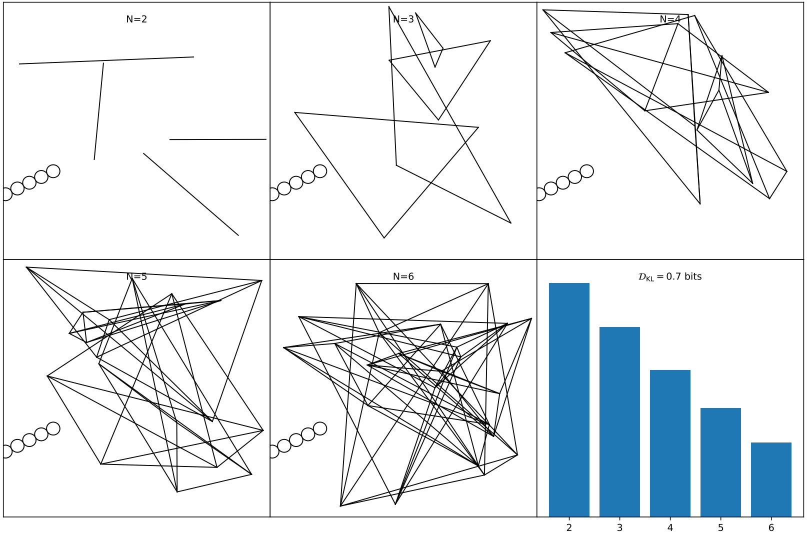

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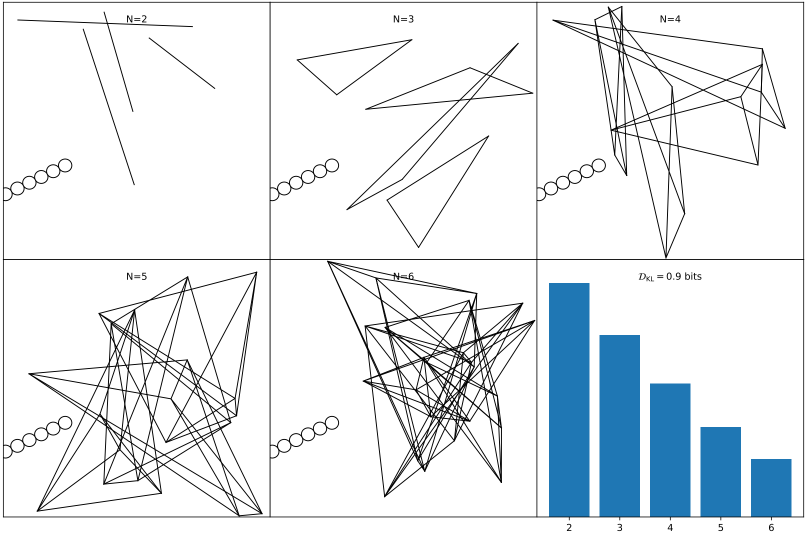

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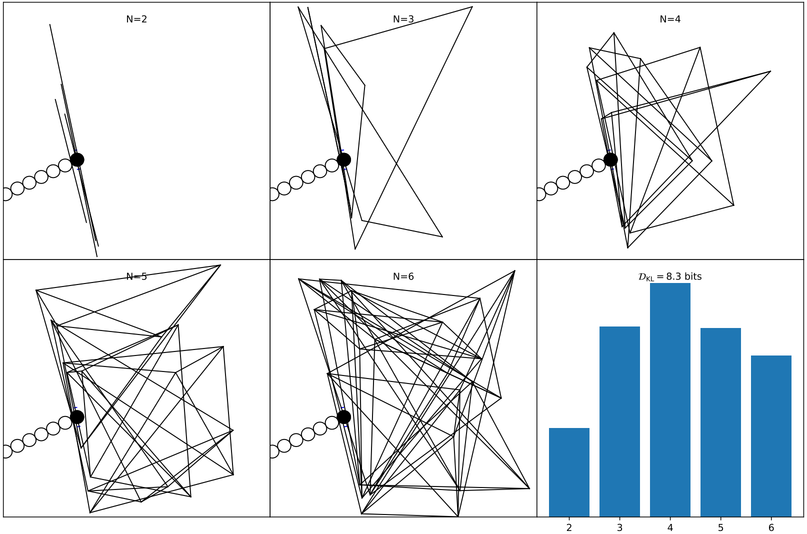

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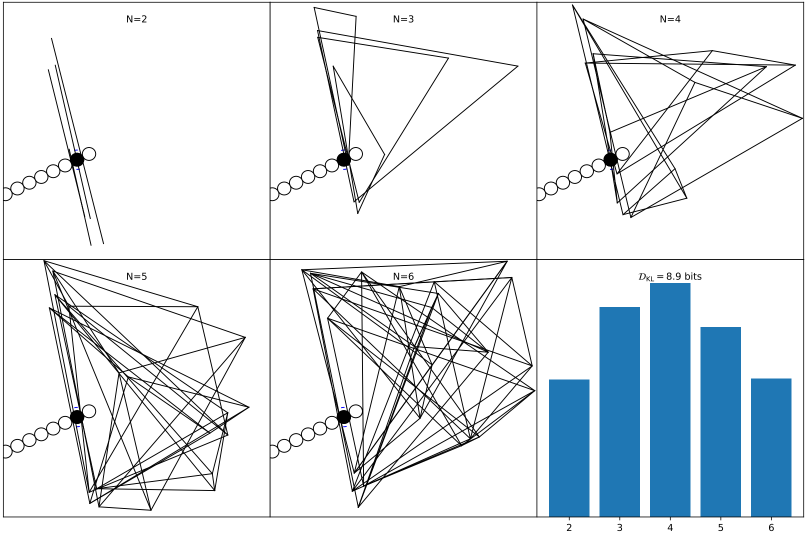

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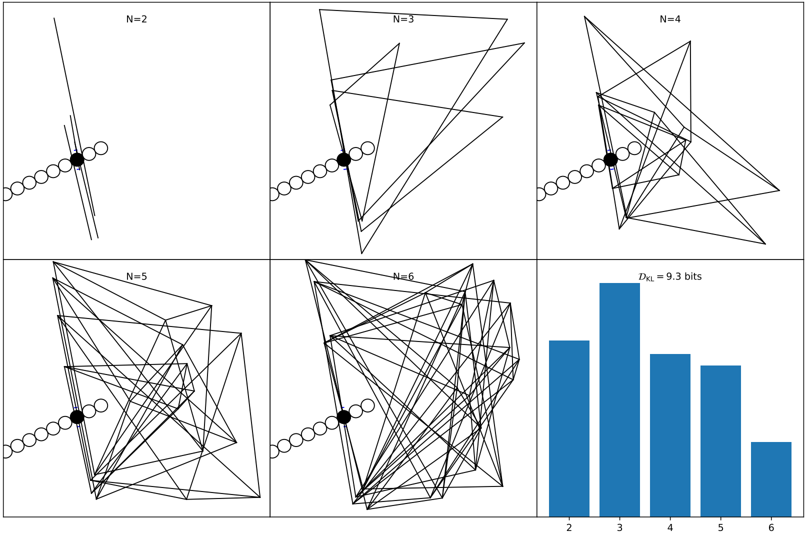

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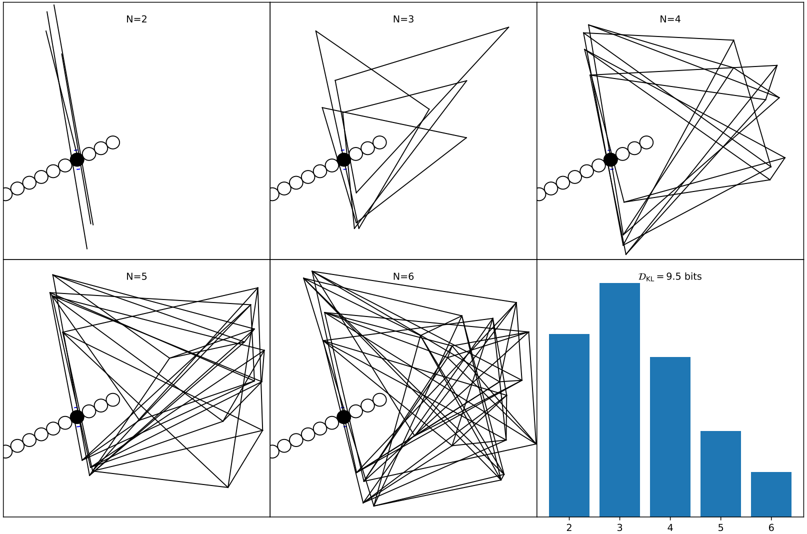

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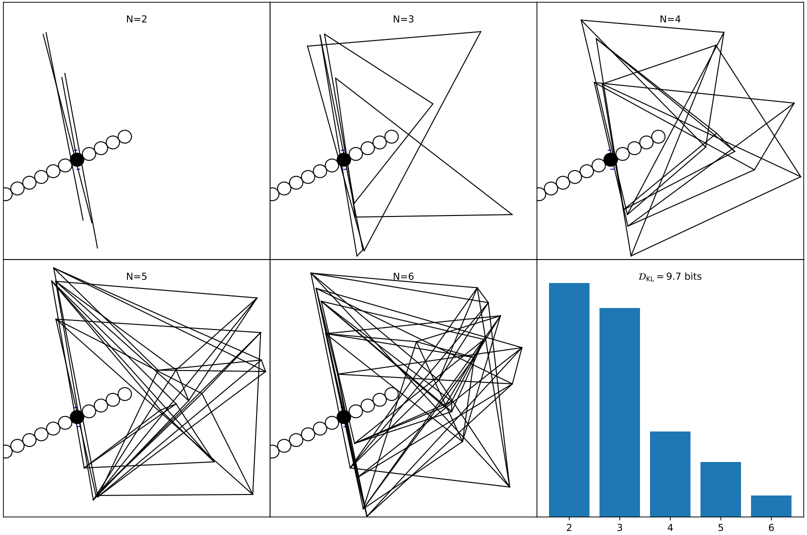

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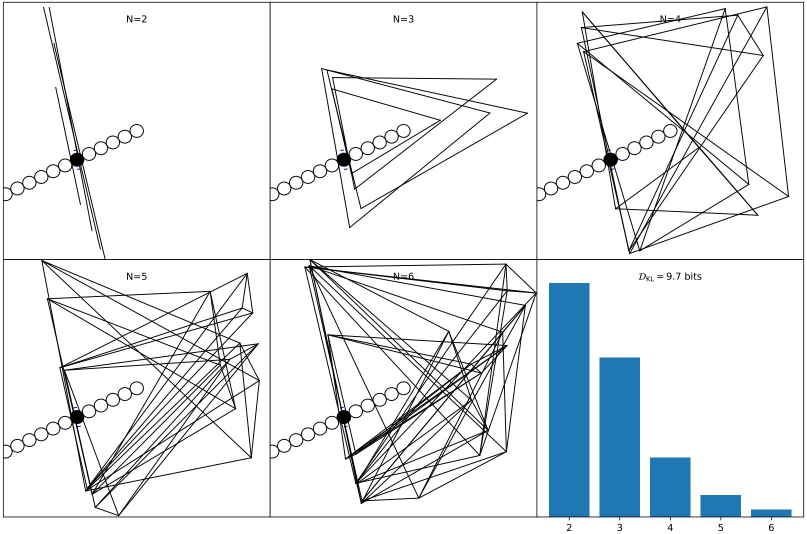

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- Extend example so that N can also vary.
- Marginal posterior on $P(N|D)$ (bottom right).
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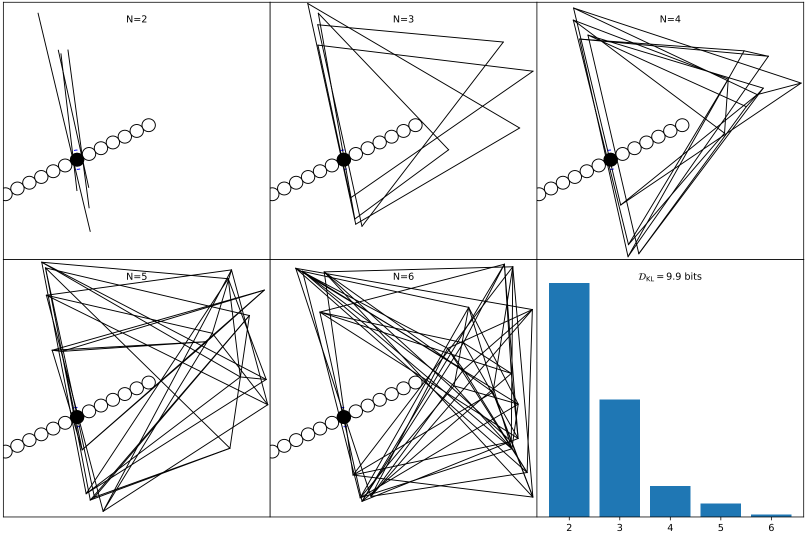

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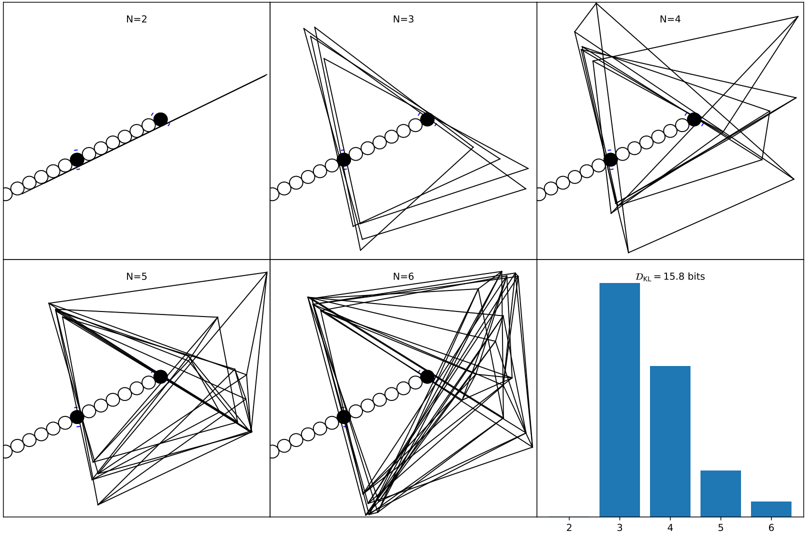

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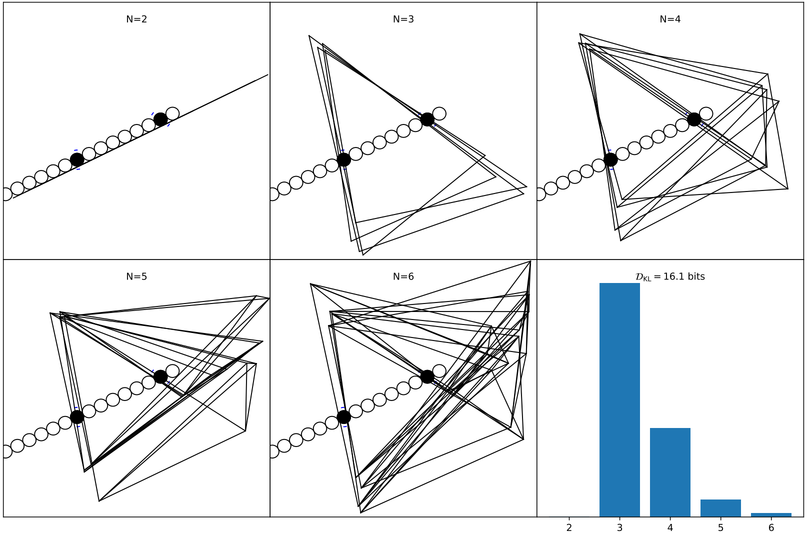

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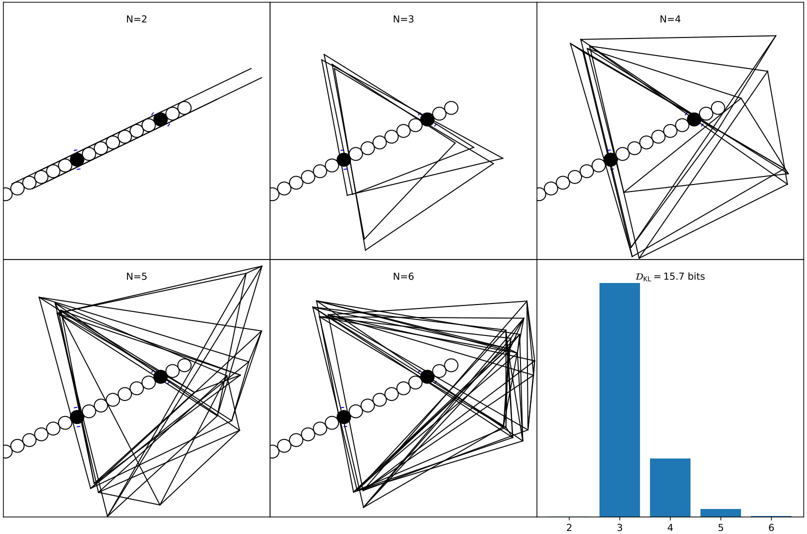

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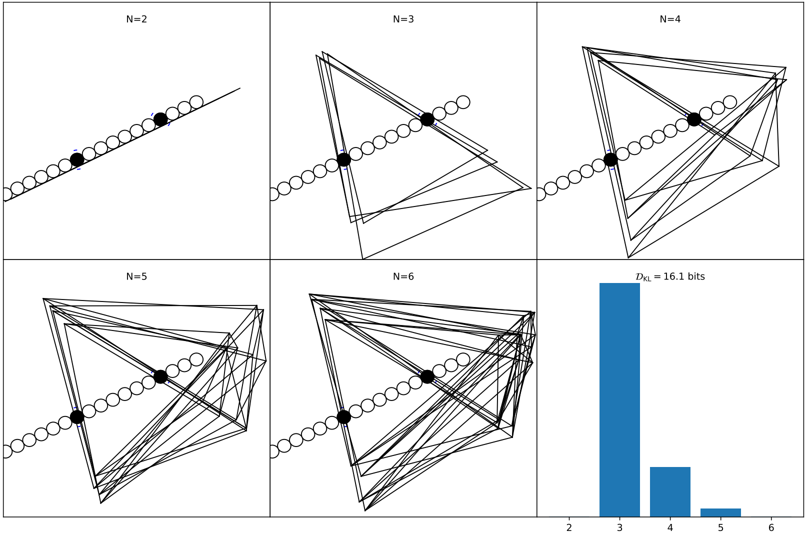

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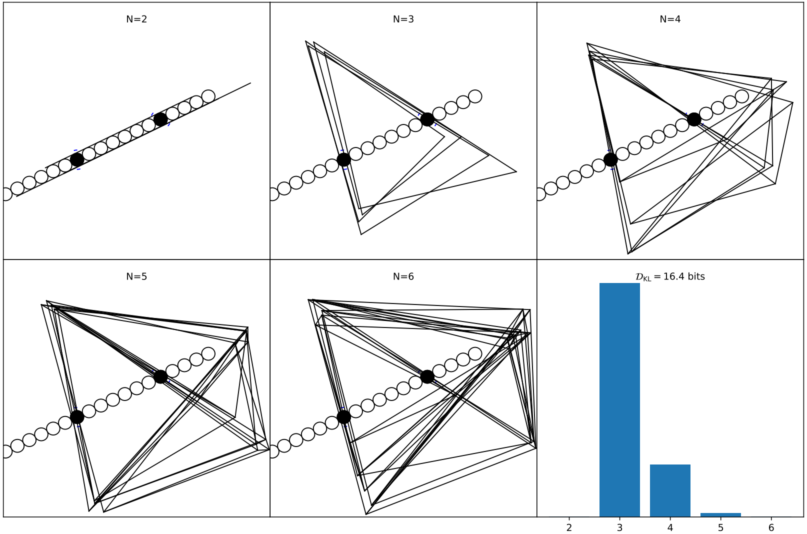

will.handley@polychord.co.uk

PolyChord

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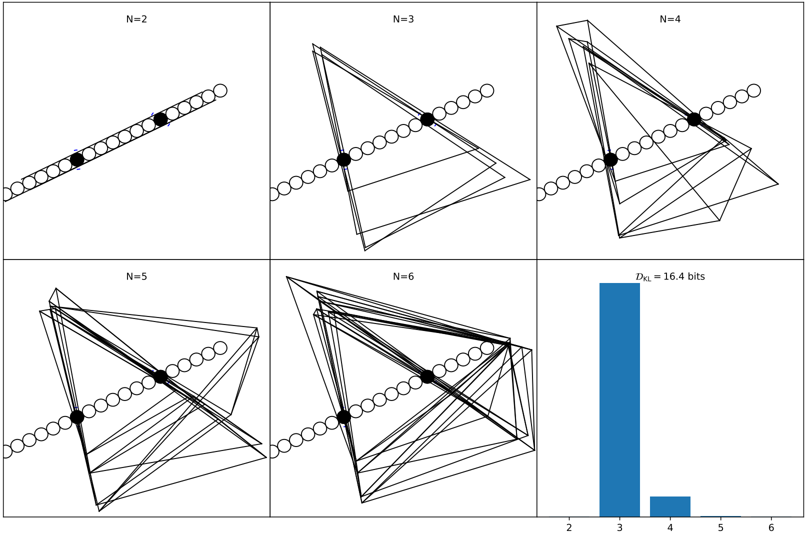

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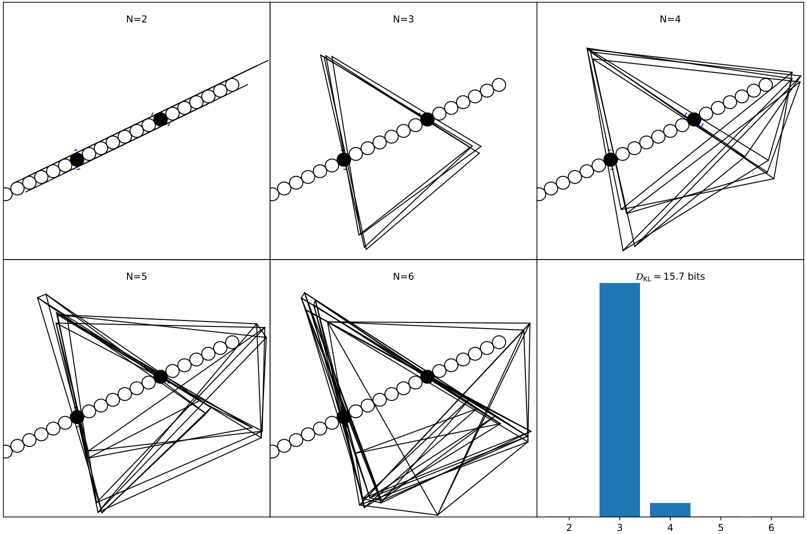

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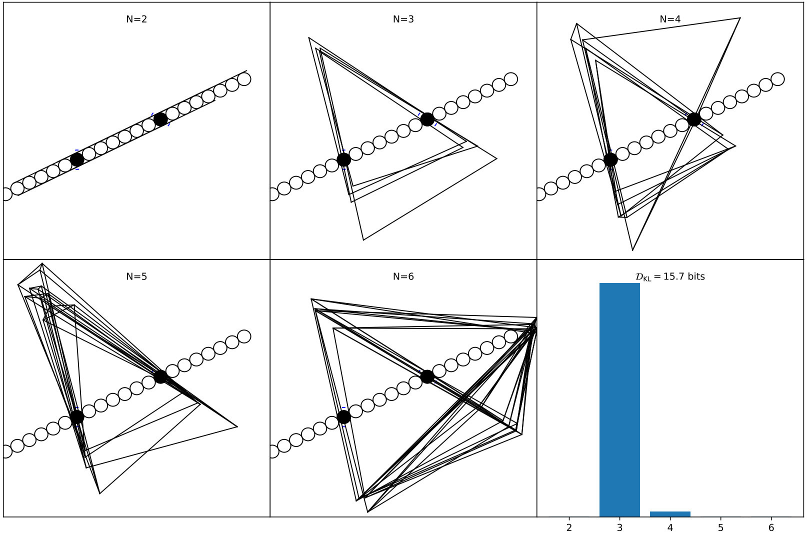

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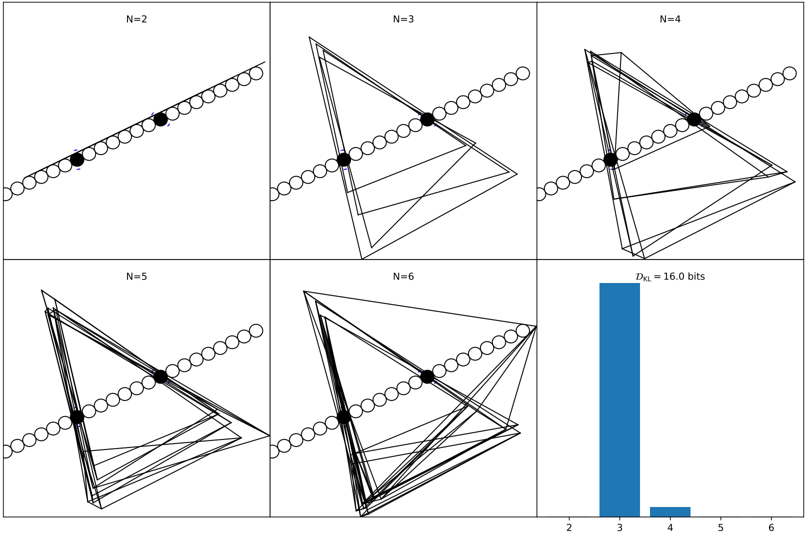

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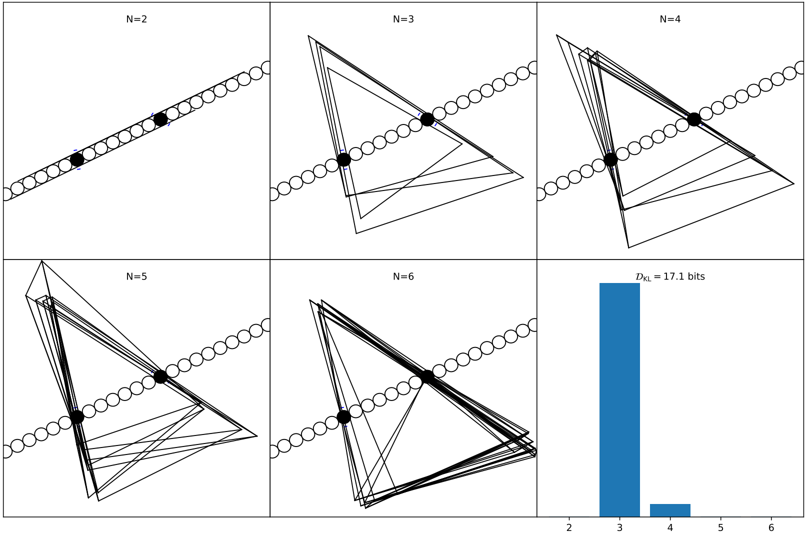
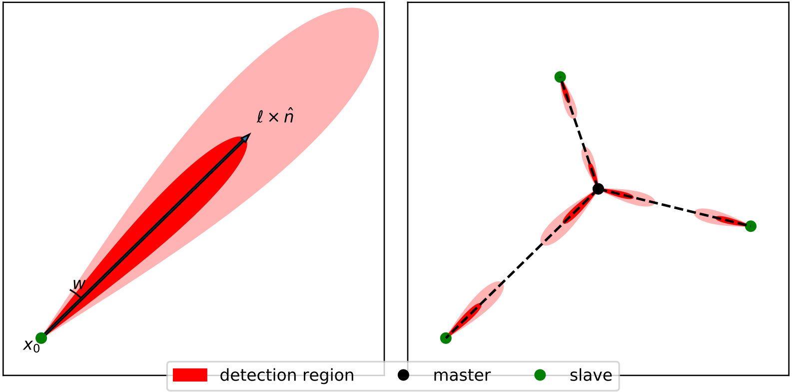


Problem refinement: broken star networks

Working under the guidance of PA consulting (James Matthews & Rob Lambert) & DSTL (Emily Russell, Emma Bowley, Jay Almond & Simon Angove), specified problem to:

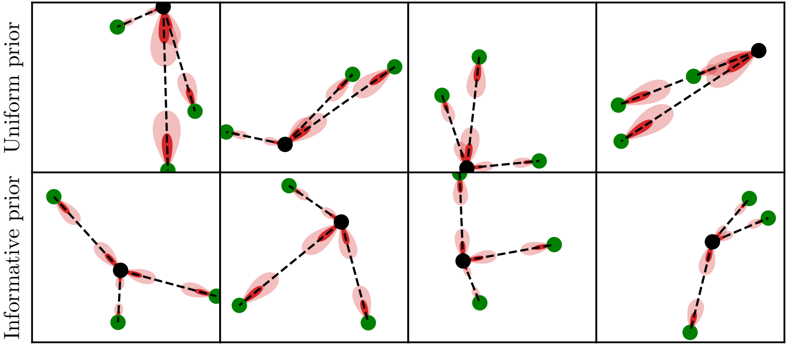
- **Broken beams:** Only detectable relatively near to transmitter
- **Star network:** $N = 4$, laid out in a topology with master node.
- Only need single detection to track down master node.



Prior adjustments

A prior distribution which is uniform in node locations results in some pathologies

- Dirichlet prior on angles prevents "squeezed" modes
- Hard prior on radii prevent nodes being too close or too far apart.



Numerical Bayesian techniques from Astrophysics (nested sampling & `PolyChord`) can encode nonlinear prior constraints such as these without increasing problem complexity.

Predictive posterior distributions for choosing next step

- **The critical part:** From our current state of knowledge $P(\Theta|D)$ of the network Θ given the data collected D, **where should we search next?**
- Taking the likelihood for new data $P(\hat{D}|\Theta)$, one can use posterior samples to compute the predictive posterior distribution $P(\hat{D}|D)$ [PPD].
- We use the PPD to choose the next path as the path which *maximises the probability of making a detection.*
- Plot on right shows:
  - Red contour: marginal posterior contours indicating knowledge of location of the central master node,
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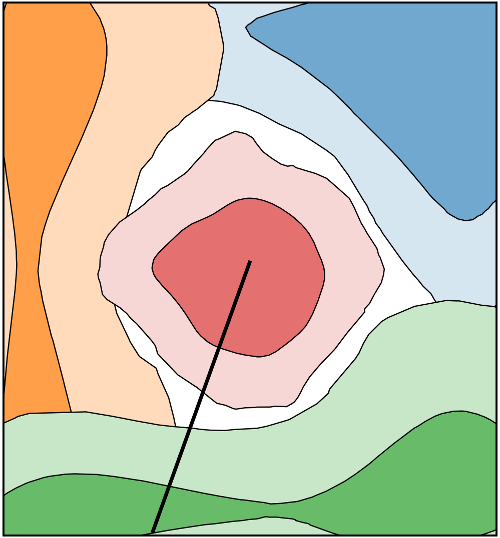

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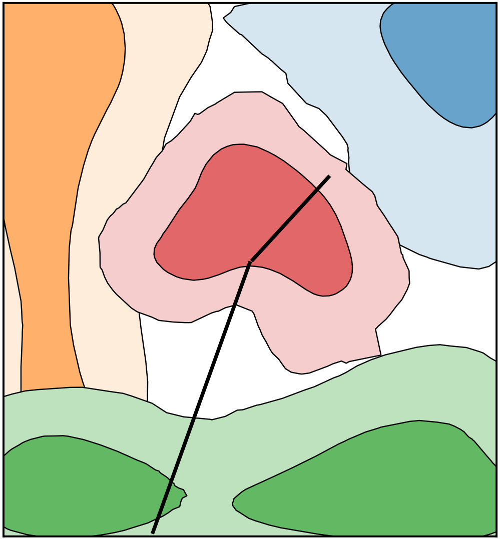

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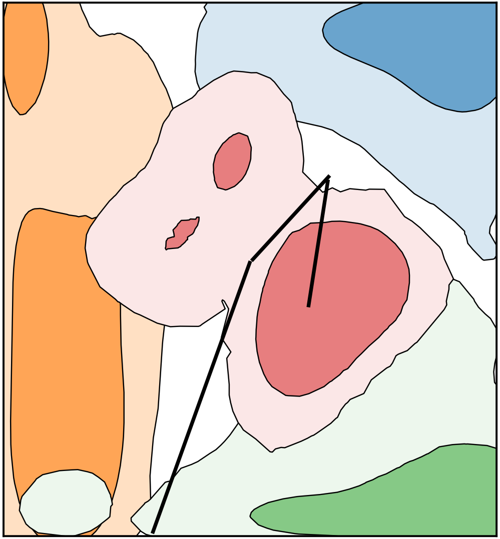

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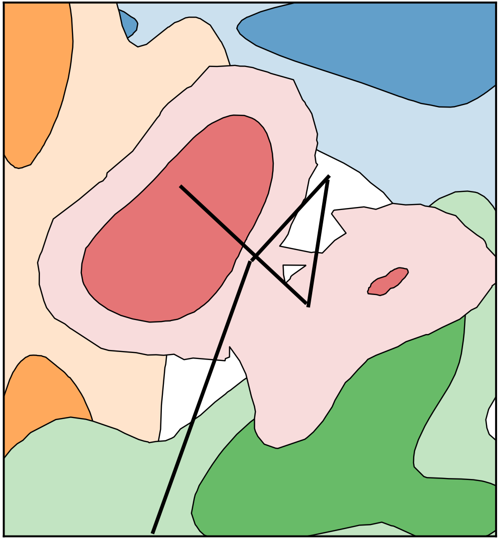

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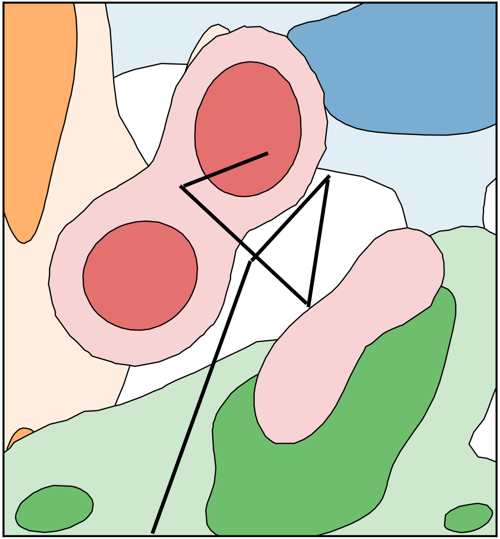

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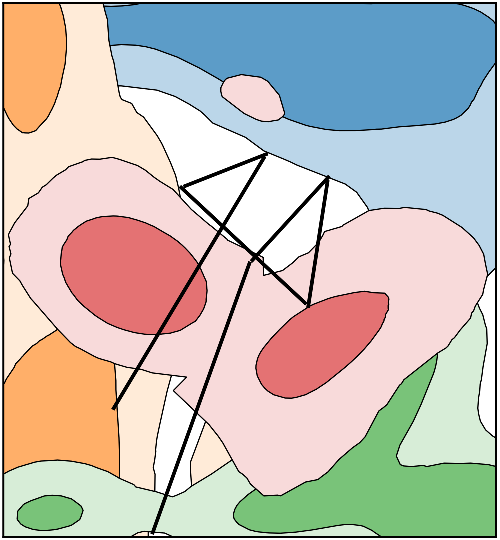

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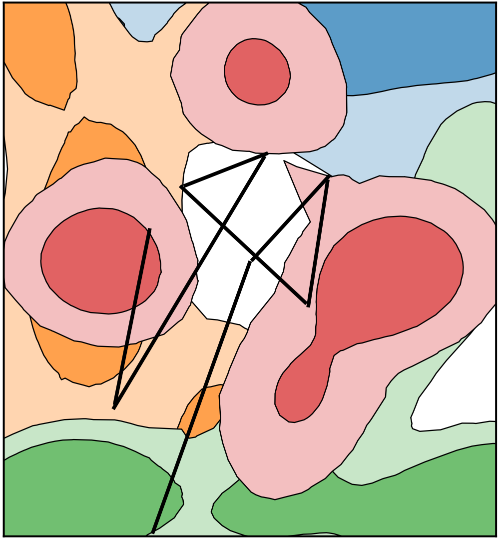

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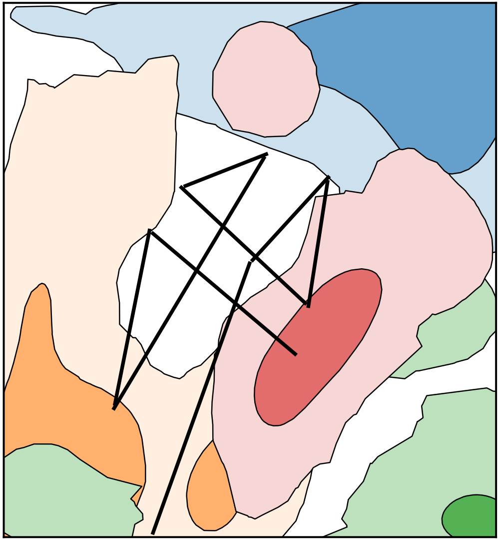

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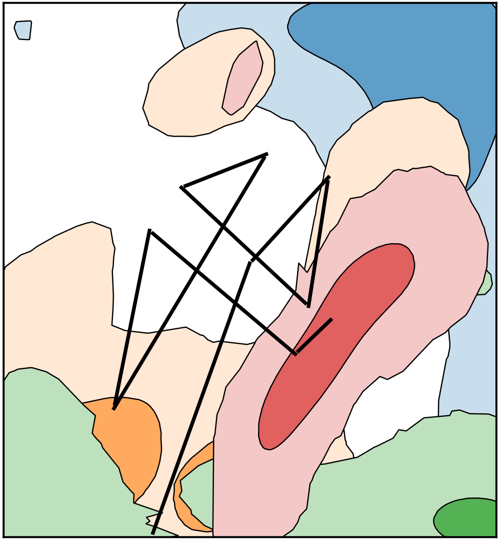
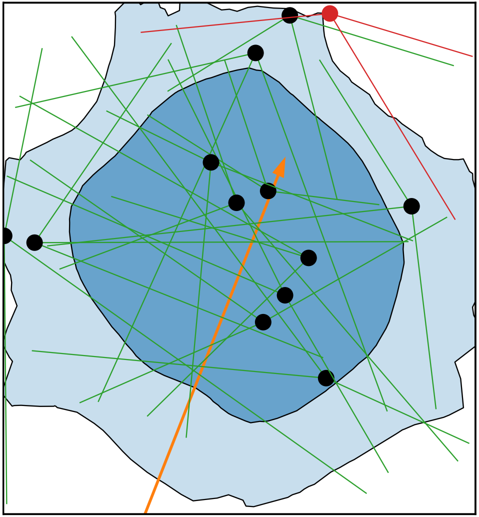


A difficult example

- Similar to previous slide, but now plotting
  - Blue contours: marginal posterior for central node
  - Red network: True network (deliberately chosen to be difficult to find)
  - Green network: Sample networks from the current posterior (non-marginal representation).
  - Orange arrow: path which maximises chance of detection
  - Probability at top: cumulative number of networks “ruled out”
- Note: optimum path does not always chase the central node location.
- A network this unlikely in real life would indicate an unrealistic prior, although the strategy still finds it



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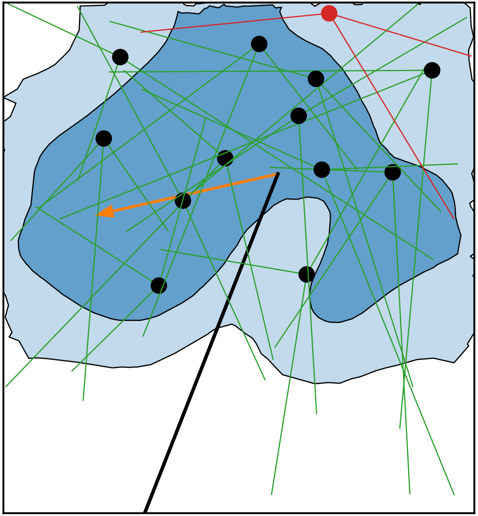
Step 2, Probability 89.68%



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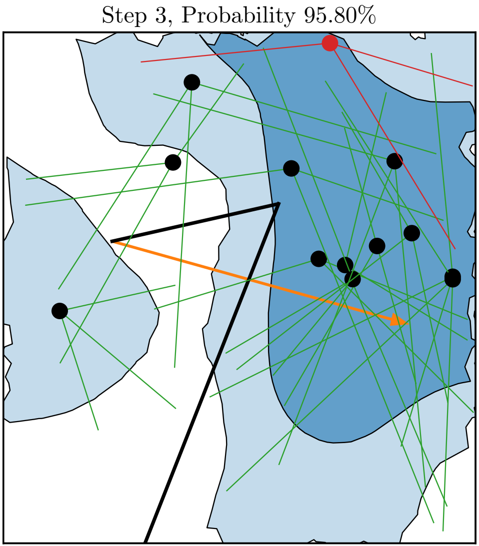

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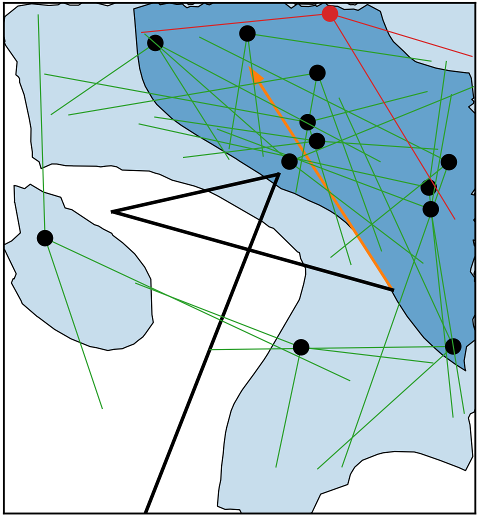
Step 4, Probability 98.25%



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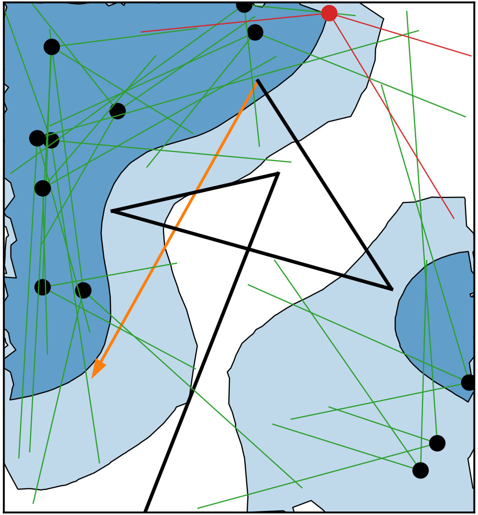
Step 5, Probability 99.10%



PolyChord

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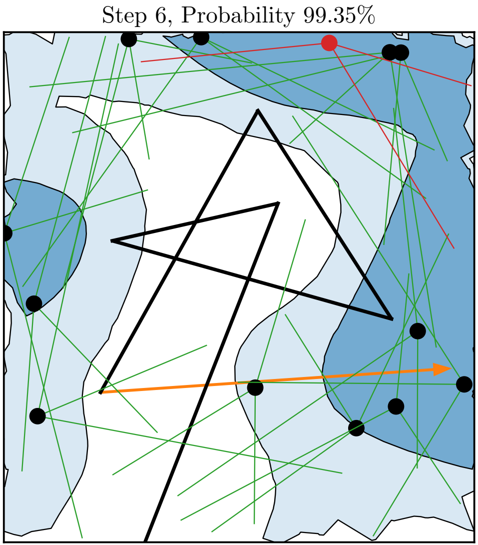

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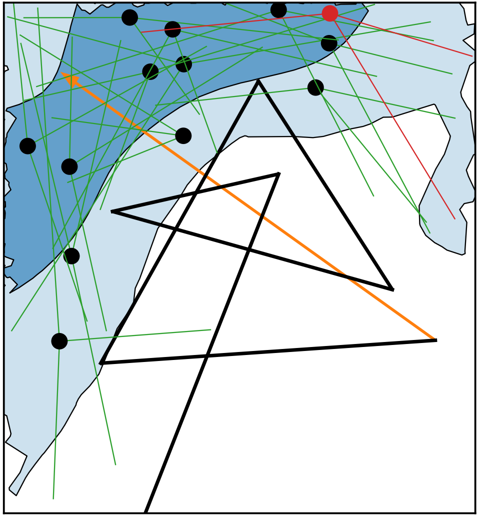
Step 7, Probability 99.67%



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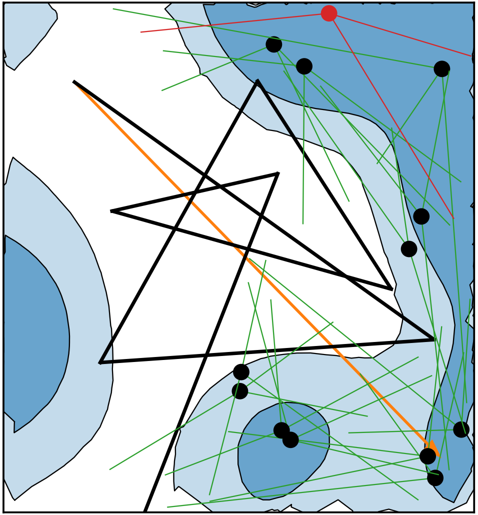
Step 8, Probability 99.82%



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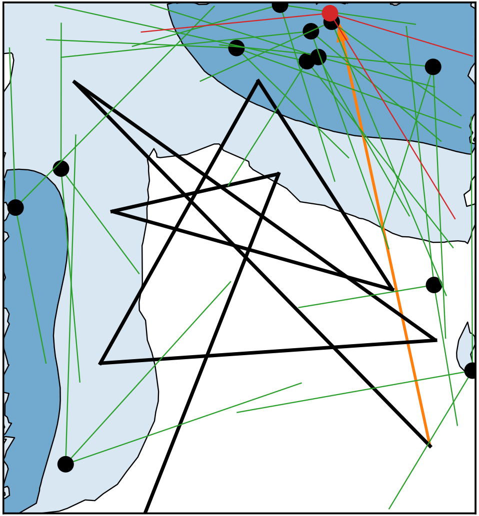
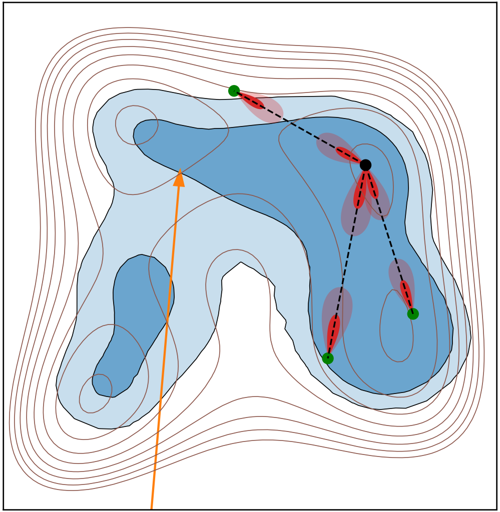
Step 9, Probability 99.86%



Himmelblau

- As before, but now with
  - a non-trivial prior indicated by contours
  - True network also indicates beams
- This could be interpreted as a hillside where e.g. it is more likely to find transmitters at the peak of the hill.

Step 1, Probability 89.25%



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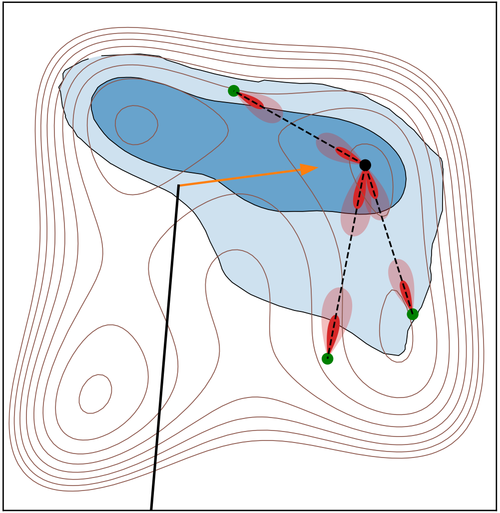
Step 2, Probability 98.12%



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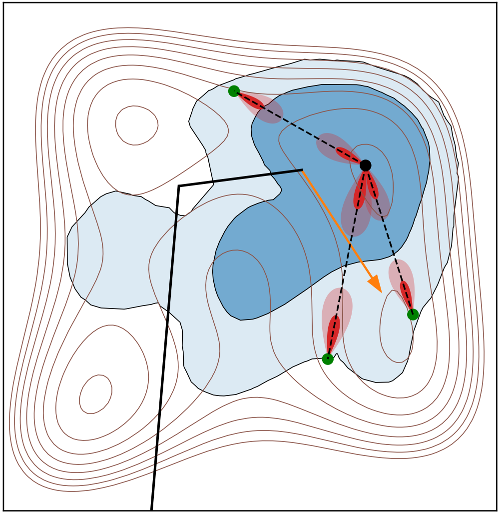
Step 3, Probability 99.73%



Himmelblau

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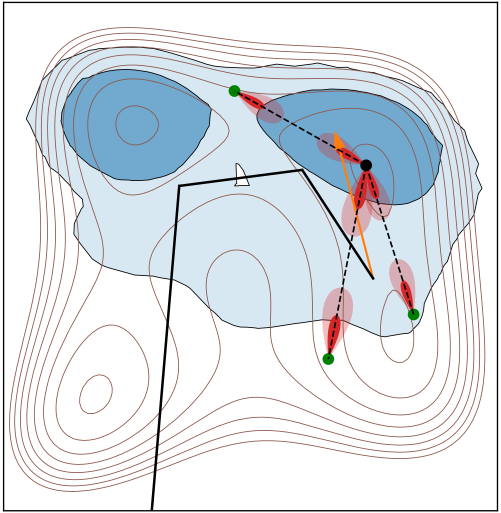
Step 4, Probability 99.96%



Conclusions

Possible extensions:

1. Increasing realism of setup (technique is not limited by complexity of forward model)
2. Three (or 2.5) dimensions
3. Optimising over topology

References:

- Figures produced under `anesthetic` `github.com/williamjameshandley/anesthetic`
- Numerial Bayesian calculations performed using `PolyChord` `polychord.co.uk`
- Original paper on `PolyChord` `arxiv.org/abs/1506.00171`

Next evolution of technique in upcoming paper:

MIDAS: Maximum Information Data Acquisition Strategies, Handley & Hobson 2021