Success Story 2 --- Optimum Sensor Placement

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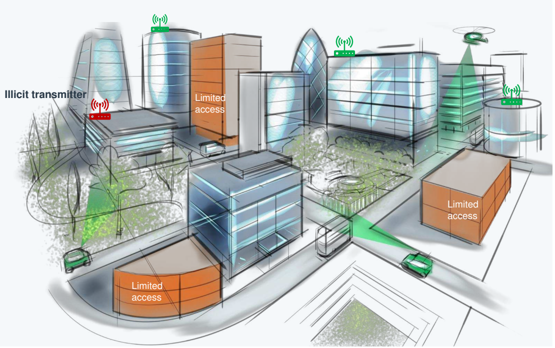
Challenges in the electromagnetic environment 2: Challenge 3

Arose at second CEME workshop as:

``Challenge 3: Locating an illicit transmitter in a dense, reflective urban environment"

- At initial scoping, determined we could assume:
  - Transmitter and sensors are static
  - Problem is offline i.e.

    cost of optimisation $\ll$ cost of deployment
  - Where should we put the sensors?
  - Which sensors should we use?
  - How can we deal with reflections?
- Continued as mini-project over Jan--Apr 2021, with James Matthews (PA) & DSTL (Emily Russell, Ben Jackson & Ben Gear)

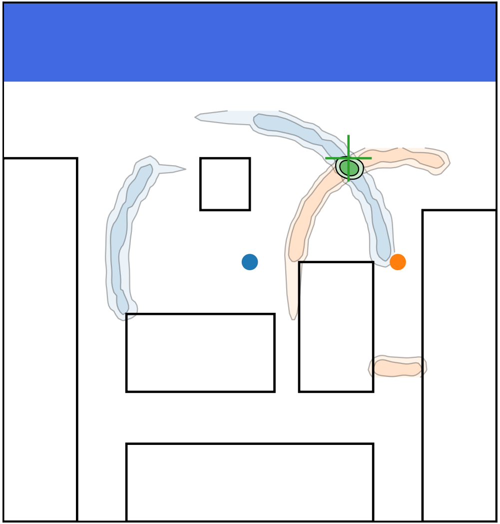


Incorporates content from third CEME workshop as:

``Challenge 1: Identifying the performance limit of a heterogeneous sensor system"

Range finding sensors

- Sensors: blue and orange dots
- Transmitter: green cross
- Contours: posterior distributions
  - individual posteriors in blue and orange (faint circular arcs)
  - combined posterior in green (solid peak)
- Measure transmitter distance with Gaussian error
- Both sensors detect: good localisation



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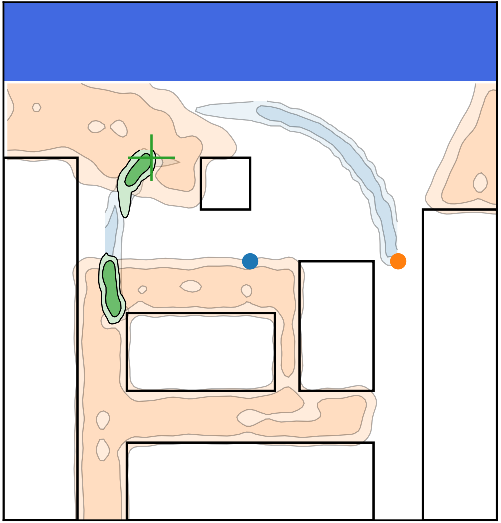

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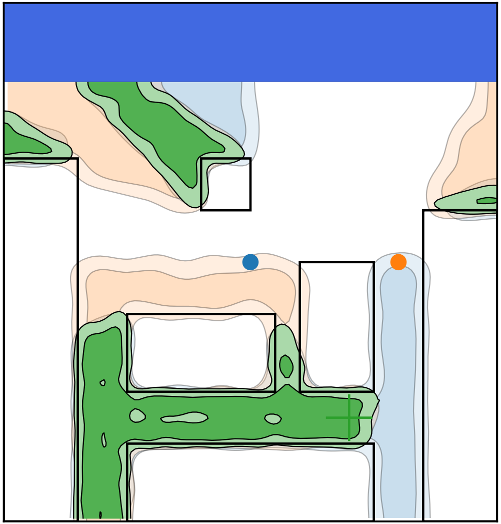
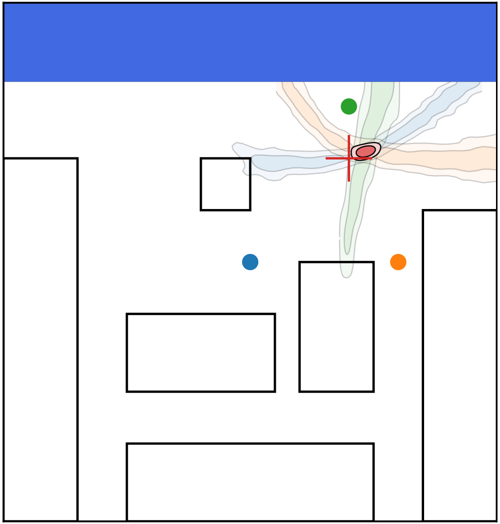
- No sensors detect: poor localisation



Time delay on arrival sensors

- Sensors: blue, orange & green dots
- Transmitter: red cross
- Contours: posterior distributions
  - pairwise posteriors excluding one sensor in blue, orange & green (faint parabolic arcs)
  - combined posterior in green (solid peak)
- Sensors measure time of arrival of signal, and use time delay-based likelihood to determine location
- All sensors detect: good localisation



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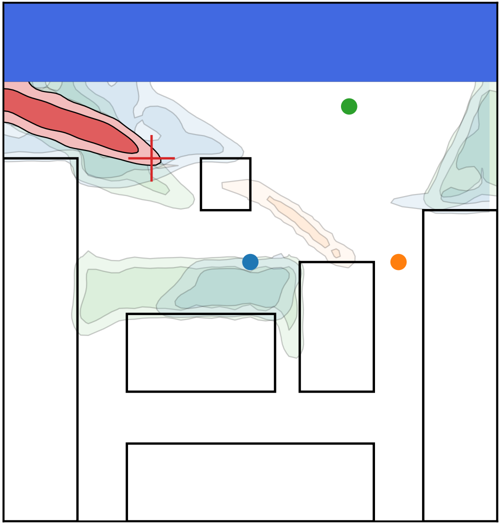

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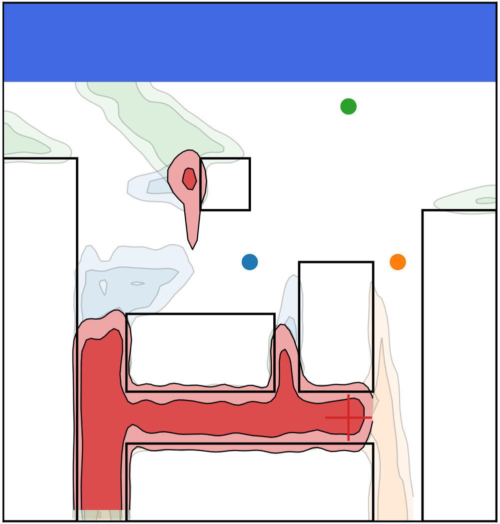
- Two sensor detection: reasonable localisation
- No sensors detect: poor (independent of sensor type)
- Extendable to heterogenous sensors (CEME3.1)



Bayes theorem, and the Kullback--Leibler divergence $\mathcal{D}_{\mathrm{KL}}$

Key question: What should we optimise in order to determine best sensor location?

- We have a forward model/generative likelihood of $P(D|\theta, X)$, where
  - D is the sensor reading (including non-detections)
  - θ is the location of the transmitter
  - X is the locations of the sensors
- Can use this to
  1. Generate mock data $D(\theta, X)$ given a transmitter location θ & sensor setup X
  2. compute posterior on sensor location $P(\theta|D, X)$ given sensor reading D and sensor set up X
- Can quantify degree of transmitter posterior localisation with the Kullback--Leibler divergence

$$\mathcal{D}_{\mathrm{KL}}(D, X) = \left\langle \log \frac{P(\theta|D)}{P(\theta)} \right\rangle_{P(\theta|D)} \approx \log \frac{\mathrm{Volume}(\mathrm{prior})}{\mathrm{Volume}(\mathrm{posterior})}$$

The Kullback Leibler divergence as localisation metric

- KL divergence measures posterior localisation
- $\mathcal{D}_{\mathrm{KL}} \approx \log \frac{\mathrm{Volume(prior)}}{\mathrm{Volume(posterior)}}$
- It is a function of
  - sensor reading D
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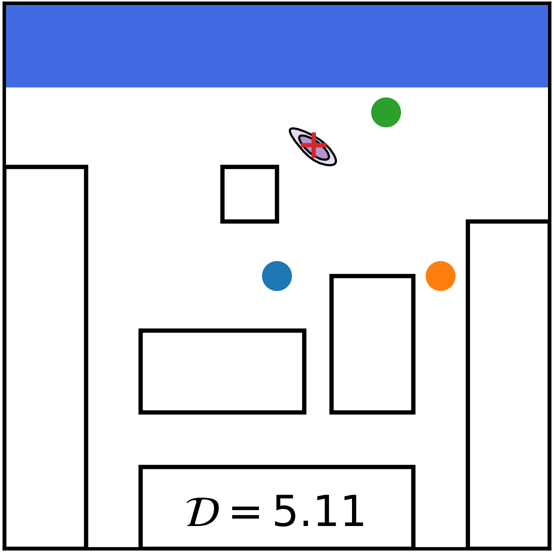

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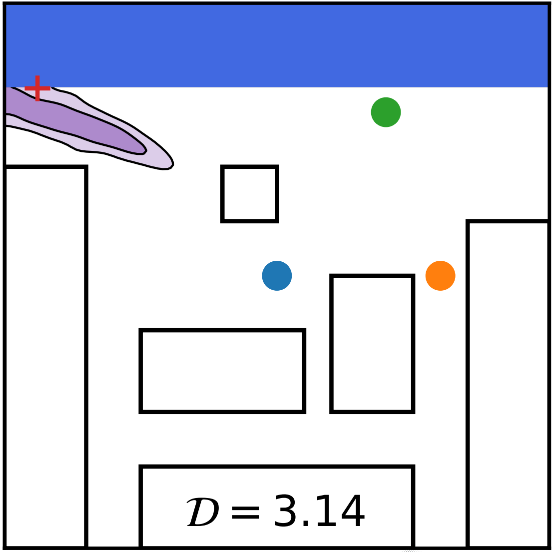

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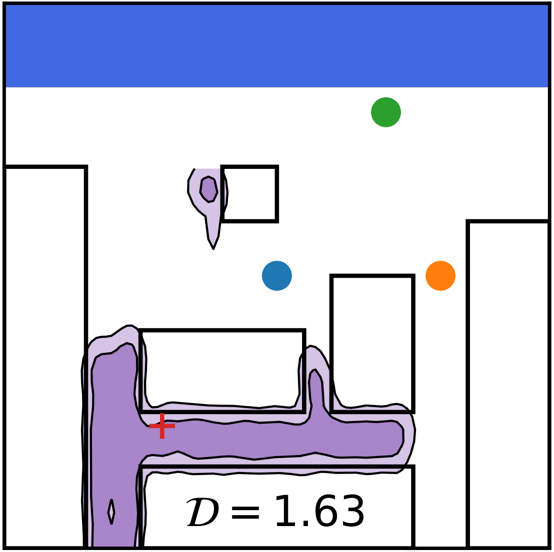

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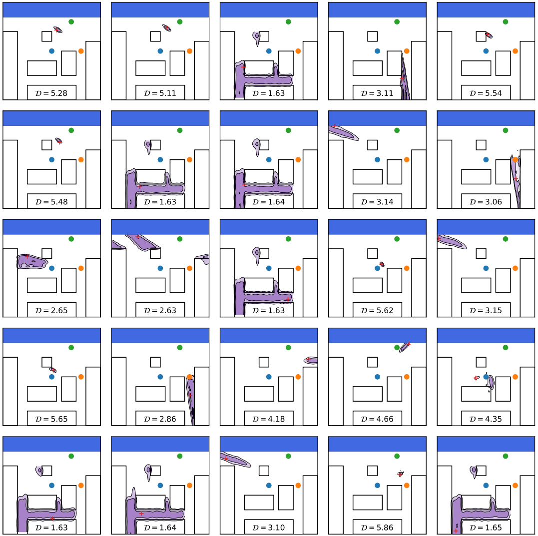

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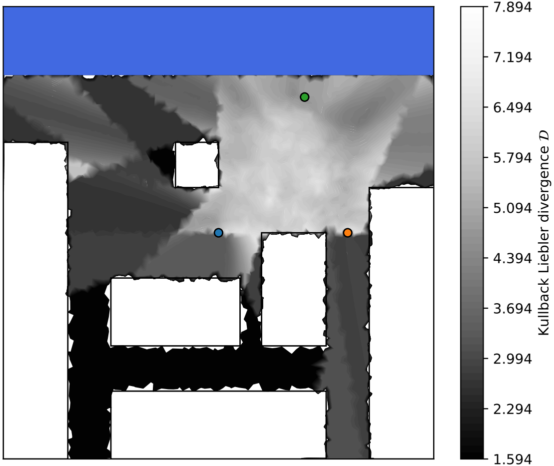
- Plot as a function of true transmitter location
- Can compress this via desired statistical function
  - mean value: $\hat{\mathcal{D}}$
  - minimum value: $\min \mathcal{D}$
  - percentiles, standard deviations etc…
- What remains is a function of sensor setup X…
- …can be optimised e.g. using `PolyChord` for the full $2N$-dimensional optimisation



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PolyChord

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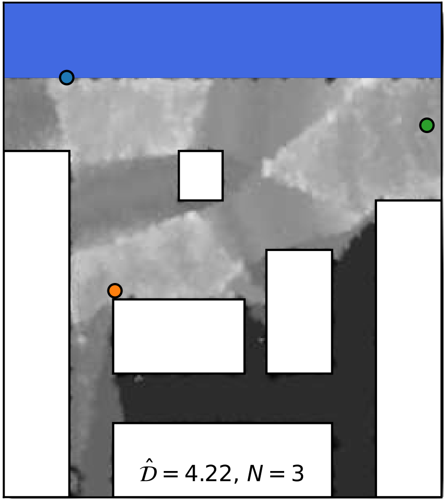

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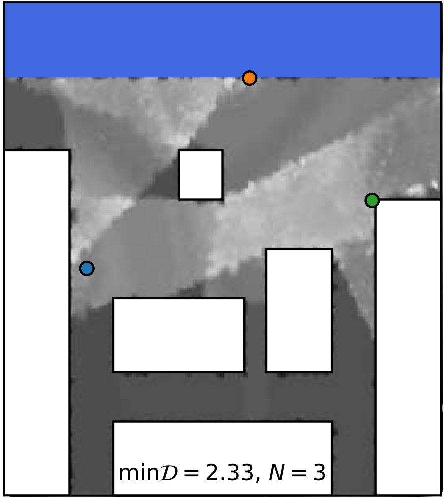
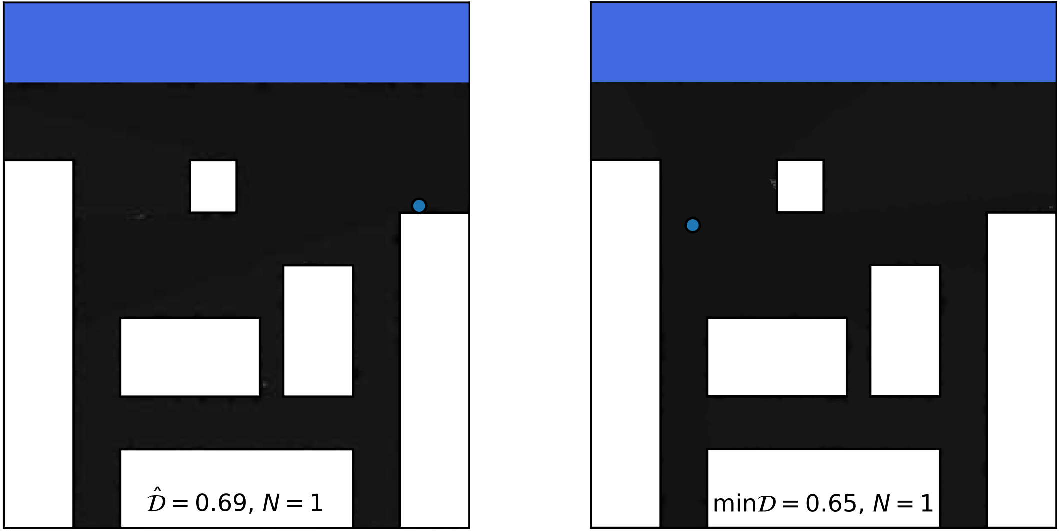


Optimising KL divergence with PolyChord for N sensors



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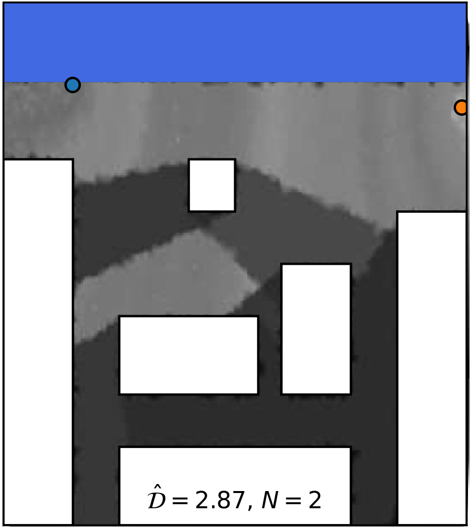

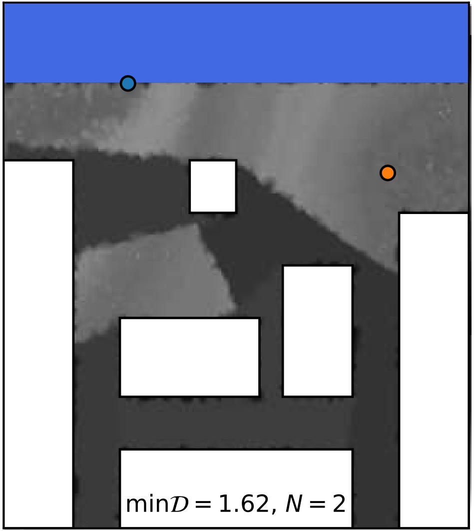

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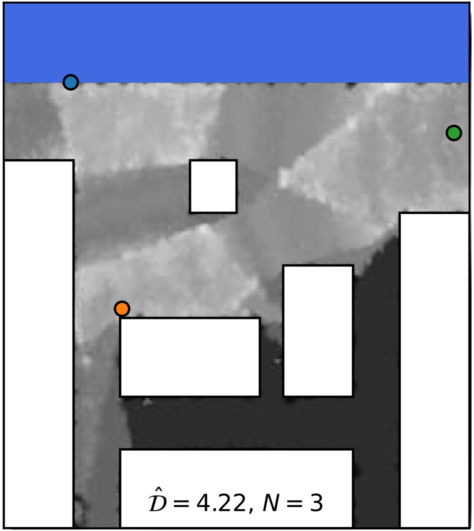

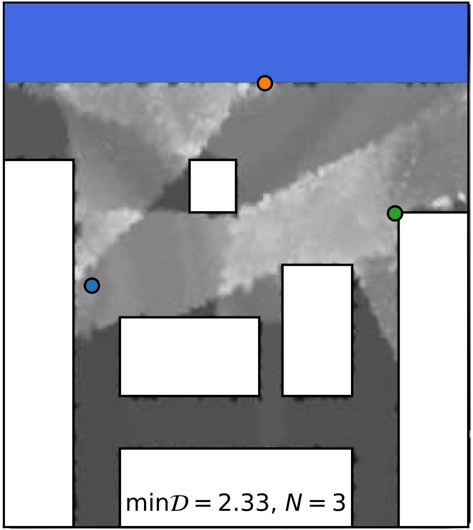

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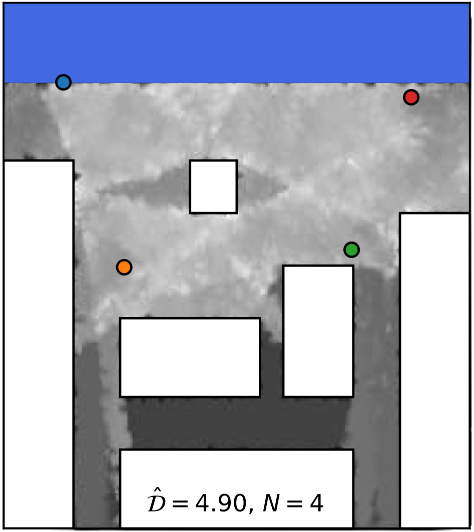

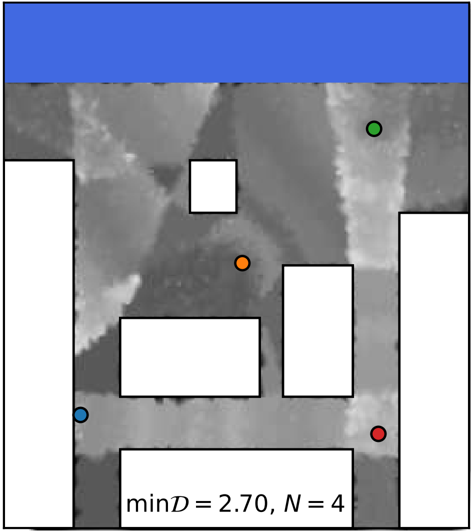

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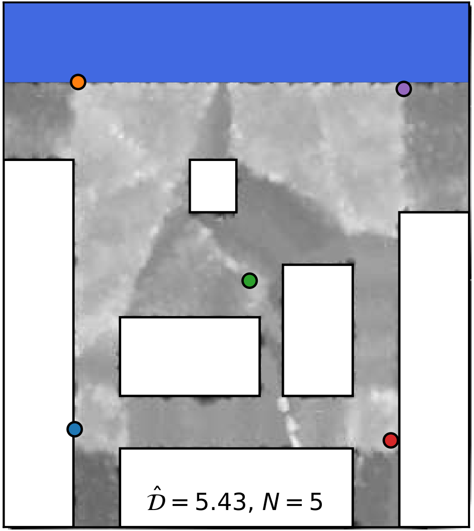

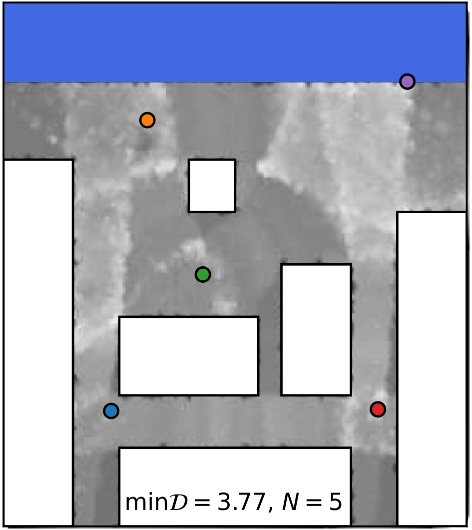

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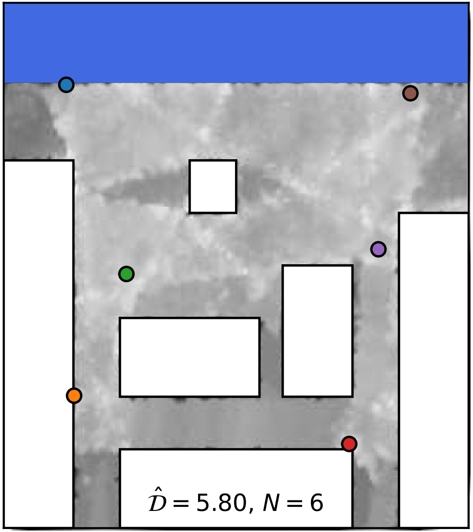

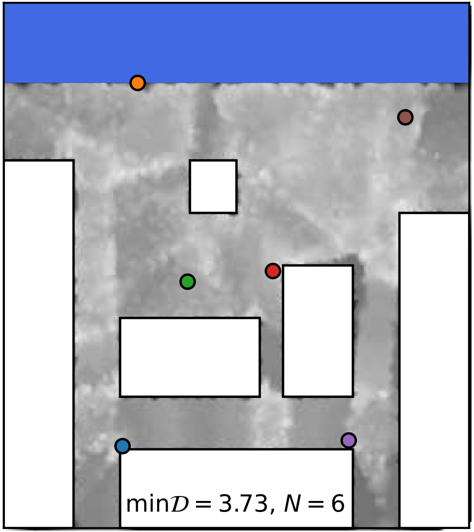

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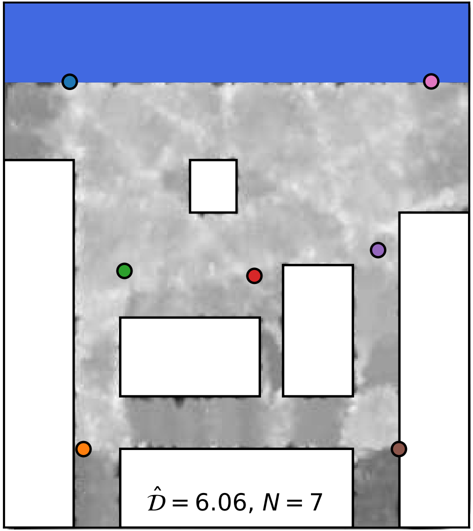

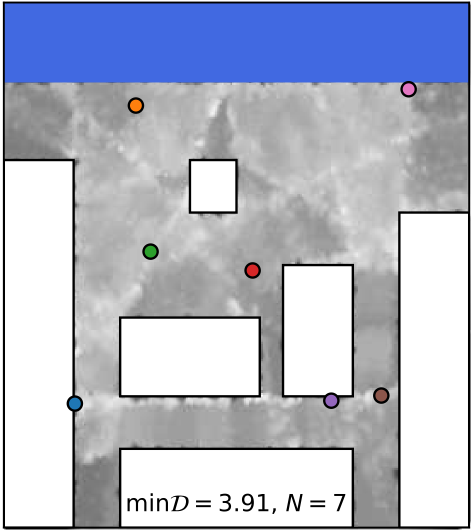

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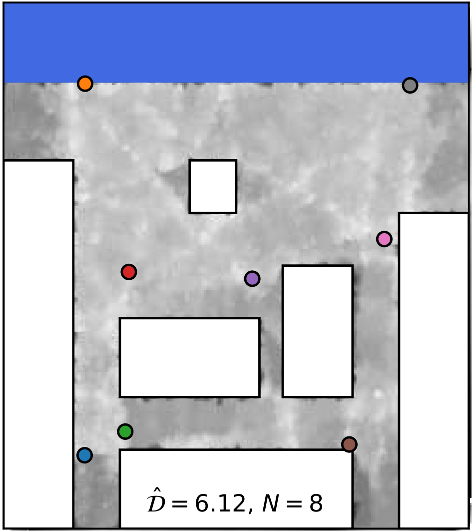

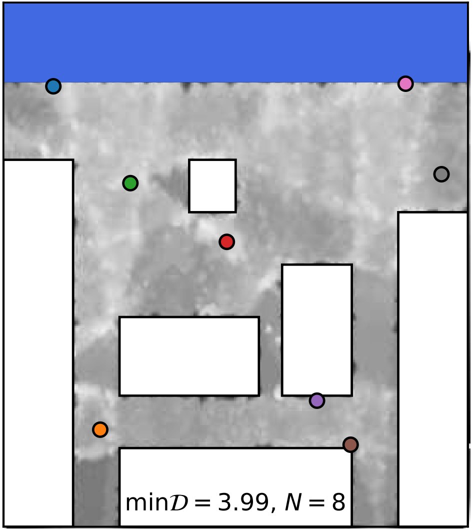

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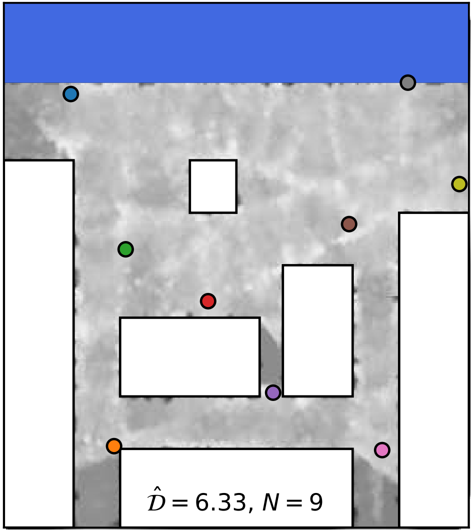
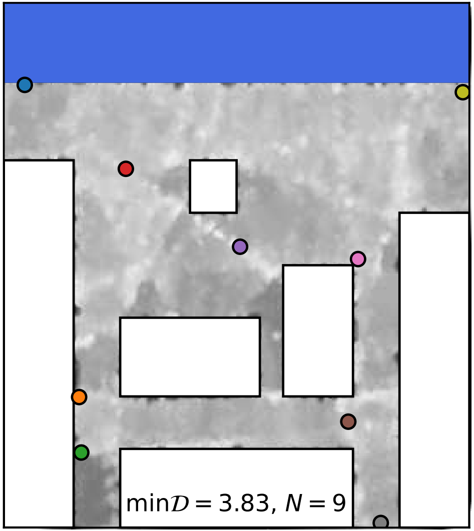
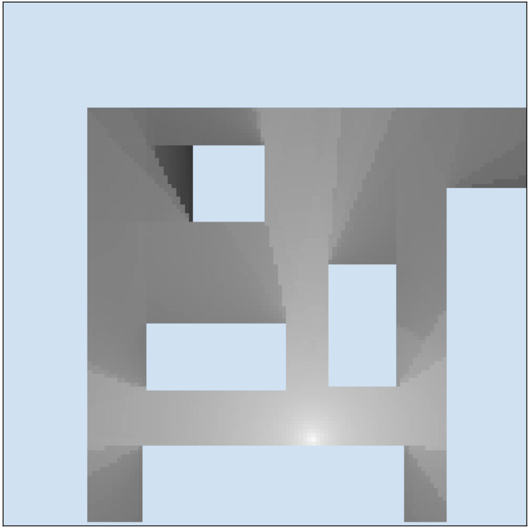




Reflections simulations

- Exploring the ability to deal with a reflective landscape was a key goal of the project
- Our metric and optimisation methodology is independent of the complexity of the forward model.
- Data files of full ray tracing simulations provided by James Matthews as an example of complexity increase
- Plot shows attenuating EM signal in a reflective, diffractive environment, with location of transmitter in bright white.



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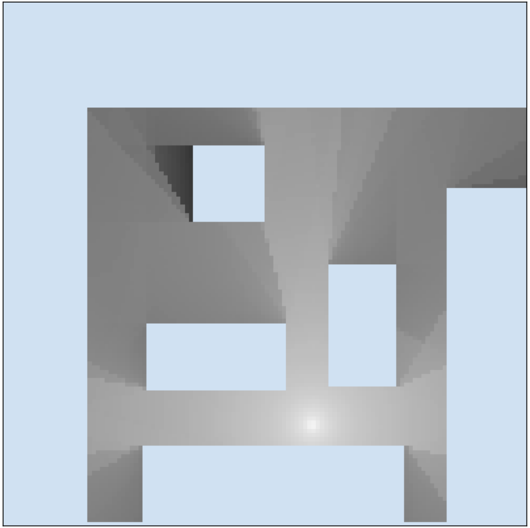

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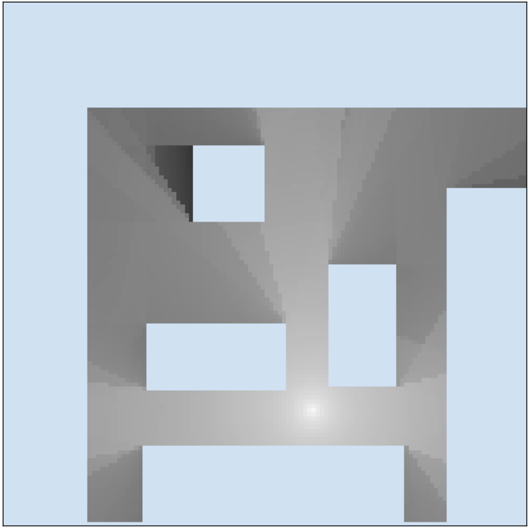

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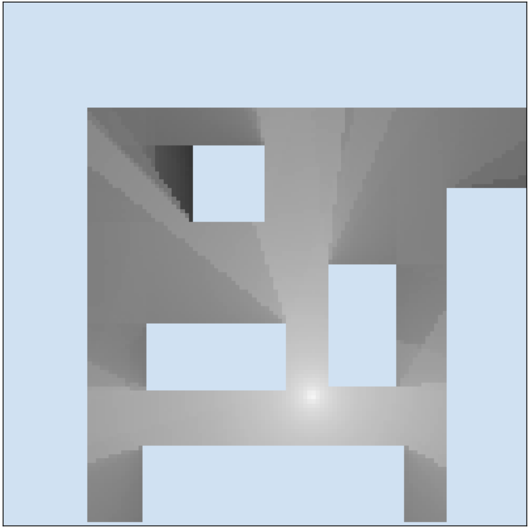

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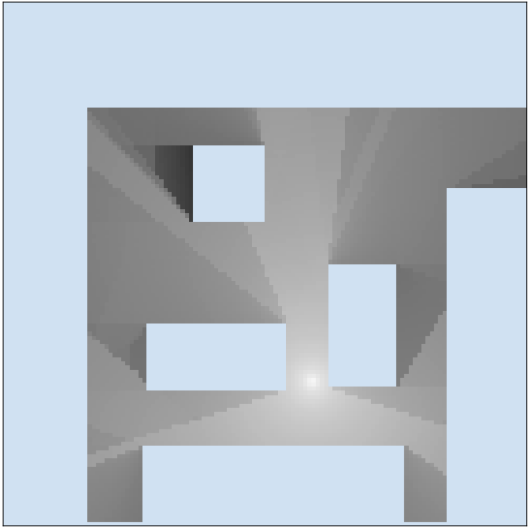

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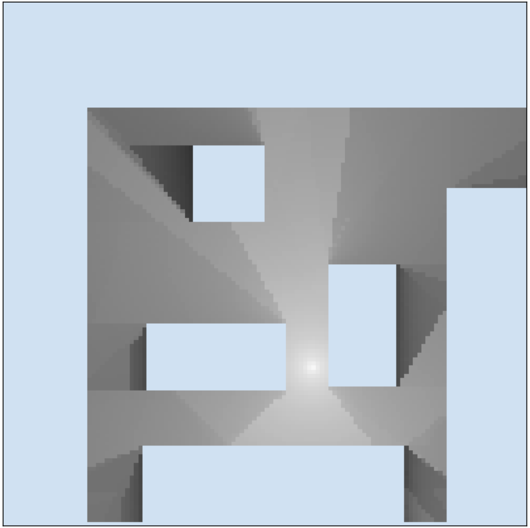

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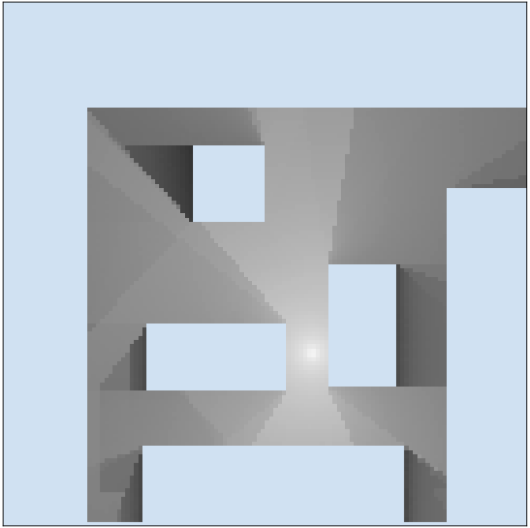

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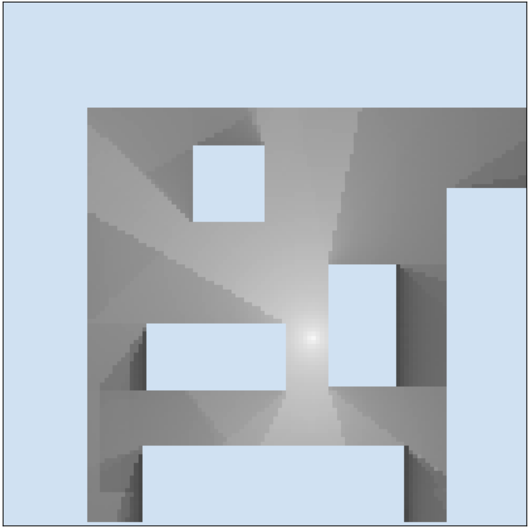

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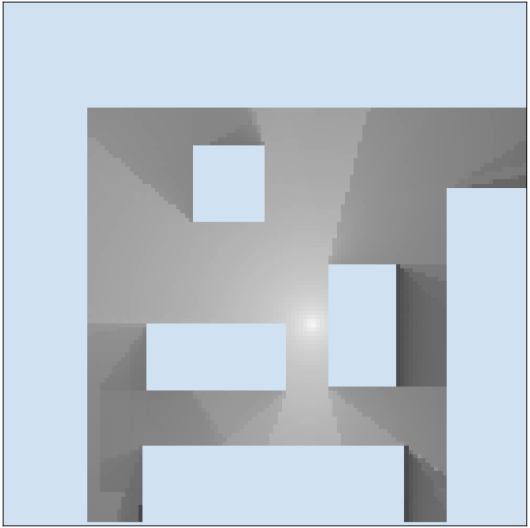

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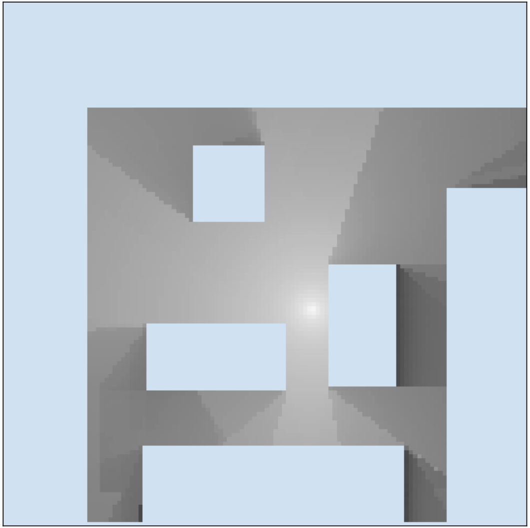

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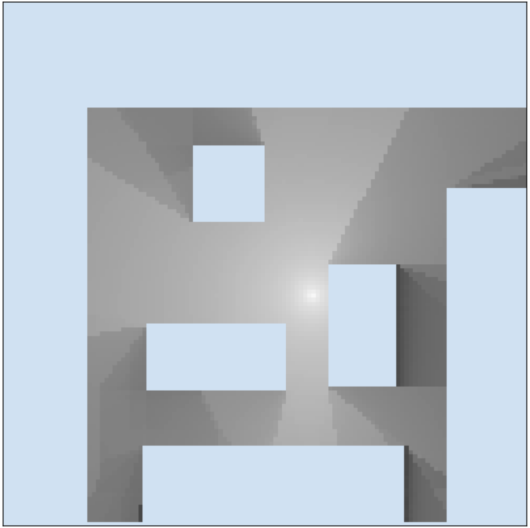

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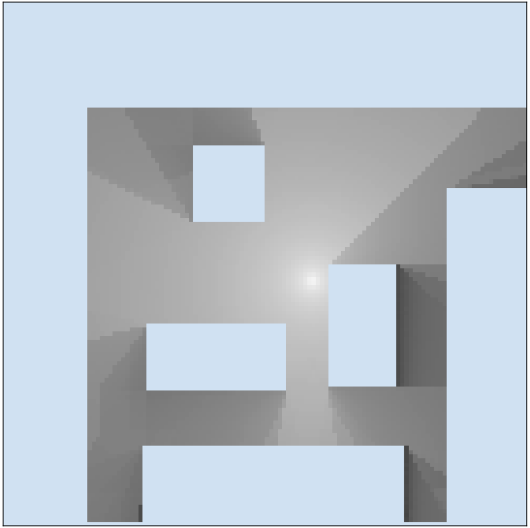

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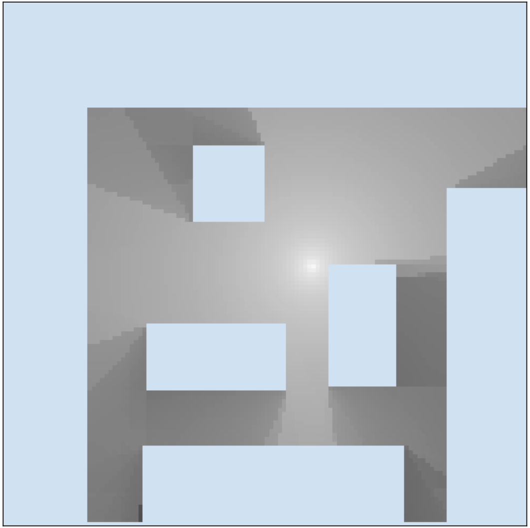

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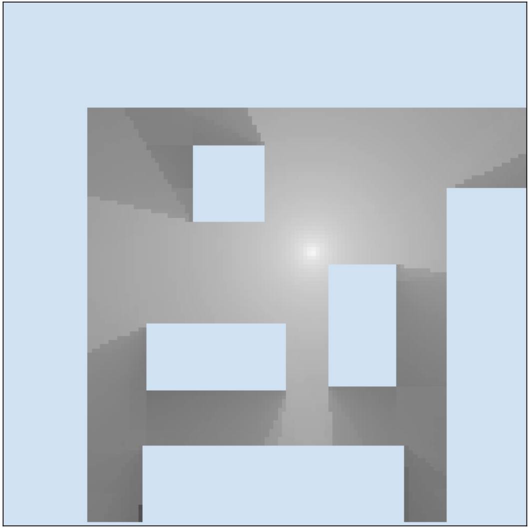

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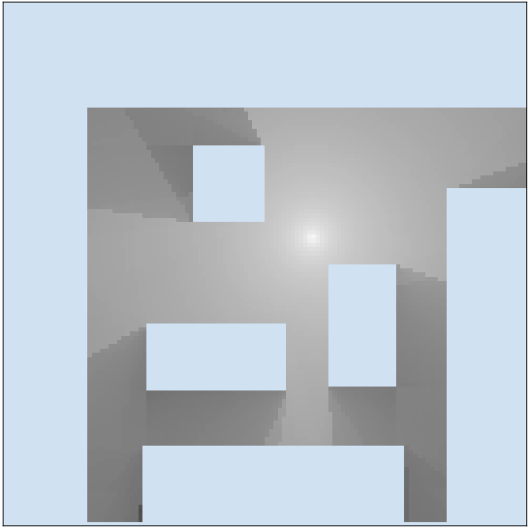

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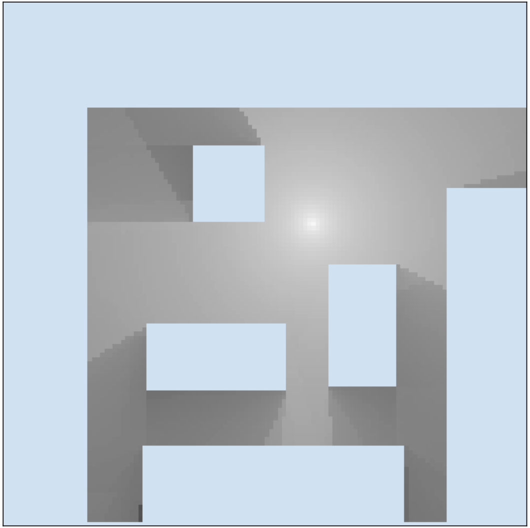

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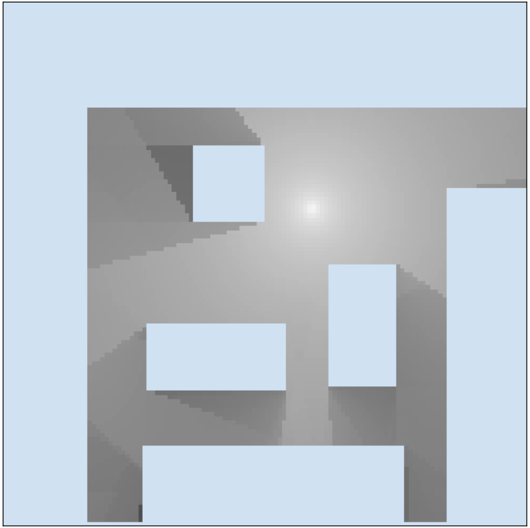

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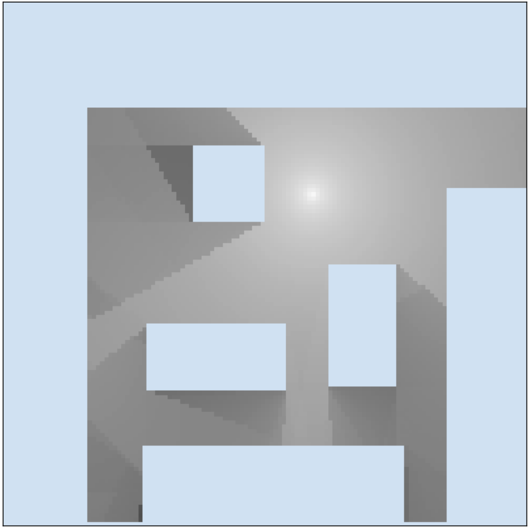

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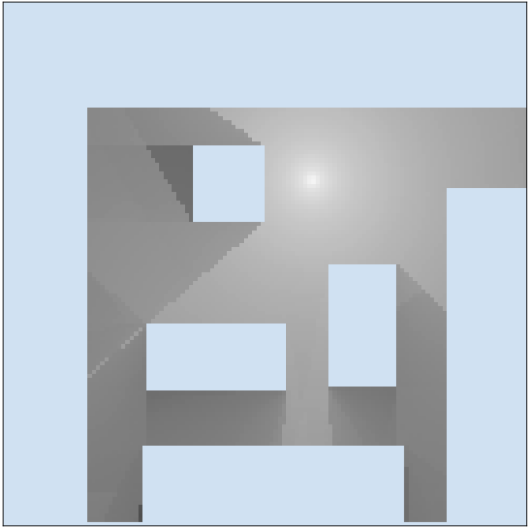

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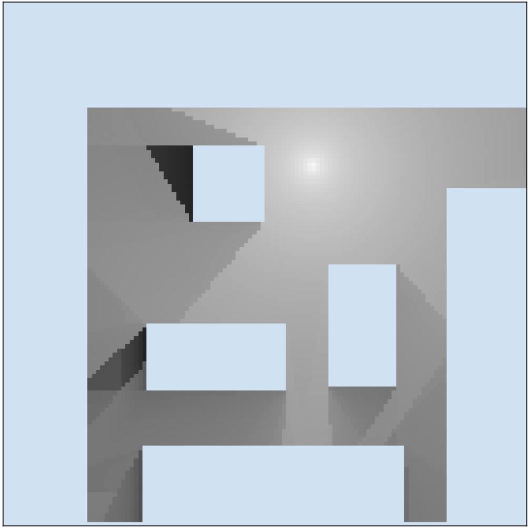

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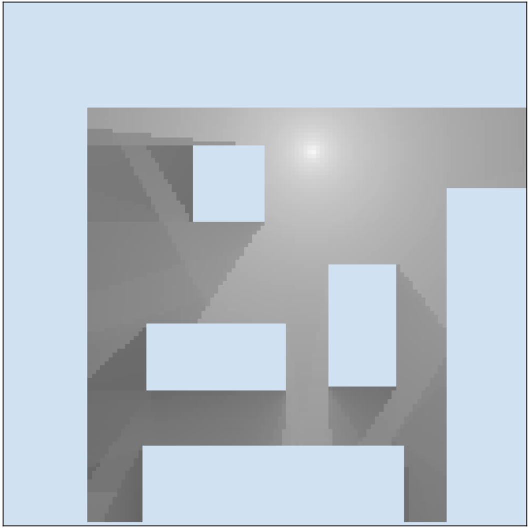

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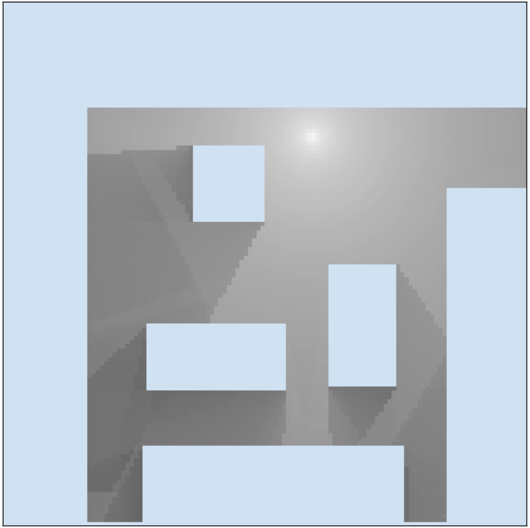

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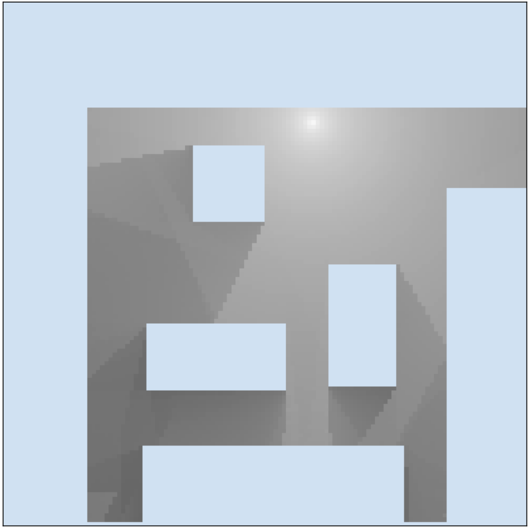


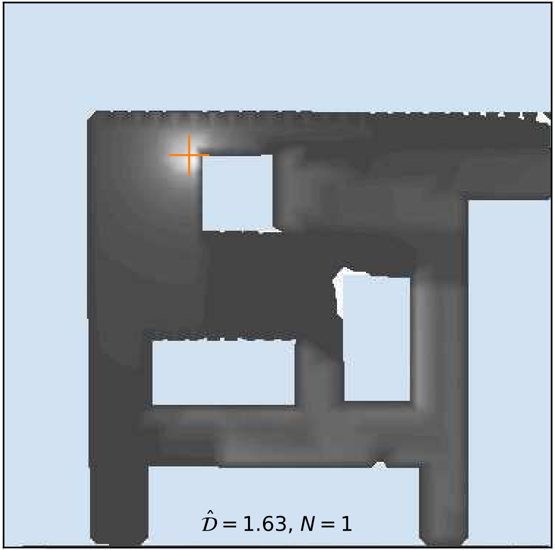
Maximising mean KL divergence in reflections

As before:

- can calculate our chosen optimisation metric (e.g. $\hat{\mathcal{D}}$, min $\mathcal{D}$,...) for the more complicated forward model including reflections
- can use `PolyChord` to optimise as a function of sensor setup $\max_X \hat{\mathcal{D}}(X)$

Extensions:

- Heterogenous sensors
- Incoporate a financial cost constraint to also allow N and type of sensor to vary.



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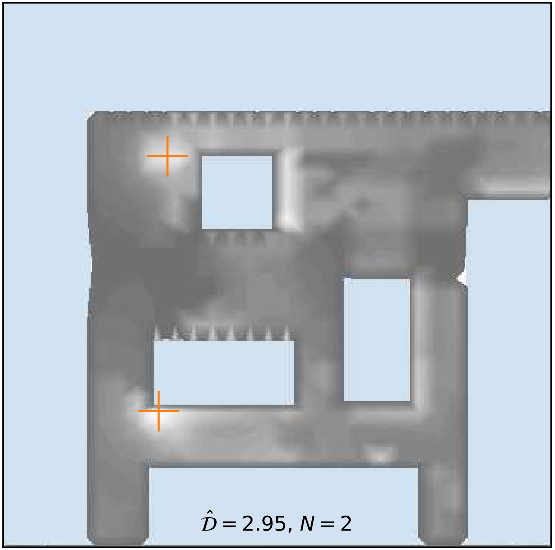

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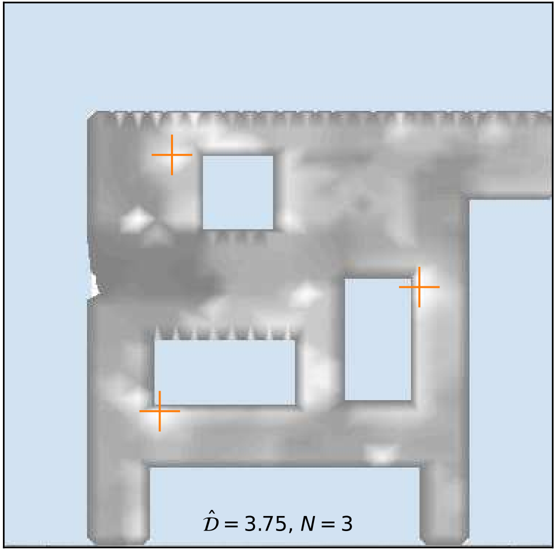

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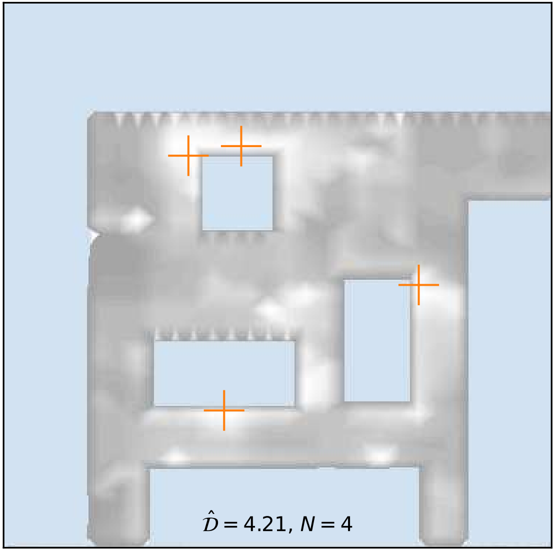

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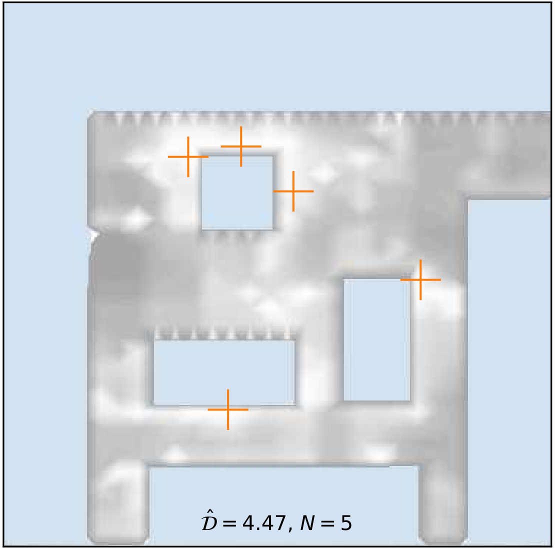

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