

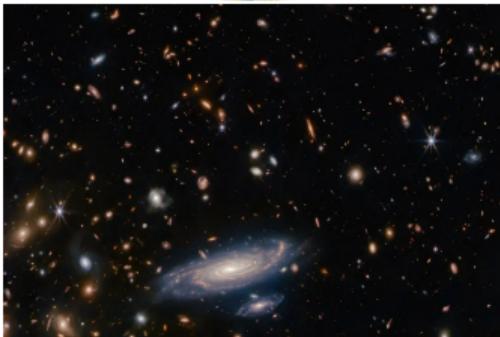
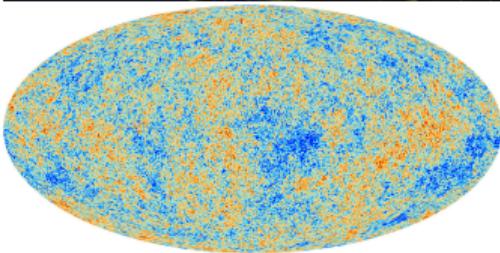
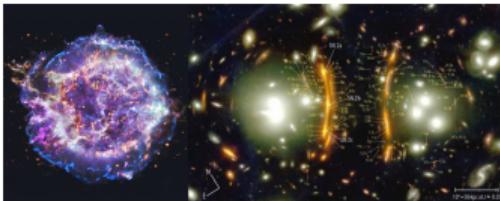
Bayesian OODA loops with MIDAS: Augmented decision making in a complex future electromagnetic environment

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University of Cambridge Royal Society University Research Fellow
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20th November 2023

Background: Bringing Bayesian Astrostatistics back to Earth



Astrophysicists interested in:

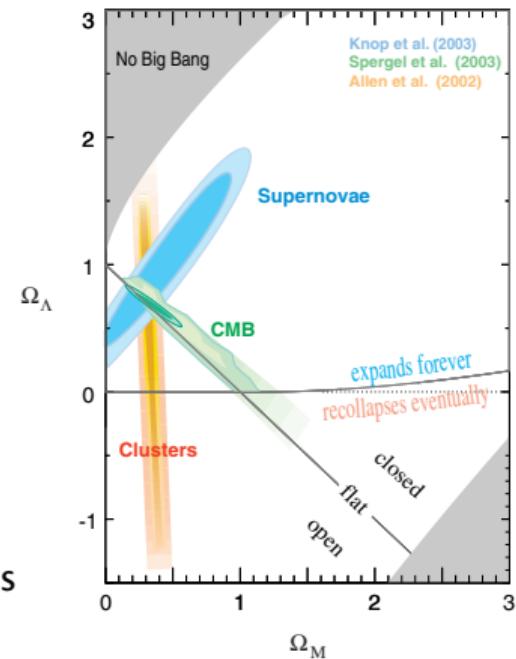
- ▶ Quantify uncertainty
- ▶ Fusion of diverse measurements
- ▶ Biggest data

Typically use Bayesian methods:

- ▶ Incorporation of prior info
- ▶ Quantifies updating knowledge
- ▶ Bayes theorem: unifying framework

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)} \quad \begin{aligned} H &: \text{Hypothesis} \\ D &: \text{Data} \end{aligned}$$

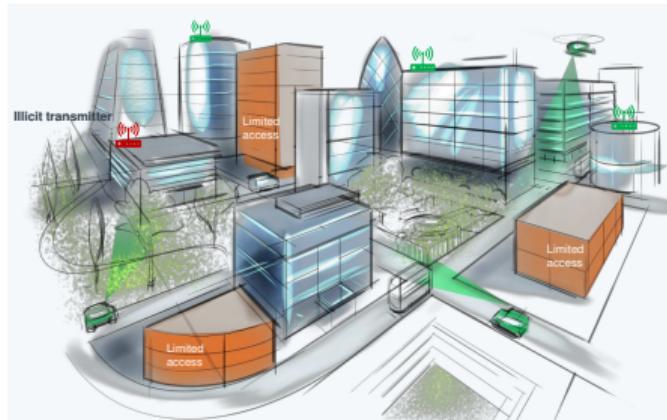
Complementary to machine learning



Background: Challenges in the Electromagnetic Environment (CEME)

Since 2020 participated in 6 Maths of CEME workshops, with 5 DSTL-funded projects

1. Optimising a search route to discover networks in a landscape of constraints
(CEME1.2, [Jan20])
2. Optimisation of sensor location
(CEME2.3 [Sep20])
3. Further optimisation of sensor location
(CEME4 [Sep21])
4. MIDAS: Maximum information data acquisition strategies
(CEME6.4) [Jan23])
5. Optimal dynamic manoeuvring & adaptation of communications networks driven by the MIDAS information-advantage mathematical framework
(DASA GAN [Oct23])



DSTL: Olly Gage, Emily Russell, Ben Jackson, Ben Gear, Emma Bowley

PA: James Matthews, Richard Claridge, Emily Morrison, Rob Lambert

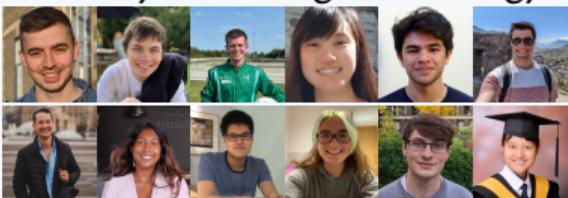
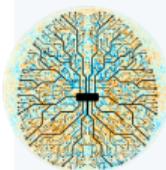
PC Ltd: Catherine Watkinson, Thomas Mcaloone, Parul Janagal, Adam Ormondroyd

UCAM/QML: Mike Hobson, Justin Ward, Oscar Bandtlow

Background: PolyChord Ltd & Nested Sampling

Nested sampling

- ▶ Framework of numerical algorithms for performing Bayesian analysis
- ▶ Performs three tasks:
 1. Optimisation $\max_x f(x)$
 2. Exploration $x \sim f$
 3. Integration $\int f(x) dx$on a-priori unknown-functions
- ▶ Key algorithm called polyChord
- ▶ Developed in my Cambridge cosmology lab



PolyChord Ltd

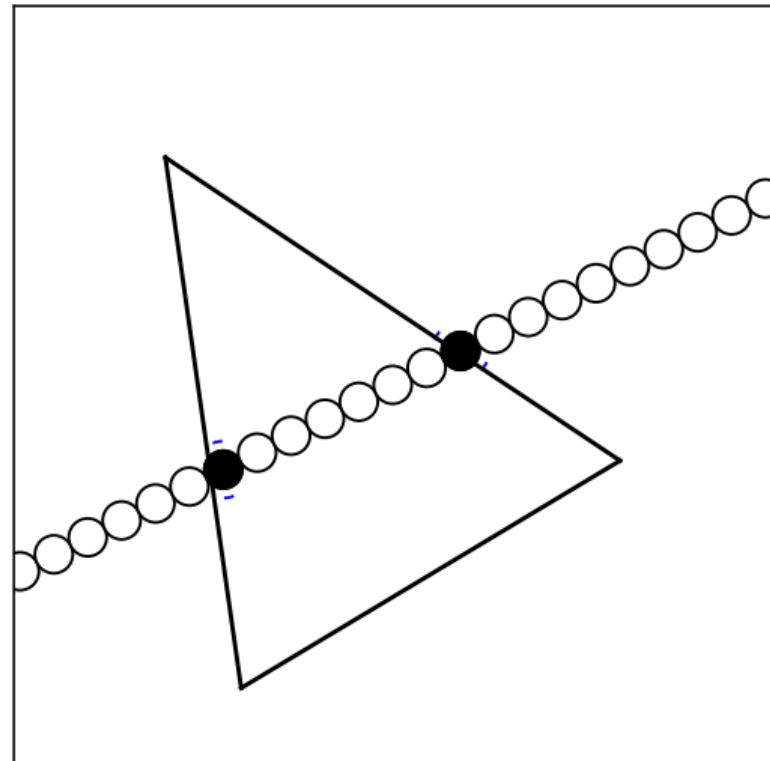
- ▶ Data Science SME spun out of Astro group
- ▶ Applies nested sampling & Bayesian machine learning to industry problems
- ▶ Working with PA/DSTL for three years
- ▶ Protein folding, Nuclear fusion, Battery optimisation, predictive maintenance.



Example 1: Mesh networks (CEME1.2)

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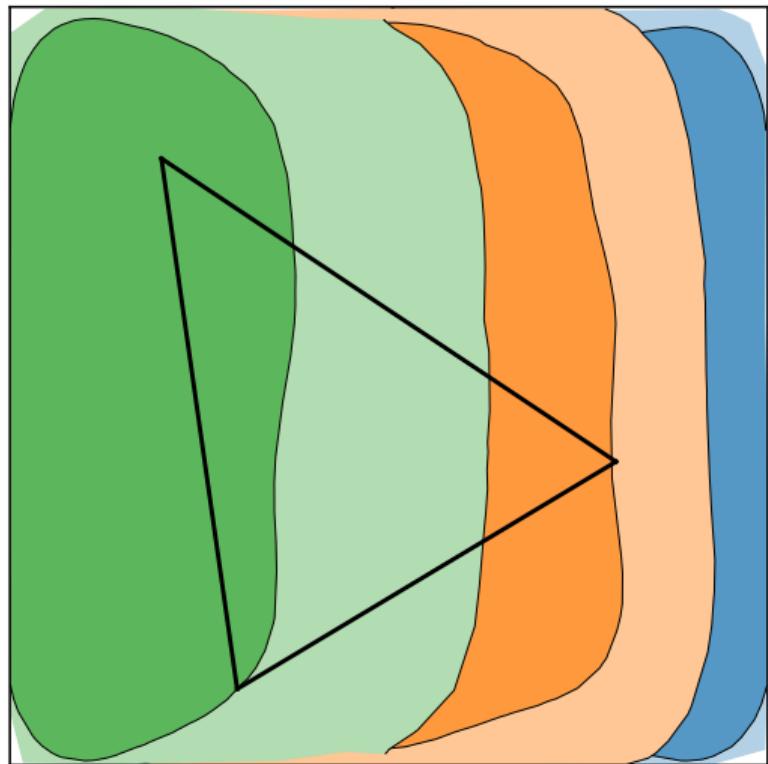
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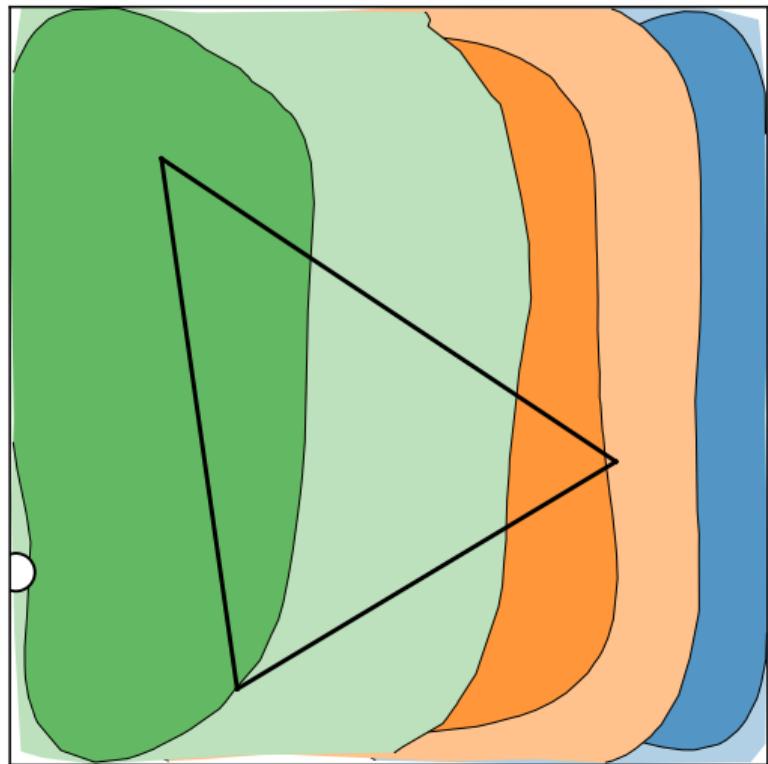
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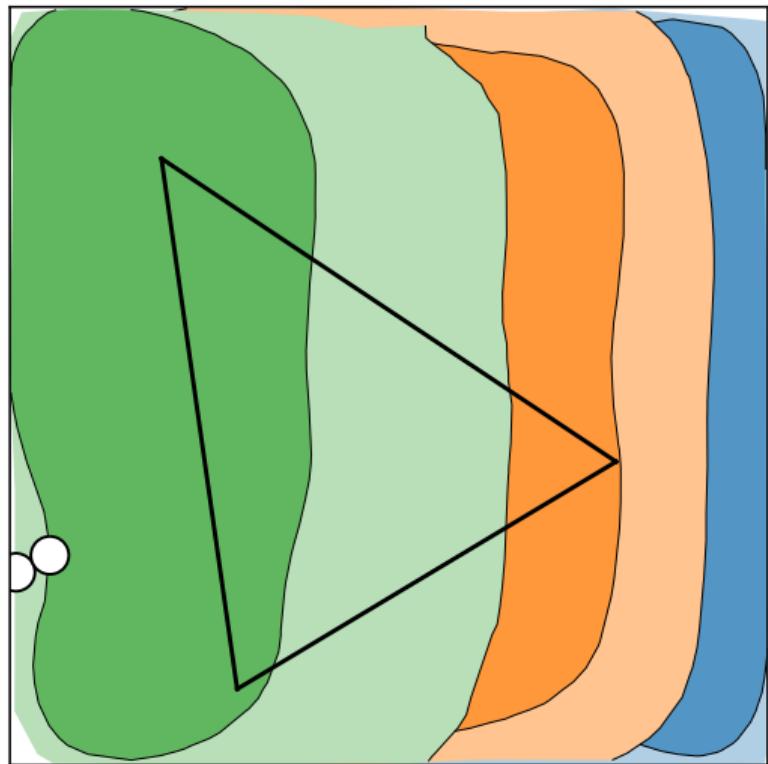
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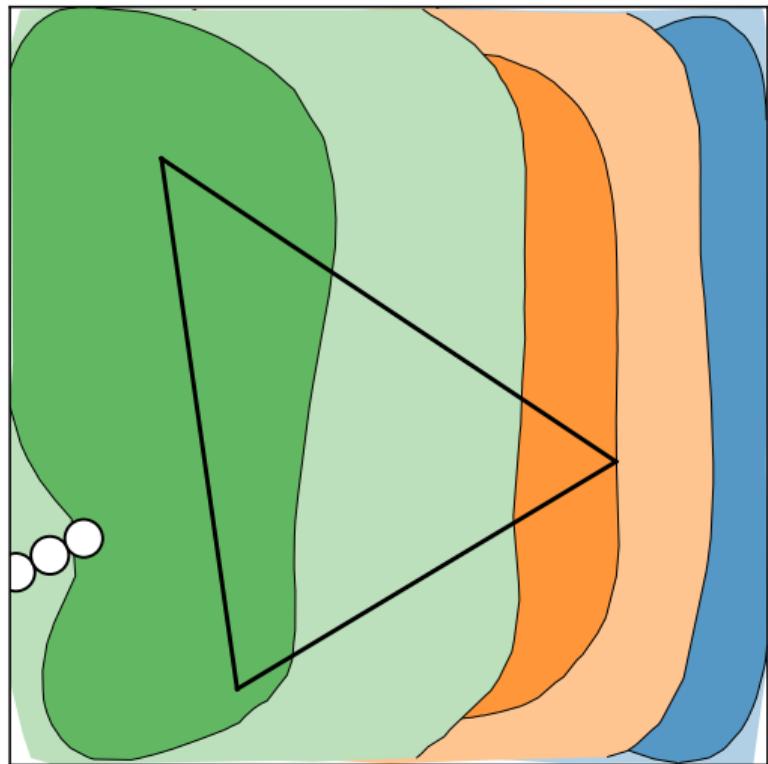
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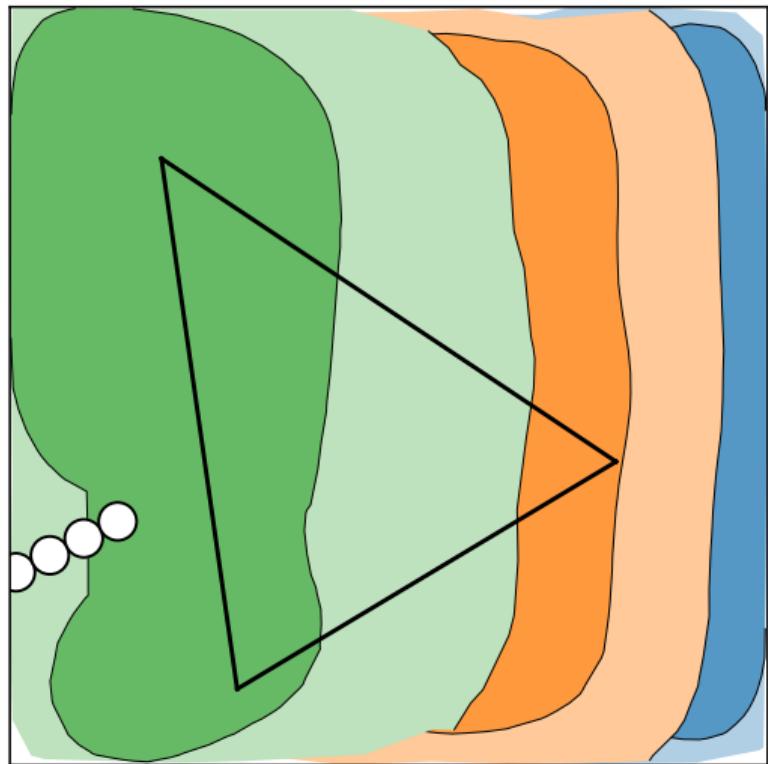
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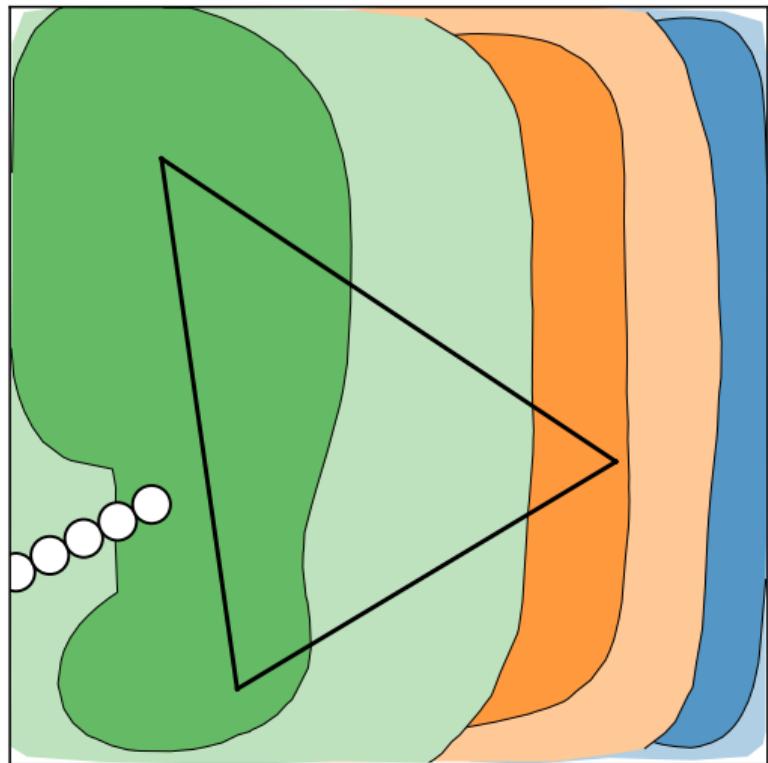
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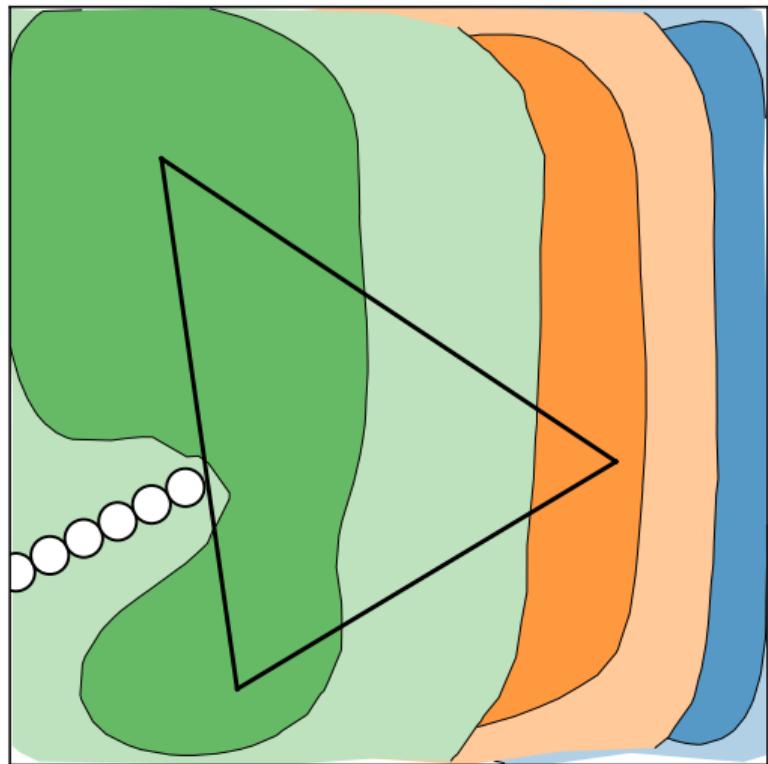
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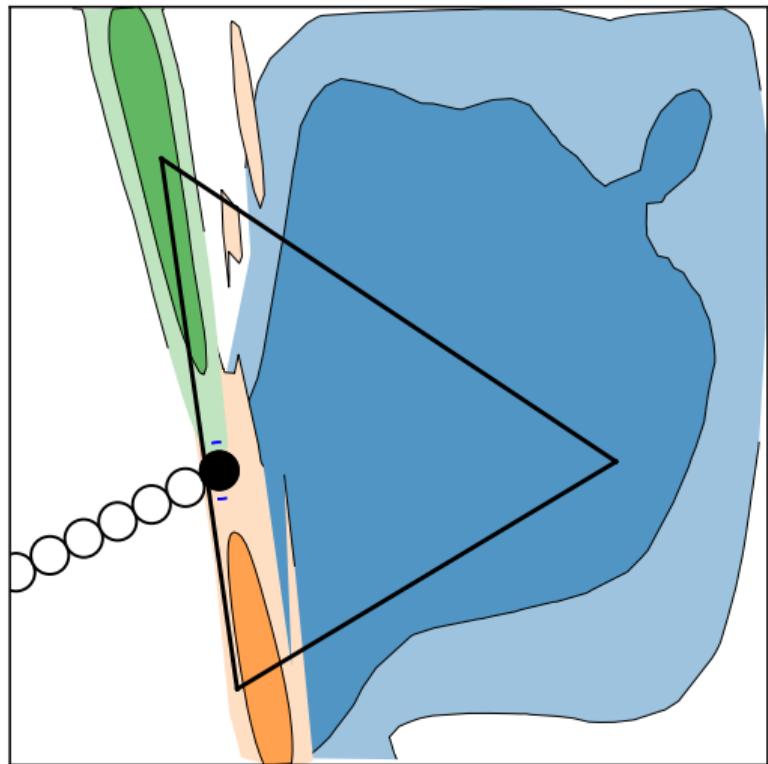
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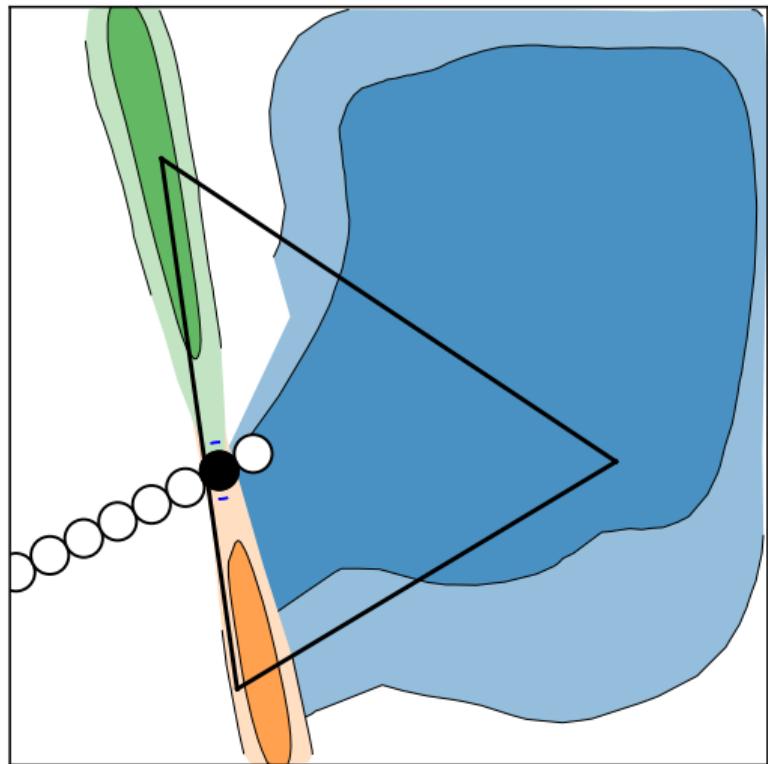
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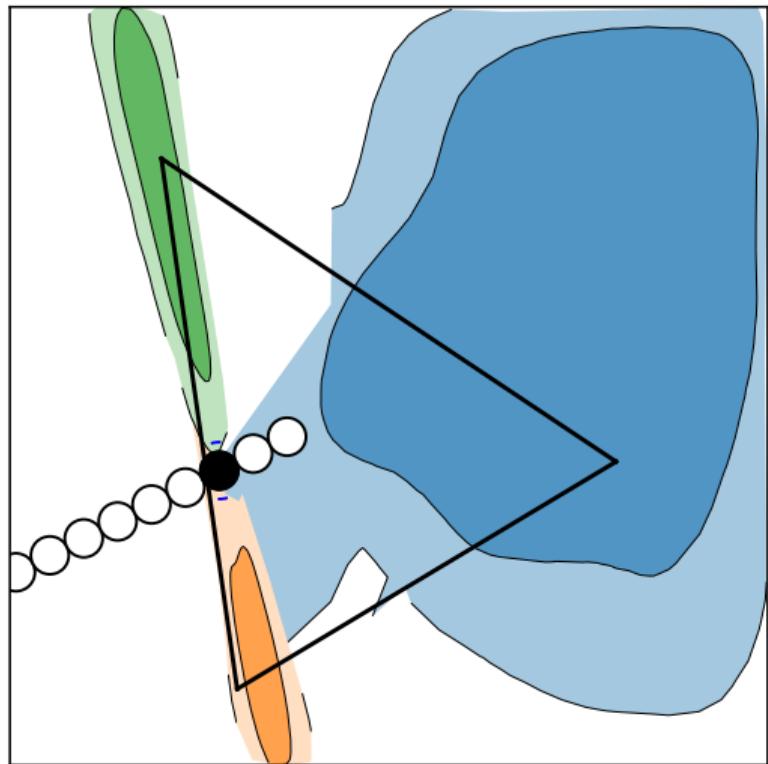
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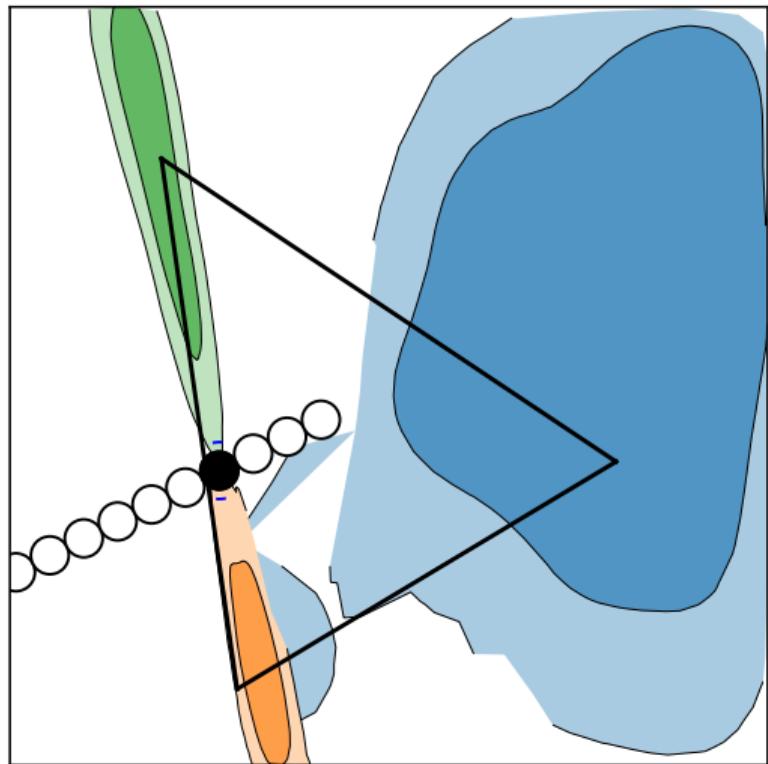
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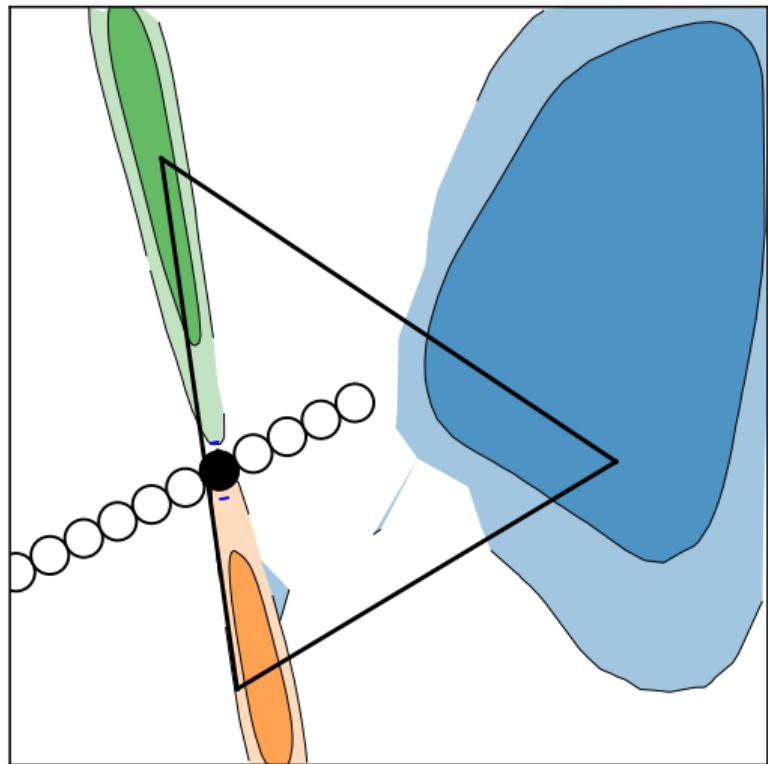
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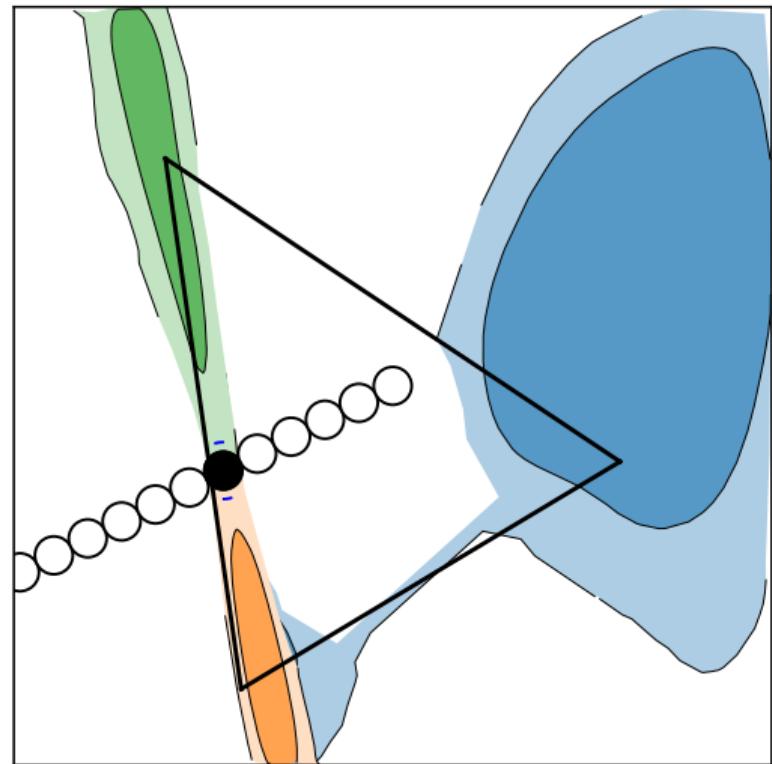
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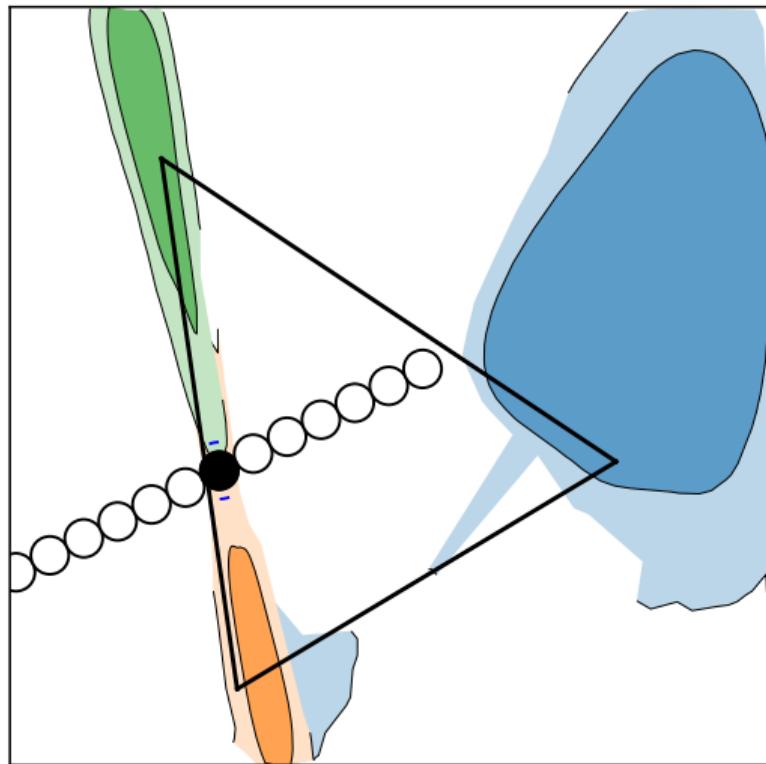
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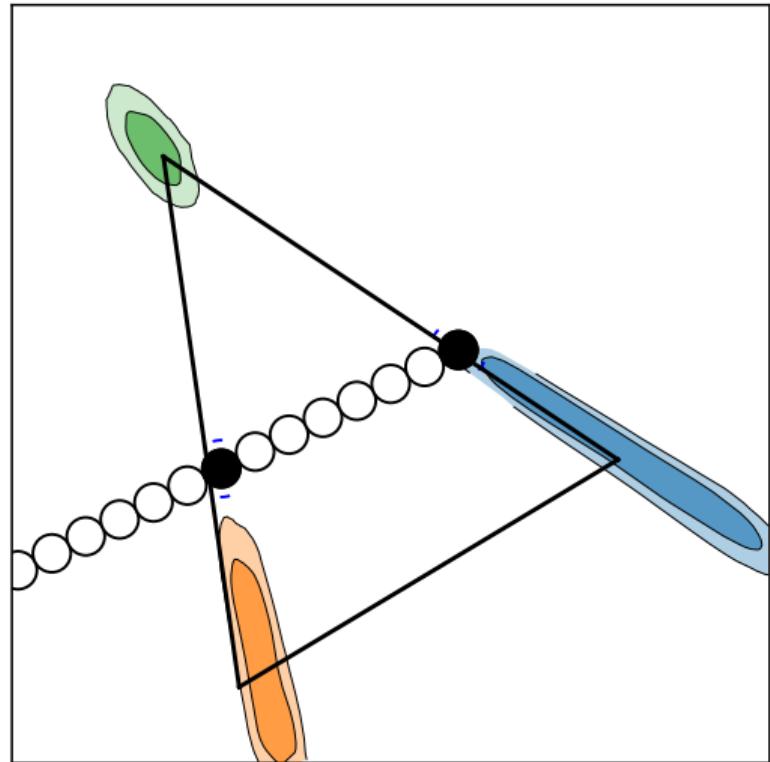
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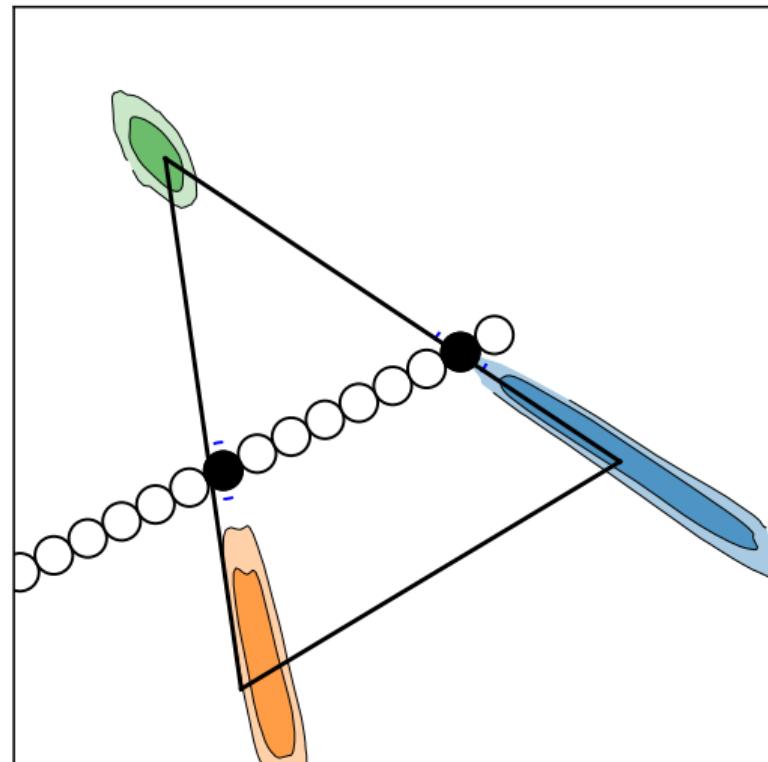
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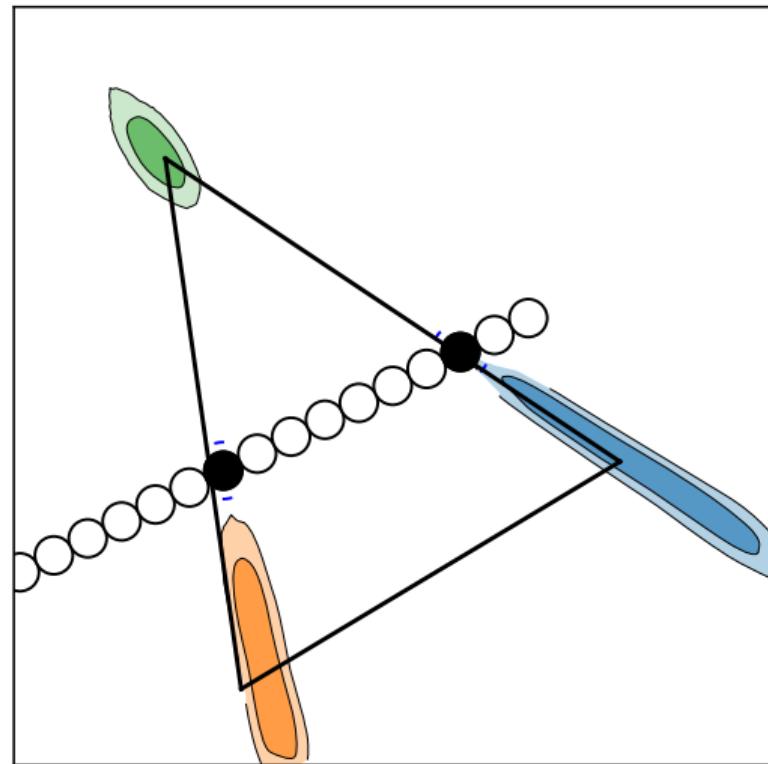
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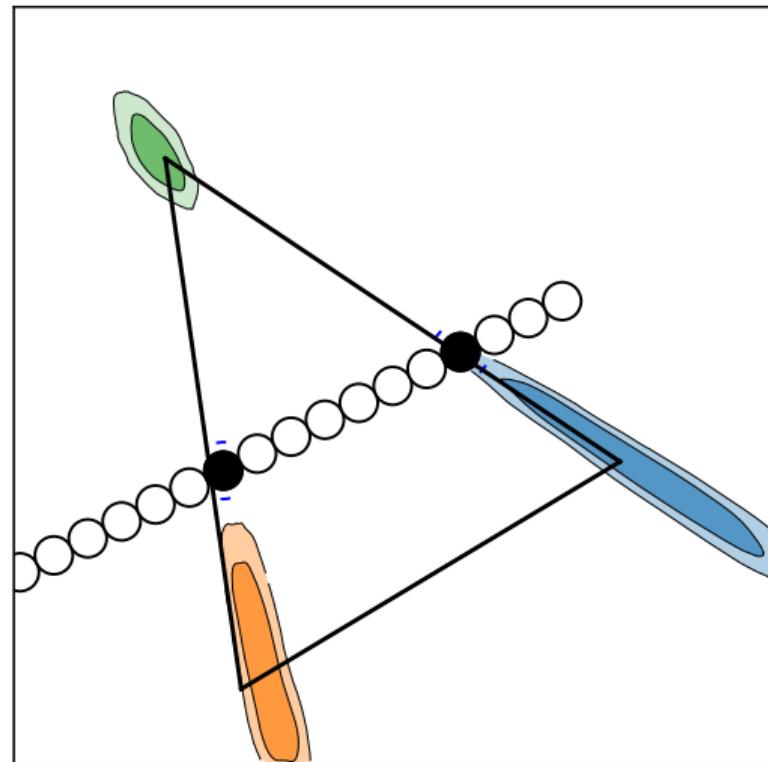
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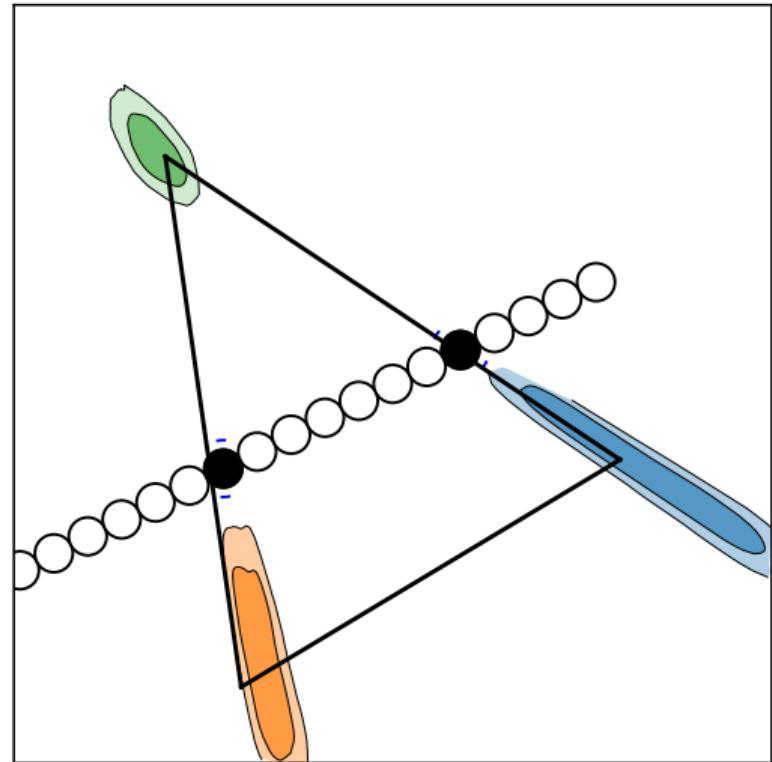
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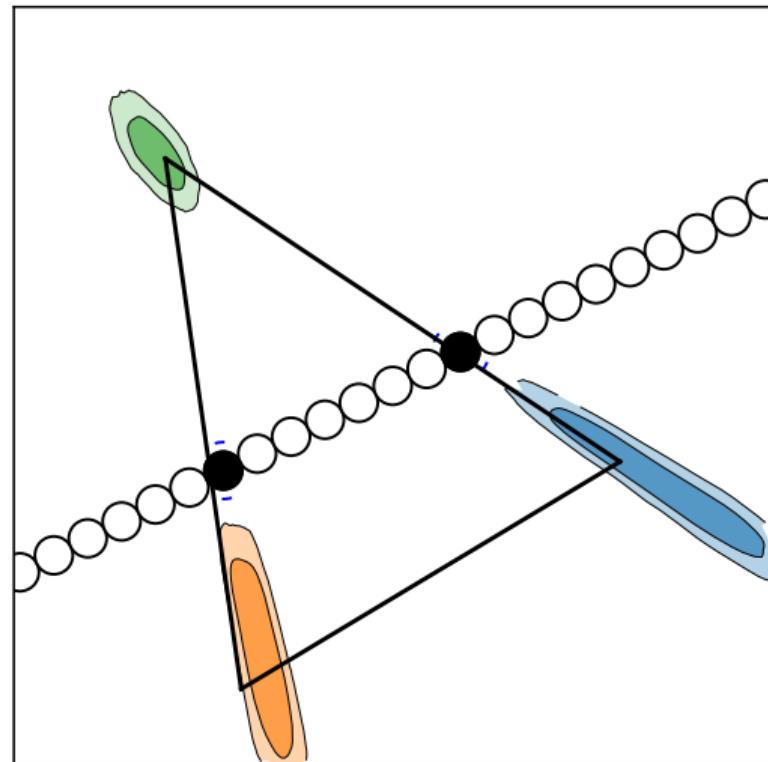
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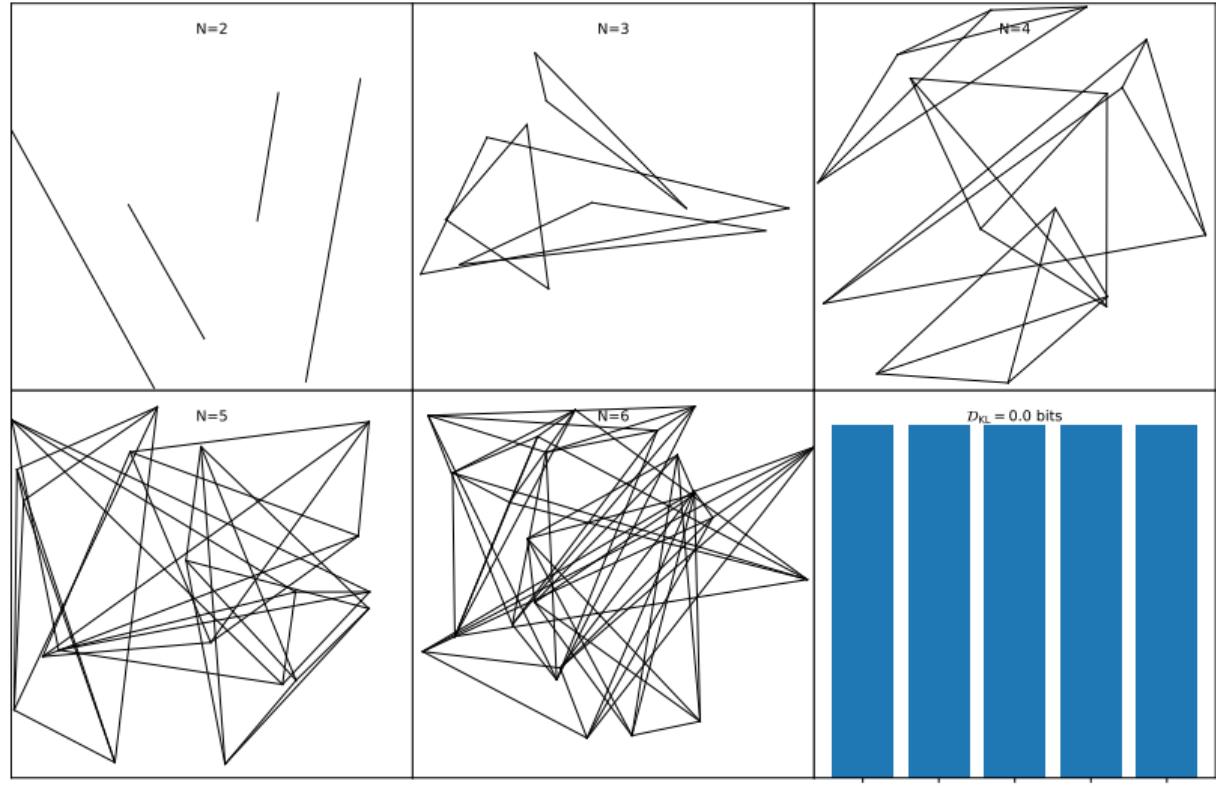
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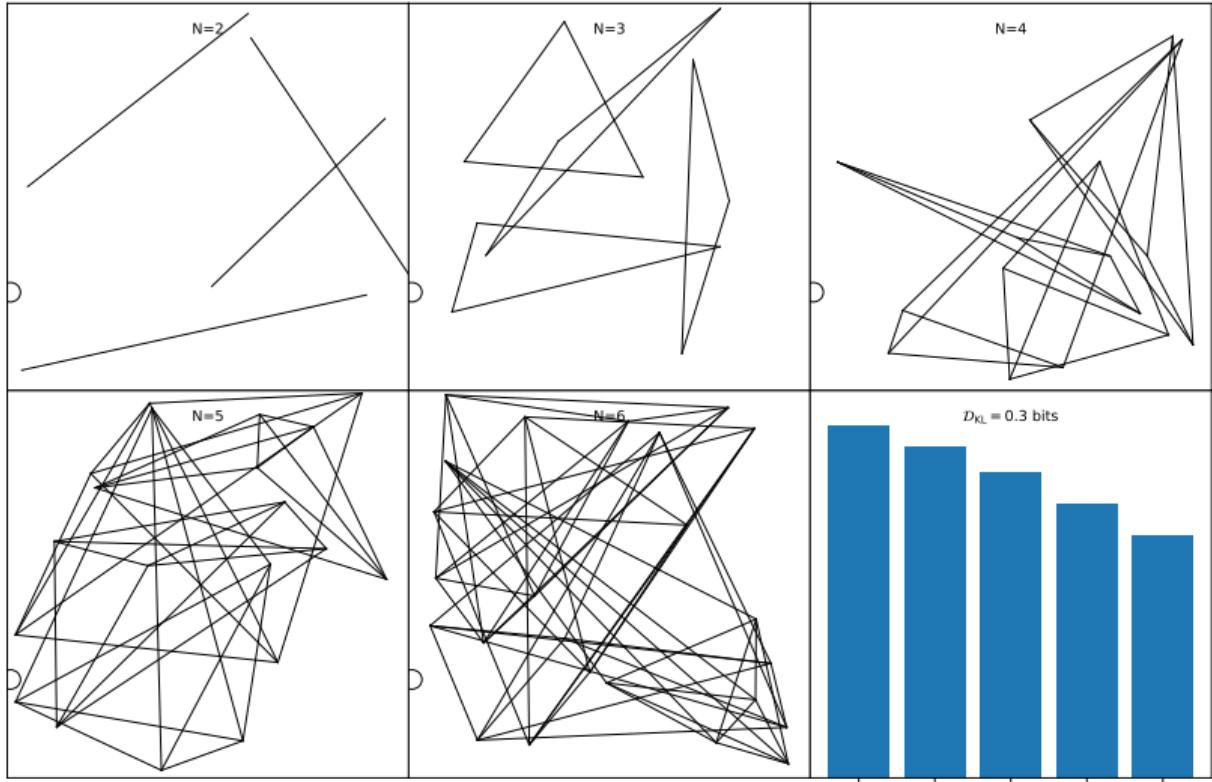
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- ▶ Extend example so that N can also vary.
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- ▶ Instead of plotting posterior *contours*, we plot *samples* from the full posterior distribution.
- ▶ Least compressed representation of the posterior.



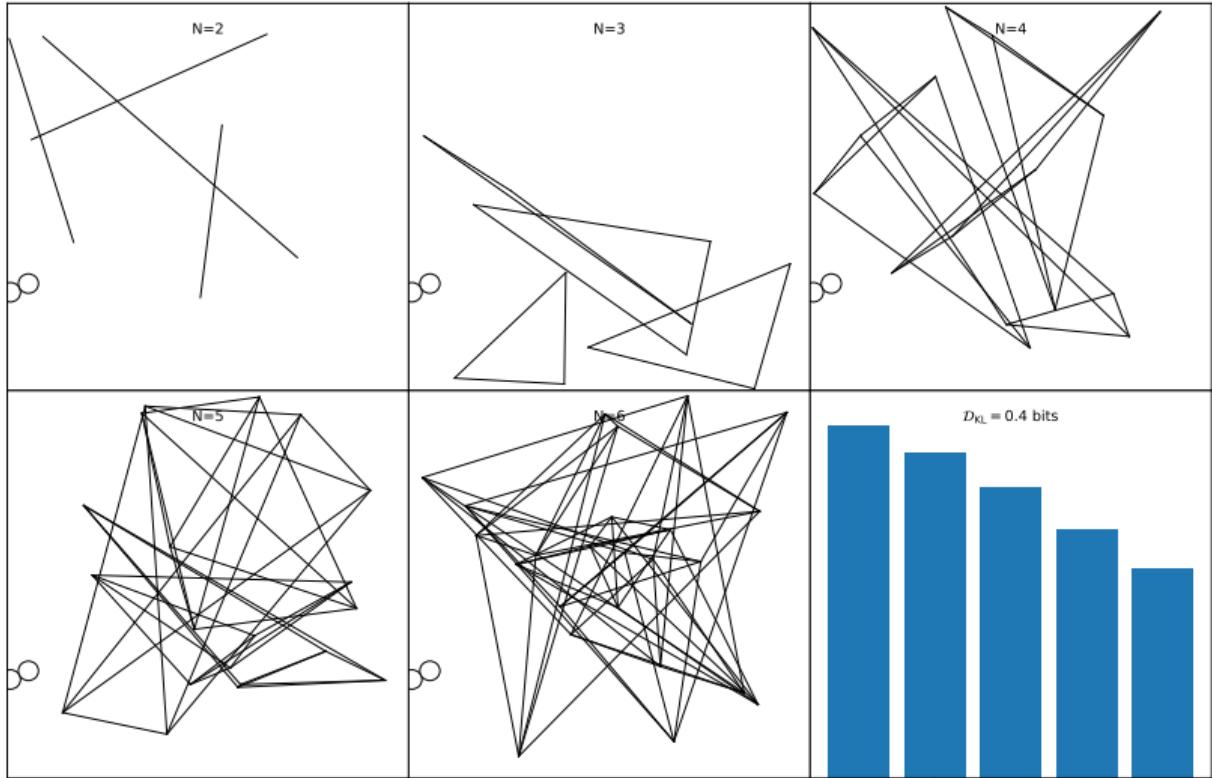
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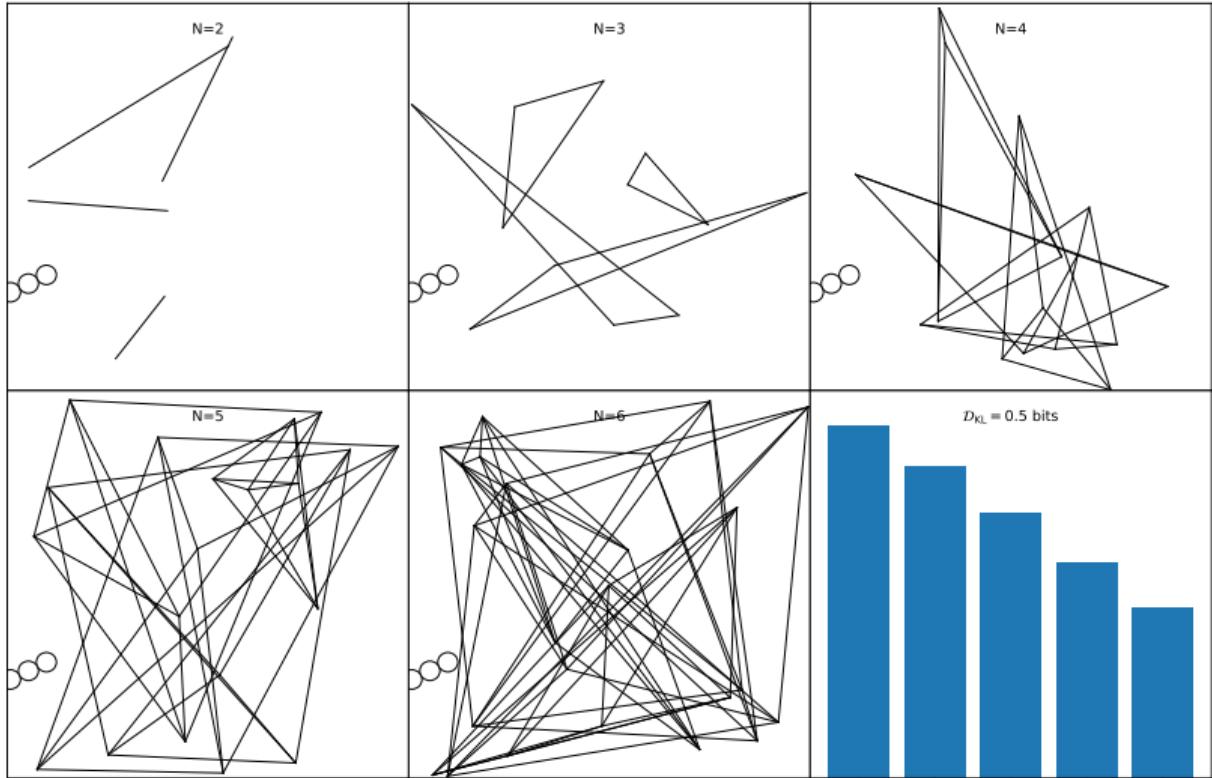
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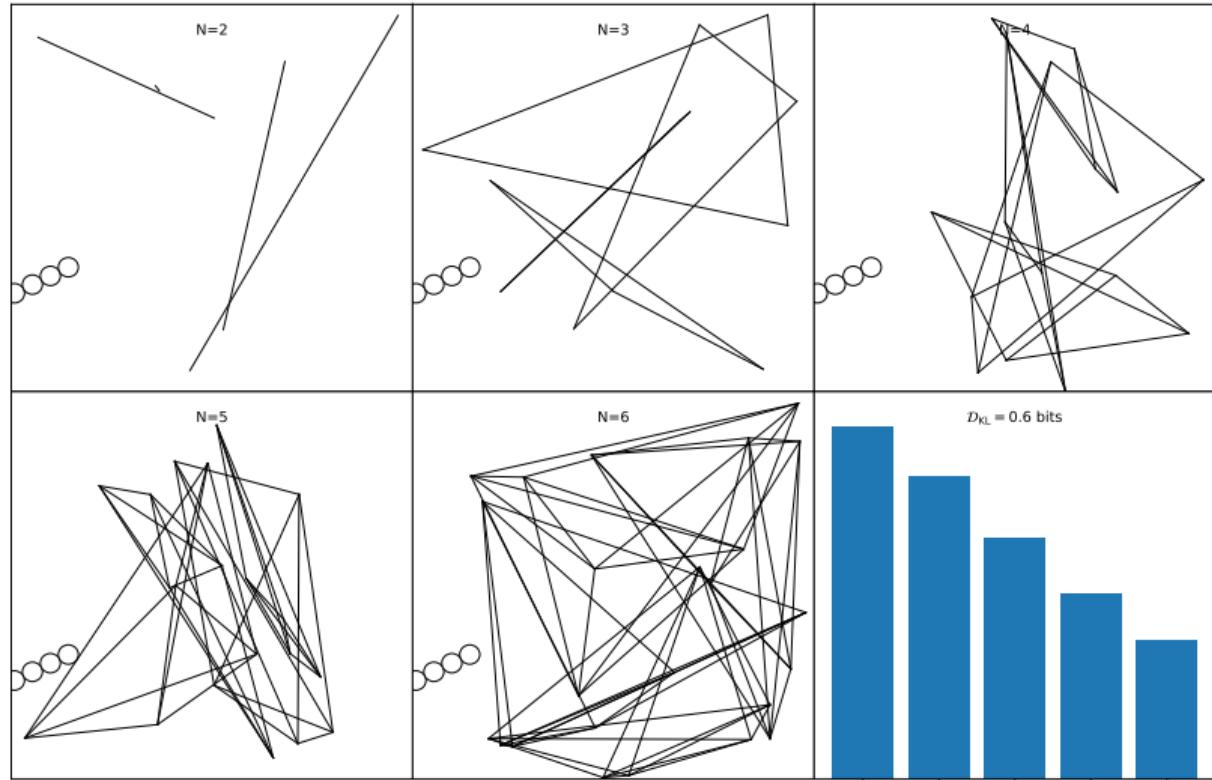
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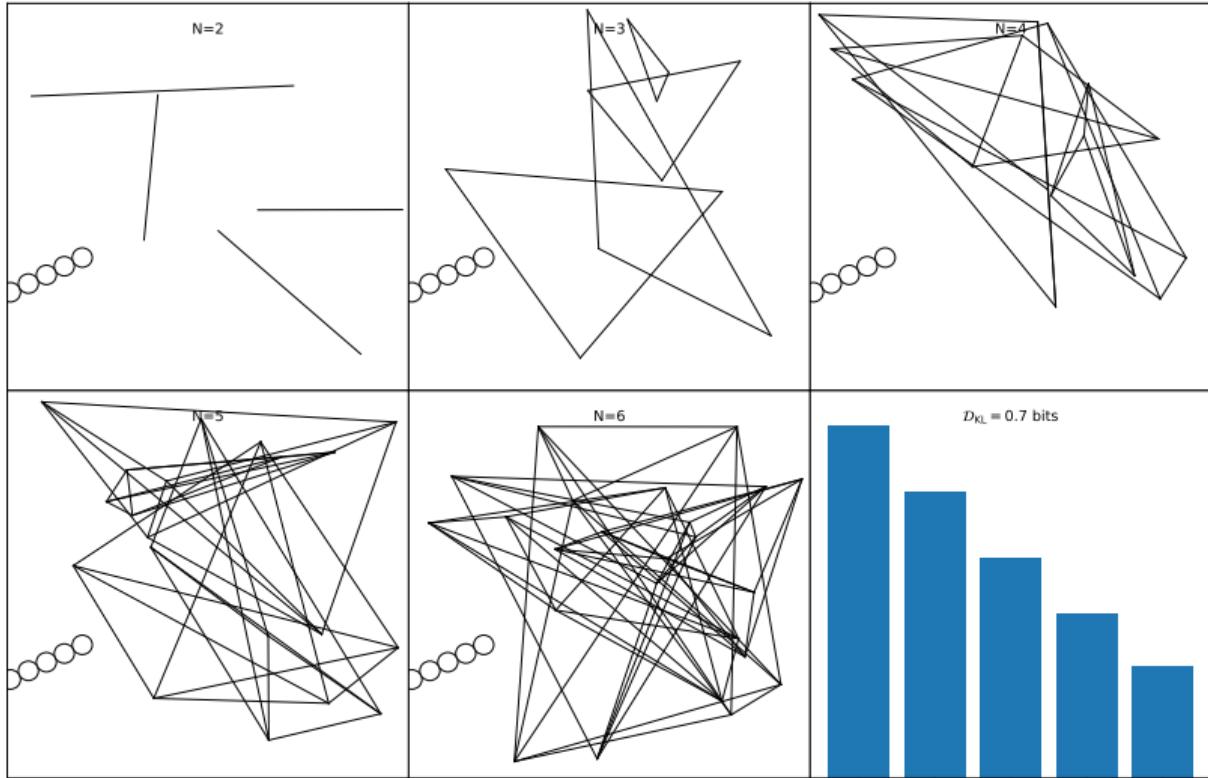
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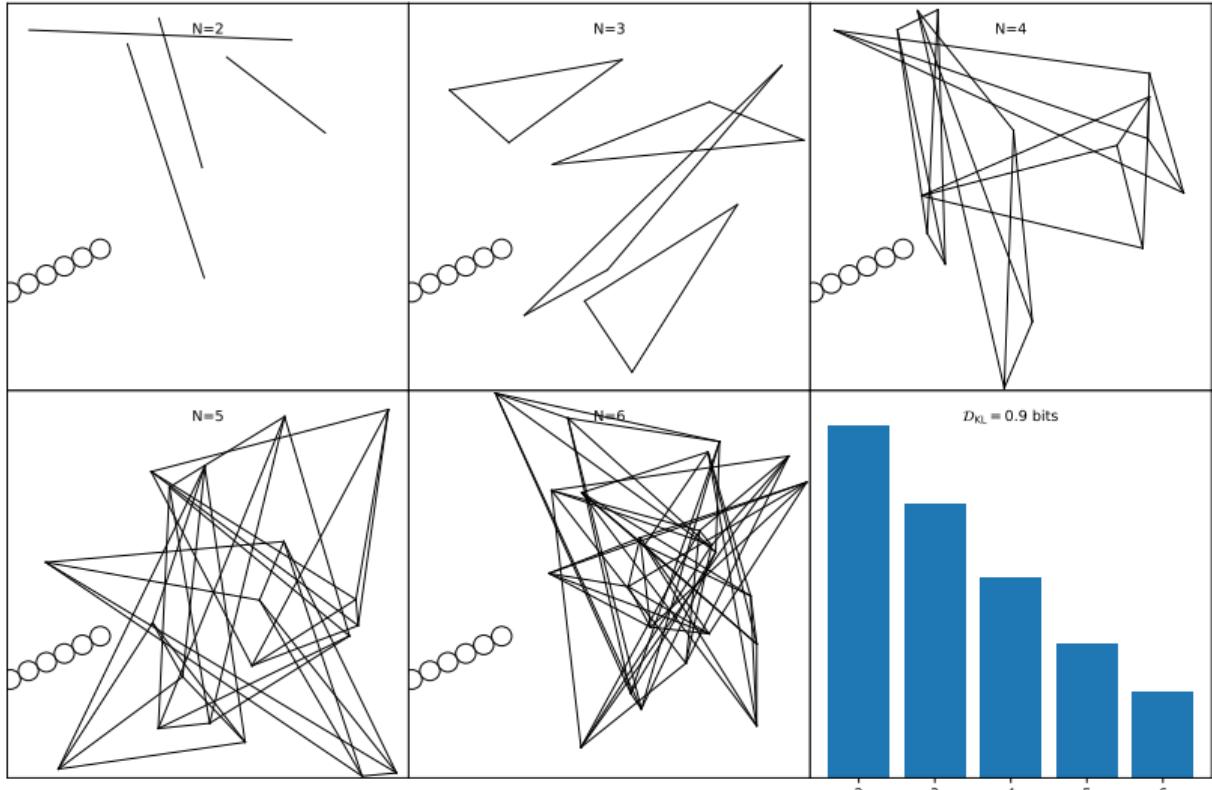
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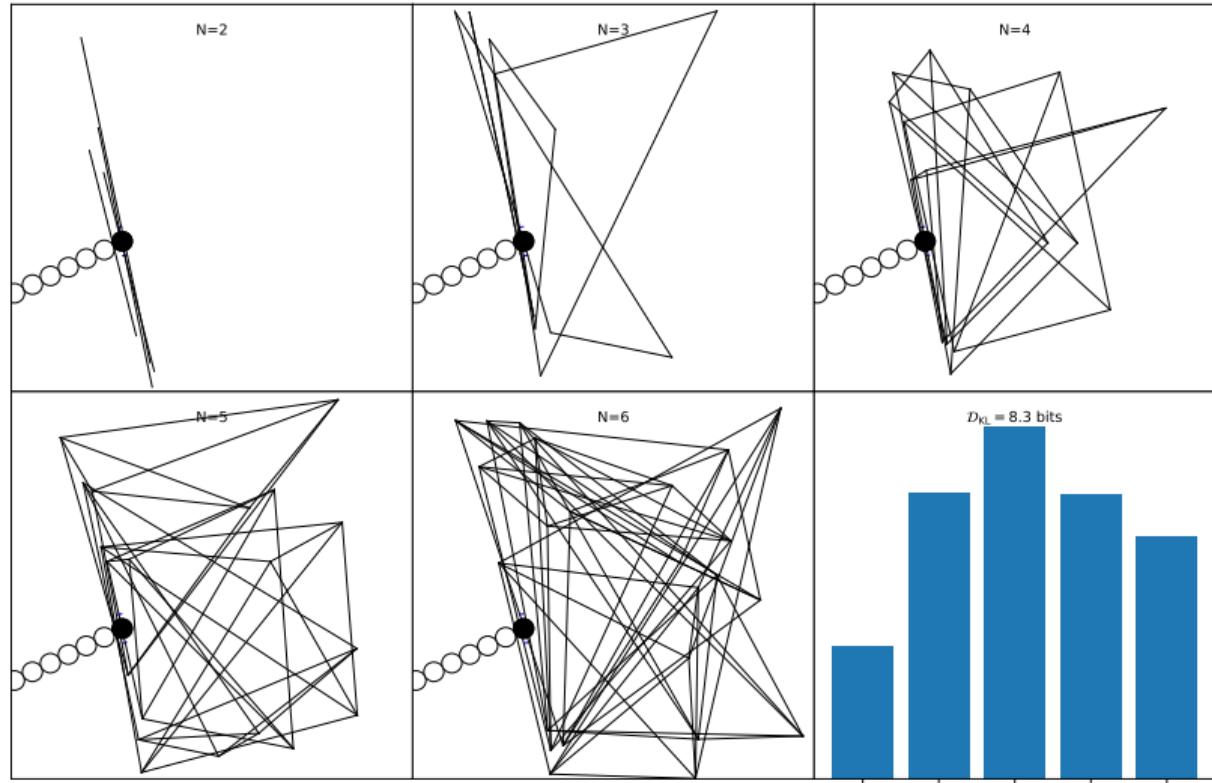
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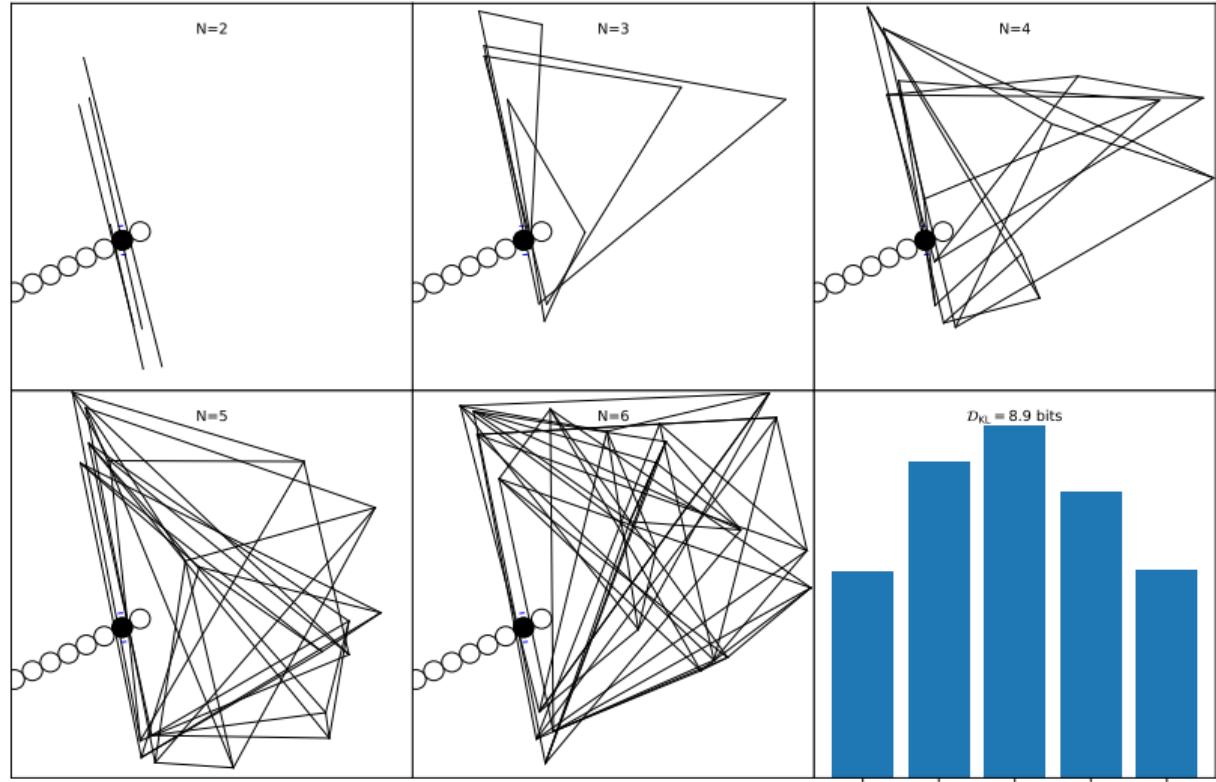
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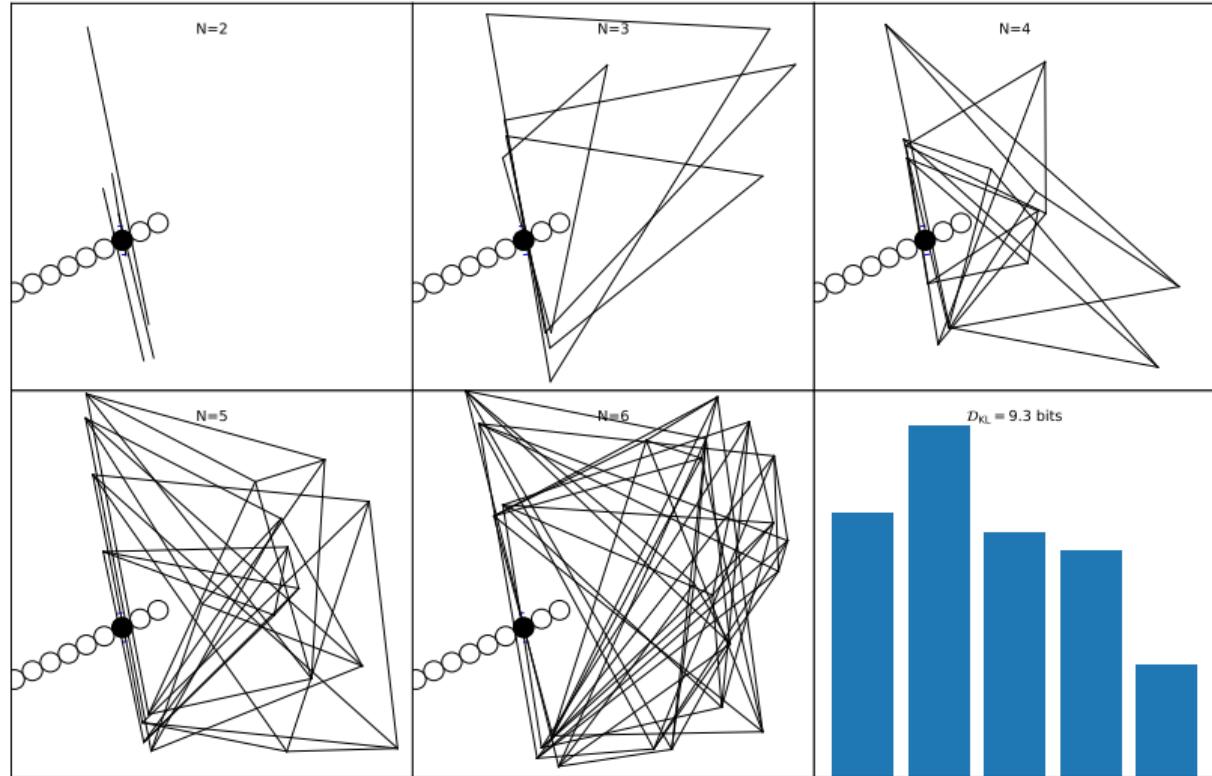
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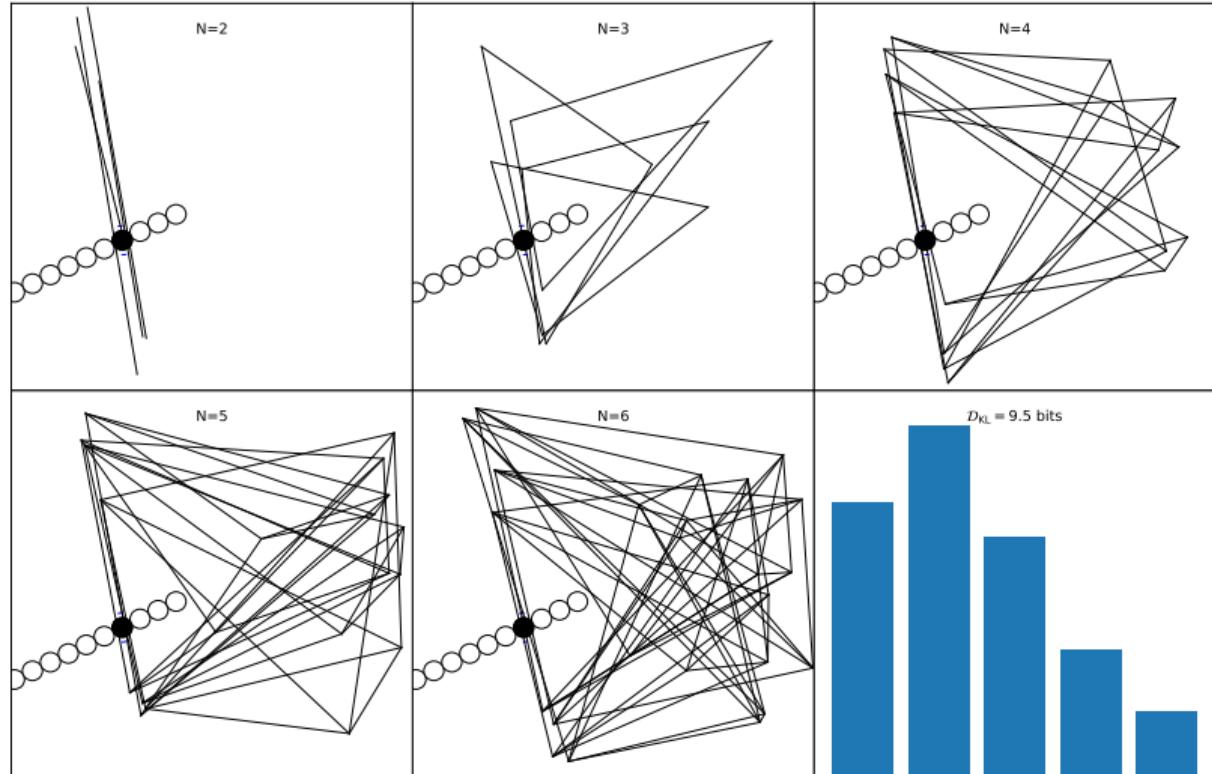
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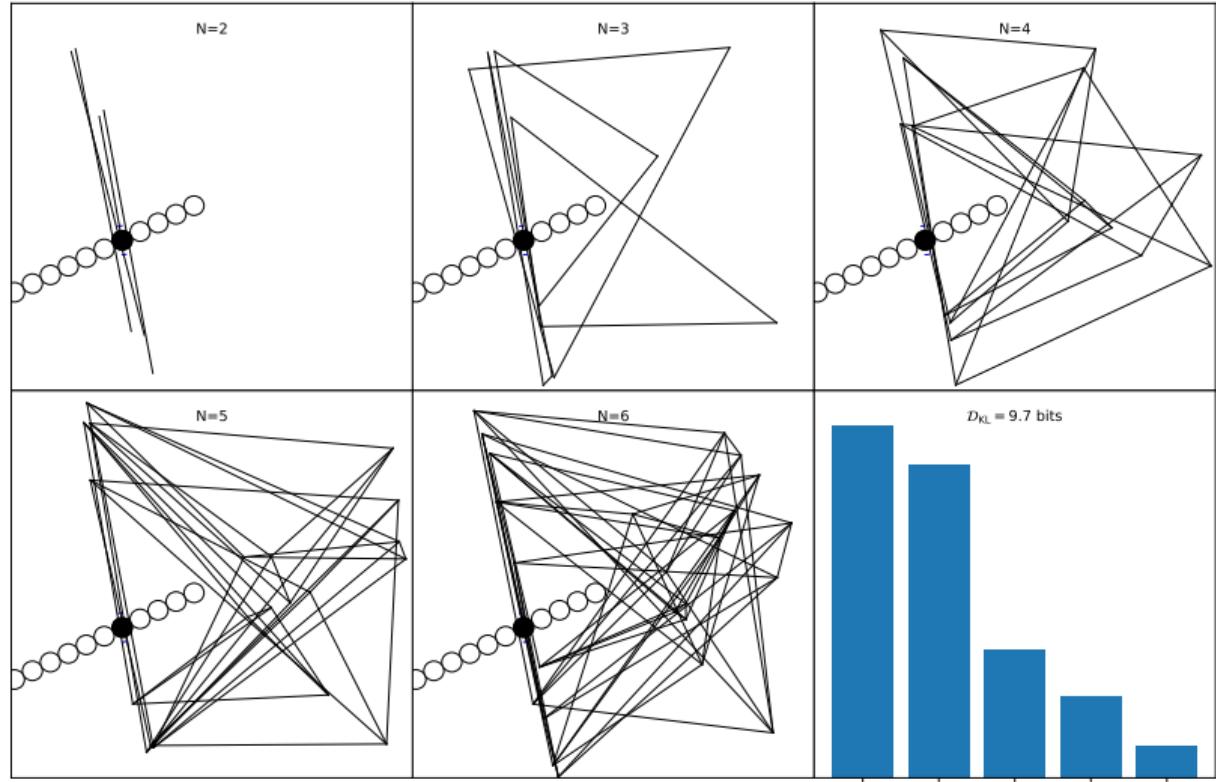
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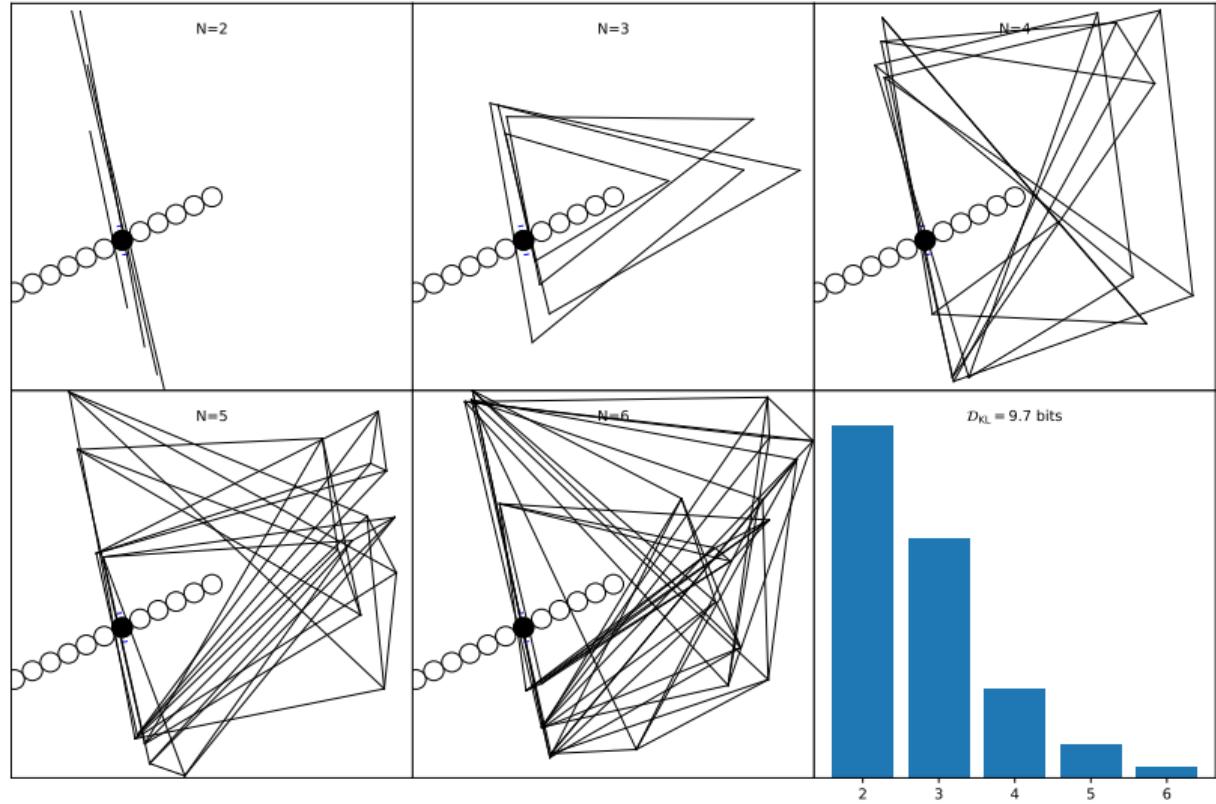
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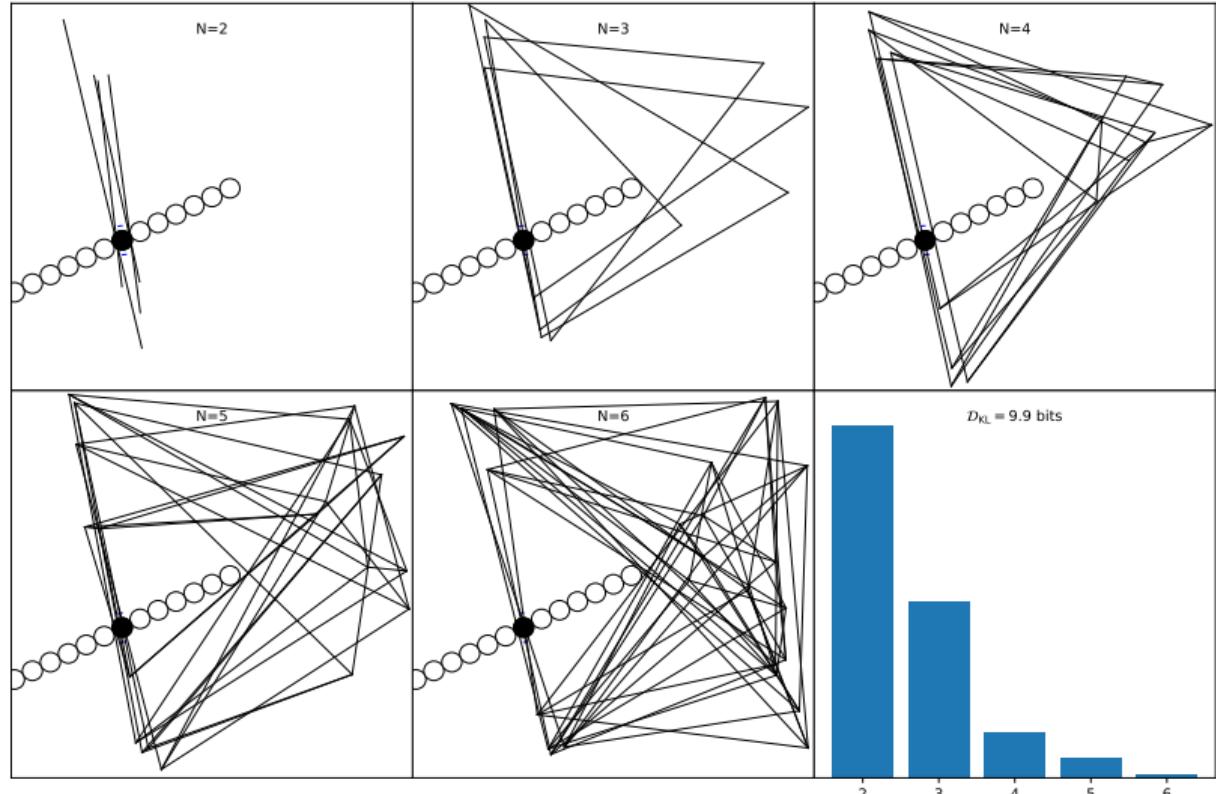
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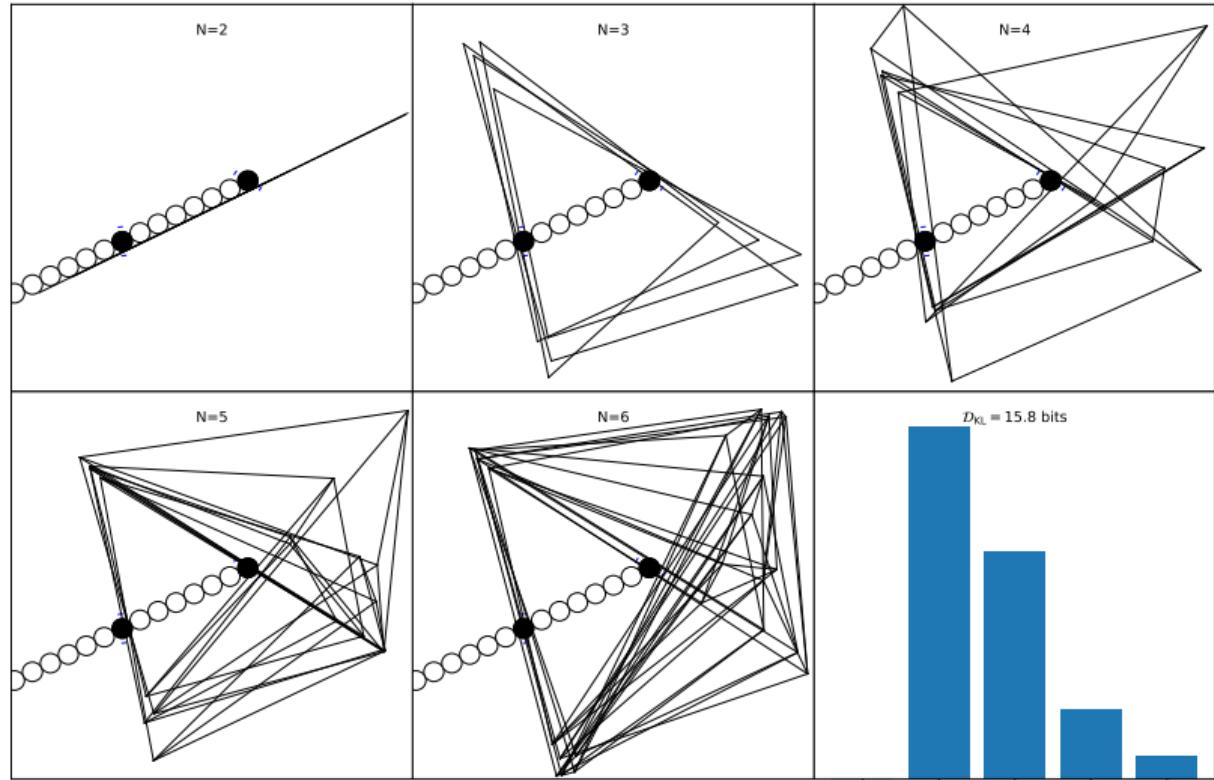
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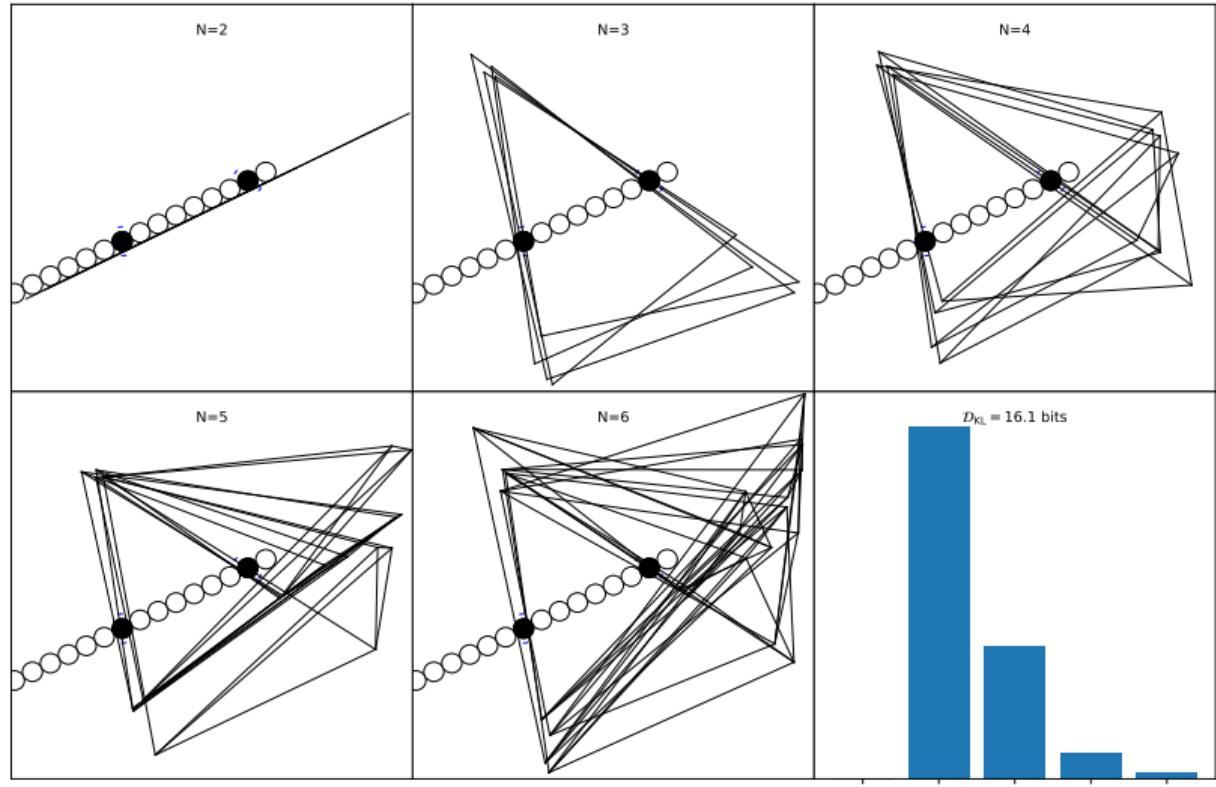
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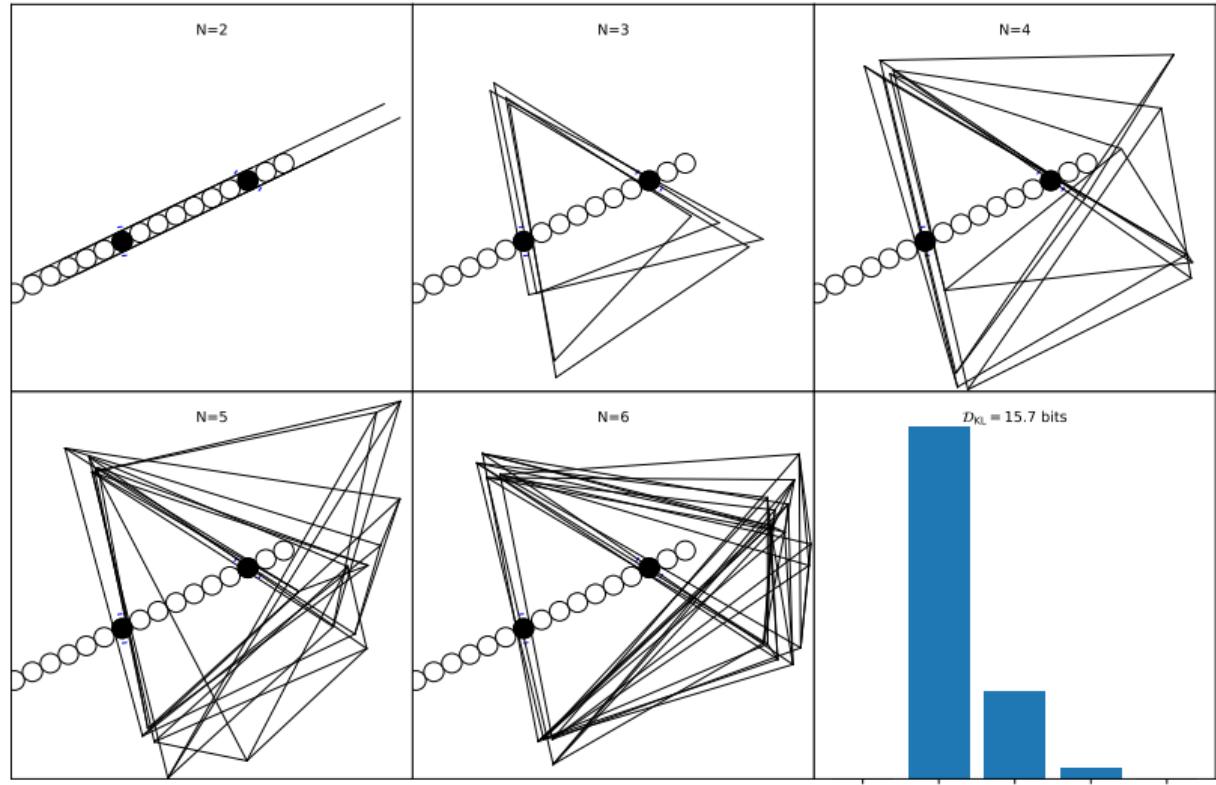
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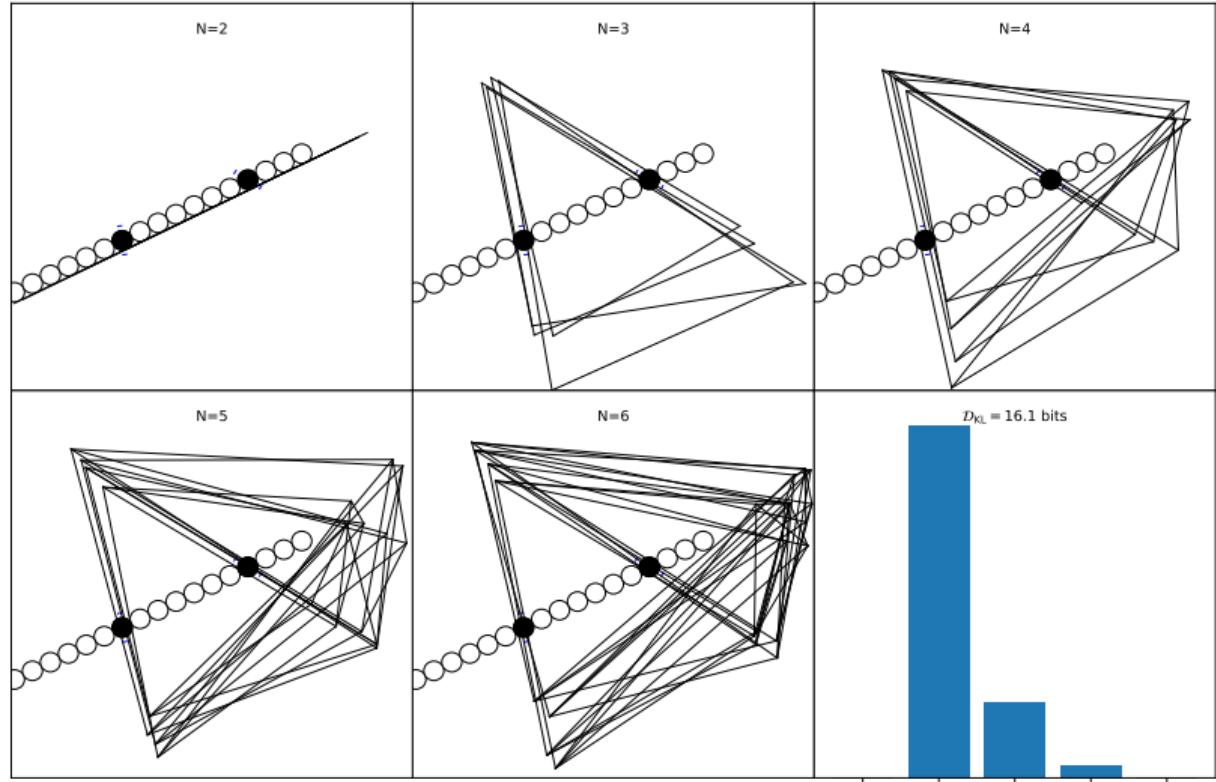
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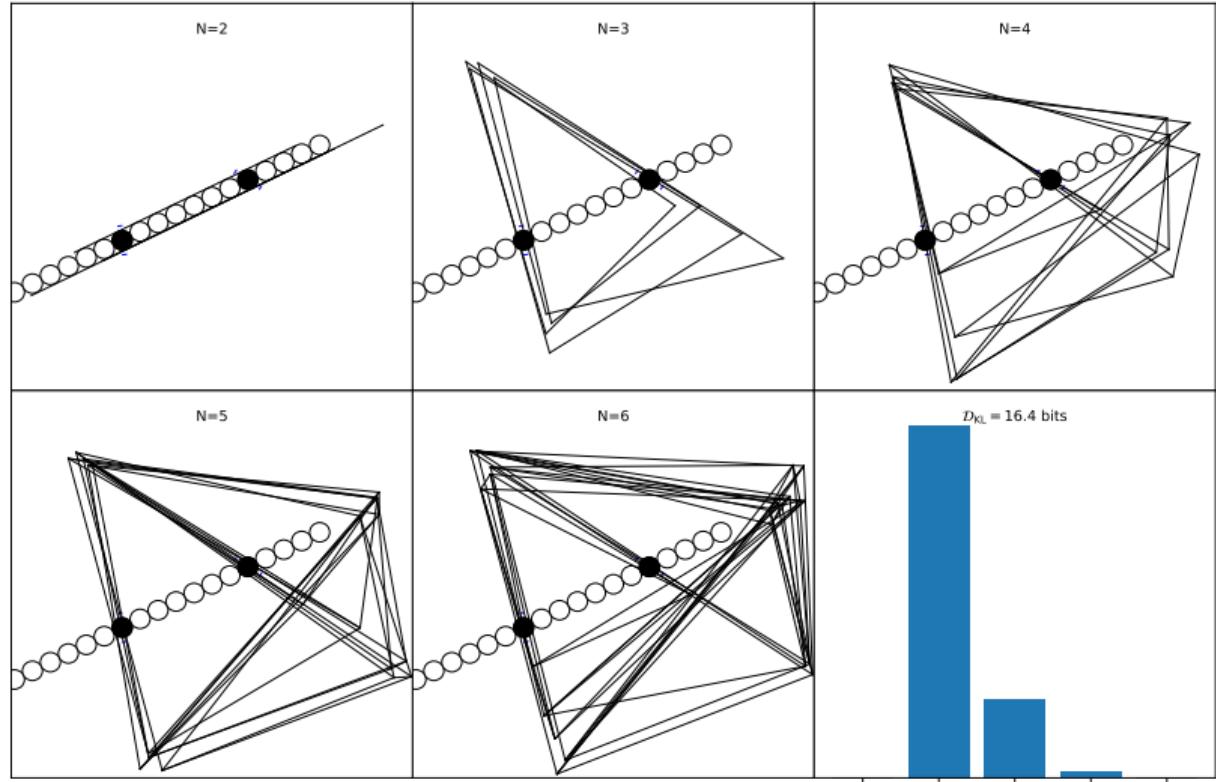
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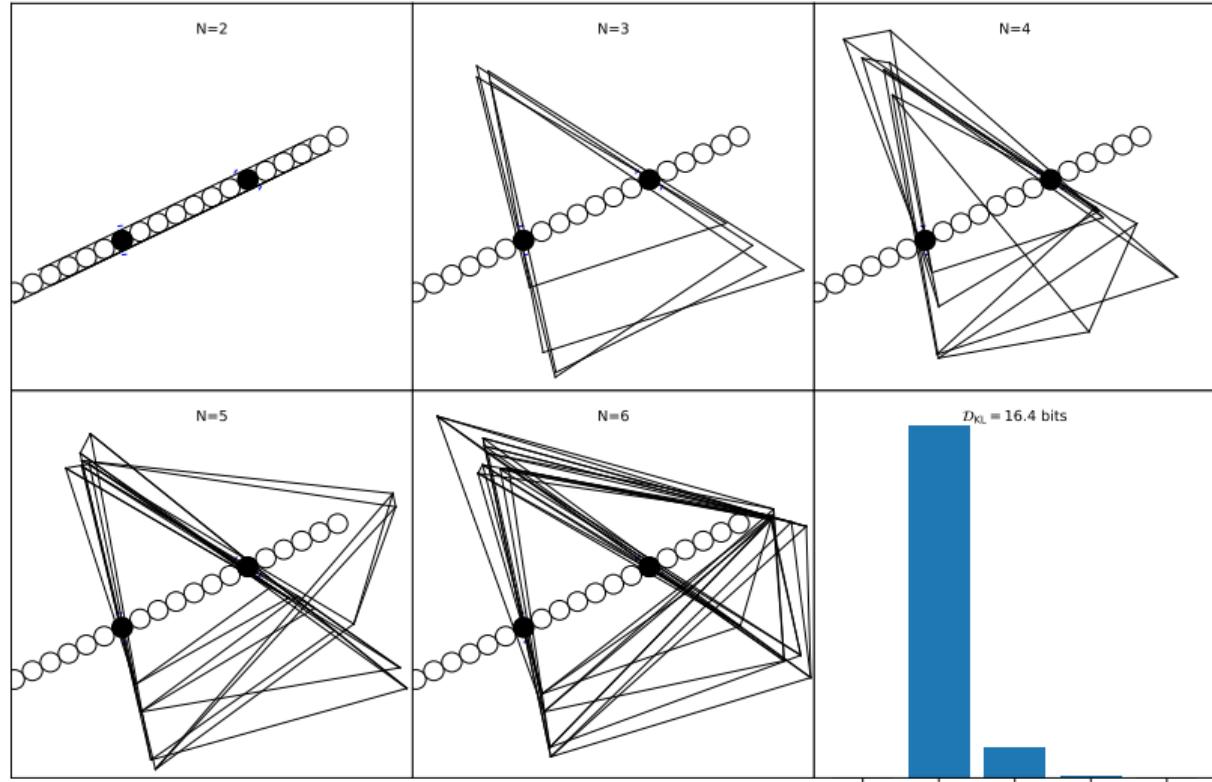
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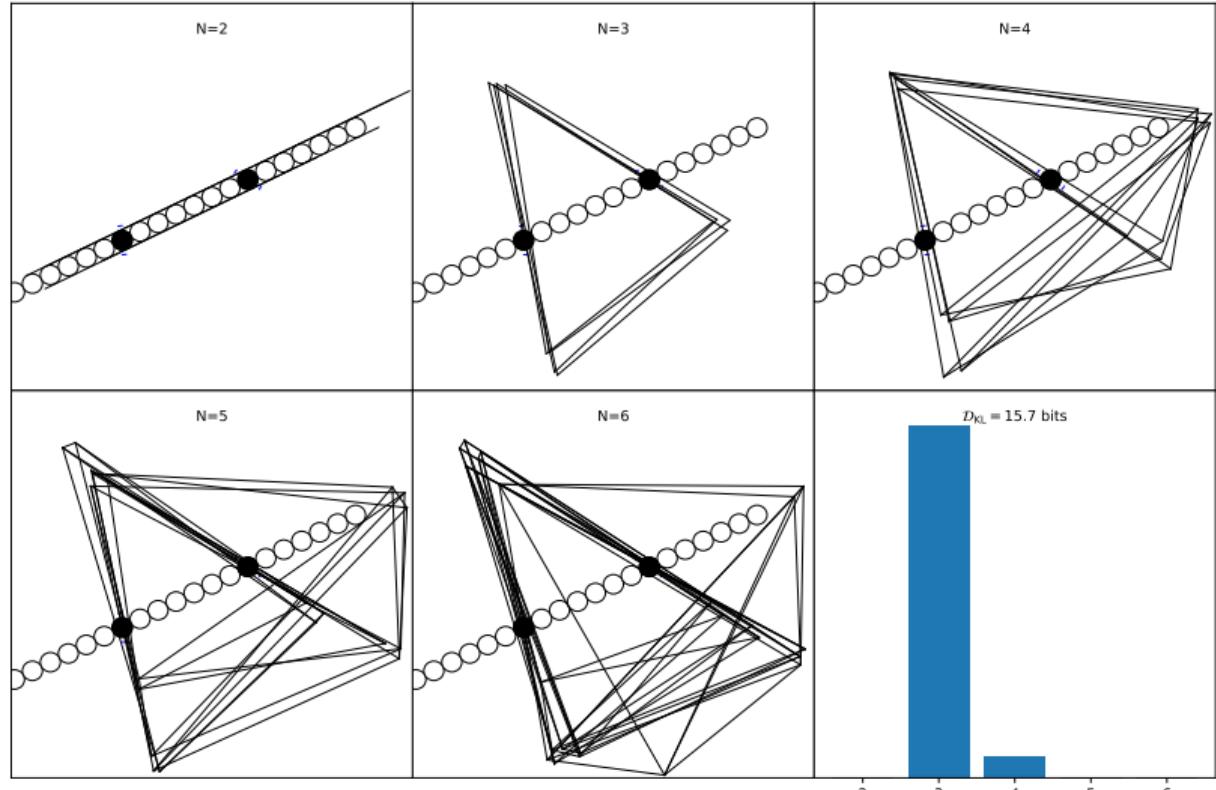
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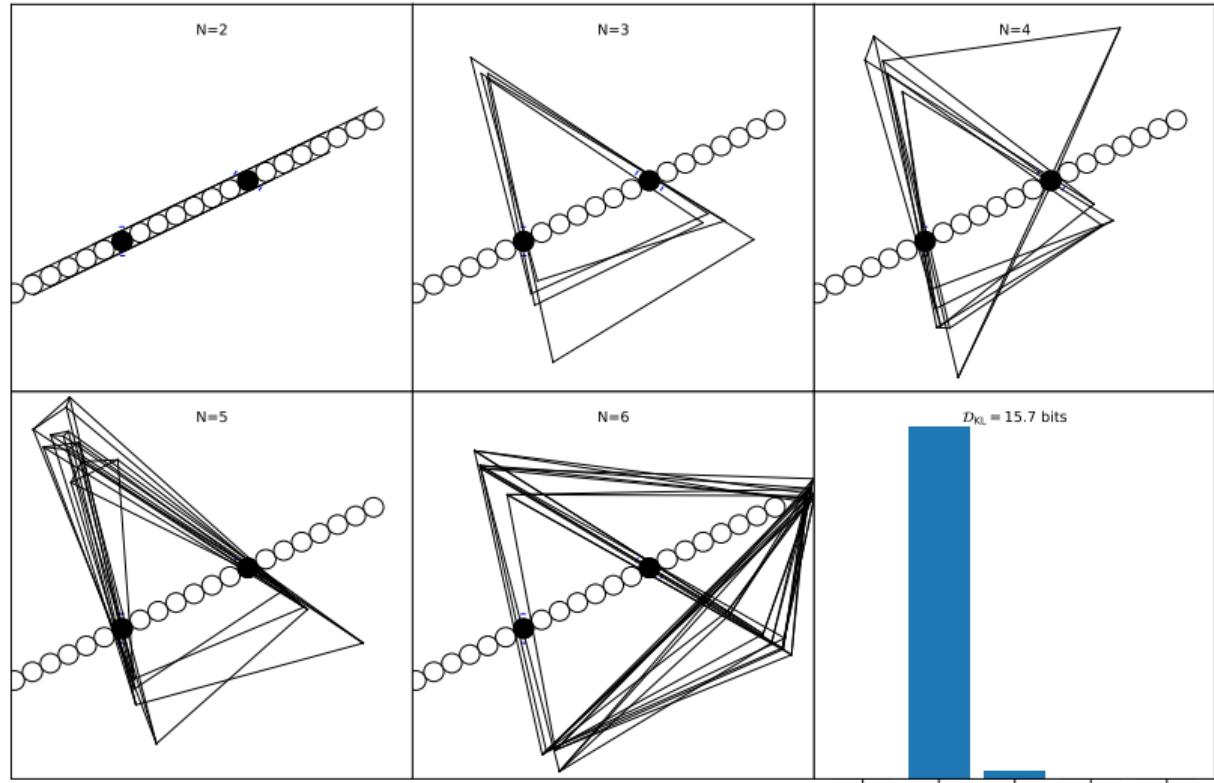
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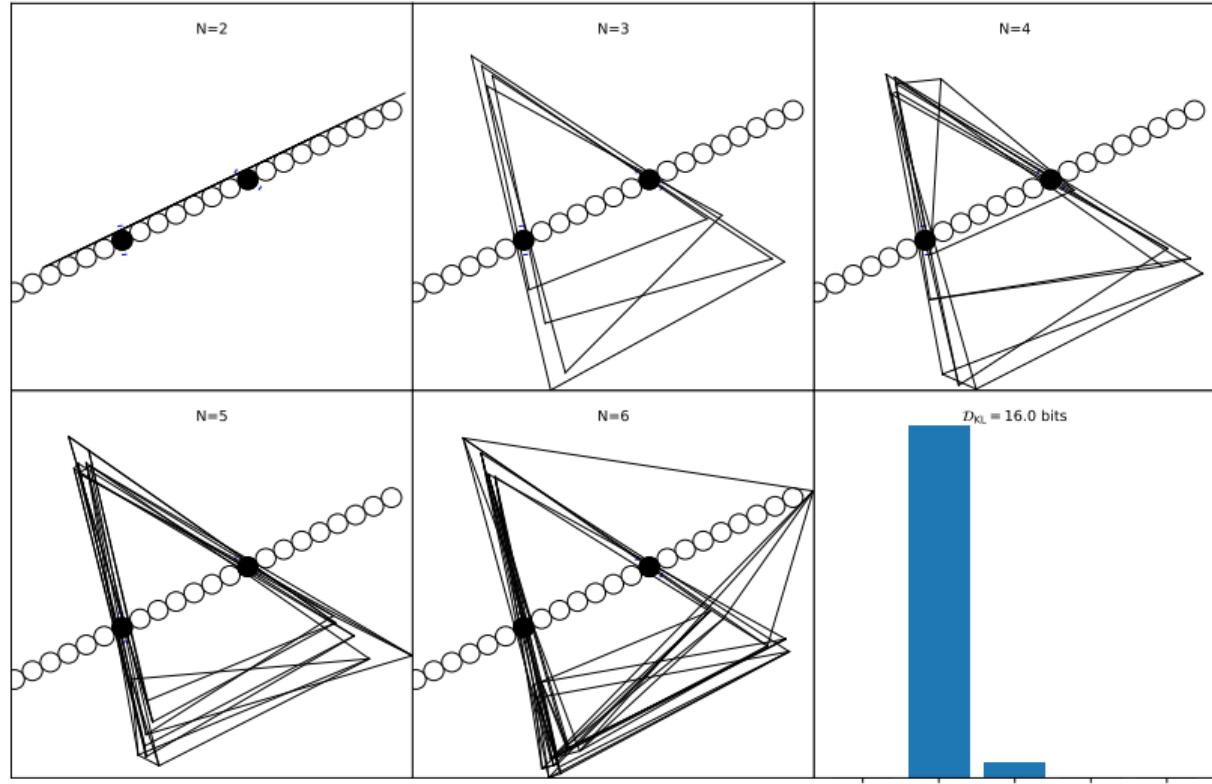
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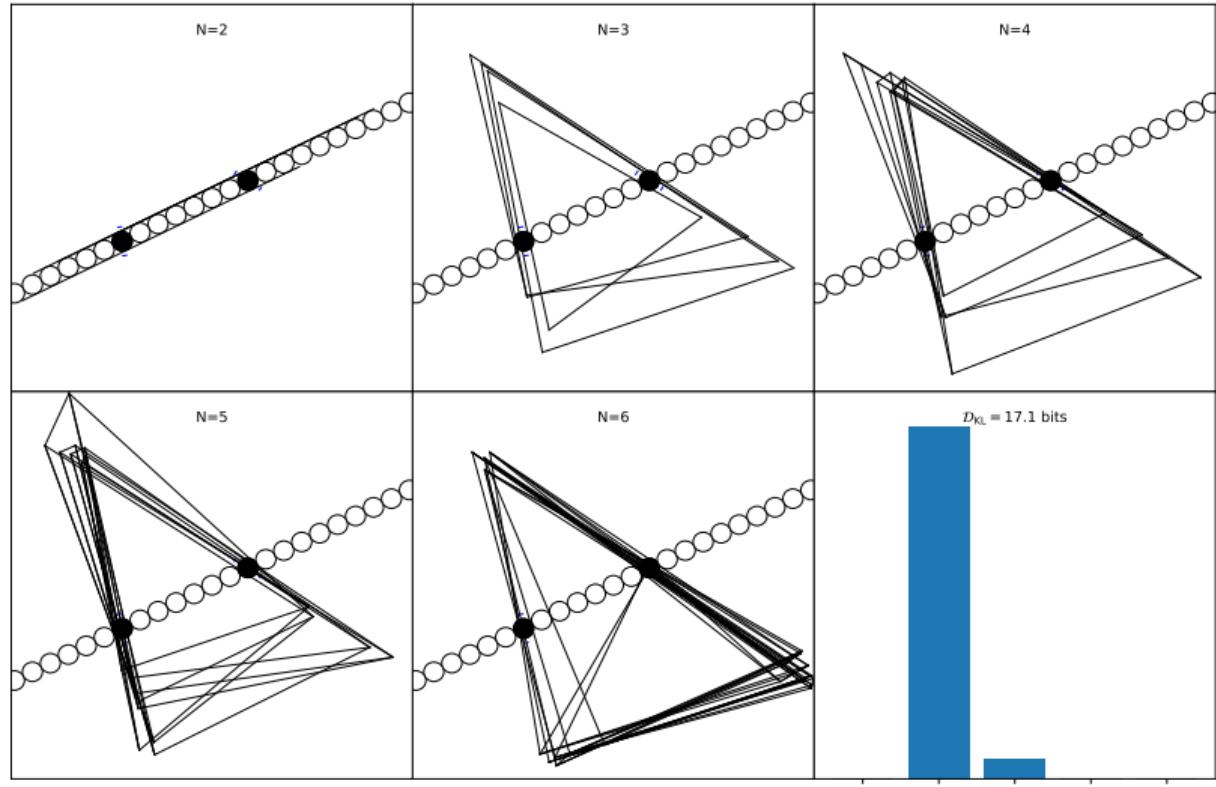
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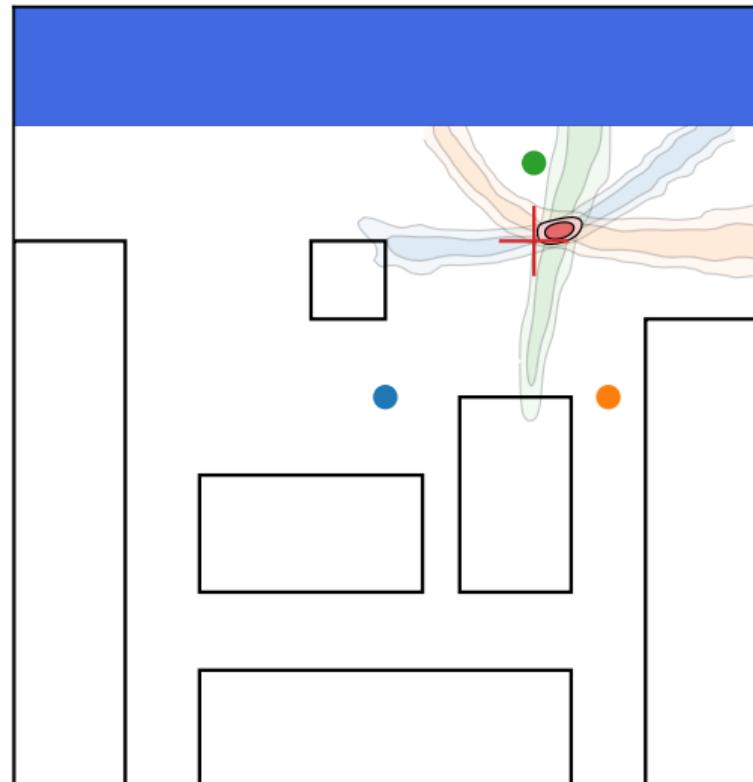
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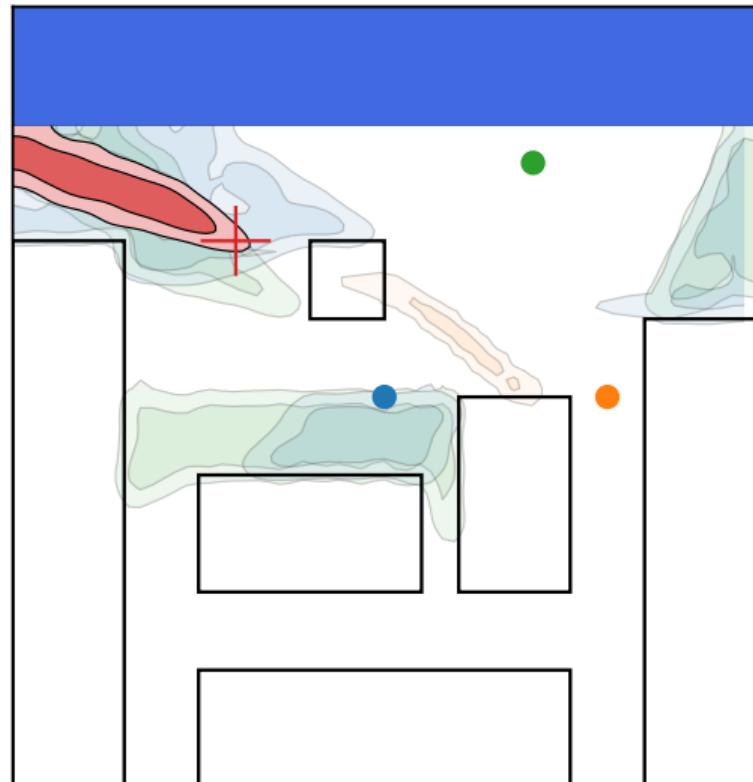
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- ▶ Transmitter: red cross
- ▶ Contours: posterior distributions
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 - ▶ combined posterior in red (solid peak)
- ▶ Where to put sensors?
- ▶ Compute the localisation (information gain \mathcal{D}_{KL}), distributed over all uncertainties:
 - ▶ transmitter location,
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 - ▶ reflections
- ▶ Find collection of good solutions
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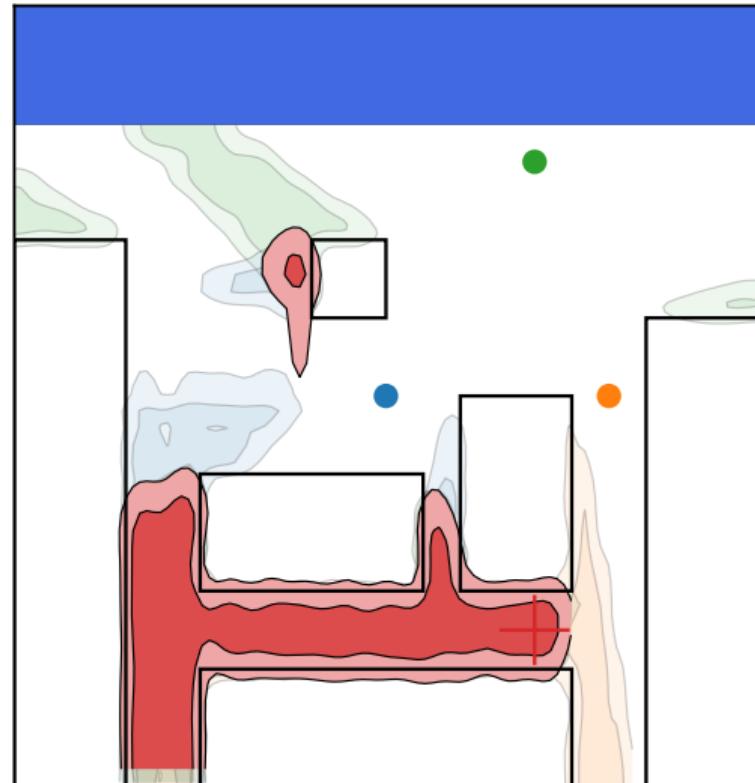
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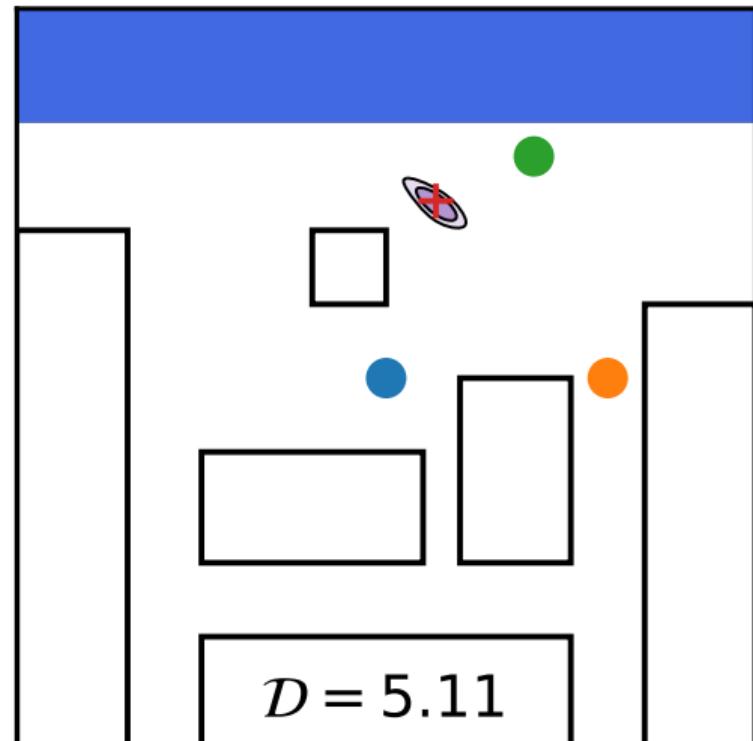
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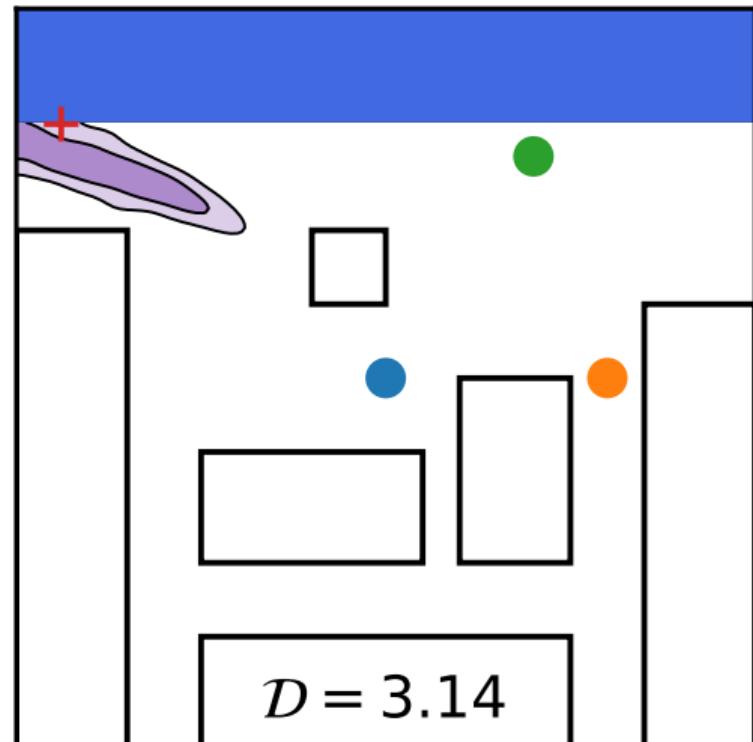
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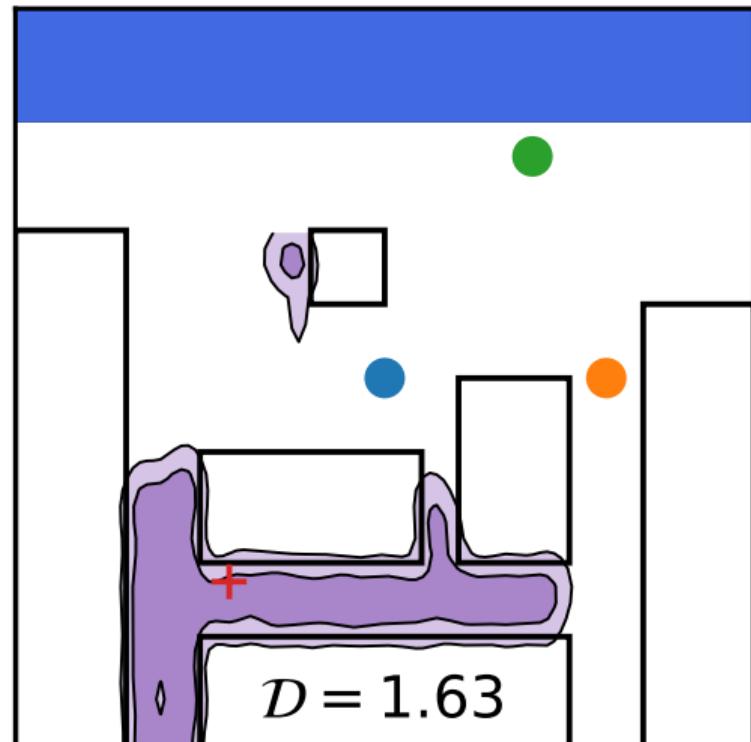
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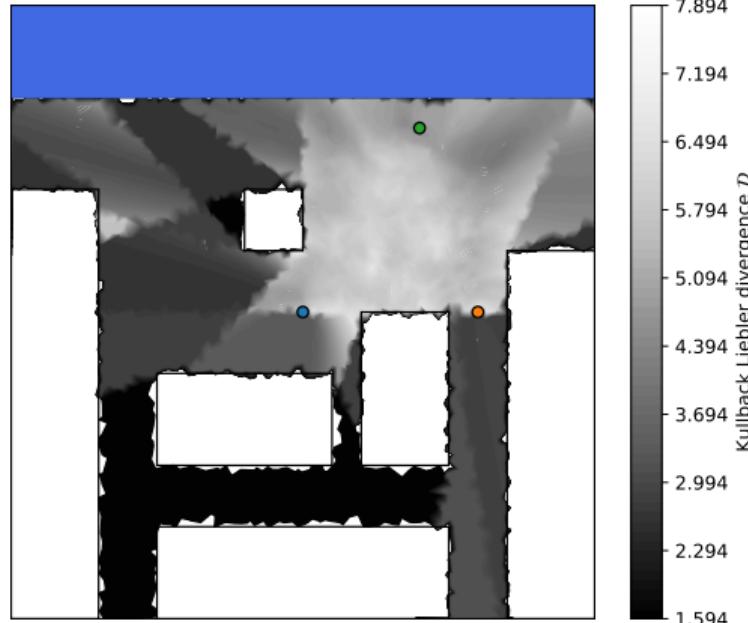
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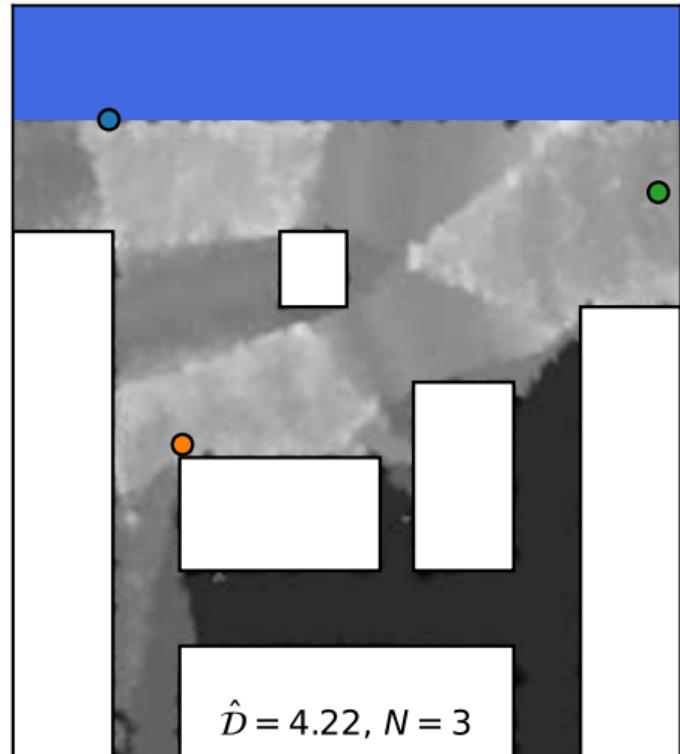
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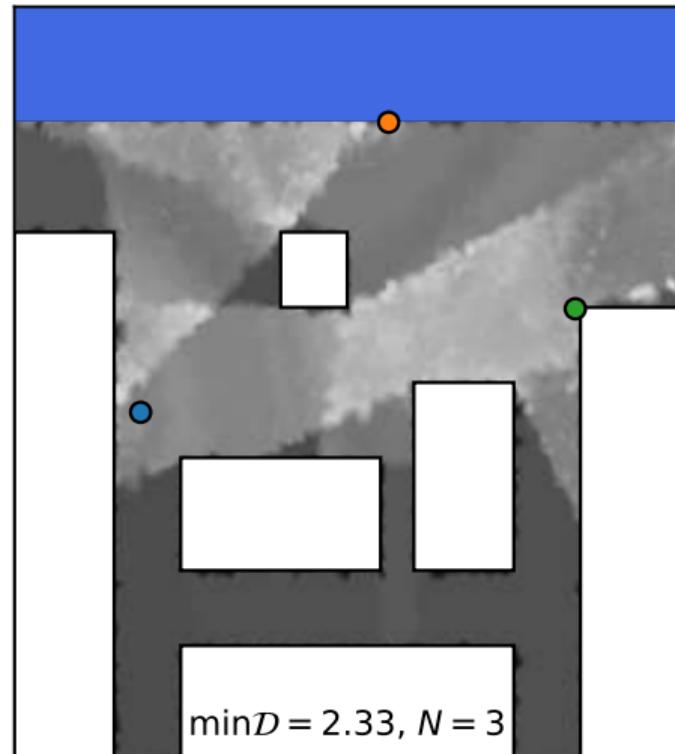
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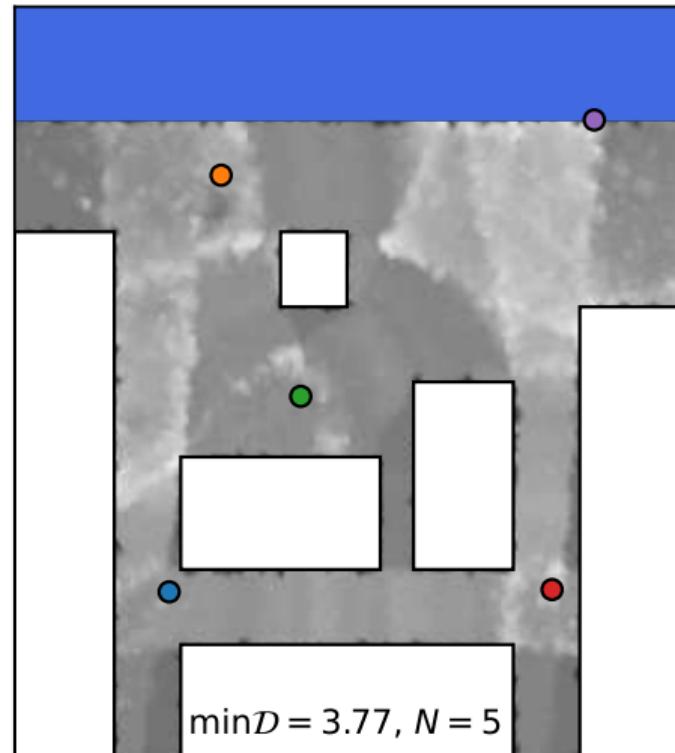
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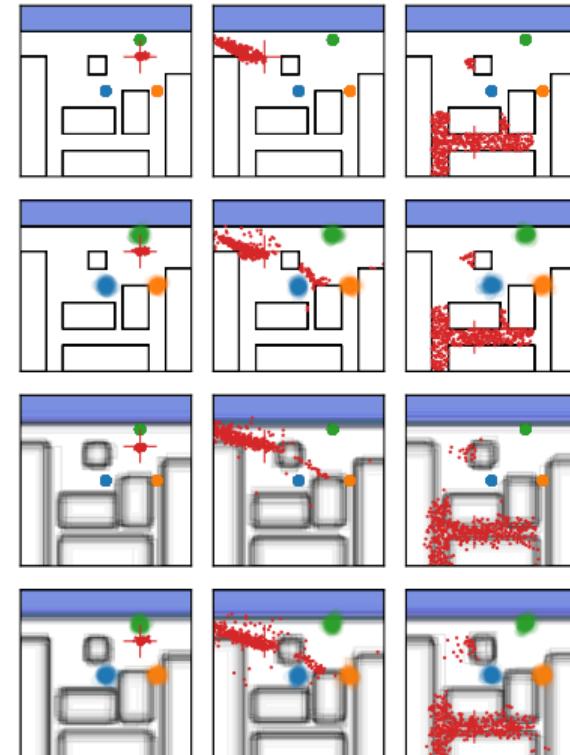
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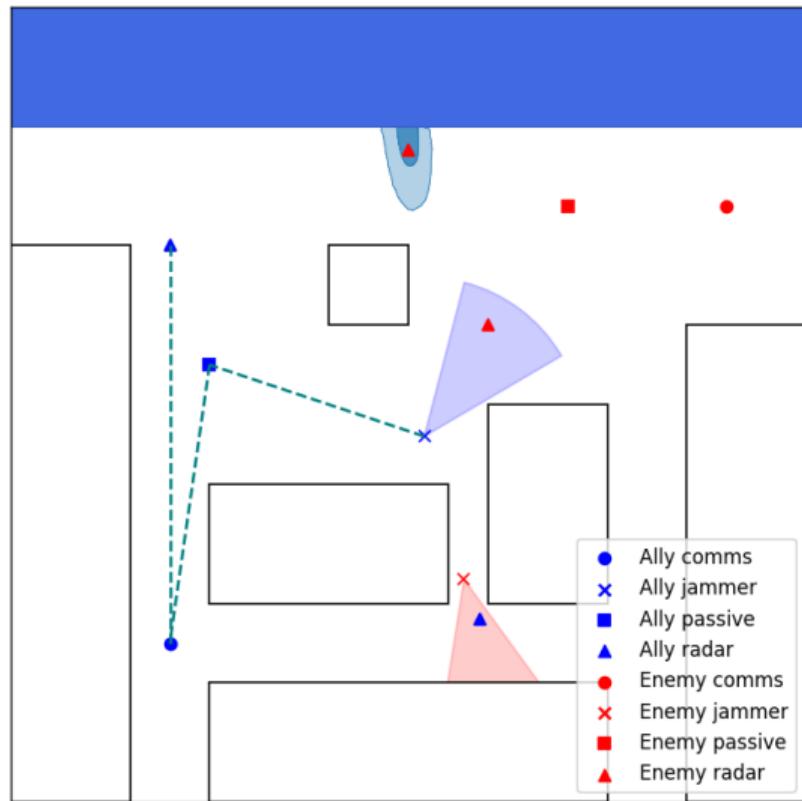
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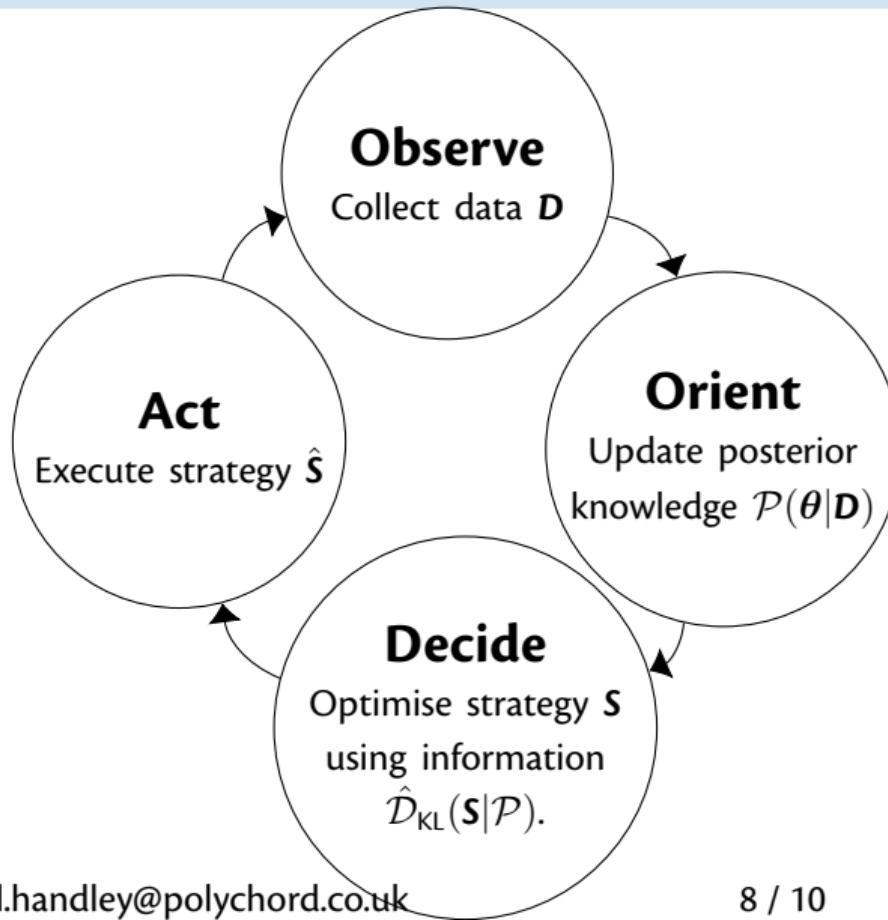


Example 3: MIDAS (CEME6.4)

- ▶ MIDAS: Maximum information data acquisition strategies
- ▶ Generalise ``best sensor location" to ``best data acquisition strategy"
- ▶ Use this to decide what data to take next.
- ▶ Add in adversarialism (two competing teams)
- ▶ Networks of communicating allied sensors
- ▶ Adversarial capacity (jamming)
- ▶ Ability re-roll (e.g. UAV) capability
- ▶ Use same approach to decide when & how to re-roll.



MIDAS & the Mathematical OODA loop (DASA GAN)



- ▶ Mathematical instantiation of John Boyd's OODA loop.
- ▶ Tight coupling between **Bayesian statistics** (updating knowledge) and **Information theory** (how/when to gather more)
- ▶ Nested sampling (PolyChord) is used at both **Orient** and **Decide** steps, in Bayesian & Optimisation mode.
- ▶ However, given that all of these will only ever be *Models* of the real world, the **Act** step will need to be **human-in-the-loop**.

So what?

Current research & future directions

- ▶ DSTL+PC/UCAM+PA have shown that Bayesian methods capture natural data fusion at scale.
- ▶ We know the complex future EME is going to be too complex for unaugmented humans.
- ▶ The missing piece is the **Act** step.
- ▶ For **human-in-the-loop** decision-making the rest of the loop needs to present and compress the information in a way that is **actionable** and **explainable**.
- ▶ This is the frontier of our current research
 - ▶ Designing optimal theoretical system that assists in the decision cycle
 - ▶ Giving human agents what they need, when you need it
 - ▶ Should get out of the way given human insight

- ▶ Astrostatisticians continue to innovate at the frontier of inference.
- ▶ These techniques currently require laptop- to high-performance computing power.
- ▶ For generation-after-next techniques, we can assume they will be in our pocket/clothing.
- ▶ The Bayesian OODA loop quantifies optimal human-in-the-loop decision-making.
- ▶ Science & Technology explored here is not challenge-specific.
- ▶ These techniques are complementary to machine learning/AI