Risk Neutral Pricing, Key Concepts in Stochastic Calculus, and Black-Scholes

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Abstract

This paper explains the foundation for option pricing and an introduction to the Black-Scholes model. Part I discusses the base key concepts of stochastic calculus and understanding of derivative pricing. Part II begins to elaborate on risk neutral pricing and risk neutral valuation in regards to the Black-Scholes Model. Finally, Part III begins to discuss the "Greeks" which measures option price sensitivity with various graphs that illustrate put options and call options.

Part I
Key Concepts in Stochastic Calculus

A. State prices

State prices, also called Arrow-Debreu securities, assume a state at time 0 and a future state at time 1.

To explain state price where two states are possible tomorrow: Up (U) and Down (D). The random variable represents the state as w; tomorrow's random variable as w_1. Thus, w_1 can take two values: w_1=U and w_1=D.

Let's assume that:

- There is a security that pays off $1 if tomorrow's state is "U" and nothing if the state is "W". The price of this security is q_P
- There is a security that pays off $1 if tomorrow's state is "W" and nothing if the state is "P". The price of this security is q_W

The probabilities of w_1=U and w_1=D, the more likely a move to W is, and the higher the price q_W gets, since q_W insures the agent against the occurrence of state W. The seller of this insurance would demand a higher premium in an efficient economy[1].

B. Risk-neutral probabilities

This is a continuation of state prices, where we can prove that if there is a security which has a certain payoff $1 no matter which state happens in time 1. Any payoff can be replicated with a linear combination of arrow-debreu securities; and this particular replication of a portfolio imposes a unique arbitrage-free price. Let P and A(i) denote the arbitrage-free price of a security and arrow-debreu security i respectively at time 0, and X(i) denote the payoff of the security when state i is reached, then we can write:

$$\mathrm{P} = \sum_{i=1}^{1} A(i) * X(i) \tag{1}$$

The arrow-debreu security prices are risk-neutral probabilities because it satisfies two conditions of being a probability measure: (1) the probability of a particular event is zero or positive. (2) the sum of all probabilities must equal 1. The payoff of a portfolio at time 1 is $1, and assuming interest rate is 0, the cost of the portfolio at time 0 should be $1[2]. That is:

$$\sum_{i=1}^{1} A(i) = 1 \tag{2}$$

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[1] Wikipedia: http://en.wikipedia.org/wiki/State_prices

[2] Kerry Back. A Course in Derivatives Securities, p. 12

C. Equivalent Martingale measure

Equivalent martingale measures are also known as risk-neutral probabilities. An equivalent martingale measure factors in the average investors degree of risk aversion to allow for a more straightforward calculation of the present value of a security. Equivalent martingale measures are most commonly used in the pricing of derivative securities, because this is the most common case of a security type which has several discrete, contingent payouts. An equivalent martingale measure is a probability distribution on future price paths satisfying two conditions:

1. The expected return on all assets in the model should be the same under the risk-neutral probability.
2. The prices that occur with positive probability under the risk-neutral probability should be identical to the prices that occur with positive probability in the original model.

D. Martingale

A martingale is a model of a fair game where knowledge of past events never predict the mean of the future. In particular, a martingale is a sequence of random variables (i.e., a stochastic process) for which, at a particular time in the realized sequence, the expectation of the next value in the sequence is equal to the present observed value even given knowledge of all prior observed values at a current time.

E. Markov

A mathematical system, the Markov chain is a system that undergoes transitions from one state to another on a state space. It is a random process usually characterized as memoryless[3]: the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes. It can thus be used for describing systems that follow a chain of linked events, where what happens next depends only on the current state of the system.

F. Brownian Motion

Under this model, assets have continuous prices evolving continuously in time and are driven by Brownian motion processes. This model requires an assumption of perfectly divisible assets and a frictionless market (i.e. that no transaction costs occur either for buying or selling). Another assumption is that asset prices have no jumps or surprises in the market. A Brownian motion model is a random process that changes continuously in time and has the property that its change over any time period is normally distributed with mean-zero and variance equal to the length of the time period[4].

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[3] Hull, John. Options, Futures, and Other derivatives, 8th Edition

[4] Kerry Back. A Course in Derivatives Securities, p. 28

A Geometric Brownian motion is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion with drift. It's satisfying a stochastic differential equation as indicated in the following equation and often used to model stock prices in the Black-Scholes model[5].

$$\mathrm{d}S_t = uS_t dt + \sigma S_t dW_t \tag{3}$$

where W_t is Wiener process or Brownian motion; u constant, represents the percentage drift and σ, constants, means percentage volatility.

G. Radon-Nikodym Derivative

The theorem is a result in measure theory that states that, given a measurable space (X,Σ), if a σ-finite measure V on (X,Σ) is absolutely continuous with respect to a σ-finite measure \mu on (X,Σ), then there is a measurable function f on X and taking values in [0,∞), such that

$$v(A) = \int_A f d\mu \tag{4}$$

The function f is called the Radon–Nikodym derivative[6].

The theorem is very important in extending the ideas of probability theory from probability masses and probability densities defined over real numbers to probability measures defined over arbitrary sets. It tells if and how it is possible to change from one probability measure to another. Specifically, the probability density function of a random variable is the Radon–Nikodym derivative of the induced measure with respect to some base measure (usually continuous random variables).

H. Girsanov's Theorem

The Girsanov theorem (probability theory) describes how the subtleties of stochastic processes change when the original measure is changed to an equivalent probability measure[7]. Girsanov's theorem is important in the general theory of stochastic processes since it enables the key result that if *Q* is a measure absolutely continuous with respect to *P* then every *P*-semimartingale is a *Q*-semimartingale.

Girsanov theorem state the theorem first for the special case when the underlying stochastic process is a Wiener process. This special case is sufficient for risk-neutral pricing in the Black-Scholes model and in many other models (e.g. all continuous models).

In Black-Scholes frame, we move from historic measure P to risk neutral measure Q via Radon-Nikodym derivative:

$$\frac{dQ}{dP} = \varepsilon\left(\int_0 \frac{r-u}{\sigma} dW_s\right) \tag{5}$$

where r means risk free rate, u denotes the asset's drift and σ represents its volatility.

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[5] Wikipedia: http://en.wikipedia.org/wiki/Geometric_Brownian_motion
[6] Wikipedia: http://en.wikipedia.org/wiki/Radon%E2%80%93Nikodym_theorem
[7] Wikipedia: http://en.wikipedia.org/wiki/Girsanov_theorem

I. Equivalent Probability measure

Equivalence here means the two measures agree on the possible and impossible outcomes. Two probability measures are said to be equivalent if they define the same null sets. The Girsanov theorem characterizes the transformation of semimartingales under equivalent changes of measure. For example, if one measure allocates a positive probability to an outcome, the other measure must also allocate a positive probability to that outcome in order to be an equivalent measure.

J. Ito's Lemma

Ito's lemma is an identity used in Ito calculus to find the differential of a time-dependent function of a stochastic process. It serves as the stochastic calculus counterpart of the chain rule. Typically, it is memorized by forming the Taylor series expansion of the function up to its second derivatives and identifying the square of an increment in the Wiener process with an increment in time. The lemma is widely employed in mathematical finance, and its best known application is in the derivation of the Black–Scholes equation for option values[8]. Suppose that the value of a variable x follows the Ito process,

$$\mathrm{dx} = \mathrm{a(x,t)dt} + \mathrm{b(x,t)dz} \tag{6}$$

where, dz is a Wiener process and a and b are functions of x and t. The variable x has a drift rate of a and a variance rate of b^2. Ito's lemma shows that a function G of x and t follows the process[9]

$$\mathrm{dG} = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}bdz \tag{7}$$

As we know, $\mathrm{dS} = \mu\mathrm{Sdt} + \sigma\mathrm{Sdz}$ is a reasonable model of stock price movements. From Ito's Lemma, it can be expressed as the following,

$$\mathrm{dG} = \left(\frac{\partial G}{\partial x}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}\sigma^2 S^2\right)dt + \frac{\partial G}{\partial x}\sigma S dz \tag{8}$$

[8] Wikipedia: http://en.wikipedia.org/wiki/It%C5%8D's_lemma

[9] John Hull. Options, Futures, and Other derivatives, 8th Edition

PART II

Risk Neutral Pricing

A. The Black-Scholes-Merton Model

The Black-Scholes-Merton model was developed by three economists in the 1970s in order to price options. Specifically, it is useful for finding the theoretical price of European call and put options. Several underlying assumptions provide the framework for applying the model. Some of the key assumptions are as follows[10]:

1) The options are European and can only be exercised at expiration
2) No dividends are paid out during the life of the option
3) Efficient markets (i.e., market movements cannot be predicted)
4) No commissions
5) The risk-free rate and volatility of the underlying are known and constant
6) Follows a lognormal distribution; that is, returns on the underlying are normally distributed

The necessary formulas to calculate the value of European call and put options are then provided[11]:

$$Call = S_0 N(d1) - Xe^{-rT} N(d2)$$

$$Put = Xe^{-rT} N(-d2) - S_0 N(-d1)$$

Where:

$$d1 = \frac{ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$d2 = \frac{ln\left(\frac{S_0}{X}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d1 - \sigma\sqrt{T}$$

S = Stock price
X = Strike Price
T = Time to maturity
σ = Volatility
r = Risk-free rate

Using the given inputs, the Black-Scholes model yields respective theoretical valuations for a call and put on the underlying asset:

[10] http://www.investopedia.com/university/options-pricing/black-scholes-model.asp

[11] Jabbour PhD, George, Financial Engineering Notes, Fall Semester 2012

Stock Price	$100
Strike Price	$100
r	5%
σ	20%
T	1 year
Call	$10.5
Put	$5.6

B. Black-Scholes-Merton and Risk-Neutral Valuation

It is also important to note that risk-neutral valuation methods are largely based on one key property of the BSM model. The differential equation stemming from the model does not involve any variables that include the risk preferences of investors[12]. As a result, any set of risk preferences can be used when valuing an option. Or said another way, the assumption that all investors are risk neutral can be made when applying the BSM model or utilizing binomial pricing models.

Applying this methodology, we can value any derivative that provides a payoff at one particular time. Using risk-neutral methods we would employ the following procedure[13]:

1) Assume that the expected return from the underlying asset is the risk-free interest rate (r)
2) Calculate the expected payoff from the derivative
3) Discount the expected payoff at the risk-free interest rate

[12] Hull, John C. *Options, Futures, and Other Derivatives.* Eighth Edition. 2012. p. 311.
[13] Hull. p. 312.

PART III Measuring Option Price Sensitivity

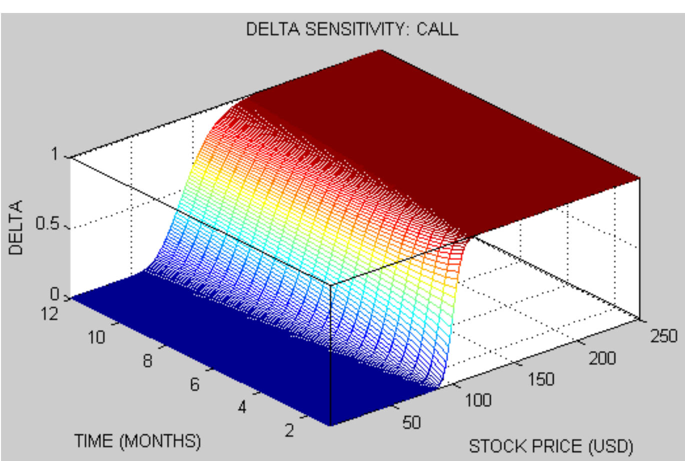
In finance, the Greek letters are used to measure how the price of an option changes with respect to other variables. Specifically, these sensitivities manifest as: price changes to the underlying asset, the magnitude of changes to the underlying asset, time to maturity, changes in volatility of the underlying asset, and changes to the risk free rate.

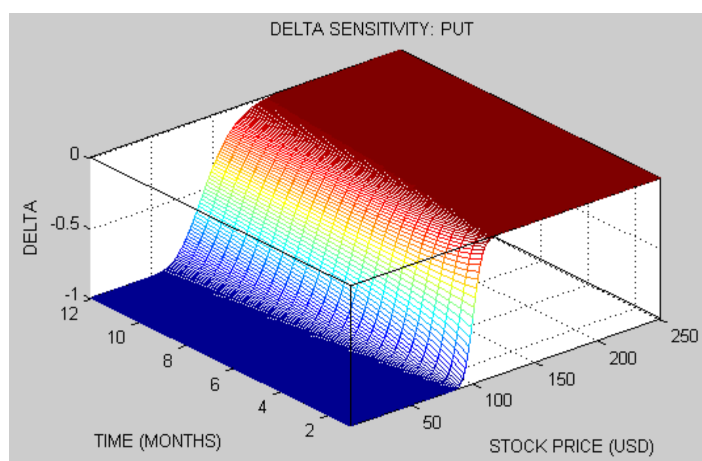
A. Delta (Δ)

$$\Delta(call) = \frac{\partial C}{\partial S} \qquad\qquad \Delta(put) = \frac{\partial P}{\partial S}$$

The delta of an option measures the amount by which the price of an option changes as compared to a change in the price of the underlying asset all other variables held constant. This can be thought of as the 1st partial derivative of the option price with respect to the underlying asset. Using the given inputs, the Delta of a call option is 0.6368 and the delta of a put option is -0.3632.

For a call option, Delta is always positive, due to a positive correlation between the price of the asset and the option price. The option price is directly related to the price of the stock so as the asset price increases so must the call price and vice versa. The Delta of a call option varies between 0 and 1; the closer it being to 1, the more the option price will move in tandem to the underlying asset price which is behaviorally indicative of an in-the-money call . Conversely, a put option price has a negative correlation to the underlying asset price. If the asset price increases, then the value of the put should decline and vice versa. The Delta of a put option varies between -1 and 0; the closer it being to -1, the more the option price will move inversely to the underlying asset price which is behaviorally indicative of an in-the-money put. As such, Delta neutrality in a portfolio provides protection against small movements in the underlying asset price.



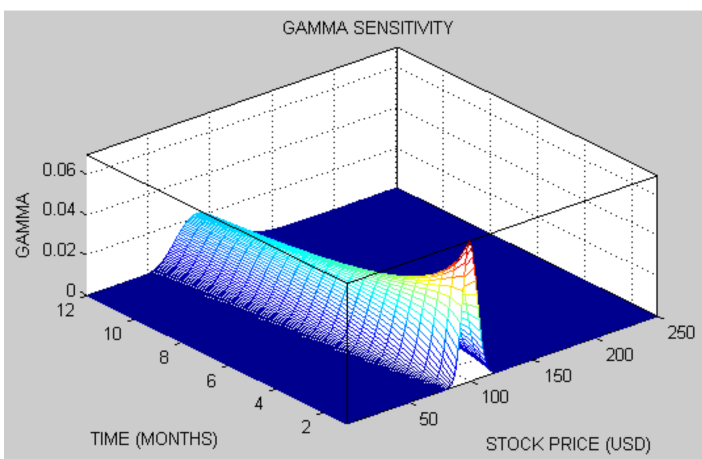


B. Gamma (Γ)

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 C}{\partial S^2}$$

The Gamma of an option is the amount by which the aforementioned option Delta changes with respect to a change in the price of the underlying asset all other variables held constant. Delta often is particularly sensitive to changes in the asset price and can be thought of as the 2^{nd} partial derivative of the option price with respect to the underlying asset. Using the given inputs, the Gamma of both the call and put options is 0.0188.

Gamma for a call and a put on the same underlying asset will be equal given the same strike and time to maturity.[14] Comparing the Delta and Gamma visualization plots, we see this relationship manifested around the strike price of S_0 = 100 (USD) for both call and put options. Taking from a physical representation, the Delta (or rate of change of the option price with respect to a change in the underlying asset) "accelerates" in the region immediately surrounding the strike. This can be seen in the Gamma visualization. If gamma is small, delta changes slowly. If gamma is highly negative or highly positive, delta is very sensitive to the price of the underlying asset.[15] As such, Gamma neutrality in a portfolio provides protection against larger movements in the underlying asset price.



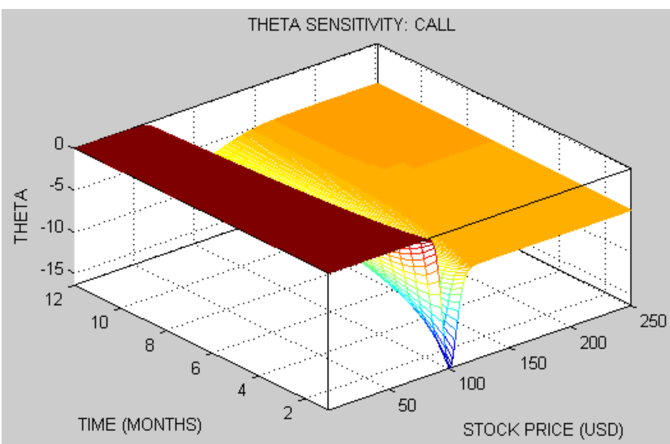
[14] Young, Stephen D. Financial Engineering Notes. p.65
[15] Hull, John C., Options, Futures, and other Derivatives. p.389

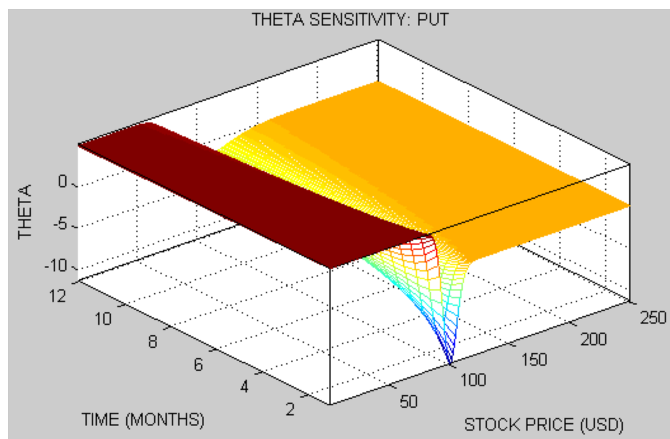
C. Theta (θ)

$$\theta(call) = -\frac{\partial C}{\partial t} \qquad \theta(put) = -\frac{\partial P}{\partial t}$$

Intuitively, it makes sense that the value of an option decreases as it approaches expiration as there is less opportunity for the asset price to deviate from the strike. The Theta of an option is the amount that the price of an option changes as with respect to the passage of time all other variables held constant. Using the given inputs, the theta of a call option is -6.4140 and the theta of a put option is -1.6579.

Theta is nearly always negative for both call and put options because the value of the option decreases as maturity approaches as the option becomes less valuable. As such, the Theta of both call and put options becomes more negative as time to maturity decreases. This is why Theta is also known of the "time decay" of an option. Also, as the asset price deviates from the strike price in either direction, Theta becomes less negative.





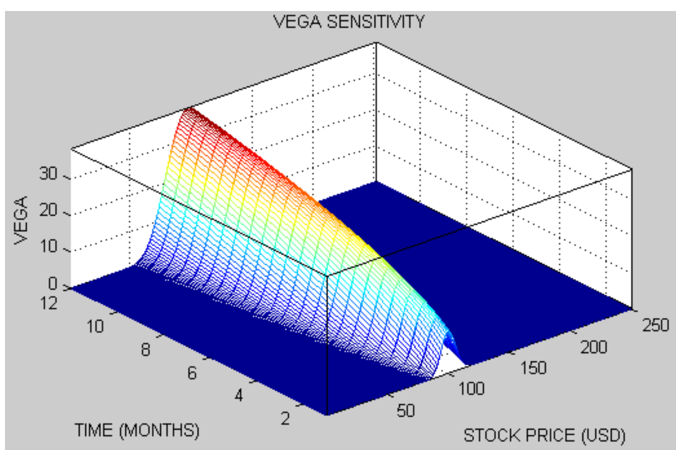
D. Vega ($\mathcal{V}$)

$$\mathcal{V} = \frac{\partial C}{\partial \sigma}$$

In practice, the volatility of the underlying asset varies with time. The Vega of an option measures the change in the option price with respect to a change to the volatility of the underlying asset all other variables held constant. Using the given inputs, the Vega of both the call and put options is 37.5240.

As with Gamma, Vega will be equal for a call and put on the same underlying asset, given the same strike and time to maturity.[16] We also see that holding all other variables constant, Vega decreases as time to maturity decreases, which indicates that option prices become less sensitive to equivalent changes in volatility (of the underlying asset) given its proximity to maturity. Said another way, Vega has the greatest impact when the contract is executed; its effect waning as maturity approaches.

As we see with the Black-Scholes model, implied volatility can be estimated given the 5 other observable inputs. Implied volatility shares a direct relationship with changes to option price. Vega therefore also measures the rate of change of an option's theoretical value in relation to changes in implied volatility.



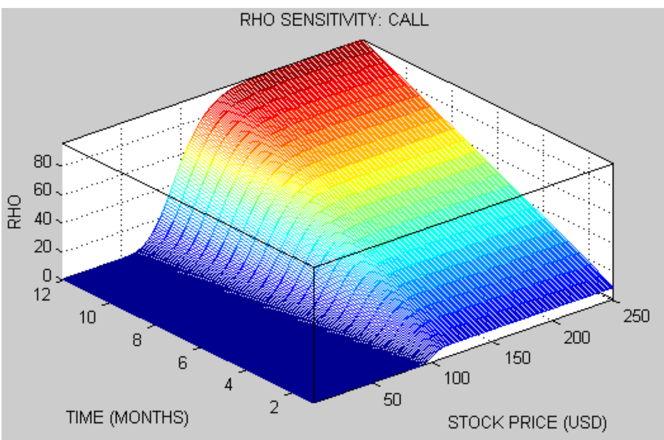
[16] Young, Stephen D., Financial Engineering Notes. p.66

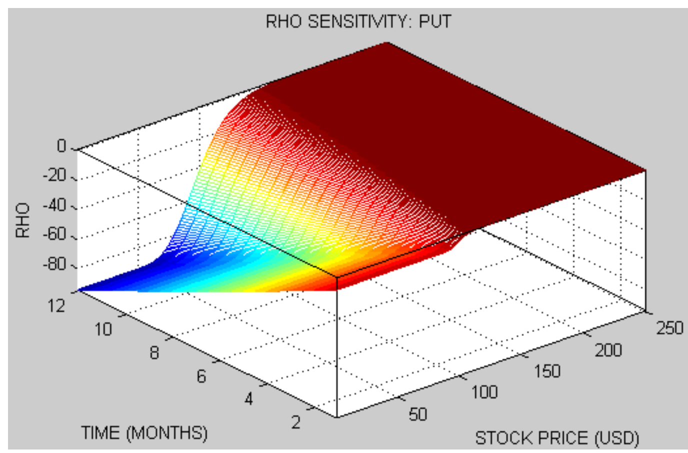
E. Rho (ρ)

$$\rho(call) = \frac{\partial C}{\partial r} \qquad \rho(put) = \frac{\partial P}{\partial r}$$

The Rho of an option measures the change in the option price with respect to a unit change in the risk free rate (i.e., short term US Treasury Bill rate), all other variables held constant. Using the given inputs, the Rho of a call option is 53.2325, while that of a put option is -41.8905.

Rho is oft considered the least impactful Greek given that many options are traded prior to maturity and/or the risk free rate is stable over the option time horizon. As such, Rho accounts for a smaller component of the overall change in the option price.





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Appendix

MATLAB SYNTAX

%INITIAL INPUTS

```
s0=100;
K=100;
r=0.05;
sigma=0.2;
T=1
d1=(log(s0/K)+(r+sigma^2/2)*T)/(sigma*sqrt(T));
d2=d1-(sigma*sqrt(T));
C=s0*normcdf(d1)-K*(exp(-r*T)*normcdf(d2)) %EUROPEAN CALL
P=K*exp(-r*T)*normcdf(-d2)-s0*normcdf(-d1) %EUROPEAN PUT
[CallDelta, PutDelta] = blsdelta(100, 100, 0.05, 1, 0.2, 0) %DELTA
[CallTheta, PutTheta] = blstheta(100, 100, 0.05, 1,0.2, 0) %THETA
[CallRho, PutRho]= blsrho(100, 100, 0.05, 1, 0.2,0) %RHO
Gamma = blsgamma(100, 100, 0.05, 1, 0.2, 0) %GAMMA
Vega = blsvega(100, 100, 0.05, 1, 0.2, 0) %VEGA
```

%1. DELTA SESITIVITY: EUROPEAN CALL

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
[CD,PD] = blsdelta(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'DELTA SENSITIVITY: CALL');
mesh(range, j, CD);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('DELTA');
title('DELTA SENSITIVITY: CALL');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%2. DELTA SENSITIVITY: EUROPEAN PUT

```
% RANGE OF STOCK PRICES
range = 1:250;
```

```
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
[CD,PD] = blsdelta(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'DELTA SENSITIVITY: PUT');
mesh(range, j, PD);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('DELTA');
title('DELTA SENSITIVITY: PUT');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%3. GAMMA SENSITIVITY: EUROPEAN OPTION

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
ga= blsgamma(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'GAMMA SENSITIVITY');
mesh(range, j, ga);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('GAMMA');
title('GAMMA SENSITIVITY');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%4. THETA SENSITIVITY: EUROPEAN CALL

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
[CT,PT]= blstheta(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'THETA SENSITIVITY: CALL');
mesh(range, j, CT);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('THETA');
title('THETA SENSITIVITY: CALL');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%5. THETA SENSITIVITY: EUROPEAN PUT

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
[CT,PT]= blstheta(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'THETA SENSITIVITY: PUT');
mesh(range, j, PT);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('THETA');
title('THETA SENSITIVITY: PUT');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%6. VEGA SENSITIVITY: EUROPEAN OPTION

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
v= blsvega(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'VEGA SENSITIVITY');
mesh(range, j, v);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('VEGA');
title('VEGA SENSITIVITY');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%7. RHO SENSITIVITY: EUROPEAN CALL

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
[CR,PR]= blsrho(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'RHO SENSITIVITY: CALL');
mesh(range, j, CR);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('RHO');
title('RHO SENSITIVITY: CALL');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```

%8. RHO SENSITIVITY: EUROPEAN PUT

```
% RANGE OF STOCK PRICES
range = 1:250;
span = length(range);
j = 1:0.5:12;

% YEARS TO OPTION MATURITY
newj = j(ones(span,1),:)'/12;
jspan = ones(length(j),1);
newrange = range(jspan,:);
pad = ones(size(newj));
[CR,PR]= blsrho(newrange, 100*pad, 0.05*pad, newj, 0.20*pad);

% PLOT OF OPTION SENSITIVITIES
figure('NumberTitle', 'off', ...
       'Name', 'RHO SENSITIVITY: PUT');
mesh(range, j, PR);
xlabel('STOCK PRICE (USD)');
ylabel('TIME (MONTHS)');
zlabel('RHO');
title('RHO SENSITIVITY: PUT');
axis([1 250  1 12  -inf inf]);
view(-40,50)
apos = get(gca,'Position');
set(gca,'Position',[apos(1) .25 apos(3) .68])
box on
```