

TDAT 2002 Matematikkprosjekt

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1 Oppgave 1

1.1 Teori

Taylor's formel med restledd:

$$R_5(x) = \frac{f^{(6)}(c)}{6!}(x - x_0)^6$$

$$EIy'''' = f(x)$$

$$y''''(x) \approx \frac{y(x-2h) - 4y(x-h) + 6y(x) - 4y(x+h) + y(x+2h)}{h^4}$$

1.2 Oppgaveløsning

1.2.1 Oppgave 5.1.21

1.

$$f(x+2h) = f(x) + 2hf'(x) + 2h^2f''(x) + \frac{4}{3}h^3f'''(x) + \frac{2}{3}h^4f''''(x) + O(h^6)$$

$$f(x-2h) = f(x) - 2hf'(x) + 2h^2f''(x) - \frac{4}{3}h^3f'''(x) + \frac{2}{3}h^4f''''(x) + O(h^6)$$

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) + O(h^6)$$

$$f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) + O(h^6)$$

2.

$$\begin{aligned} f(x+h) + f(x-h) + f(x+2h) + f(x-2h) \\ = 4f(x) + 5h^2f''(x) + \frac{17}{12}h^4f''''(x) \end{aligned}$$

3.

$$\begin{aligned} f''(x)h &= f(x-h) + f(x+h) \\ &= \\ f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) \\ &\quad + \\ f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) \\ &= f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + f(x) - hf'(x) + \frac{1}{2}h^2f''(x) \\ &= 2f(x) + h^2f''(x) + \frac{1}{2}h^4f''''(x) \end{aligned}$$

4.

$$\begin{aligned}
h^2 f''(x) &= 2f(x) - \frac{1}{12}h^4 f''''(x) + f(x-h) + f(x+h) \\
&= 4f(x) + 5(-2f(x) - \frac{1}{12}h^4 f''''(x)) + \frac{17}{12}h^4 f''''(x) \\
&= 4f(x) - 10f(x) - \frac{5}{12}h^4 f''''(x) + \frac{17}{12}h^4 f''''(x) \\
&= -6f(x) + h^4 f''''(x) + 5f(x-h) + 5f(x+h)
\end{aligned}$$

5.

$$\begin{aligned}
&f(x-2h) + f(x-h) + f(x+h) + f(x+2h) + 6f(x) - 5f(x-h) - 5f(x+h) \\
f''''(x) &= \frac{2(x-2h) - 4(x-h) + 6f(x) - 4(x+h) + f(x+h)}{h^4} = O(n^2)
\end{aligned}$$

1.2.2 Oppgave 5.1.22a

$$f(x) = f'(x) = 0$$

then

$$f''''(x+h) - \frac{16f(x+h) - 9f(x+2h) + \frac{8}{3}f(x+3h) - \frac{1}{4}f(x+4h)}{h^4} = O(h^2)$$

1.

$$\begin{aligned}
f(x-h) &= f(x) - hf'(x) + \frac{1}{2}h^2 f''(x) - \frac{1}{6}h^3 f'''(x) + \frac{1}{24}h^4 f''''(x) \\
f''''(x-h) &= f''''(x) + f^{(5)}(x) + \frac{1}{2}h^2 f^{(6)}(x) + \frac{1}{6}h^3 f^{(7)}(x) + \frac{1}{24}h^4 f^{(8)}(x) \\
f(x+h) &= f(x) + hf'(x) + \frac{1}{2}h^2 f''(x) + \frac{1}{6}h^3 f'''(x) + \frac{1}{24}h^4 f''''(x) \\
f(x+2h) &= f(x) + 2hf'(x) + 2h^2 f''(x) + \frac{4}{3}h^3 f'''(x) + \frac{2}{3}h^4 f''''(x) \\
f(x+3h) &= f(x) + 3hf'(x) + \frac{9}{2}h^2 f''(x) + \frac{9}{2}h^3 f'''(x) + \frac{27}{8}h^4 f''''(x) + \frac{81h^5 f^{(5)}(x)}{40} \\
f(x+3h) &= f(x) + 4hf'(x) + 8h^2 f''(x) + \frac{32}{3}h^3 f'''(x) + \frac{32}{3}h^4 f''''(x) + \frac{128h^5 f^{(5)}(x)}{15}
\end{aligned}$$

2.

$$f''''(x-h) = f''''(x) + f^{(5)}(x) + O(h^2)$$

3.

$$\begin{aligned}
& f''''(x) + f^{(5)} + O(h^2) - \\
& 16\left(\frac{1}{2}h^2 f''(x)\right) + \frac{1}{6}h^3 f'''(x) + \frac{1}{24}h^4 f''''(x) + \frac{1}{120}h^5 f^{(5)}(x) + O(h^6)) \\
& -9(2h^2 f'(x) + \frac{4}{3}h^3 f'''(x) + \frac{2}{3}h^4 f''''(x) + \frac{4}{15}h^5 f^{(5)}(x) + O(h^6)) \\
& + \frac{8}{3}\left(\frac{9}{2}h^2 f''(x) + \frac{9}{2}h^3 f'''(x) + \frac{27}{8}h^4 f''''(x) + \frac{81}{40}h^5 f^{(5)}(x)\right) + O(h^6) \\
& - \frac{1}{4}(8h^2 f''(x) + \frac{32}{3}h^3 + \frac{32}{3}h^4 f''''(x) + \frac{128}{15}h^5 f^{(5)}(x) + O(h^6)) \\
& =
\end{aligned}$$

$$\begin{aligned}
& \frac{f''''(x) + f^{(5)}(x) + O(h^2) - (8 - 18 + 12 - 2)h^2 f''(x) + (\frac{8}{3} + 12 - 12 - \frac{8}{3})f'''(x) + (\frac{2}{3} - 6 + 9 - \frac{8}{3})f''''(x) + (\frac{2}{15} - \frac{12}{5} + \frac{21}{5} - \frac{32}{15})f^{(5)}(x)}{h^4} \\
& = f''''(x) + f^{(5)}(x) + O(h^2) - f''''(x) - f^{(5)}(x) = O(h^2)
\end{aligned}$$

2 Oppgave 2

3 Oppgave 3

4 Oppgave 4

5 Oppgave 5

6 Oppgave 6

7 Oppgave 7