1 Teori

Taylors formel med restledd:

$$R_5(x) = \frac{f^{(6)}(c)}{6!}(x - x_0)^6$$

$$EIy'''' = f(x)$$

$$y''''(x) \approx \frac{y(x-2h) - 4y(x-h) + 6y(x) - 4y(x+h) + y(x+2h)}{h^4}$$

2 Oppgaveløsing

Oppgave 5.1.21

1.

$$f(x+2h) = f(x)+2hf'(x)+2h^2f''(x)+\frac{4}{3}h^3f'''(x)+\frac{2}{3}n^4f''''(x)+O(n^6)$$

$$f(x-2h) = f(x)-2hf'(x)+2h^2f''(x)-\frac{4}{3}h^3f'''(x)+\frac{2}{3}n^4f''''(x)+O(n^6)$$

$$f(x+h) = f(x)+hf'(x)+\frac{1}{2}h^2f''(x)+\frac{1}{6}h^3f'''(x)+\frac{1}{24}n^4f''''(x)+O(n^6)$$

$$f(x-h) = f(x)-hf'(x)+\frac{1}{2}h^2f''(x)-\frac{1}{6}h^3f'''(x)+\frac{1}{24}n^4f''''(x)+O(n^6)$$

2.

$$f(x+h) + f(x-h) + f(x+2h) + f(x-2h)$$

$$= 4f(x) + 5h^2f'''(x) + \frac{17}{12}h^4f''''(x)$$

3.

$$f''(x)h = f(x-h) + f(x+h)$$

$$=$$

$$f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}n^4f''''(x)$$

$$+$$

$$f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}n^4f''''(x)$$

$$= f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + f(x) - hf'(x) + \frac{1}{2}h^2f''(x)$$

$$= 2f(x) + h^2f''(x) + \frac{1}{2}h^4f''''(x)$$

4.

$$h^2 f''(x) = 2f(x) - \frac{1}{12}h^4 f''''(x) + f(x-h) + f(x+h)$$

$$= 4f(x) + 5(-2f(x) - \frac{1}{12}h^4 f''''(x)) + \frac{17}{12}h^4 f''''(x)$$

$$= 4f(x) - 10f(x) - \frac{5}{12}h^4 f''''(x) + \frac{17}{12}h^4 f''''(x)$$

$$= -6f(x) + h^4 f''''(x) + 5f(x-h) + 5f(x+h)$$

5.

$$f(x-2h)+f(x-h)+f(x+h)+f(x+2h)+6f(x)-5f(x-h)-5f(x+h)$$

$$f''''(x) = \frac{2(x-2h) - 4(x-h) + 6f(x) - 4(x+h) + f(x+h)}{h^4} = O(n^2)$$

Oppgave 5.1.22a

$$f(x) = f'(x) = 0$$

then

$$f''''(x+h) - \frac{16f(x+h) - 9f(x+2h) + \frac{8}{3}f(x+3h) - \frac{1}{4}f(x+4h)}{h^4} = O(h^2)$$

$$f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2 f''(x) - \frac{1}{6}h^3 f'''(x) + \frac{1}{24}h^4 f''''(x)$$

$$f''''(x-h) = f''''(x) + f^{(5)}(x) + \frac{1}{2}h^2 f^{(6)}(x) + \frac{1}{6}h^3 f^{(7)}(x) + \frac{1}{24}h^4 f^{(8)}(x)$$

$$f(x+h)$$

1.

$$f''''(x-h) = f''''(x) + f^{(5)}(x) + O(h^2)$$

2.

$$f''''(x) + f^{(5)} + O(h^2) -$$

$$16(\frac{1}{2}h^2 f''(x)) + \frac{1}{6}h^3 f'''(x) + \frac{1}{24}h^4 f''''(x) + \frac{1}{120}h^5 f^{(5)}(x) + O(h^6))$$

$$
\begin{gathered}
-9(2h^2 f'(x) + \frac{4}{3}h^3 f'''(x) + \frac{2}{3}h^4 f''''(x) + \frac{4}{15}h^5 f^{(5)}(x) + O(h^6)) \\
+\frac{8}{3}(\frac{9}{2}h^2 f''(x) + \frac{9}{2}h^3 f'''(x) + \frac{27}{8}h^4 f''''(x) + \frac{81}{40}h^5 f^{(5)}(x)) + O(h^6) \\
-\frac{1}{4}(8h^2 f''(x) + \frac{32}{3}h^3 + \frac{32}{3}h^4 f''''(x) + \frac{128}{15}h^5 f^{(5)}(x) + O(h^6)) \\
=
\end{gathered}
$$

$$
\begin{aligned}
&\tfrac{f''''(x)+f^{(5)}(x)+O(h^2)-(8-18+12-2)h^2 f''(x)+(\frac{8}{3}+12-12-\frac{8}{3})f'''(x)+(\frac{2}{3}-6+9-\frac{8}{3})f''''(x)+(\frac{2}{15}-\frac{12}{5}+\frac{21}{5}-\frac{32}{15})f^{(5)}(x)}{h^4} \\
&= f''''(x) + f^{(5)}(x) + O(h^2) - f''''(x) - f^{(5)}(x) = O(h^2)
\end{aligned}
$$