

1 Teori

Taylor's formel med restledd:

$$R_5(x) = \frac{f^{(6)}(c)}{6!}(x - x_0)^6$$

$$EIy'''' = f(x)$$

$$y''''(x) \approx \frac{y(x-2h) - 4y(x-h) + 6y(x) - 4y(x+h) + y(x+2h)}{h^4}$$

2 Oppgaveløsning

1.

$$f(x+2h) = f(x) + 2hf'(x) + 2h^2f''(x) + \frac{4}{3}h^3f'''(x) + \frac{2}{3}h^4f''''(x) + O(h^6)$$

$$f(x-2h) = f(x) - 2hf'(x) + 2h^2f''(x) - \frac{4}{3}h^3f'''(x) + \frac{2}{3}h^4f''''(x) + O(h^6)$$

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) + O(h^6)$$

$$f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) + O(h^6)$$

2.

$$\begin{aligned} f(x+h) + f(x-h) + f(x+2h) + f(x-2h) \\ = 4f(x) + 5h^2f''(x) + \frac{17}{12}h^4f''''(x) \end{aligned}$$

3.

$$\begin{aligned} f''(x)h &= f(x-h) + f(x+h) \\ &= f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + f(x) - hf'(x) + \frac{1}{2}h^2f''(x) \\ &= 2f(x) + h^2f''(x) + \frac{1}{2}h^4f''''(x) \\ h^2f''(x) &= 2f(x) - \frac{1}{12}h^4f''''(x) + f(x-h) + f(x+h) \end{aligned}$$

$$\begin{aligned}
&= 4f(x) + 5(-2f(x) - \frac{1}{12}h^4f'''(x)) + \frac{17}{12}h^4f'''(x) \\
&= 4f(x) - 10f(x) - \frac{5}{12}h^4f'''(x) + \frac{17}{12}h^4f'''(x) \\
&= -6f(x) + h^4f'''(x) + 5f(x-h) + 5f(x+h)
\end{aligned}$$

4.

$$f(x-2h)+f(x-h)+f(x+h)+f(x+2h)+6f(x)-5f(x-h)-5f(x+h)$$