

1 Teori

Taylor's formel med restledd:

$$R_5(x) = \frac{f^{(6)}(c)}{6!}(x - x_0)^6$$
$$EIy'''' = f(x)$$
$$y''''(x) \approx \frac{y(x-2h) - 4y(x-h) + 6y(x) - 4y(x+h) + y(x+2h)}{h^4}$$

2 Oppgaveløsning

Oppgave 5.1.21

1.

$$f(x+2h) = f(x) + 2hf'(x) + 2h^2f''(x) + \frac{4}{3}h^3f'''(x) + \frac{2}{3}h^4f''''(x) + O(h^6)$$

$$f(x-2h) = f(x) - 2hf'(x) + 2h^2f''(x) - \frac{4}{3}h^3f'''(x) + \frac{2}{3}h^4f''''(x) + O(h^6)$$

$$f(x+h) = f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) + O(h^6)$$

$$f(x-h) = f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x) + O(h^6)$$

2.

$$f(x+h) + f(x-h) + f(x+2h) + f(x-2h)$$
$$= 4f(x) + 5h^2f''(x) + \frac{17}{12}h^4f''''(x)$$

3.

$$f''(x)h = f(x-h) + f(x+h)$$
$$=$$
$$f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x)$$
$$+$$
$$f(x) - hf'(x) + \frac{1}{2}h^2f''(x) - \frac{1}{6}h^3f'''(x) + \frac{1}{24}h^4f''''(x)$$
$$= f(x) + hf'(x) + \frac{1}{2}h^2f''(x) + f(x) - hf'(x) + \frac{1}{2}h^2f''(x)$$
$$= 2f(x) + h^2f''(x) + \frac{1}{2}h^4f''''(x)$$

4.

$$\begin{aligned}
 h^2 f''(x) &= 2f(x) - \frac{1}{12}h^4 f''''(x) + f(x-h) + f(x+h) \\
 &= 4f(x) + 5(-2f(x) - \frac{1}{12}h^4 f''''(x)) + \frac{17}{12}h^4 f''''(x) \\
 &= 4f(x) - 10f(x) - \frac{5}{12}h^4 f''''(x) + \frac{17}{12}h^4 f''''(x) \\
 &= -6f(x) + h^4 f''''(x) + 5f(x-h) + 5f(x+h)
 \end{aligned}$$

5.

$$\begin{aligned}
 &f(x-2h) + f(x-h) + f(x+h) + f(x+2h) + 6f(x) - 5f(x-h) - 5f(x+h) \\
 f''''(x) &= \frac{2(x-2h) - 4(x-h) + 6f(x) - 4(x+h) + f(x+h)}{h^4} = O(n^2)
 \end{aligned}$$

Oppgave 5.1.22a

$$f(x) = f'(x) = 0$$

then

$$f''''(x+h) - \frac{16f(x+h) - 9f(x+2h) + \frac{8}{3}f(x+3h) - \frac{1}{4}f(x+4h)}{h^4} = O(h^2)$$

1.