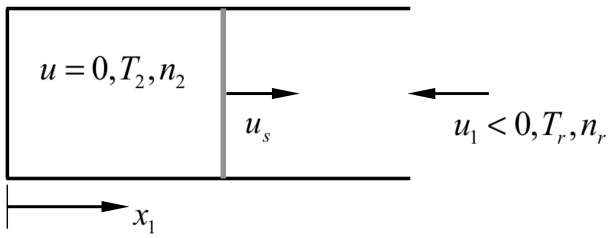
Semester Project



A monatomic gas, initially has a uniform scaled velocity $\hat{u}_1 = -1$ (independent of $\hat{x}_1$), and at $\hat{t} = 0^+$, a reflecting wall is inserted at $\hat{x}_1 = 0$. I have posted a Fortran computer program ProblD.f to solve the reflected shock problem (a one-dimensional transient flow) using the BGK approximation to the Boltzmann equation. All calculations are performed using scaled variables with the inflow gas conditions providing the reference values of density n_{ref}, and temperature, T_{ref}. The program uses $\eta_{ref} \equiv (kT_{ref}/m)^{1/2}$ as the reference velocity and the reference mean free path $\lambda_{ref} \equiv 1/n_{ref}\sigma_{ref}$ as the reference length, where σ_{ref} is a reference cross-section. As discussed in class this corresponds to $Kn = 1$ flow. The program solves the BGK equation numerically using time-splitting, with first order time-stepping and first-order upwind differences for the spatial (convective) derivative. It assumes the gas has a cross-section inversely proportional to velocity (pseudo-Maxwell molecules) so that $\hat{\nu} = \hat{n}$.

Input to the program is provided by the file 1Dinp.txt. You will need to vary input parameters via this file. There are some comments in the file but I summarize the input variables here.

Line 1: `nspace, ivxmin, ivxmax, ivymin, ivymax, ivzmin, ivzmax`

`nspace` = # of space points along the x-axis

`ivxmin` = negative integer specifying minimum index for x-velocity

`ivxmax` = positive integer specifying maximum index for x-velocity

`ivymin, ivymax, ivzmin, ivzmax` = corresponding variables for y - and z-velocity.

Line 2: `alphax, betav, deltat`

`alphax` = scaled x-distance between discrete points.

`betav` = scaled velocity space distance between discrete points (same for all components).

`deltat` = scaled discrete time step.

Line 3: `nd f, uf, Tf`

`ndf` = scaled freestream density

`uf` = scaled freestream velocity

item `Tf` = scaled freestream temperature

Line 4: `ntstep, nsplot, npr`

`ntstep` = total number of timesteps in calculation

`nsplot` = index of space location where distribution function is printed out.

`npr` = print interval in computational time steps for output of property profiles. The program is currently set-up so that maximum number of profiles that can be stored and then output is 11 (initial profile+10 others)

I recommend a computational domain of $\hat{x}_1 = [0, 50]$ as a minimum for final plots, though you should make it smaller when testing and debugging code. Verify that the values of `alphax, betav, deltat` and the range of velocity in velocity space (`ivxmin, ivxmax, ...`) you use in your calculations give converged and accurate results, i.e. the computed profiles do not change significantly when `alphax, betav, deltat` are decreased or `ivxmin, ivxmax ...` increased.

a. Plot scaled density ($\hat{n}$), x_1-velocity ($\hat{u}_1$), and temperature ($\hat{T}$) profiles at $\hat{t} = 10$, 20, 30, 40, 50.
b. Plot the cuts through the distribution $\hat{\varphi}(\hat{\eta}_1, \hat{\eta}_2, 0)$ at $\hat{t} = 40$, and $\hat{x}_1$=40, 45, 50. These are 3-D plots.
c. Plot the scaled heat flux $\hat{q}_1$ as function of $\hat{x}_1$ for $\hat{t} = 40$. Verify that $\hat{q}_{2,3} = 0$
d. Plot the scaled pressure $\hat{p} = \hat{n}\hat{T}$, and stress tensor component $\hat{\tau}_{11}$ as a function of $\hat{x}_1$ for $\hat{t} = 40$
e. If we assume hard sphere molecules $\hat{\nu} = \hat{n}\hat{T}^{1/2}$. Compare the shock density and temperature profiles for the two different cross-section models at $\hat{t} = 30$
f. Re-run the code with the upstream gas velocity, $\hat{u} = -2$. You may have to adjust the bounds in velocity space, and modify computational parameters appropriately. Compare the profiles of density and temperature for these two cases and comment on the differences.
g. Now modify the program to compute the collision integral using the ES-BGK equation for Maxwell molecules. Rerun the computations with $\hat{u} = -1$. Compare the scaled density, temperature, heat flux $\hat{q}_1$, and stress tensor $\hat{\tau}_{11}$ profiles at $\hat{t} = 40$. Compare these profiles with those obtained from the simple BGK equation and comment on the differences.
h. Estimate the shock wave speed for pseudo-Maxwell molecules and specify the uncertainty in your estimate.

Note:

From continuum gas dynamics it can be shown that the scaled shock velocity $\hat{u}_s$ in the lab reference frame is given by the positive root of the quadratic equation:

$$\hat{u}_s^2 + \frac{3-\gamma}{2}\hat{u}_1\hat{u}_s - \left[\gamma + \frac{(\gamma-1)}{2}\hat{u}_1^2\right] = 0.$$

Noting that $M = \hat{u}/\sqrt{\gamma}$, the Mach number of the flow into the shock is $M = (\hat{u}_1 + \hat{u}_s)/\sqrt{\gamma}$, you can use the property ratios across a normal shock from continuum gas dynamics to compare your computations of scaled pressure, density, and temperature, in the region behind the shock relative to upstream values. Remember that $\gamma = 5/3$ for a monatomic gas.

In your project report please comment on and interpret the results shown on each plot presented - I don't just want to see plots with the correct numerical values. Specify the values of computational parameters used to obtain any results presented.