Sand Processing

Sanderson and Sons are distributors of sand and similar materials. One part of their process involves receiving deliveries of sand which are fed downwards onto a conveyor belt, which moves with constant speed. The sand piles up and is transported across the conveyor belt, before falling off the end to continue processing. It is thought that vibrations in the conveyor, and possibly other effects, cause the sand to 'diffuse', as well as being transported by the conveyor, although this diffusion is likely small. Sanderson and Sons have experienced some problems with this process as the sand appears to pile up steeply towards one end of the conveyor belt, which can then interfere with the delivery process. They are principally interested in understanding how and why this accumulation occurs, and if the rate of delivery can be altered to prevent or suppress this; at the moment it is thought that the delivery occurs almost uniformly across the conveyor belt at an approximately constant rate in time.

The main aims of this study are to:

- Identify a suitable model for transport of sand on a conveyor belt;
- Explain why the sand appears to reach a 'steady state' with a large pile at one end of the conveyor;
- Determine how the height of the pile varies with how quickly the sand is delivered, the length of the conveyor belt, and its speed;
- Assuming that the conveyor belt can be between 5m and 10m long with a range of speeds between 0.25m/s and 0.5m/s, determine a range of delivery rates such the the sand pile remains below a given height of, say, 1m. Is this particular choice of height important?
- Investigate if there are 'optimal' ways of delivering the sand, for example, non-uniformly in space or time, where 'optimal' means minimizing the height of the pile whilst still ensuring that the sand is delivered to the next part of the process at a reasonable rate. You may also consider varying the speed of the conveyor belt over time.

Hints:

- You likely want to start with a 1D advection-diffusion equation and consider possible boundary conditions.
- Try to find an analytic solution to your problem, even if you have to make some reasonable approximations. Remember that the diffusion is supposed to be small, but compared to what?
- For numerics, you could use something simple such as forward Euler combined with finite difference, but be sure to validate your choice of step size and other parameters.
- It is likely that you will need to use your numerical simulations for the later aims of the project.