Industrial Mathematics Group Project 2: Traffic Networks

Group 1

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Executive Summary

It is commonly observed that road closures for maintenance and imperfect driving cause traffic congestion. The purpose of this report is to investigate how we can best model real-life road networks, the effect of roadworks on these networks, and assist Perthshire Road Agency in making effective road closure decisions to reduce traffic congestion.

This report builds on an existing single-lane road model by extending it to two and multiple-lane roads, also introducing a method of closing lane segments to simulate roadblocks. Furthermore we consider an extension to our model by specifying a maximum acceleration and deceleration and taking into account other cars' velocities to avoid over-breaking.

We validate our improvements by comparing simulated data to real-life data from the A31, M25, and M6, for which we see our model produces qualitatively accurate results. We also find the accuracy of our model decreases with the number of lanes, but increases with the length of the road. Importantly, we can see how increasing the number of lanes reduces traffic congestion, along with how it increases with road closure.

Our model can assist in visualising the impact of the road closure on the speeds of cars on an n-lane road but more work should be done using our model to determine an optimal road closure strategy. We recommend investigating 'warning distances' for road closure which maximises the total flow of cars.

1 The model

Agent-based models are commonly used in industry, e.g. population model [9], epidemiology [14] and pedestrian flow prediction [1]. We begin modelling traffic flow with an open-boundary Agent-based model in [10] as a suitable approximation for a single-lane road.

We define a road as a fixed array with L sites, where each site can be either empty or occupied by a single car. Every vehicle is restricted to move forward (in only one direction) with respective velocity $v \in [0, v_{max}]$, where v_{max} is a 'speed limit'. On each time step each car will move forward by v sites. For each car, if the next car is $j \leq v$ sites ahead, a driver will change the current speed from v to $j-1$ or to $v+1$ otherwise. To make the model more realistic, cars randomly reduce their velocity to $\max(v-1,\, 0)$ with probability p. We introduce a term called car density ρ defined as the number of cars N over the length of the road L, with $\rho = \frac{N}{L}$. At $t = 0$, we set the initial number of cars on a road by $N = \rho \cdot L$ and place them at random positions with random speeds $v \leq v_{max}$.
Boundary conditions can be specified by the first site and the length of the road. If the first site is empty, we uniformly generate an $\alpha \in [0, 1]$. If $\alpha \leq \rho\bar{v}$ we choose to place a car at the first site with a random $v < v_{max}$. Moreover, we drop cars out of the system if their next position is greater than L. We iterate these rules over a number of time steps to model the movement of traffic on a single-lane road.

When considering our model in a real-life scenario, using [10] we define one time-step to be one second, the length of a site to be 7.5m, and v_{max} to be 5 sites per second, which corresponds to 37.5 m/s (135 km/h). Using real motorway data [7], 135 km/h seems to be a realistic maximum speed, even though it exceeds the national speed limit of 112 km/h [8].

Note that the animations of each simulation can be found separately from the report.

2 Multi-lane roads

2.1 Two-lane roads

With regards to developing our model, we first implement a two-lane road. The model is comprised of two adjacent single-lane roads as seen in Section 1, however, we introduce more rules to allow for lane changing. We found in [12] the easiest way to do this is to have cars instantaneously change lanes and then move forward on two different steps. They encourage further work to look at the effect of combining these two steps, which is what we have decided to do. The lane-changing algorithm considers the distance in front of the car for the current lane, the distance in front in the target lane and the distance behind the car in the target lane - this is to ensure that we can safely switch lanes without the cars behind us in the target lane crashing into us or cause them to brake to a stop.

This gives us an algorithm for a car i in position $x(i)$, with velocity $v(i)$, proven to be theoretically valid in [12]. Let $j_a(i)$ be the gap in front of i and $j_{ab}(i)$ be the gap behind i in the next lane respectively. If the current position in adjacent lane $x_a(i)$ is free, we define $\text{weight}_1 = 1$ if the space in front in current lane $j(i) < v(i)$ and $j_a(i) < v$. Otherwise $\text{weight}_1 = 0$. Then we characterise the gaps in next lane by $\text{weight}_2 = v - j_a(i)$ and $\text{weight}_3 = v_{max} - j_{ab}(i)$. If $\text{weight}_1 > \text{weight}_2$ and $\text{weight}_1 > \text{weight}_3$, the car moves to the targeted lane. We implement this algorithm to give us Figure 1.

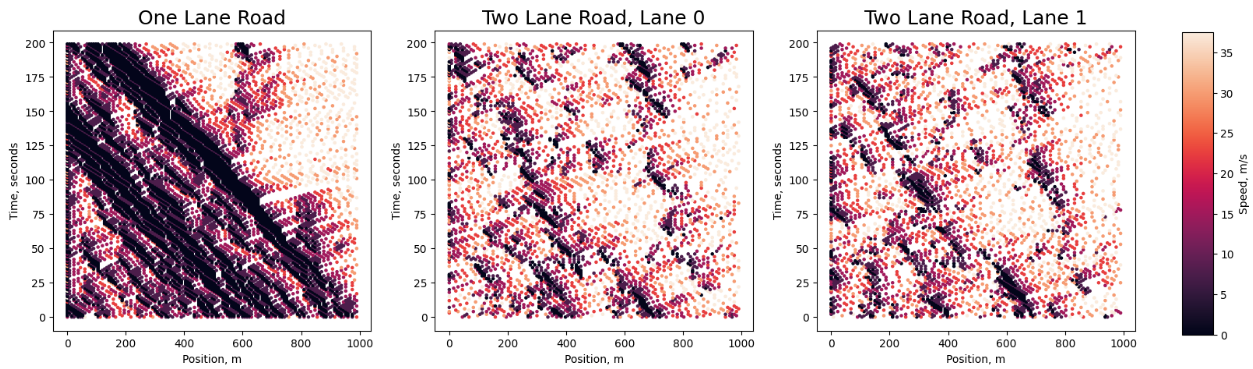


Figure 1: One-lane vs two-lane road, with consistent $N \approx 50$ and $p = 0.3$

In Figure 1 (and Animations 1 and 2), the darker regions are where the traffic is slow and the lighter regions are where the traffic is fast. With the same number of cars on the road, we see far more traffic build up in the one-lane model, which can be expected. We also notice that the two lanes on the same road are effectively equivalent because there is no bias toward one lane over another unlike in the real world where we may have a 'slow' lane. Thus the formation of traffic jams in the two-lane model should follow the same pattern as in the single-lane model. We aim to repeat this result for more than two lanes.

2.2 More than two lanes

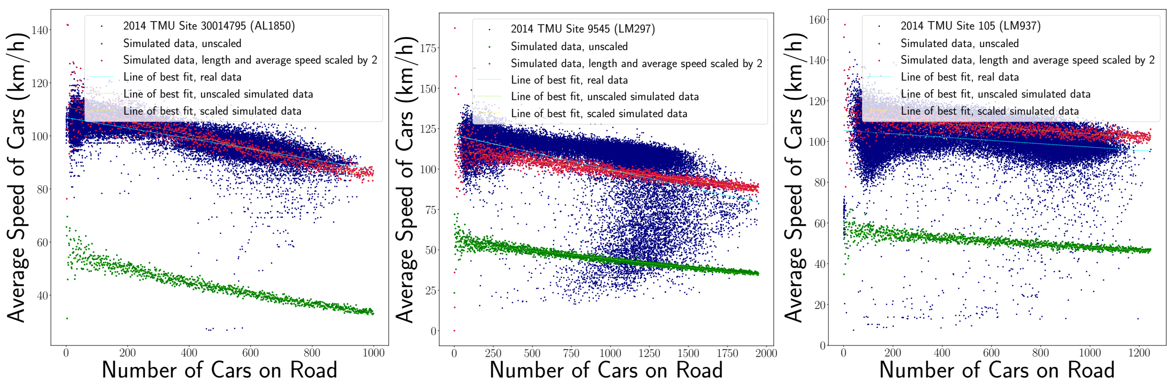
When extending this lane-switching algorithm to $n > 2$ lanes we must exercise some caution. Consider the following scenario for 3 lanes: a position becomes available in the middle lane, a car currently in the left-most lane may wish to lane change into it, but so may a car currently in the right-most lane. If they both switch at the same time then they would crash. To avoid this we introduce an additional rule; cars can only switch to lanes on their left and right on odd and even time steps respectively.

3 Real-world data

In this section, we compare our multi-lane model to real roads with data in 2014 from Highways England [6, 7, 5]. We choose two-lane data from the A31 between M27, J1 and A338 (TMU Site 30014795), four-lane data from the M25 between J9 and J10 (TMU Site 9545), and six-lane data from the M6 between J16 and J15 (TMU Site 105). Using journey time data from Highways England [4, 3, 2], we know the A31 segment is a primary A-road dual carriageway with a length of 16760m (2235 sites); and the M25 and M6 segment are motorway segments with a length of 9360m (1248 sites) and 15320m (2043 sites) respectively. We will use the real-life speed and site length definitions from Section 1.

To validate our model with real-world data, we run our n-lane weighted model over different car densities and find the average speed of all the cars on the road after a limiting number of time steps (a few thousand). When testing this method, we find p does not make much of a qualitative impact on our results, so we arbitrarily pick $p = 0.1$ in Figure 2.

The green data points in Figure 2 is the data we get from our model using the same parameters as the real-world data, and the red data points are found by doubling the length and the resultant average speeds. We see the red data lines up much better over all the simulations. We hypothesise this difference is due to the over-breaking of cars in our model - a car treats a car ahead as stationary so it will slow down to stop, rather than match the speed of the car in front. Increasing the road length absorbs this difference, as cars will generally be further apart, so this is something to consider when improving the model.



(a) A31 segment (two-lanes) (b) M25 segment (four-lanes) (c) M6 segment (six-lanes)

Figure 2: Average speed against the number of cars for roads with different numbers of lanes

Even though our model was out by a scale factor, importantly the qualitative patterns that emerge are the same. We see a small negative regression, meaning the average speed drops with more cars on the road, as would be expected on a real road. We can see that our simulated data has a relatively low variance, and there are more outliers in the real data. This is especially noticeable for the four-lane motorway, for which there is a bell curve shape - this may be due to other real-world factors such as crashes or vehicles running at different maximum speeds, above or below the speed limit.

By comparing Figure 2a and 2b, we observe our approximation is slightly less accurate for more lanes, even under similar-length roads. We can also observe from Figure 2c and 2b that the approximation is more accurate for a longer road under the same density. Nevertheless, both estimations of multi-lane roads can also be improved by taking advantage of scaling. Therefore we can make conclusion that our model works qualitatively well for all lanes, but is visibly more accurate for longer roads and two-lane roads.

4 Road Closure

To meet Perthshire Road Agency's request regarding the effects of road closures on their road network, we will update the multi-lane model to allow for closures in specified lanes. Within our model, we select a 'block' in one or more of the lanes by specifying a fixed start and end site. Cars perceive roadblocks as stationary cars, and thus the lane-changing algorithm is still relevant here. However, there is a slight tweak to be made; if there is a roadblock that ends at position i in the adjacent lane, the car i will treat j_{ab} as infinite when it looks behind in the lane-changing algorithm, because switching onto that lane will not cause the car behind it (a roadblock) to crash into the back of the switching car. This is best understood with the following illustration:

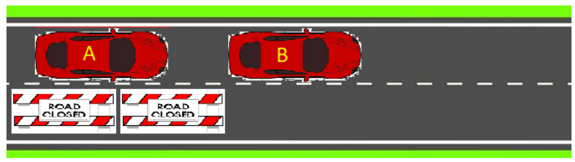


Figure 3: Cars Switching Lanes after a roadblock ends

Car B in Figure 3 will see the gap behind it as infinite, which ensures the algorithm only cares about the forward gaps and $j_{ab}(i)$ is not a limiting factor when we are at the end of a roadblock. Car A will also see the gap behind it in the adjacent lane as infinity, but since the adjacent spot $x_a(i)$ is not free it will not consider a lane change.

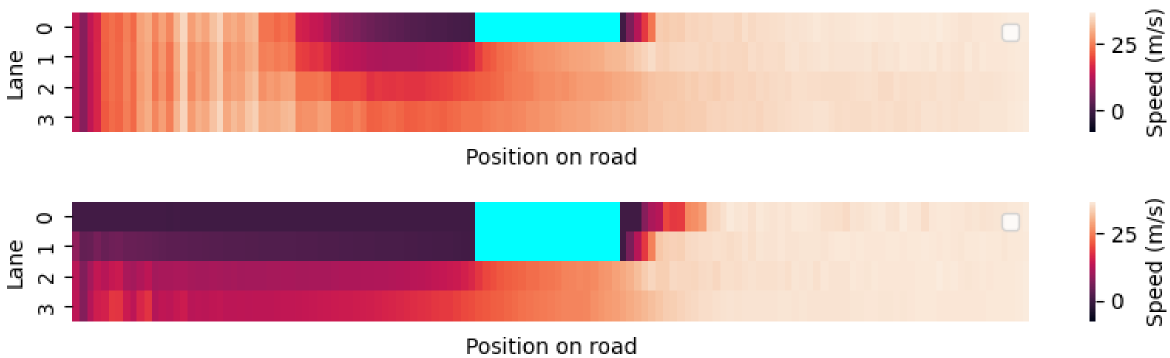


Figure 4: Average car speeds on a 1000m road with a roadblock (blue) at 425m-575m

Under the same road density ρ, it is evident from the example that traffic jams are more likely to occur when a road closure is present. An example of a four-lane road taking $\rho = 0.25$ and $p = 0.1$ with a road closure on one lane (Animation 3) and two lanes (Animation 4) can be seen in Figure 4. We average the speed of all the cars that appear on that site over a large number of time steps. The darker sections indicate where cars are the slowest on the road, while the lighter parts show where cars are the fastest. As expected, we see the slowest speeds occur in the lane(s) just before the roadblock, and once cars move back into those lanes they speed up significantly. We see how the effect of the roadblock on speed dissipates with the number of lanes away from the roadblock, e.g., cars in lane 3 move quicker than the lane adjacent to the roadblock. Lastly, we see that shutting two lanes rather than one decreases the speed of each lane 2 and 3 dramatically, which is expected, as more cars must travel through a smaller bottleneck. We would therefore suggest that it is best to schedule non-emergency roadworks on only one lane at a time. Perth Road Agency will be able to use this model to simulate the effects of the lane closure on their roads, knowing the length of the road and the typical number of cars on that road in a time period. Notice that we can also place a roadblock on the last site of a road lane to simulate a merging lane junction. This is not entirely efficient but allows us to model a wider range of real-life road networks.

5 Model Extensions

The current base model for agent decisions ensures that no crashes occur, using instantaneous deceleration. This assumption comes at a cost: drivers are more conservative than in reality, and cars can decelerate at unrealistic rates. This difference between simulation and reality is notable on the highways chosen for real-world data.

To extend the model, space and time can be discretised further (into Δx and Δt respectively). As a consequence, cars need to be assigned a length l. Cars are given a maximum acceleration ($a_{max} = 3\,\mathrm{m\,s^{-2}}$) and deceleration ($a_{min} = 5\,\mathrm{m\,s^{-2}}$) [13]. Agents also include nearby cars' velocity in their decisions (not only distance), still in an attempt not to crash. The updated agents' decisions can be described as follows:

1. **Acceleration:** $[v \to min(v_{max}, v + \Delta ta), a \in \{1, 2, ..., a_{max}\}]$.
2. **Deceleration:** $[v \to max(0, v - \Delta ta), a \in \{1, 2, ..., a_{min}\}]$.
3. **Randomisation: Deceleration** with probability p.
4. **Motion:** Each vehicle advances $v\Delta t$ sites.

Cars will accelerate if they are below some critical speed, and decelerate if they are above it. In our case, we have defined critical speed as a speed at which a driver will be able to react (with reaction time t_r) and brake in time to not hit the driver ahead. This can be expressed as

$$v_{crit} = \frac{d - l_a + n\Delta t v_a + \frac{1}{2}(\Delta t)^2 a_{min}(n(n+1) - m(m+1))}{m\Delta t + t_r}, \quad n = \left\lfloor \frac{v_a}{\Delta t a_{min}} \right\rfloor, \quad m = \left\lfloor \frac{v}{\Delta t a_{min}} \right\rfloor.$$

where d is the distance to the car ahead and subscript $_a$ is used to denote the car ahead. This change has been implemented in the single-lane model (Animation 5), but more adaptation to lane-switching decisions would be necessary to expand the model. This is a potential avenue for future research.

6 Conclusion and Future Work

In conclusion, our model can be used by Perth Road Agency to give a good approximation on multiple-lane traffic flow, which is backed up by real data, and can be used to investigate the effects of a specific lane closure, should they wish to carry out maintenance on one of their own roads where they know the length and typical traffic flow rates.

However, there are many modifications that can be done to make our model more logically sound. Cars currently merge by treating a roadblock as a stationary car, but in real life cars may be warned via signage ahead of time. We can find an optimal strategy inspired by [11] for road closure by defining a concept called warning distance d to implement the lane-changing resulting from the road closure. If a car is less than d sites away from the block, the driver will decide to leave the lane. We can then decide on an optimal strategy by finding a warning distance that maximises the total flow on the road.

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