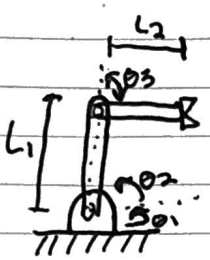
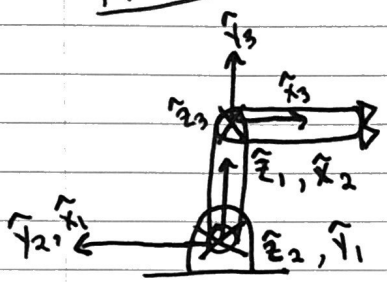
3 DOF Robotic manipulator



θ_1 is spinning perpendicular to θ_2. θ_1 rotates base, θ_2 rotates base-Link1 joint.

Frames:



Link parameters:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	L_1	0	θ_3

$$
{}^0_3T = \begin{bmatrix} -S_3C_1 + C_1C_2C_3 & -S_2C_1C_3 - S_3C_1C_2 & -S_1 & L_1C_1C_2 \\ -S_1S_2S_3 + S_1C_2C_3 & -S_1S_2C_3 - S_1S_3C_2 & C_1 & L_1S_1C_2 \\ -S_2C_3 - S_3C_2 & S_2S_3 - C_2C_3 & 0 & -L_1S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$X = L_1\cos(\theta_1)\cos(\theta_2) \quad Y = L_1\sin(\theta_1)\cos(\theta_2) \quad Z = -L_1S_2$

$$X^2+Y^2 = L_1^2\cos^2(\theta_1)\cos^2(\theta_2) + L_1^2\sin^2(\theta_1)\cos^2(\theta_2)$$

$$X^2+Y^2 = L_1^2\cos^2(\theta_2) \qquad \cos^2(\theta_2) = \frac{X^2+Y^2}{L^2}$$

$$\sin^2(\theta_2) = 1 - \cos^2(\theta_2)$$

$$\sin^2(\theta_2) = 1 - \frac{X^2+Y^2}{L^2}$$

$$\frac{\sin^2(\theta_2)}{\cos^2(\theta_2)} = \frac{1 - \frac{X^2+Y^2}{L^2}}{\frac{X^2+Y^2}{L^2}} \qquad \tan^2(\theta_2) = \frac{L_1^2}{X^2+Y^2} - 1$$

$$\tan(\theta_2) = \sqrt{\frac{L_1^2}{x^2+y^2} - 1}$$

$$\theta_2 = \arctan\left(\sqrt{\frac{L_1^2}{x^2+y^2} - 1}\right)$$

$$\frac{y}{x} = \frac{L_1 S_1 C_2}{L_1 C_1 C_2} = \frac{\sin(\theta_1)}{\cos(\theta_1)} \qquad \frac{y}{x} = \tan(\theta_1)$$

$$\theta_1 = \arctan\left(\frac{y}{x}\right)$$

$$\theta_3 = \phi - \theta_1 - \theta_2$$