Product Market Power as a Labor Distortion: the Case of Home Production Substitutes

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PRELIMINARY AND IN PROGRESS

Abstract

Imperfect competition in markets for home inputs constrains households' ability to outsource or automate home production and, when women bear the burden of home production, restricts female labor supply. Perhaps surprisingly, this channel has not been incorporated or quantified in classic models in family macroeconomics. We fill this gap by introducing monopolistic competition in the market for home production substitutes into a simple model of household time allocation. Households consume market goods and home-produced goods. The latter are produced using a combination of the wife's time and home inputs, where the market for home inputs is monopolistically competitive. We show that markups on home inputs reduce female hours worked relative to the efficient benchmark. We also show that market power for home inputs can dampen or amplify the response of female hours worked to other economic forces, such as technological change lowering the physical cost of these inputs and the narrowing of the gender wage gap.

Keywords: Market power, home production, female labor supply, gender inequality, antitrust

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Introduction

Despite the large gender convergence in labor force participation and other labor market outcomes over the last century, significant gender gaps remain.[1] Although a vast literature has put forth multiple explanations for this gender convergence as well as remaining obstacles, the ability to outsource or automate home production has recently emerged as a major explanatory factor. Because the bulk of home production has historically been and continues to be borne by women, time-saving technology or marketization of home activities lowers the opportunity cost of female labor force participation.[2] Perhaps surprisingly, the effect of *market structure* in these markets on female labor force participation has thus far been almost completely absent from the analysis.

This is a surprising and important omission for several reasons. First, there has been a well-documented rise in markups overall (see e.g. De Loecker, Eeckhout, and Unger 2020), which are understood theoretically to be an important potential determinant of labor supply. Second, some of the markets most directly relevant to home production are well-known to feature imperfect competition, such as childcare (Bastos and Cristia 2012 and Berlinski, Ferreyra, Flabbi, and Martin 2024) or infant formula (Federal Trade Commission 2024 and Wang 2024). If prices of home production substitutes deviate from their marginal cost, this may constitute an additional obstacle to rising female labor force participation. Moreover, there may be important interactions between market power and other economic forces influencing female labor supply. For example, technological progress in home production substitutes, as documented e.g. by Greenwood, Seshadri, and Yorukoglu (2005) and Albanesi and Olivetti (2016), may not fully pass through to prices if the relevant markets are imperfectly competitive. Likewise, market trends or policies reducing the gender wage gap may have a muted effect on female labor force participation if the latter is constrained by high prices of home production substitutes.

In this paper, we study the connection between product market power and female labor supply. To do so, we incorporate imperfect competition into a simple model of household time allocation, following a modeling approach similar to Albrecht, Phelan, and Pretnar (2025). The model considers a household consisting of a husband, who works in the market full time, and a wife, who divides time between market work and home production. The household consumes market goods and home-produced goods where the latter can be produced using a combination of home production time and household inputs. The crucial departure from

[1] See e.g. Goldin (2014) and Olivetti and Petrongolo (2016) for details.

[2] The importance of home production substitutes has already been recognized by a large literature, such as Greenwood, Seshadri, and Yorukoglu (2005) in the context of home appliances, and Albanesi and Olivetti (2016) in the context of infant formula.

the traditional family economics framework[3] is that we assume imperfect competition in *both* market goods and home production substitutes. On the market good side, we assume that the household has preferences over a continuum of varieties, each produced by a monopolistic firm as in Dixit and Stiglitz (1977). On the home production side, we assume that the household consumes a continuum of varieties of home goods, where each variety can in turn be produced using a combination of time and an input supplied by a monopolistic firm.[4] Finally, there is an exogenous gender gap in wages when working in the market. Importantly, the model nests the workhorse framework in family economics (Greenwood, Guner, and Vandenbroucke 2017) as well as standard applications of the Dixit and Stiglitz (1977) monopolistic competition framework.

We first characterize the equilibrium. The model is tractable and yields expressions for the equilibrium levels of consumption of the two types of goods and time allocation. We derive markups on market goods as well as home production substitutes. We show that the markup on home production substitutes is endogenous and depends on the elasticity of substitution between time and inputs within a variety as well as the elasticity of substitution across varieties. The markup also depends on the gender wage gap, because the wife's market wage determines the opportunity cost of time, which is a substitute for the home input.

We then use the model to ask several questions about the implications of market power for home inputs. First, we examine the efficiency properties of equilibrium and the effects of markups on labor supply. Not surprisingly, efficiency requires for both market goods and home inputs to be priced at marginal cost. In the unregulated market equilibrium, markups in the two sectors drive consumption in opposite directions: markups on market goods lead households to substitute away from market consumption and toward home-produced goods, whereas markups on household inputs do the opposite. However, both markups increase home production time and thereby reduce female hours worked. The markup on market goods does so through a standard mechanism, by essentially acting as a sales tax (Edmond, Midrigan, and Xu 2023). The markup on home production substitutes does so by making these substitutes more expensive, causing the household to rely more on its own time in producing home goods.

Second, we conduct comparative statics with respect to several key parameters, among them the physical cost of producing home inputs and the gender wage gap. In doing so, we seek to relate to classic work in the macroeconomics of the family, which identifies the narrowing gender wage gap (Jones, Manuelli, and McGrattan 2015) and technological progress

[3] See Greenwood, Guner, and Vandenbroucke (2017) for a survey.

[4] In that regard, the production side for home inputs resembles a monopolist with a competitive fringe as in e.g. Ferraro, Ghazi, and Peretto (2024), except that here the household's own time plays the role of a competitive fringe.

in home production substitutes (Greenwood, Seshadri, and Yorukoglu 2005) as key factors behind rising female labor force participation. We show that endogenous markups on home production substitutes not only affect the *level* of female labor supply, but also affect its *response* to these exogenous factors. The endogenous response of markups can serve as either a dampening or an amplifying force, depending on the relative size of the elasticities of substitution across and within home good varieties. When the degree of substitutability across varieties is relatively low, market power for home inputs dampens the response of prices to marginal costs. In other words, market power results in imperfect pass-through of costs to prices. The reason is that lower costs of home production substitutes make households more reliant on these products; this lowers the elasticity of demand and raises the markup. As a result, falling costs of home production substitutes have a muted effect on labor supply.

Perhaps more interestingly, when the degree of substitutability across varieties is relatively low, market power also dampens the response of labor supply to a rising female wage. When the wage rises, so does the opportunity cost of home production. This makes households substitute towards purchased home inputs and away from home production time. Since substitutability across varieties is low, firms face a less elastic demand curve and can thus raise their markups. The rising markup partially offsets the effect of a higher wage.

Our results have important implications both for interpreting observed trends and for policy design. In terms of interpreting observed trends, our results suggest that quantifying the effect of exogenous technical change or falling gender wage gap should be done taking markups into account. Moreover, rising markups themselves may play an important role in explaining patterns in female labor supply. With regard to policy, our results suggest that a given policy, e.g. one narrowing the gender wage gap, may be less effective in the presence of market power. On the other hand, the results also suggest that policies aimed at reducing product market power, e.g. antitrust regulation, can play an important role in facilitating female labor supply.

Related Literature. A vast literature has identified time-saving technology and markets as major contributors to increasing female labor force participation. This research documents how falling costs of home production substitutes—from home appliances and electrification to childcare and infant formula—have reduced the time burden of home production and enabled greater female labor market participation (see e.g., Greenwood, Seshadri, and Yorukoglu, 2005, Attanasio, Low, and Sánchez-Marcos, 2008, Cavalcanti and Tavares, 2008, Coen-Pirani, León, and Lugauer, 2010, Dinkelman, 2011, and Albanesi and Olivetti, 2016). A related strand of this literature also shows that such technological progress helps explain broader demographic trends including marriage, divorce, and fertility patterns (Greenwood and Guner, 2008, Greenwood, Guner, Kocharkov, and Santos, 2016, Santos and Weiss, 2016, and Bar, Hazan, Leukhina, Weiss, and Zoabi, 2018, and Cerina, Moro, and Rendall, 2021).

All of these studies treat the relevant product markets as perfectly competitive, implying, in particular, that the falling costs of these products are passed through completely to prices. Relative to this literature, our contribution is to study the effect of market power for home inputs.[5] We argue that deviating from the perfectly competitive benchmark in these markets has important implications. First, the trend in prices of home inputs can no longer be interpreted as stemming from costs only. Instead, the trend in markups of these products has to be taken into account. This matters for counterfactual experiments, e.g. inferring the effect on female labor supply if the technological progress had not occurred. Second, market power interacts with other driving forces such as the falling gender wage gap. For example, consider the exercise of disentangling the contributions of the falling gender wage gap and the falling price of home appliances. A standard way of doing so would assume that a fall in the gender wage gap would leave the prices of home appliances unaffected. This is not the case in our model, since the prices of home inputs respond to demand through monopolistic pricing. Third, the analysis opens the door to additional policy avenues, namely regulation of markets for home inputs, for impacting female labor supply.

This paper also contributes to a large and growing literature on the implications of market power for labor supply and aggregate output, much of it building on the monopolistic competition framework of Dixit and Stiglitz (1977). The documented rise in markups in the U.S. (De Loecker, Eeckhout, and Unger 2020) has in recent years spurred a vibrant literature, among which De Loecker, Eeckhout, and Mongey (2021) and Edmond, Midrigan, and Xu (2023) examine the implications for labor supply. These studies focus on "neutral" changes in markups, which act as a sales tax on products and therefore distort labor supply. Moreover, the focus is on male labor supply in these papers. In contrast, our model features markups on products that are substitutes to home production. We show that the effects are qualitatively similar in some respects but different in others, and the framework is suitable for thinking about markets of particular relevance for household time allocation and therefore female labor supply. Our framework is closest to the innovative work by Albrecht, Phelan, and Pretnar (2025), who likewise allow household time to be substitutable with market-purchased goods and show that this has important implications for the inefficiency of markup heterogeneity. We see our work as closely complementary and addressing a different research question. In

[5] While some work has certainly documented market power in specific home input markets – such as childcare (Bastos and Cristia, 2012 and Berlinski, Ferreyra, Flabbi, and Martin, 2024), infant formula (Wang, 2024), and ready-to-eat foods (Nevo, 2001), we undertake a macro approach by considering market power for such products collectively, and, most importantly, our focus is on its effects on female labor supply rather than consumer welfare in each specific market.

Albrecht, Phelan, and Pretnar (2025), the focus is on how households' ability to use time as a substitute affects markups on products and the resulting allocation of resources across firms. Conversely, the focus of our analysis is on how market power for certain products affects the household's allocation of time. Furthermore, unlike Albrecht, Phelan, and Pretnar (2025), we explicitly interpret these products as home production substitutes and focus on the effect on female labor supply.

Since our focus is ultimately on labor supply distortions, our work is also related to the growing literature on the welfare losses from labor market power, exemplified by Berger, Herkenhoff, and Mongey (2022) and Jungerman (2023). Importantly, the existing work on monopsony has not stressed the gender dimension (with recent work by Sharma (2023) being an important exception). Even more importantly, our mechanism here is different and novel, as we argue that *product* market power leads to a particular distortion in *labor* markets when the products are home production substitutes.

Finally, our analysis of market power and its regulatory implications likewise makes our paper relevant to recent developments in antitrust law, as emphasized in Sanders (2024). The first development is an increasing focus on gender inequality issues as they relate to antitrust regulation, a connection that has increasingly attracted the attention of competition authorities around the world.[6] Despite the close reliance of antitrust law on economic theory, theoretical foundations for connecting market regulation to gender inequality have not yet been put forth. Our analysis of markets for home production substitutes is motivated by the recent work of Sanders (2024), who argues that prioritizing these markets can promote gender equality simultaneously with efficiency improvements. The second development is an increasing focus of antitrust on labor markets, as exemplified e.g. in Marinescu and Hovenkamp (2019), Marinescu and Posner (2019), and Berger, Hasenzagl, Herkenhoff, Mongey, and Posner (2023). As explained above, our approach is distinct in emphasizing that product market power can act as a wedge suppressing efficiency of labor supply, and corresponding regulation of product markets therefore has labor market effects.

1 Model

We study a static representative-household model where the household derives utility from consumption of market and non-market goods. Non-market goods can be produced using a combination of home time as well as intermediate inputs, which we term *home production*

[6] In 2018, the OECD hosted a global forum examining the intersection of gender and competition. Two years later, with support from the Canadian government, the OECD launched the Gender Inclusive Competition Policy Project, which focuses on developing new evidence to guide a gender conscious antitrust policy.

substitutes. Both market goods and intermediate inputs into non-market goods are produced using labor. The household chooses how to allocate its time between labor and home production, and how to allocate its income between market goods and intermediate inputs. The key departure relative to the existing family economics literature is that both the market good and the intermediate input for home production are produced by monopolistic firms à la Dixit and Stiglitz (1977) that charge endogenous markups.

1.1 Setup

Environment. The economy is static and populated by a representative household and two types of firms: a continuum of firms producing market goods and a continuum of firms producing intermediate inputs into home production. The representative household consists of a husband and a wife and maximizes joint utility.

Preferences. The household's utility is given by

$$\alpha \ln(C) + (1-\alpha)\ln(X), \tag{1}$$

where $\alpha \in (0,1)$, C denotes consumption of market goods (taken to be the numéraire), and X denotes consumption of non-market goods.

Technology. Total consumption of market goods is a CES aggregator

$$C = \left(\int_0^1 c_j^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\eta > 1$ is the elasticity of substitution across varieties j. Similarly, total consumption of non-market goods is a CES aggregator

$$X = \left(\int_0^1 x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where σ is the elasticity of substitution across varieties i. Each variety i of non-market goods can be produced using a combination of time ℓ_i and intermediate inputs d_i according to

$$x_i = \left(\phi \ell_i^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) d_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

where $\phi \in (0,1)$ and we assume $\epsilon, \sigma > 1$.

The household is endowed with one unit of male time and one unit of female time. We assume that the husband works in the market full-time, whereas the wife's time is split between working in the market and home production. The husband supplies one unit of efficiency units of labor inelastically. There is a gender wage gap: each unit of male time supplies 1 efficiency unit of labor, whereas 1 unit of female time supplies $\gamma \in (0, 1)$ efficiency units of labor. For the purposes of this analysis, γ can be interpreted either as a gap in productivity or as taste-based discrimination by employers.[7] Denote the male wage by w_m and the female wage by w_f. We assume labor markets are competitive which implies that $w_f = \gamma w_m$.

Both market good varieties j and non-market good varieties i use a linear technology, but we allow for different labor requirements for their production. In particular, we assume that one efficiency unit of labor produces one unit of each market good variety, while one efficiency unit of labor produces $1/\zeta$ units of each intermediate input, where $\zeta > 0$.

Market Structure. The market structure for both types of goods is monopolistically competitive à la Dixit and Stiglitz (1977). There is a monopolist firm producing each market good variety j. Similarly, there is a monopolist firm producing each intermediate input variety i. We assume that the household owns both types of firms and receives their profits as lump-sum dividends.

Discussion. The key parameters of the model are ϵ, the elasticity of substitution between time and home inputs, and σ, the elasticity of substitution between different varieties of home inputs. The former can be thought of as a technological parameter (see. e.g. McGrattan, Rogerson, and Wright 1997 or Fang, Hannusch, and Silos 2020). The latter captures the degree of product differentiation, product-specific preferences, or switching costs present in the economy. Home inputs are imperfect substitutes for home production time ($\epsilon < \infty$) and for each other ($\sigma < \infty$). Like Albrecht, Phelan, and Pretnar (2025) (and unlike, e.g., Atkeson and Burstein 2008 or De Loecker, Eeckhout, and Mongey 2021) the model does not assume a ranking between ϵ and σ a priori. To give an example, this implies that two different brands of frozen foods may be more easily substitutable for each other than either one is for cooking at home. On the other hand, commuting costs can make two daycare centers in different geographic regions very poor substitutes with one another, relative to either provider's substitutability with home-provided childcare. Since our stylized model is intended to capture a range of non-market goods and home activities, we allow for either possibility, and the magnitude of substitutability is ultimately an empirical question which

[7] Both interpretations lead to the same results.

we discuss in the quantitative application of the model.

1.2 Household

The household maximizes (1) subject to Equations (2) to (4), as well as the budget constraint,

$$\int_0^1 p_j c_j \mathrm{d}j + \int_0^1 r_i d_i \mathrm{d}i = w_m + w_f \left(1 - \int_0^1 \ell_i \mathrm{d}i\right) + \Pi \tag{5}$$

and feasibility constraints on hours worked,

$$\int_0^1 \ell_i \mathrm{d}i \leq 1, \tag{6}$$

and

$$\ell_i \geq 0 \ \forall i, \tag{7}$$

where p_j denotes the price of the market-good variety j, r_i denotes the price of the intermediate input i, w_m and w_f denote the male and female wage, respectively, and $\Pi := \int_0^1 \pi_i^d \mathrm{d}i + \int_0^1 \pi_j^c \mathrm{d}j$ is the sum of rebated firm profits. The left-hand side of Equation (5) consists of the household's expenditures on market goods and intermediate inputs into home production. The right-hand side captures the household's income, noting that the female time spent working equals the time left over from home production. Equation (6) is a feasibility constraint on the total time spent in home production, where we normalize the endowment of time to 1. Equation (7) states that time in each home production activity cannot be negative. The solution to the household's problem gives a tuple of policy functions

$$\{x_i(\mathbf{p}, \mathbf{r}, w_m, w_f, \Pi), d_i(\mathbf{p}, \mathbf{r}, w_m, w_f, \Pi), \ell_i(\mathbf{p}, \mathbf{r}, w_m, w_f, \Pi), c_j(\mathbf{p}, \mathbf{r}, w_m, w_f, \Pi)\},$$

where $\mathbf{p} = \{p_j\}_{j\in[0,1]}$ is the vector of prices for market goods and $\mathbf{r} = \{r_i\}_{i\in[0,1]}$ is the vector of prices for home inputs.

1.3 Firms

Market good producer j chooses the price of its variety p_j given prices of its competitors $\mathbf{p_{-j}}$, prices of the home substitute goods $\mathbf{r}$, and wages $\{w_m, w_f\}$ to maximize profits

$$\pi_j^c := \max_{p_j} (p_j - w_m) c_j(p_j; \mathbf{p_{-j}}, \mathbf{r}, w_m, w_f, \Pi) \tag{8}$$

where $c_j(\cdot)$ is the demand function for good j taken from the household problem. To conserve notation, we will suppress dependence on the aggregate price vectors and profits and write simply $c_j(p_j)$. Similarly, home good producer i chooses the price of its variety r_i to maximize its profits

$$\pi_i^d := \max_{r_i}(r_i - \zeta w_m) d_i(r_i; \mathbf{p}, \mathbf{r_{-i}}, w_m, w_f, \Pi) \tag{9}$$

where, again, we write $d_i(r_i)$ from now on.

1.4 Market clearing

The market clearing condition for this economy is simply the aggregate resource constraint,

$$1 + \gamma \left(1 - \int_0^1 \ell_i \mathrm{d}i\right) = \int_0^1 c_j \mathrm{d}j + \zeta \int_0^1 d_i \mathrm{d}i, \tag{10}$$

which states that total labor supply must equal total labor demand.

1.5 Equilibrium

We are now ready to define an equilibrium of this economy:

Definition 1 (Equilibrium)**.** *An equilibrium is a list of household consumption rules and policy functions* $\{x_i(\cdot), d_i(\cdot), \ell_i(\cdot), c_j(\cdot)\}$ *as well as a vector of prices* $\{\mathbf{p}, \mathbf{r}, w_m, w_f\}$ *and household rebated profits* $\Pi := \int_0^1 \pi_i^d \mathrm{d}i + \int_0^1 \pi_j^c \mathrm{d}j$ *such that*

1. *the household's policies maximize* (1) *subject to the constraints in Equations* (2) *to* (7)*,*

2. *the prices* p_j *and* r_i *and the equilibrium profits* $\{\pi_j^c\}$ *and* $\{\pi_i^d\}$ *are solutions to Equations* (8) *and* (9)*, respectively, subject to the household demand functions* $\{x_i(\cdot), d_i(\cdot), \ell_i(\cdot), c_j(\cdot)\}$*,*

3. *the wages* w_m *and* w_f *are pinned down in a competitive labor market with*

$$w_f = \gamma w_m,$$

4. *and the market clearing condition* (10) *is satisfied.*

2 Equilibrium characterization

2.1 Household decisions

To characterize labor supply and demand for home inputs, we solve the household problem taking prices as given. First, we note that constraint (7) will not bind because of Inada conditions on the functional forms. Constraint (6) binds if the wife chooses not to work, and is slack otherwise. We focus throughout on parameter values such that the equilibrium is interior, i.e. (6) does not bind and the wife's labor hours are strictly positive.[8] We can then characterize the household's optimal choice as follows:

Proposition 1. *In an interior equilibrium, the household's optimal choices satisfy*

$$c_j = \alpha \frac{p_j^{-\eta}}{P^{1-\eta}}(w_m + w_f + \Pi) \tag{11}$$

$$x_i = (1-\alpha)\frac{q_i^{-\sigma}}{Q^{1-\sigma}}(w_m + w_f + \Pi) \tag{12}$$

$$d_i = (1-\phi)^{\epsilon}(1-\alpha)\frac{r_i^{-\epsilon} q_i^{\epsilon-\sigma}}{Q^{1-\sigma}}(w_m + w_f + \Pi) \tag{13}$$

$$\ell_i = \phi^{\epsilon}(1-\alpha)\frac{w_f^{-\epsilon} q_i^{\epsilon-\sigma}}{Q^{1-\sigma}}(w_m + w_f + \Pi) \tag{14}$$

where we define the price indices q_i, Q, and P to be

$$q_i := \left[\phi^{\epsilon} w_f^{1-\epsilon} + (1-\phi)^{\epsilon} r_i^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{15}$$

$$Q := \left(\int_0^1 q_i^{1-\sigma} \mathrm{d}i\right)^{\frac{1}{1-\sigma}} \tag{16}$$

$$P := \left(\int_0^1 p_j^{1-\eta} \mathrm{d}j\right)^{\frac{1}{1-\eta}} \tag{17}$$

Proof. See Appendix A.1. □

The nested Cobb-Douglas and CES structures of household preferences implies a standard characterization of optimal choices, whereby the expenditure shares on each product depend on its price relative to an aggregate price index. The household's problem can be thought of as proceeding in two steps. For any amount of non-market goods, the household chooses the cost-minimizing combination of time and home inputs to obtain those non-market goods.

[8] Since the model is calibrated to an economy with positive female labor force participation, this will be the empirically relevant case. A necessary and sufficient condition on parameters for an interior equilibrium, as well as the characterization of equilibrium with non-participation, are available on request.

The good-specific price index q_i captures this minimal cost. The household then optimally chooses the mix of market and non-market goods. The aggregate price indices P and Q capture the relative prices of market vs. non-market goods.

2.2 Firm pricing decisions

Market Goods Producers. Firm j chooses p_j to maximize the objective in Equation (8), where $c_j(p_j)$ is given by Equation (11). Standard derivations yield that the price p_j is a markup of the marginal cost of labor w_m,

$$p_j = \frac{\eta}{\eta - 1} w_m \tag{18}$$

where the markup is the usual Lerner index (Lerner 1934) of the elasticity of substitution.

Home Goods Producers. Firm i chooses r_i to maximize the objective in Equation (9), where $d_i(r_i)$ is given by Equation (13). Substituting Equation (13) into the firm's objective, we get

$$\pi_i^d = \max_{r_i} (r_i - \zeta w_m)(1-\phi)^\epsilon (1-\alpha) \frac{r_i^{-\epsilon} q_i^{\epsilon - \sigma}}{Q^{1-\sigma}} (w_m + w_f + \Pi) \tag{19}$$

We obtain the following characterization of the solution:

Proposition 2. *The firm's problem in Equation* (19) *has a unique solution* r_i*, which satisfies the equation*

$$\frac{\phi^\epsilon w_f^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}} \cdot \epsilon + \frac{(1-\phi)^\epsilon r_i^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}} \cdot \sigma = \frac{r_i}{r_i - \zeta w_m} \tag{20}$$

Proof. See Appendix A.2. □

2.3 Equilibrium allocations

Because of symmetry, in equilibrium we have $c_j = C$, $\forall j$, $x_i = X$, $d_i = d$, and $\ell_i = \ell$, $\forall i$. Similarly, we can now impose $p_j = P$ and $q_i = Q$ for all i, j. Normalizing $P = 1$, the male wage w_m is then determined from Equation (18) as

$$w_m = \frac{\eta - 1}{\eta}, \tag{21}$$

and the equilibrium female wage must equal

$$w_f = \gamma \frac{\eta - 1}{\eta}. \tag{22}$$

Given w_m and w_f, we can solve for the equilibrium price r from Equation (20), which in turn determines the equilibrium Q via Equation (15). It will be convenient to define the markup on home inputs,

$$\mu := \frac{r}{\zeta w_m} \tag{23}$$

The markup μ is then pinned down from Equation (20) as

$$\frac{\phi^\epsilon (\mu\zeta)^{\epsilon-1}}{\phi^\epsilon (\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon} + (1-\phi)^\epsilon \gamma^{\epsilon-1}} \cdot \epsilon + \frac{(1-\phi)^\epsilon \gamma^{\epsilon-1}}{\phi^\epsilon (\mu\zeta)^{\epsilon-1} + (1-\phi)^\epsilon \gamma^{\epsilon-1}} \cdot \sigma = \frac{1}{1-1/\mu} \tag{24}$$

The equilibrium quantity Q follows from Equation (15) as

$$Q = \frac{\eta-1}{\eta}[\phi^\epsilon \gamma^{1-\epsilon} + (1-\phi)^\epsilon (\mu\zeta)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \tag{25}$$

Turning to quantities, the equilibrium values of C, X, d, and ℓ are uniquely determined by the symmetric version of the resource constraint

$$\gamma\ell + C + \zeta d = 1 + \gamma, \tag{26}$$

together with the symmetric versions of Equations (11) to (14). From these, we obtain expressions for all the equilibrium variables:

Proposition 3. *The symmetric equilibrium $C, X, d,$ and ℓ are given by*

$$C = \frac{\phi^\epsilon \gamma^{1-\epsilon} + (1-\phi)^\epsilon (\mu\zeta)^{1-\epsilon}}{\left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^\epsilon\gamma^{1-\epsilon} + \left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^\epsilon(\mu\zeta)^{1-\epsilon}}(1+\gamma) \tag{27}$$

$$X = \frac{\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}[\phi^\epsilon \gamma^{1-\epsilon} + (1-\phi)^\epsilon (\mu\zeta)^{1-\epsilon}]^{\frac{\epsilon}{\epsilon-1}}}{\left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^\epsilon\gamma^{1-\epsilon} + \left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^\epsilon(\mu\zeta)^{1-\epsilon}}(1+\gamma) \tag{28}$$

$$d = \frac{\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}(1-\phi)^\epsilon (\mu\zeta)^{1-\epsilon}}{\left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^\epsilon\gamma^{1-\epsilon} + \left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^\epsilon(\mu\zeta)^{1-\epsilon}}\frac{1+\gamma}{\mu\zeta} \tag{29}$$

$$\ell = \frac{\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}\phi^\epsilon \gamma^{1-\epsilon}}{\left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^\epsilon\gamma^{1-\epsilon} + \left[1 + \frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^\epsilon(\mu\zeta)^{1-\epsilon}}\frac{1+\gamma}{\gamma} \tag{30}$$

where μ is determined by Equation (24).

Proof. See Appendix A.3. □

The markup on home inputs μ creates a wedge that distorts equilibrium allocations, particularly the allocation of time between market work and home production. The key insight comes from examining equation (24). At the extremes, the formula simplifies to standard cases, which we further discuss below in Section 2.4. When $\phi = 1$ (households rely entirely on time), we get $\frac{\mu}{\mu-1} = \epsilon$, reflecting the elasticity of substitution between home time and purchased home inputs. When $\phi = 0$ (households rely entirely on purchased inputs), we obtain $\frac{\mu}{\mu-1} = \sigma$, reflecting the elasticity of substitution between non-market good varieties.

More generally, $\frac{\mu}{\mu-1}$ represents a weighted average of these two elasticities, where the weights are endogenously determined by the parameters ϕ, ζ, and γ. When households are highly reliant on their own time for provision of non-market goods – either because home time is very productive (ϕ is high), intermediate inputs require substantial labor to produce (ζ is high), or female wages are relatively low (γ is low) – each intermediate input firm behaves as if household time is its key competitor. In this case, the relevant elasticity is ϵ, and $\frac{\mu}{\mu-1}$ approaches this value. Conversely, when households are not very reliant on their home time and rely heavily on purchased home inputs, intermediate input firms face competition primarily from other input varieties rather than from home time. The relevant elasticity becomes σ, and $\frac{\mu}{\mu-1}$ approaches this value accordingly.

An implication of this analysis is that an increase in the reliance on home inputs relative to household time may raise or lower the markup on home inputs. We confirm this in the comparative statics results of Section 3.

2.4 Nested cases

Our model combines monopolistic competition with a relatively standard framework from family economics. In particular, the model nests several special cases prominent in the literature. When $\phi = 1$, home-produced goods use time only, and the model then reduces to the standard Dixit and Stiglitz (1977) monopolistic competition framework with endogenous labor supply. In that case, the markup on home inputs is irrelevant and the markup on market goods acts as a conventional distortionary tax.

At the other extreme, when $\phi = 0$, the model reduces to a Dixit and Stiglitz (1977) monopolistic competition framework with two classes of goods with different elasticities of substitution, which determine each product's respective markup.. In this case, the markup on market goods is equal to $\frac{\eta}{\eta-1}$ and the markup on home goods is equal to $\mu = \frac{\sigma}{\sigma-1}$.

Next, if the elasticity of substitution in either sector is taken to infinity, the respective product market approaches perfect competition. Taking the limits $\eta \to \infty$ and $\sigma \to \infty$

recovers the usual framework in family macroeconomics (Greenwood, Guner, and Vandenbroucke 2017). In this case, the markup on home inputs is $\mu = 1$, and so their price equals marginal cost, $r = \zeta w_m$. Our framework therefore permits very straightforward comparisons to well-known benchmarks, which we will elaborate on below in the quantitative analysis.

2.5 Efficiency

We consider the problem of an unconstrained social planner who takes γ and ζ as given and chooses the allocations $\{x_i, c_j, \ell_i, d_i\}$ to maximize household utility given by Equation (1) subject to Equations (2) to (4) and (10). We summarize the efficient allocation in the following result:

Proposition 4 (Efficient allocation)**.** *The solution to the social planner's problem is*

$$C^* = \alpha(1+\gamma) \tag{31}$$

$$X^* = \frac{1}{\left[\phi^{\epsilon}\gamma^{1-\epsilon} + (1-\phi)^{\epsilon}\zeta^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}}(1-\alpha)(1+\gamma) \tag{32}$$

$$d^* = \frac{(1-\phi)^{\epsilon}\zeta^{1-\epsilon}}{\phi^{\epsilon}\gamma^{1-\epsilon} + (1-\phi)^{\epsilon}\zeta^{1-\epsilon}}(1-\alpha)\frac{1+\gamma}{\zeta} \tag{33}$$

$$\ell^* = \frac{\phi^{\epsilon}\gamma^{1-\epsilon}}{\phi^{\epsilon}\gamma^{1-\epsilon} + (1-\phi)^{\epsilon}\zeta^{1-\epsilon}}(1-\alpha)\frac{1+\gamma}{\gamma} \tag{34}$$

Proof. See Appendix A.4. □

Comparing the efficient allocation characterized in Equations (31) to (34) to the equilibrium allocation in Equations (27) to (30) gives the straightforward result that efficiency is only achieved when prices are equal to marginal costs in both markets:

Corollary 1. *The equilibrium allocation is inefficient whenever $\eta < \infty$ or $\sigma < \infty$.*

3 Comparative statics

In this section, we study how market power for home production substitutes distorts labor supply and interacts with other outcomes in the model.

Direct effects of market power. We first confirm the intuitive result that market power in *either* market distorts labor supply.

Lemma 1. $\frac{d\ell}{d\eta} < 0$ *and* $\frac{d\ell}{d\sigma} < 0$.

Proof. This follows directly from differentiating Equation (30) with respect to each parameter. □

As is well known, market power in the market goods sector distorts labor supply, acting as a distortionary tax on market consumption. In addition, market power for home production substitutes creates a novel, analogous distortion: by inducing substitution away from purchased home inputs and instead towards home production time, it induces households to substitute toward home production and away from market work.

We next revisit classic questions in family macroeconomics, namely how time allocation, captured here by the female time in home production ℓ, depends on the narrowing gender wage gap, captured by an increase in γ, and technological progress in home substitutes, captured by a fall in ζ. The key insight here is that market power in the home input market interacts with these economic forces in a way that market power in the market goods sector does not. Moreover, the endogenous response of markups can either dampen or amplify the effects of other economic trends, depending on the relative substitutability of time and household inputs.

Technological progress in home inputs. We first analyze the effect of a decline in the cost of producing home inputs due to technical change. Differentiating Equation (24) with respect to ζ, we get the following result:

Lemma 2. *The comparative statics properties of the equilibrium markup with respect to ζ are as follows:*

1. *If $\epsilon > \sigma$, then $-\frac{\mu}{\zeta} < \frac{d\mu}{d\zeta} < 0$.*
2. *If $\epsilon = \sigma$, then $\frac{d\mu}{d\zeta} = 0$.*
3. *If $\epsilon < \sigma$, then $\frac{d\mu}{d\zeta} > 0$.*

Proof. See Appendix A.5. □

A decrease in the marginal cost ζ of producing home inputs makes households more reliant on home inputs and less reliant on home time in equilibrium. If $\epsilon > \sigma$, this means that households are more reliant on a less easily substitutable product. Producers of home inputs then face a lower elasticity of demand and optimally raise their markups. Lemma 2 establishes that prices still fall on net, but the fall is dampened by the rise in markups. If $\epsilon < \sigma$, home inputs are very easily substitutable with one another. The increased reliance on home inputs then raises the elasticity of demand and lowers markups. In this case, the fall in the price is amplified by the endogenous response of markups.

Next, we examine the implications for time allocation. We can decompose

$$\frac{d\ell}{d\zeta} = \frac{\partial \ell}{\partial \zeta} + \frac{\partial \ell}{\partial \mu}\frac{d\mu}{d\zeta},$$

where $\frac{d\ell}{d\zeta}$ is the total derivative of ℓ with respect to ζ, $\frac{\partial \ell}{\partial \zeta}$ is the direct effect of ζ keeping μ fixed, and $\frac{\partial \ell}{\partial \mu}\frac{d\mu}{d\zeta}$ is the effect operating through the equilibrium markup. It is straightforward to verify from Equation (30) that $\frac{\partial \ell}{\partial \zeta} > 0$, i.e., the direct effect of a decrease in the cost of home inputs is to lower home production time and raise labor supply. This is the standard result in family economics emphasized, e.g., in Greenwood, Seshadri, and Yorukoglu (2005). The novelty is that the fall in ζ is not perfectly passed through to prices, and as shown in Lemma 2, the response of the markup can be either dampening or amplifying. It is straightforward to verify that $\frac{\partial \ell}{\partial \mu} > 0$, and so the direction of the equilibrium effect depends on the sign of $\frac{d\mu}{d\zeta}$. We summarize this result in the following proposition:

Proposition 5. *We have* $\frac{d\ell}{d\zeta} > 0$ *and*

1. $\frac{d\ell}{d\zeta} < \frac{\partial \ell}{\partial \zeta}$ *if* $\epsilon > \sigma$, *or*
2. $\frac{d\ell}{d\zeta} = \frac{\partial \ell}{\partial \zeta}$ *if* $\epsilon = \sigma$, *or*
3. $\frac{d\ell}{d\zeta} > \frac{\partial \ell}{\partial \zeta}$ *if* $\epsilon < \sigma$.

As stressed in the family economics literature, a falling cost of home inputs has the effect of raising female labor supply. Here, the cost is not passed through to prices one-for-one, and the rise in labor supply may be either dampened or amplified by the endogenous markup response, following the logic of Lemma 2.

Narrowing the gender wage gap. We similarly consider the effect of a smaller gender wage gap, captured here by an increase in γ. Differentiating Equation (24) with respect to γ, we get the result:

Lemma 3. *The comparative statics properties of the equilibrium markup with respect to* γ *are as follows:*

1. *If* $\epsilon > \sigma$, *then* $0 < \frac{d\mu}{d\gamma} < \frac{\mu}{\gamma}$.
2. *If* $\epsilon = \sigma$, *then* $\frac{d\mu}{d\gamma} = 0$.
3. *If* $\epsilon < \sigma$, *then* $\frac{d\mu}{d\gamma} < 0$.

Proof. See Appendix A.5. □

The logic of Lemma 3 parallels that of Lemma 2. A higher γ raises the opportunity cost of home production, making households relatively more reliant on home inputs. The effect on markups depends on whether home inputs are highly substitutable with each other. Similarly to the above discussion of technological progress, we use this result to examine comparative statics of ℓ with respect to γ. The overall effect of a rise in γ on ℓ can be decomposed into the direct effect $\frac{\partial \ell}{\partial \gamma}$ and the equilibrium effect through μ:

$$\frac{d\ell}{d\gamma} = \frac{\partial \ell}{\partial \gamma} + \frac{\partial \ell}{\partial \mu}\frac{d\mu}{d\gamma},$$

The direct effect $\frac{\partial \ell}{\partial \gamma}$ is negative, as can be verified from Equation (30). Intuitively, this result follows for two reasons. First, a higher γ raises the opportunity cost of home production, which lowers ℓ. Second, a higher γ raises household income, which further lowers ℓ since a richer household can purchase more home inputs in the market. The direct effect is thus necessarily to reduce time spent in home production. The equilibrium effect through μ can be dampening or amplifying, similarly to the discussion above. We summarize this result in the following proposition:

Proposition 6. *We have* $\frac{d\ell}{d\gamma} < 0$ *and*

1. $\frac{d\ell}{d\gamma} > \frac{\partial \ell}{\partial \gamma}$ *if* $\epsilon > \sigma$*;*
2. $\frac{d\ell}{d\gamma} = \frac{\partial \ell}{\partial \gamma}$ *if* $\epsilon = \sigma$*;*
3. $\frac{d\ell}{d\gamma} < \frac{\partial \ell}{\partial \gamma}$ *if* $\epsilon < \sigma$*.*

This simple result has potentially non-trivial policy implications. Policies that reduce the gender wage gap may be more or less effective, in a quantitative sense, depending on the market power in the home inputs market. Intuitively, a policy reducing the gender wage gap would, all else equal, draw more women into the labor force; but this effect may be offset by endogenously rising prices of home production substitutes, which in turn make it more difficult to enter the labor force despite higher wages.

4 Discussion and next steps

This paper has argued theoretically that *product* market power can act as a *labor* market distortion. When the products are home production substitutes, the distortionary effects of product market power are qualitatively different from the usual monopoly distortion in Dixit and Stiglitz (1977) and the work emanating from it. In particular, market power for home production substitutes interacts in non-trivial ways with other documented forces affects female labor supply, and may either dampen or amplify them.

In ongoing work, we calibrate the model by combining strategies from household economics and the markup literature. The model can then be used to conduct counterfactual experiments regarding the trends in technological progress in the home sector, the narrowing gender wage gap, and rising markups, as well as interactions between these forces. We seek to understand how the various trends jointly contribute to the observed patterns in female labor supply. Finally, the model can be used to study the general equilibrium impact of regulations in the market for home inputs.

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A Proofs

A.1 Proofs for the household problem

Proof of Proposition 1. As stated in the main text, we focus here on parameter configurations such that Equation (6) does not bind. Denote by λ the Lagrange multiplier on Equation (5). The derivation of the demand functions proceeds in three steps. We first derive the demand equation for market goods. We then derive the demand equation for home inputs. Finally, we aggregate expenditures to obtain the expressions in the text.

Demand for market goods. The first-order condition for c_j reads

$$\lambda p_j = c_j^{-\frac{1}{\eta}} C^{\frac{1}{\eta}} \frac{\alpha}{C} \tag{35}$$

It then follows that for any $j, k \in [0, 1]$,

$$\frac{p_j}{p_k} = \frac{c_j^{-\frac{1}{\eta}}}{c_k^{-\frac{1}{\eta}}},$$

or, rearranging,

$$p_j c_k^{-\frac{1}{\eta}} = p_k c_j^{-\frac{1}{\eta}}$$

Raising both sides to the power $1 - \eta$, we have

$$p_j^{1-\eta} c_k^{\frac{\eta-1}{\eta}} = p_k^{1-\eta} c_j^{\frac{\eta-1}{\eta}}$$

Integrating both sides with respect to k and using the expressions for C and P in Equations (2) and (17), we get

$$p_j^{1-\eta} C^{\frac{\eta-1}{\eta}} = P^{1-\eta} c_j^{\frac{\eta-1}{\eta}}$$

Raising both sides to the power $\frac{\eta}{\eta-1}$, we get

$$p_j^{-\eta} C = P^{-\eta} c_j$$

Rearranging yields the demand function

$$c_j = \frac{p_j^{-\eta}}{P^{-\eta}} C \tag{36}$$

From Equation (35) it then also follows that

$$\lambda = \frac{\alpha}{PC} \tag{37}$$

Demand for home inputs and home production time. We use a similar procedure to derive the demand for home inputs. The first-order conditions for ℓ_i and d_i, respectively, are

$$\phi \ell_i^{-\frac{1}{\epsilon}} x_i^{\frac{1}{\epsilon}} x_i^{-\frac{1}{\sigma}} X^{\frac{1}{\sigma}} \frac{1-\alpha}{X} = w_f \lambda, \tag{38}$$

and

$$(1-\phi) d_i^{-\frac{1}{\epsilon}} x_i^{\frac{1}{\epsilon}} x_i^{-\frac{1}{\sigma}} X^{\frac{1}{\sigma}} \frac{1-\alpha}{X} = r_i \lambda \tag{39}$$

Combining the two first-order-conditions Equations (38) and (39) gives

$$d_i = \left(\frac{1-\phi}{\phi} \cdot \frac{w_f}{r_i} \right)^{\epsilon} \ell_i \tag{40}$$

Substituting this expression into Equation (4) yields

$$d_i = (1-\phi)^{\epsilon} r_i^{-\epsilon} q_i^{\epsilon} x_i, \tag{41}$$

$$\ell_i = \phi^{\epsilon} (w_f)^{-\epsilon} q_i^{\epsilon} x_i, \tag{42}$$

where q_i is defined in Equation (15). Substituting back into the first-order conditions Equations (38) and (39), we get

$$\lambda q_i = x_i^{-\frac{1}{\sigma}} X^{\frac{1}{\sigma}} \frac{1-\alpha}{X} \tag{43}$$

It follows that for any $i, k \in [0, 1]$,

$$\frac{q_i}{q_k} = \frac{x_i^{\frac{1}{\sigma}}}{x_k^{\frac{1}{\sigma}}},$$

or, rearranging,

$$q_i x_k^{\frac{1}{\sigma}} = q_k x_i^{\frac{1}{\sigma}}$$

Raising both sides to the power $1-\sigma$, we have

$$q_i^{1-\sigma} x_k^{\frac{\sigma-1}{\sigma}} = q_k^{1-\sigma} x_i^{\frac{\sigma-1}{\sigma}}$$

Integrating both sides with respect to k and using the expressions for X and Q in Equations (3) and (16), we get

$$q_i^{1-\sigma} X^{\frac{\sigma-1}{\sigma}} = Q^{1-\sigma} x_i^{\frac{\sigma-1}{\sigma}}$$

Raising both sides to the power $\frac{\sigma}{\sigma-1}$ gives

$$q_i^{-\sigma} X = Q^{-\sigma} x_i$$

and rearranging gives

$$x_i = \frac{q_i^{-\sigma}}{Q^{-\sigma}} X \tag{44}$$

From Equation (43), it then also follows that

$$\lambda = \frac{1-\alpha}{QX} \tag{45}$$

To get demand functions for d_i and ℓ_i, we substitute Equation (44) back into Equations (41) and (42) to obtain

$$d_i = (1-\phi)^\epsilon r_i^{-\epsilon} q_i^{\epsilon-\sigma} Q^\sigma X, \tag{46}$$

$$\ell_i = \phi^\epsilon (\theta w_f)^{-\epsilon} q_i^{\epsilon-\sigma} Q^\sigma X \tag{47}$$

Aggregation. First, we aggregate expenditures on market goods. From Equation (36), it is immediate that

$$\int_0^1 p_j c_j \mathrm{d}j = PC \tag{48}$$

Next, we aggregate expenditures on home goods. From Equations (46) and (47), we have

$$r_i d_i + w_f \ell_i = q_i^{1-\sigma} Q^\sigma X$$

and so

$$\int_0^1 r_i d_i \mathrm{d}i + w_f \int_0^1 \ell_i \mathrm{d}i = QX \tag{49}$$

Adding Equations (48) and (49) and using the budget constraint Equation (5) gives

$$PC + QX = w_m + w_f + \Pi \tag{50}$$

Using Equations (37) and (45) in Equation (50), we get

$$C = \frac{\alpha}{P}(w_m + w_f + \Pi), \tag{51}$$

$$X = \frac{1-\alpha}{Q}(w_m + w_f + \Pi) \tag{52}$$

Substituting Equations (51) and (52) into Equations (36), (44), (46) and (47) yields Equations (11) to (14) in Proposition 1.

□

A.2 Proofs for the firm's problem

Proof of Proposition 2. After removing the multiplicative constants and plugging in our definition of q_i from Equation (15), the firm's problem is equivalent to solving,

$$\max_{r_i}(r_i - \zeta w_m) r_i^{-\epsilon} \left[\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}\right]^{\frac{\epsilon-\sigma}{1-\epsilon}}$$

The firm's objective can be rewritten in log form as

$$\ln(r_i - \zeta w_m) - \epsilon \ln(r_i) + \frac{\epsilon - \sigma}{1-\epsilon} \ln\left(\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}\right)$$

The necessary first-order condition for r_i is then

$$0 = \frac{1}{r_i - \zeta w_m} - \epsilon \frac{1}{r_i} + (\epsilon - \sigma) \frac{(1-\phi)^\epsilon r_i^{-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}}$$

which, after multiplying through by r_i, becomes

$$0 = \frac{r_i}{r_i - \zeta w_m} - \epsilon + (\epsilon - \sigma) \frac{(1-\phi)^\epsilon r_i^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}} := F(r_i) \tag{53}$$

To show uniqueness, we follow a procedure similar to Albrecht, Phelan, and Pretnar (2025) and prove that $F'(r_i) < 0$ whenever $F(r_i) \geq 0$, where F is defined to be the right-hand side of Equation (53). Differentiating F yields

$$F'(r_i) = -\frac{\zeta w_m}{(r_i - \zeta w_m)^2} + (\epsilon - \sigma)(1-\epsilon) \frac{(1-\phi)^\epsilon \phi^\epsilon w_f^{1-\epsilon} r_i^{-\epsilon}}{[\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}]^2}$$

If $\epsilon > \sigma$, then $F'(r_i) < 0$ is guaranteed, so the firm's problem is concave everywhere. If $\epsilon \leq \sigma$, we note that $F(r_i) \geq 0$ implies

$$\frac{r_i}{r_i - \zeta w_m} \geq (\sigma - \epsilon) \frac{(1-\phi)^\epsilon r_i^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}} + \epsilon > (\sigma - \epsilon) \frac{(1-\phi)^\epsilon r_i^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}}$$

and

$$\frac{\zeta w_m}{r_i - \zeta w_m} \geq (\sigma - \epsilon)\frac{(1-\phi)^\epsilon r_i^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}} + \epsilon - 1 > (\epsilon - 1)\frac{\phi^\epsilon w_f^{1-\epsilon}}{\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}}$$

and so

$$r_i F'(r_i) = -\frac{\zeta w_m r_i}{(r_i - \zeta w_m)^2} + (\sigma - \epsilon)(\epsilon - 1)\frac{(1-\phi)^\epsilon \phi^\epsilon w_f^{1-\epsilon} r_i^{-\epsilon}}{[\phi^\epsilon w_f^{1-\epsilon} + (1-\phi)^\epsilon r_i^{1-\epsilon}]^2} < 0$$

as desired.

□

A.3 Proofs for equilibrium characterization

Proof of Proposition 3. The symmetric versions of Equations (11) to (14) are

$$C = \alpha(w_m + w_f + \Pi) \tag{54}$$

$$X = (1-\alpha)\frac{1}{Q}(w_m + w_f + \Pi) \tag{55}$$

$$d = (1-\phi)^\epsilon (1-\alpha) r^{-\epsilon} Q^{\epsilon-1}(w_m + w_f + \Pi) \tag{56}$$

$$\ell = \phi^\epsilon (1-\alpha) w_f^{-\epsilon} Q^{\epsilon-1}(w_m + w_f + \Pi) \tag{57}$$

since $c_j = C, x_i = X, d_i = d, \ell_i = \ell, p_j = P = 1, q_i = Q$. By substituting Equation (54) into Equations (55) to (57), we obtain expressions for X, d, and ℓ as functions of C:

$$X = \frac{(1-\alpha)}{\alpha}\frac{1}{Q}C \tag{58}$$

$$d = (1-\phi)^\epsilon \frac{(1-\alpha)}{\alpha} r^{-\epsilon} Q^{\epsilon-1} C \tag{59}$$

$$\ell = \phi^\epsilon \frac{(1-\alpha)}{\alpha} w_f^{-\epsilon} Q^{\epsilon-1} C \tag{60}$$

We can then use these, along with our expressions for w_m, w_f, r, and Q given by Equations (21) to (23) and (25), respectively, in Equation (26), obtaining

$$\gamma^{1-\epsilon}\phi^\epsilon \frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}[\phi^\epsilon \gamma^{1-\epsilon} + (1-\phi)^\epsilon (\mu\zeta)^{1-\epsilon}]^{-1} C + C$$
$$+ \zeta^{1-\epsilon}(1-\phi)^\epsilon \frac{(1-\alpha)}{\alpha}\mu^{-\epsilon}\frac{\eta}{\eta-1}[\phi^\epsilon \gamma^{1-\epsilon} + (1-\phi)^\epsilon (\mu\zeta)^{1-\epsilon}]^{-1} C = 1 + \gamma$$

which we can simplify and rearrange to solve for C:

$$\begin{aligned}
C &= \frac{1+\gamma}{1+\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}[\phi^{\epsilon}\gamma^{1-\epsilon}+(1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}]^{-1}(\gamma^{1-\epsilon}\phi^{\epsilon}+\zeta^{1-\epsilon}(1-\phi)^{\epsilon}\mu^{-\epsilon})} \\
&= \frac{(1+\gamma)[\phi^{\epsilon}\gamma^{1-\epsilon}+(1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}]}{[\phi^{\epsilon}\gamma^{1-\epsilon}+(1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}]+\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}(\gamma^{1-\epsilon}\phi^{\epsilon}+\zeta^{1-\epsilon}(1-\phi)^{\epsilon}\mu^{-\epsilon})} \\
&= \frac{[\phi^{\epsilon}\gamma^{1-\epsilon}+(1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}]}{\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^{\epsilon}\gamma^{1-\epsilon}+\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}}(1+\gamma)
\end{aligned}$$

Substituting this back into our expressions for X, d, and ℓ in Equations (58) to (60), we get

$$\begin{aligned}
X &= \frac{\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}[\phi^{\epsilon}\gamma^{1-\epsilon}+(1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}]^{\frac{\epsilon}{\epsilon-1}}}{\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^{\epsilon}\gamma^{1-\epsilon}+\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}}(1+\gamma) \\
d &= \frac{\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}(1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}}{\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^{\epsilon}\gamma^{1-\epsilon}+\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}}\frac{1+\gamma}{\mu\zeta} \\
\ell &= \frac{\frac{(1-\alpha)}{\alpha}\frac{\eta}{\eta-1}\phi^{\epsilon}\gamma^{1-\epsilon}}{\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\right]\phi^{\epsilon}\gamma^{1-\epsilon}+\left[1+\frac{1-\alpha}{\alpha}\frac{\eta}{\eta-1}\frac{1}{\mu}\right](1-\phi)^{\epsilon}(\mu\zeta)^{1-\epsilon}}\frac{1+\gamma}{\gamma}
\end{aligned}$$

as desired. □

A.4 Proof for the efficient allocation

Proof of Proposition 4. The social planner solves

$$\max_{x_i, c_j, \ell_i, d_i} \alpha \ln(C) + (1-\alpha)\ln(X)$$

$$\text{s.t. } C = \left(\int_0^1 c_j^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} \tag{2}$$

$$X = \left(\int_0^1 x_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

$$x_i = \left(\phi \ell_i^{\frac{\epsilon-1}{\epsilon}} + (1-\phi) d_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

$$1+\gamma\left(1-\int_0^1 \ell_i \mathrm{d}i\right) = \int_0^1 c_j \mathrm{d}j + \zeta\int_0^1 d_i \mathrm{d}i \tag{10}$$

Substituting Equations (2) to (4) into the objective and letting Λ^* be the Lagrange multiplier on Equation (10), we have the first-order conditions for c_j, d_i, and ℓ_i, respectively, as

$$\Lambda^* = \frac{\alpha}{C} C^{\frac{1}{\eta}} c_j^{-\frac{1}{\eta}} \tag{61}$$

$$\Lambda^* \zeta = \frac{1-\alpha}{X} X^{\frac{1}{\sigma}} x_i^{-\frac{1}{\sigma}} (1-\phi) x_i^{\frac{1}{\epsilon}} d_i^{-\frac{1}{\epsilon}} \tag{62}$$

$$\Lambda^* \gamma = \frac{1-\alpha}{X} X^{\frac{1}{\sigma}} x_i^{-\frac{1}{\sigma}} \phi x_i^{\frac{1}{\epsilon}} \ell_i^{-\frac{1}{\epsilon}} \tag{63}$$

Combining Equations (62) and (63), one derives (following the same logic as the proof of Proposition 1)

$$d_i = (1-\phi)^{\epsilon} \frac{\zeta^{-\epsilon}}{(Q^*)^{-\epsilon}} x_i \tag{64}$$

$$\ell_i = \phi^{\epsilon} \frac{\gamma^{-\epsilon}}{(Q^*)^{-\epsilon}} x_i \tag{65}$$

where we defined

$$Q^* := \left[\phi^{\epsilon} \gamma^{1-\epsilon} + (1-\phi)^{\epsilon} \zeta^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{66}$$

Substituting Equation (64) back into Equation (62), using Equation (61), and imposing symmetry, we have

$$X = \frac{1-\alpha}{\alpha} \frac{C}{Q^*}$$

and therefore

$$d = (1-\phi)^{\epsilon} \zeta^{-\epsilon} (Q^*)^{\epsilon-1} \frac{1-\alpha}{\alpha} C$$
$$\ell = \phi^{\epsilon} \gamma^{-\epsilon} (Q^*)^{\epsilon-1} \frac{1-\alpha}{\alpha} C$$

Substituting these expressions back into Equation (10), we have

$$\left[1 + \frac{1-\alpha}{\alpha} (Q^*)^{\epsilon-1} (\phi^{\epsilon} \gamma^{1-\epsilon} + (1-\phi)^{\epsilon} \zeta^{1-\epsilon})\right] C = 1 + \gamma$$

Simplifying using the definition of Q^* yields the desired expression for C which we can then use to obtain the expressions for X, d, and ℓ,

$$\begin{aligned}
C &= \alpha(1+\gamma) \\
X &= \frac{1}{[\phi^\epsilon\gamma^{1-\epsilon}+(1-\phi)^\epsilon\zeta^{1-\epsilon}]^{\frac{1}{1-\epsilon}}}(1-\alpha)(1+\gamma) \\
d &= \frac{(1-\phi)^\epsilon\zeta^{1-\epsilon}}{\phi^\epsilon\gamma^{1-\epsilon}+(1-\phi)^\epsilon\zeta^{1-\epsilon}}(1-\alpha)\frac{1+\gamma}{\zeta} \\
\ell &= \frac{\phi^\epsilon\gamma^{1-\epsilon}}{\phi^\epsilon\gamma^{1-\epsilon}+(1-\phi)^\epsilon\zeta^{1-\epsilon}}(1-\alpha)\frac{1+\gamma}{\gamma}
\end{aligned}$$

which completes the proof. □

A.5 Proofs of comparative statics results

Proof of Lemma 2. Equation (24) can be written as:

$$\epsilon-(\epsilon-\sigma)\frac{(1-\phi)^\epsilon}{\phi^\epsilon(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon}+(1-\phi)^\epsilon}=\frac{1}{1-1/\mu}$$

Fully differentiating with respect to ζ, we have

$$\frac{(\epsilon-1)(\epsilon-\sigma)\phi^\epsilon(1-\phi)^\epsilon}{[\phi^\epsilon(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon}+(1-\phi)^\epsilon]^2}\left(\mu^{\epsilon-2}\zeta^{\epsilon-1}\gamma^{1-\epsilon}\frac{d\mu}{d\zeta}+\mu^{\epsilon-1}\zeta^{\epsilon-2}\gamma^{1-\epsilon}\right)=-\frac{1}{(\mu-1)^2}\frac{d\mu}{d\zeta}$$

Multiplying both sides and rearranging, we have

$$\left[\frac{(\epsilon-1)(\sigma-\epsilon)\phi^\epsilon(1-\phi)^\epsilon}{[\phi^\epsilon(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon}+(1-\phi)^\epsilon]^2}\mu^{\epsilon-1}\zeta^{\epsilon-1}\gamma^{1-\epsilon}-\frac{\mu}{(\mu-1)^2}\right]\frac{d\mu}{d\zeta}=\frac{(\epsilon-1)(\epsilon-\sigma)\phi^\epsilon(1-\phi)^\epsilon}{[\phi^\epsilon(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon}+(1-\phi)^\epsilon]^2}\mu^\epsilon\zeta^{\epsilon-2}\gamma^{1-\epsilon} \tag{67}$$

If $\epsilon>\sigma$, the right-hand side of Equation (67) is strictly positive, and the expression in brackets on the left-hand side is strictly negative, so that $\frac{d\mu}{d\zeta}<0$. It is also straightforward to show, from Equation (67), that $\frac{\zeta}{\mu}\frac{d\mu}{d\zeta}>-1$. If $\epsilon=\sigma$, it is trivial that $\frac{d\mu}{d\zeta}=0$. If $\epsilon<\sigma$, the expression on the right-hand side of Equation (67) is strictly negative, so it remains to sign the expression in brackets on the left-hand side. From Equation (24), it follows that

$$\frac{\mu}{\mu-1}>(\sigma-\epsilon)\frac{(1-\phi)^\epsilon}{\phi^\epsilon(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon}+(1-\phi)^\epsilon} \tag{68}$$

and

$$\frac{1}{\mu - 1} = \epsilon - 1 + (\sigma - \epsilon)\frac{(1-\phi)^{\epsilon}}{\phi^{\epsilon}(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon} + (1-\phi)^{\epsilon}} > (\epsilon - 1)\frac{\phi^{\epsilon}(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon}}{\phi^{\epsilon}(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon} + (1-\phi)^{\epsilon}} \quad (69)$$

so that

$$\frac{(\epsilon-1)(\sigma-\epsilon)\phi^{\epsilon}(1-\phi)^{\epsilon}}{[\phi^{\epsilon}(\mu\zeta)^{\epsilon-1}\gamma^{1-\epsilon} + (1-\phi)^{\epsilon}]^2}\mu^{\epsilon-1}\zeta^{\epsilon-1}\gamma^{1-\epsilon} - \frac{\mu}{(\mu-1)^2} < 0$$

so that the expression in brackets is strictly negative. It follows that $\frac{d\mu}{d\zeta} > 0$ when $\epsilon < \sigma$. □

Proof of Lemma 3. The proof mirrors the proof of Lemma 2. □