1) 2.

$$P(\text{predict win} \mid \text{win}) = 0.766$$

$$P(\text{predict loss} \mid \text{loss}) = 0.892$$

$$P(\text{win} \mid \text{predict win}) = \frac{P(\text{predict win} \mid \text{win})\, p(\text{win})}{P(\text{predict win})} = \frac{(0.766)(0.045)}{P(\text{predict win})}$$

$$P(\text{loss} \mid \text{predict win}) = \frac{P(\text{predict win} \mid \text{loss})\, P(\text{loss})}{P(\text{predict win})} = \frac{(1-0.892)(1-0.045)}{P(\text{predict win})}$$

$$1 = P(\text{win} \mid \text{predict win}) + P(\text{loss} \mid \text{predict win})$$

$$1 = \frac{0.03447 + 0.10314}{P(\text{predict win})}$$

$$\Rightarrow P(\text{predict win}) = 0.13761$$

$$\Rightarrow P(\text{win} \mid \text{predict win}) = \frac{0.766\,(0.045)}{0.13761} = \boxed{0.25}$$

$$P(\text{win} \mid \text{predict loss}) = \frac{P(\text{predict loss} \mid \text{win})\, P(\text{win})}{P(\text{predict loss})} = \frac{(1-0.766)(0.045)}{P(\text{predict loss})}$$

$$P(\text{loss} \mid \text{predict loss}) = \frac{P(\text{predict loss} \mid \text{loss})\, P(\text{loss})}{P(\text{predict loss})} = \frac{(0.892)(1-0.045)}{P(\text{predict loss})}$$

$$P(\text{win} \mid \text{predict loss}) + P(\text{loss} \mid \text{predict loss})$$

$$= \frac{0.01053 + 0.85186}{P(\text{predict loss})} = 1 \Rightarrow P(\text{predict loss}) = 0.86239$$

$$\Rightarrow P(\text{win} \mid \text{predict loss}) = \frac{(1-0.766)(0.045)}{0.86239}$$

$$= \boxed{0.0122}$$

b)

$$P(\text{win} \mid \text{predict loss}) = \frac{P(\text{predict loss} \mid \text{win})\, P(\text{win})}{P(\text{predict loss})} = \frac{[1 - P(\text{predict win} \mid \text{win})]\, P(\text{win})}{P(\text{predict loss})}$$

$$P(\text{loss} \mid \text{predict loss}) = \frac{P(\text{predict loss} \mid \text{loss})\, P(\text{loss})}{P(\text{predict loss})} = \frac{[0.892][1 - P(\text{win})]}{P(\text{predict loss})}$$

$$P(\text{win} \mid \text{predict loss}) + P(\text{loss} \mid \text{predict loss})$$

$$\Rightarrow \frac{[1 - P(\text{predict win} \mid \text{win})]\, P(\text{win})}{P(\text{predict loss})} + \frac{(0.892)(1 - P(\text{win}))}{P(\text{predict loss})} = 1$$

$$0.234 \cdot P(\text{win}) + 0.892 - 0.892\, P(\text{win}) = P(\text{predict loss})$$

$$-0.658\, P(\text{win}) + 0.892 = P(\text{predict loss})$$

$$\Rightarrow P(\text{win} \mid \text{predict loss}) = \frac{0.234\, P(\text{win})}{0.892 - 0.658\, P(\text{win})} = 0.5$$

$$\Rightarrow 0.234\, P(\text{win}) = 0.446 - 0.329\, P(\text{win})$$

$$P(\text{win}) = \frac{0.446}{0.234 + 0.329} = \boxed{0.79}$$

** Plot on Word Doc **

** See ipynb code & notebook pdf shows 0.79 **

c)

$$P(\text{win} \mid \text{predict win}) = \frac{P(\text{predict win} \mid \text{win})\, P(\text{win})}{P(\text{predict win})} = \frac{0.766\,(0.15)}{P(\text{predict win})}$$

$$P(\text{loss} \mid \text{predict win}) = \frac{P(\text{predict win} \mid \text{loss})\, P(\text{loss})}{P(\text{predict win})} = \frac{(1-0.892)(1-0.15)}{P(\text{predict win})}$$

$$\Rightarrow \quad P(\text{predict win}) = (0.766)(0.15) + (1-0.892)(1-0.15)$$

$$= 0.2067$$

$$\Rightarrow \quad P(\text{win} \mid \text{predict win}) = \frac{0.766\,(0.15)}{0.2067} = \boxed{0.556}$$

d)

$$\text{Accuracy} = P(\text{predict win} \mid \text{win}) \cdot P(\text{win}) + P(\text{predict loss} \mid \text{loss}) \cdot P(\text{loss})$$

$$= (0.766)(0.15) + (0.892)(0.85)$$

$$= \boxed{0.8731 \text{ or } 87.31\%}$$

e)

$$P(\text{win} \mid \text{predict win}) = \frac{P(\text{predict win} \mid \text{win})\, P(\text{win})}{P(\text{predict win})} = \frac{(0.766)(0.15)}{P(\text{predict win})}$$

$$P(\text{loss} \mid \text{predict win}) = \frac{P(\text{predict win} \mid \text{loss})\, P(\text{loss})}{P(\text{predict win})} = \frac{(1-\text{Specificity})(1-0.15)}{P(\text{predict win})}$$

$$\Rightarrow \quad \frac{0.1149 + 0.85 - 0.85\,\text{Specificity}}{P(\text{predict win})} = 1$$

$$\Rightarrow \quad P(\text{predict win}) = 0.9649 - 0.85\,\text{Specificity}$$

$$\Rightarrow \quad P(\text{win} \mid \text{predict win}) = \frac{0.1149}{0.9649 - 0.85\,\text{Specificity}}$$ → Plot to Python

** See Plot on Doc **

f) ** See Doc **

2) a. Set $MSE = \sum_{i=1}^{N} (x_i - a)^2$ as the mean squared error

$$= \sum_{i=1}^{N} x_i^2 - 2ax_i + a^2$$

$$\frac{d}{da}(MSE) = \sum_{i=1}^{N} -2x_i + 2a$$

$$= 2\sum_{i=1}^{N}(a - x_i)$$

Setting $\frac{dMSE}{da} = 0$ to find critical points.

$$\Rightarrow \quad 2\sum_{i=1}^{N}(a - x_i) = 0$$

$$\sum_{i=1}^{N} a - \sum_{i=1}^{N} x_i = 0$$

$$aN - \sum_{i=1}^{N} x_i = 0$$

$$\Rightarrow \boxed{a^*_{critical} = \frac{\sum_{i=1}^{N} x_i}{N} = \bar{x} \text{ (Mean)}}$$

To check whether or not this is a value that provides MSE_{min},

take $\frac{d^2 MSE}{da^2} = \frac{d}{da}\left(2\sum_{i=1}^{N}(a - x_i)\right)$

$$= 2\sum_{i=1}^{N} 1$$

$$= 2N > 0 \quad \text{for } N > 0$$

↳ Given since N == # of terms

$\Rightarrow$ $a_{critical}$ gives MSE_{min}

QED

2) b. $\frac{d}{da}\left(\log p(x|a,\sigma^2)\right) = \frac{d}{da}\left(-\frac{1}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}(x-a)^2\right)$

$= -\frac{1}{2\sigma^2}(2)(-1)(x-a)$

$= \frac{1}{\sigma^2}(x-a) = 0$

$\Rightarrow \underline{x_{crit} = a} = \text{Mean (Same to } a)$

2) c. $\text{Bernoulli}(y;\sigma(z)) = \sigma(z)^y(1-\sigma(z))^{(1-y)}$

$-\log \text{Bernoulli}(y;\sigma(z)) = -\log\left(\sigma(z)^{(y)}(1-\sigma(z))^{(1-y)}\right)$

$= -\log(\sigma(z)^y) - \log(1-\sigma(z))^{(1-y)}$

$= -y\log(\sigma(z)) - (1-y)\log(1-\sigma(z))$

$= \ell(y,\sigma(z))$

QED