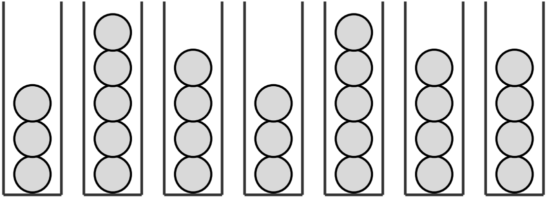
History-Independent Load Balancing



Michael A. Bender — Stony Brook University

William Kuszmaul — CMU

Elaine Shi — CMU

Rose Silver — CMU

History-Independent Data Structures

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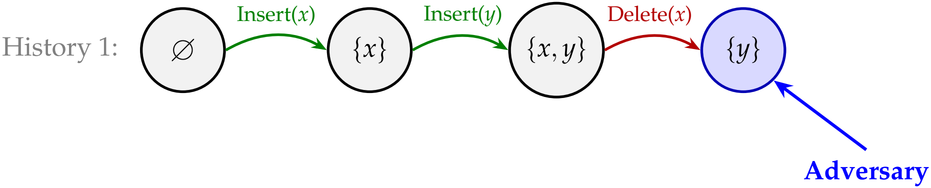
History 1: $\varnothing \xrightarrow{\text{Insert}(x)} \{x\} \xrightarrow{\text{Insert}(y)} \{x, y\} \xrightarrow{\text{Delete}(x)} \{y\}$

History-Independent Data Structures



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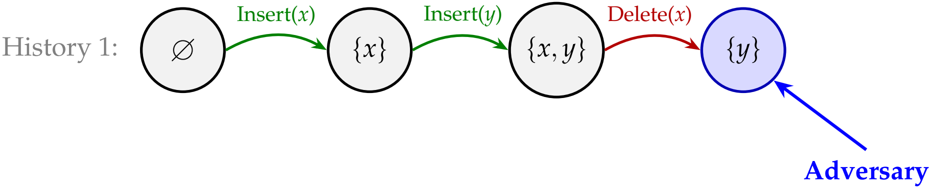

History Independence (Micciancio '97, Naor & Teague '01)

- The state reveals only the current elements—**not the history of operations.**

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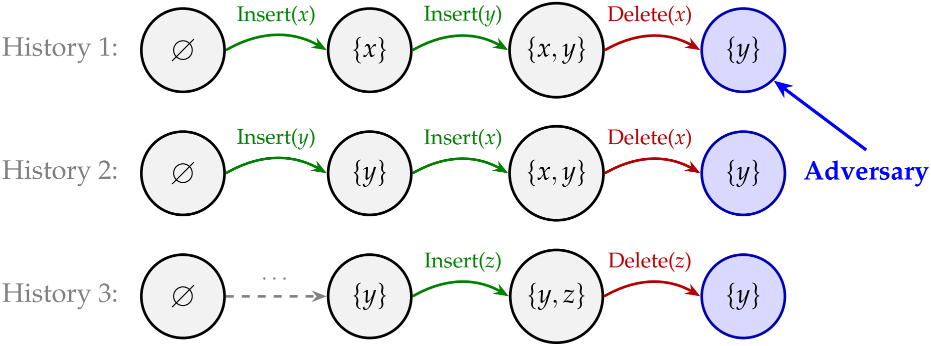

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History Independent Data Structures

A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

Micciancio '97, Naor & Teague '01, Buchbinder & Petrank '03, Molnar et al. '06, Blelloch & Golovin '07, Moran et al. '07, Naor et al. '08, Golovin '08–'10, Bajaj & Sion '13, Roche et al. '15, Bender et al. '16

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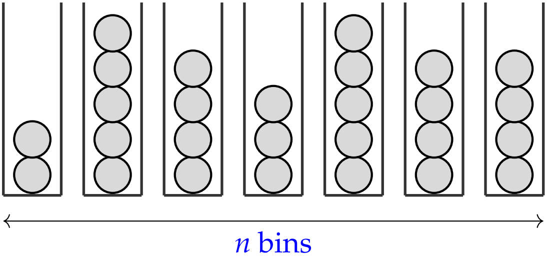
Yet some basic questions remain open.

This work: History-Independent Load Balancing

Two-Choice Load Balancing



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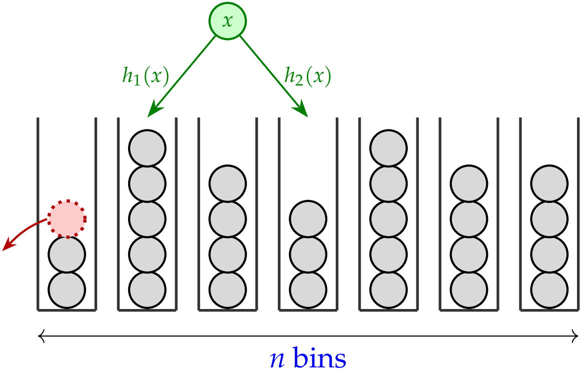

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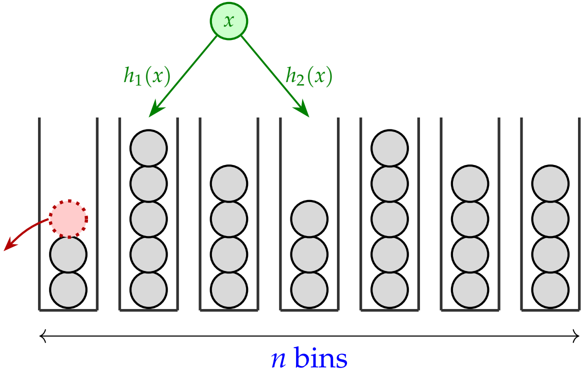

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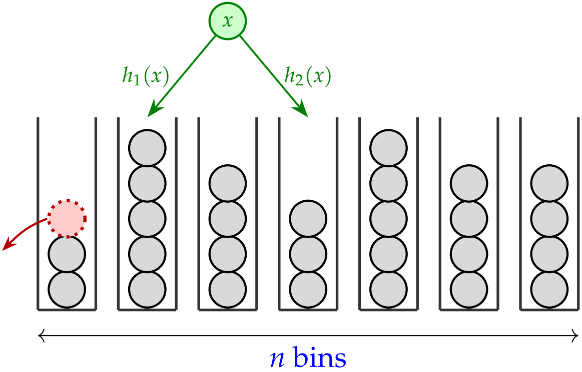
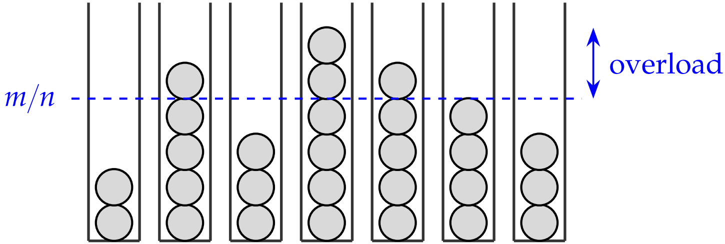


- Balls are inserted/deleted, with up to m present at a time.
- Each ball has two random bins where it can go.
- We must maintain a valid assignment of balls to bins.

Two Goals



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- i.e., the amount by which the fullest bin exceeds m/n.

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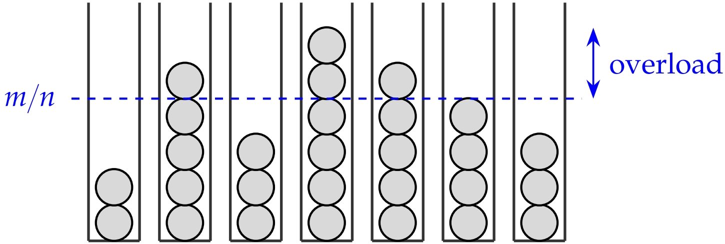

Minimize Overload:

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Minimize Recourse:

- i.e., the number of balls moved around on any given insertion/deletion.

Putting it All Together

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Question: Does there exist a history-independent solution with small recourse and small overload?

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Question: Does there exist a history-independent solution with small recourse and small overload?

Our Main Result: There exists a history-independent solution with:

- High probability overload $O(1)$
- Expected recourse $O(\log \log(m/n))$

Past Work (Not History Independent)

Overload	Recourse	Reference	Caveats
$O(\log\log n)$	0	[ABKU '94] [BCSV '00]	insertion-only
$O(1)$	$O(\log(m/n))$	[Dietzfelbinger, Weidling '07]	insertion-only
$\tilde{O}(\sqrt{m/n})$	$O(1)$	[Frieze, Petti '18]	insertion-only
$O(\log(m/n))$	0	[Bansal, Kuszmaul '22]	no reinsertions
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If we want overload $O(1)$, our result is a new state of the art!

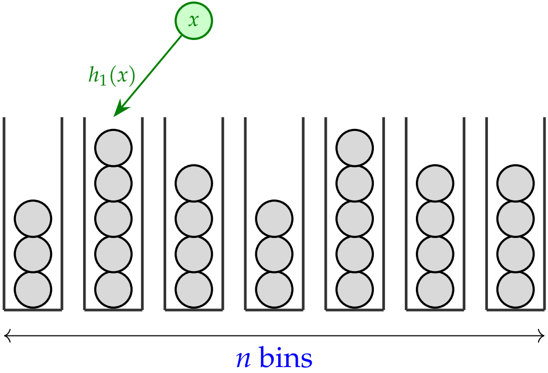
Rest of Talk: A Simple Warmup

Theorem: There exists a history-independent solution with:

- High-probability overload $\cancel{O(1)}$ $O(\log\log n)$.
- Expected recourse $\cancel{O(\log\log(m/n))}$ $O(m/n)$.

Baseline 1: The Single-Choice Strategy

To insert a ball x, just put it in bin $h_1(x)$:



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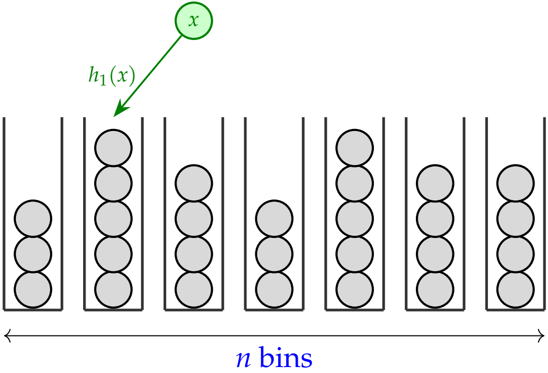

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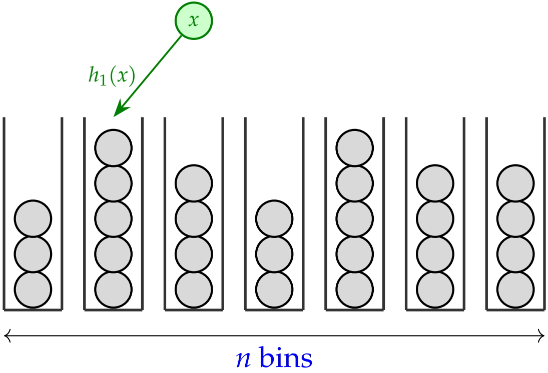

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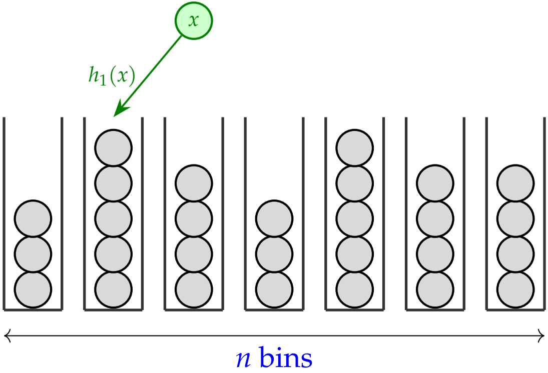
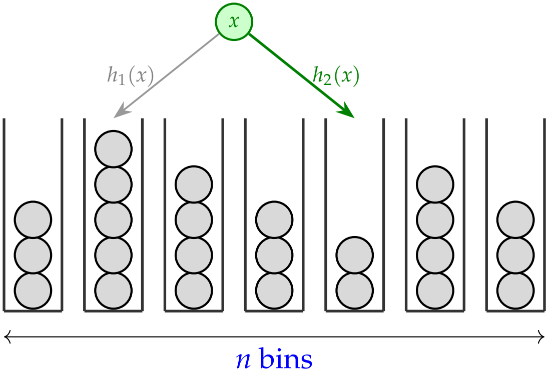


- This is history-independent ✓
- The recourse is 0 ✓
- But... the overload is huge, roughly $\sqrt{m/n}$ ✗

Baseline 2: Greedy Insertions

To insert a ball x, put it in the emptier of its choices:



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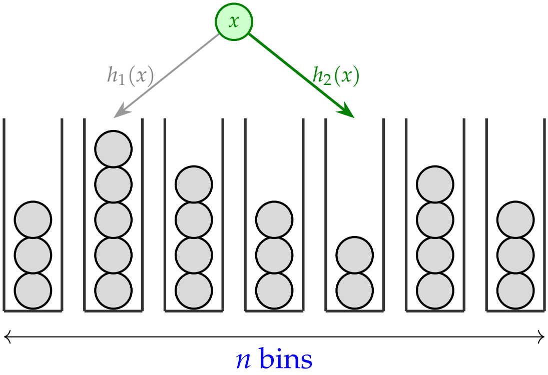

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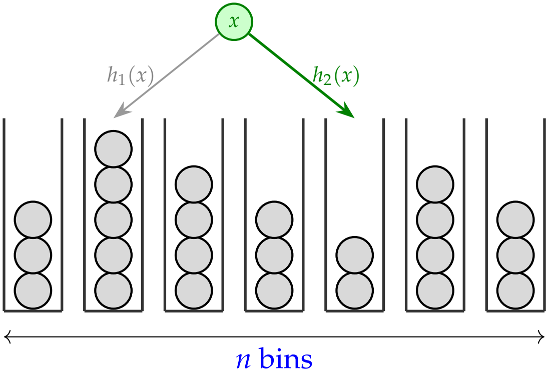

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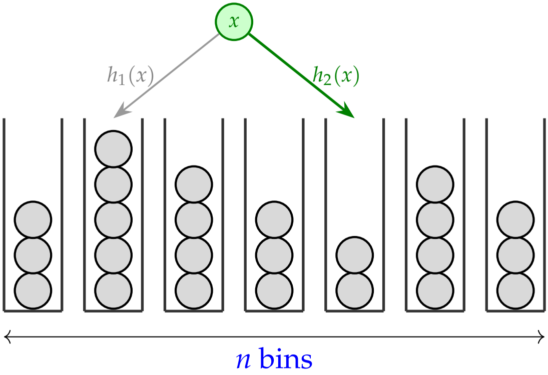

- This is **not** history-independent ✗
- The recourse is 0 ✓
- In the insertion-only case, the overload is $O(\log \log n)$ ✓

[Berenbrink, Czumaj, Steger, and Vöcking '00]

Warmup: History-Independent Greedy

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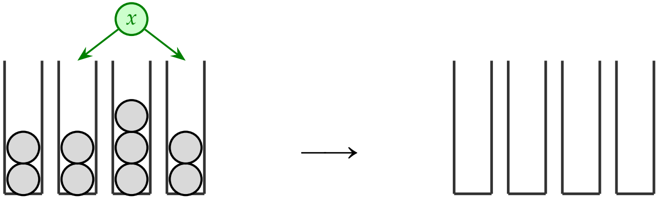

Current state A_S $\longrightarrow$ Computing $A_{S \cup \{x\}}$

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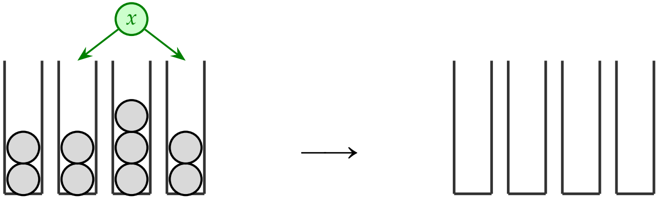

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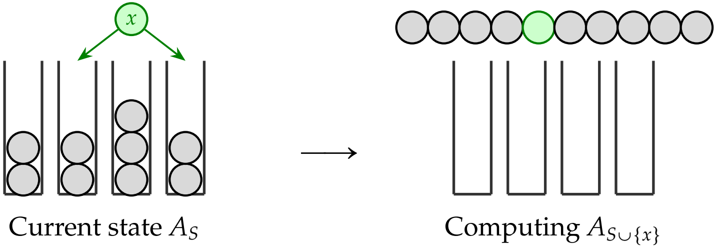

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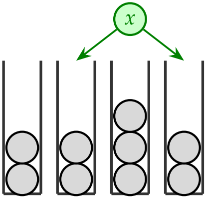

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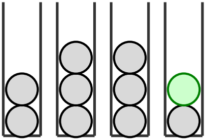

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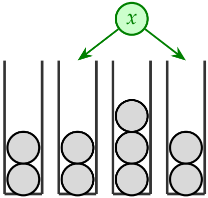

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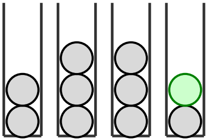
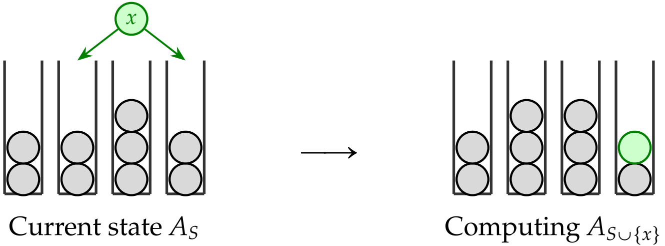
$\longrightarrow$



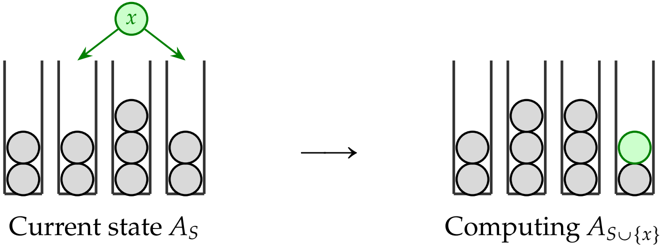
Computing $A_{S \cup \{x\}}$

ANALYZING HISTORY-INDEPENDENT GREEDY



- The algorithm is history independent ✓

Analyzing History-Independent Greedy



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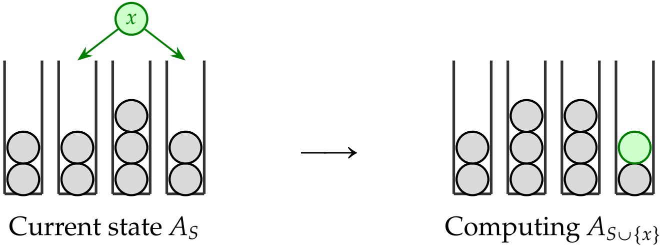
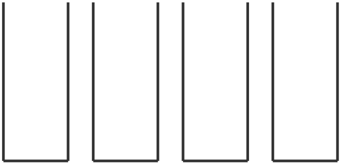
ANALYZING HISTORY-INDEPENDENT GREEDY



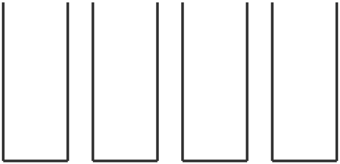
- The algorithm is history independent ✓
- The overload is $O(\log \log n)$ ✓
- What is the recourse?

Analyzing the Recourse

Recourse $= 0$



Computing A_S



Computing $A_{S \cup \{x\}}$

How many balls change assignments between A_S and $A_{S \cup \{x\}}$?

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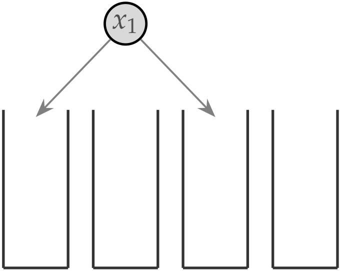

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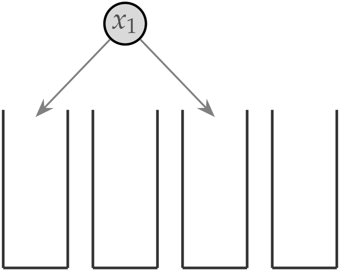

Computing $A_{S \cup \{x\}}$

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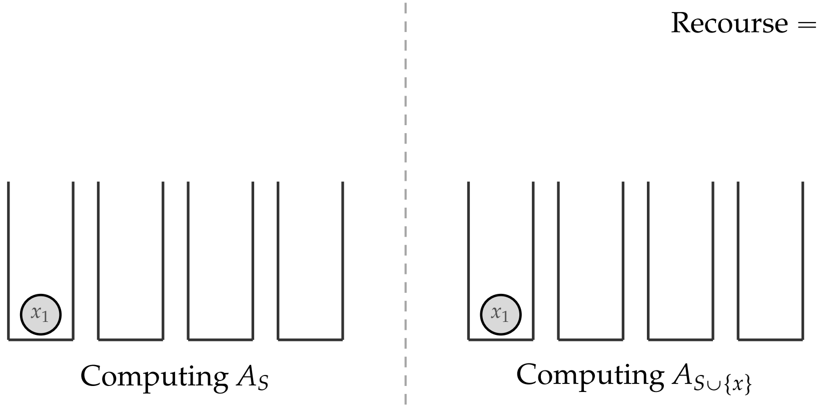

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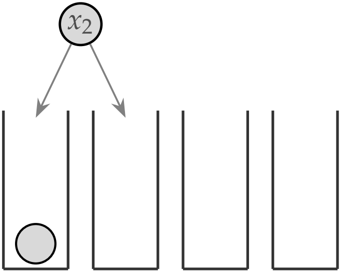

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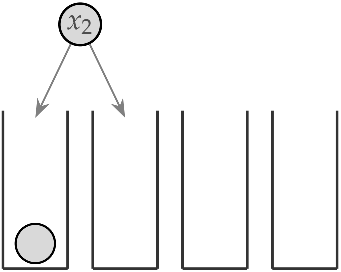

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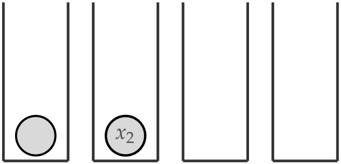

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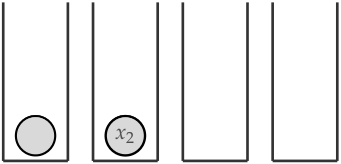

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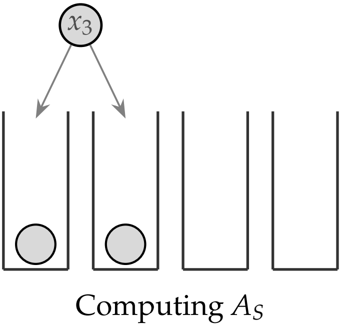

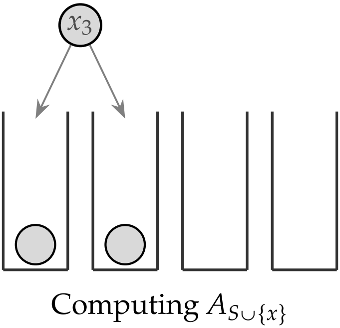

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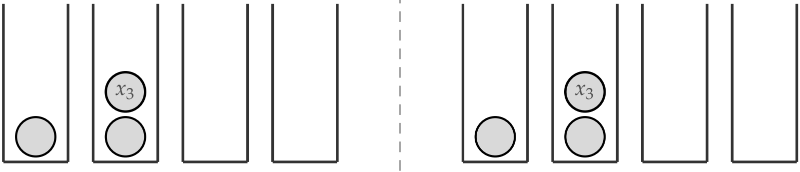

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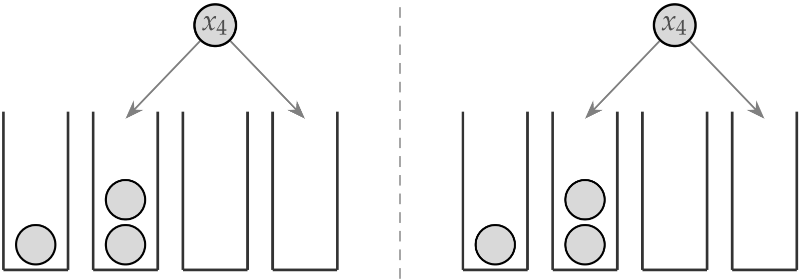

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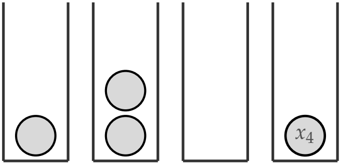

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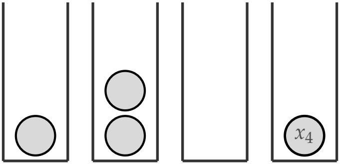

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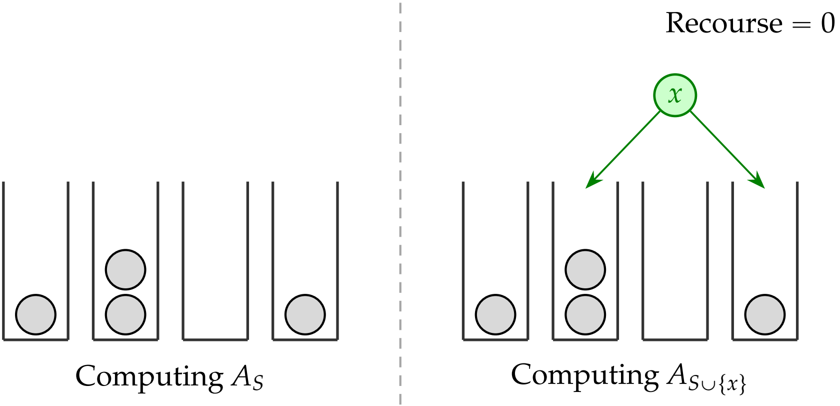

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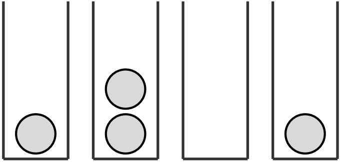

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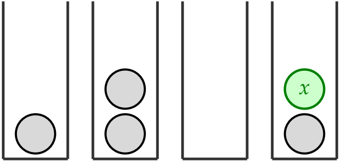

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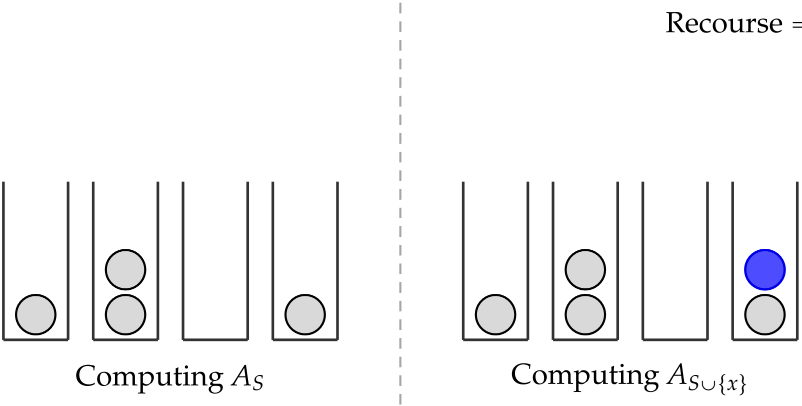
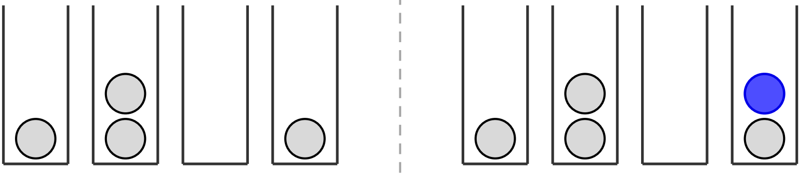


Computing A_S | Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

Analyzing the Recourse

Recourse $= 0$

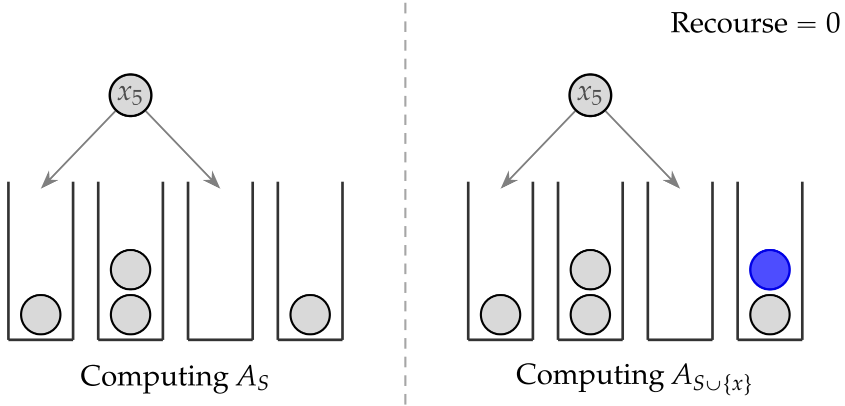


Computing A_S | Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

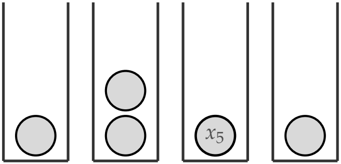


Future insertions will experience either:

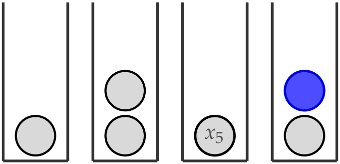
1. No recourse

Analyzing the Recourse

Recourse $= 0$



Computing A_S



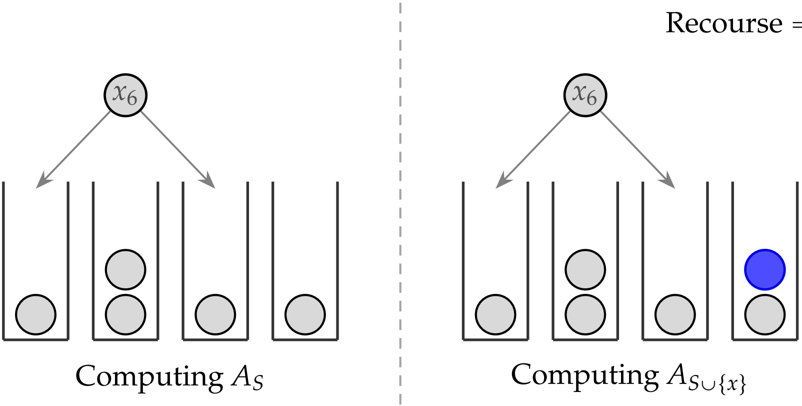
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

Recourse = 0

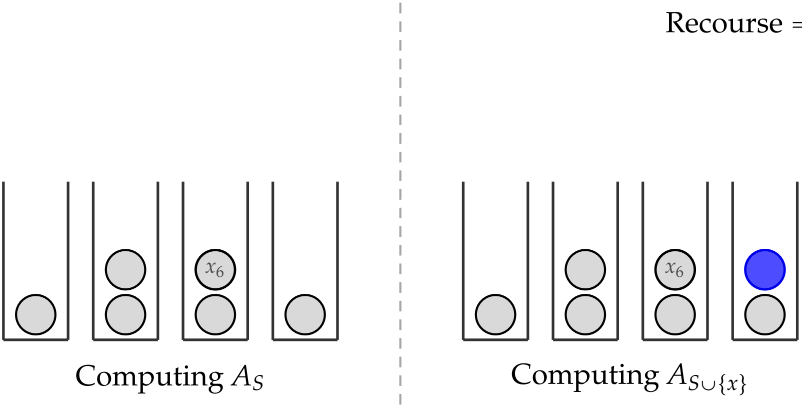


Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

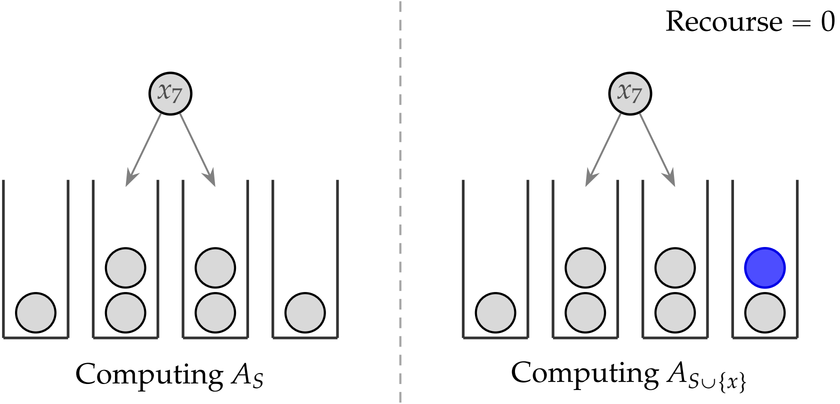
Recourse = 0



Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

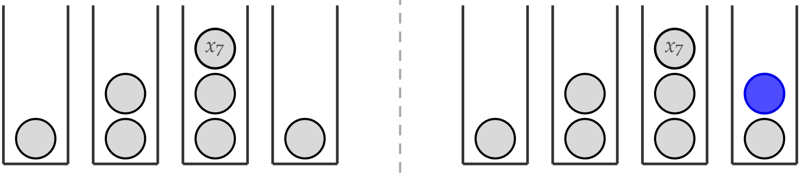


Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

Recourse = 0

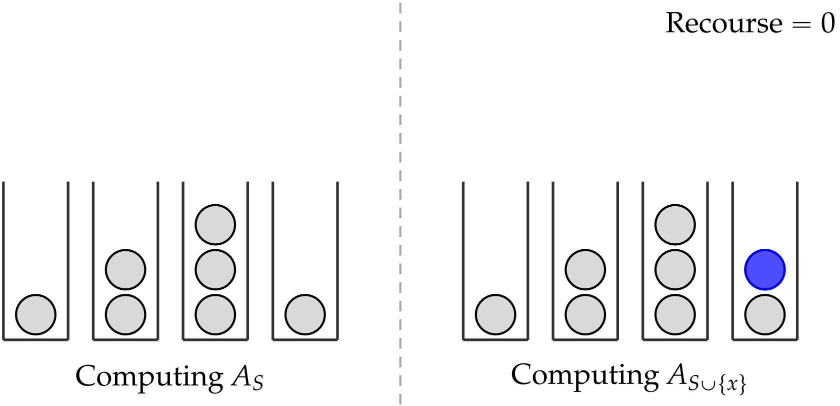


Computing A_S Computing $A_{S\cup\{x\}}$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

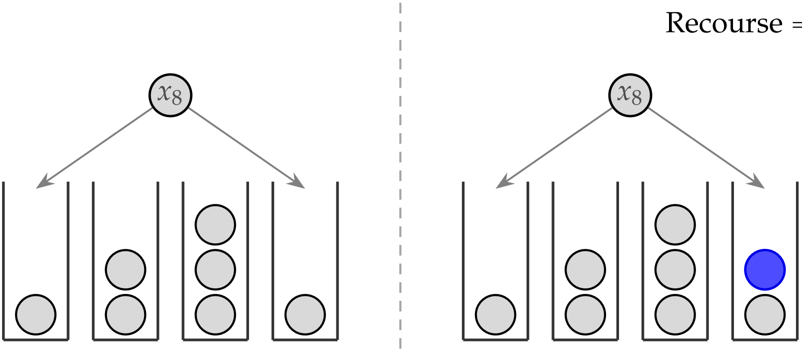


Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 0



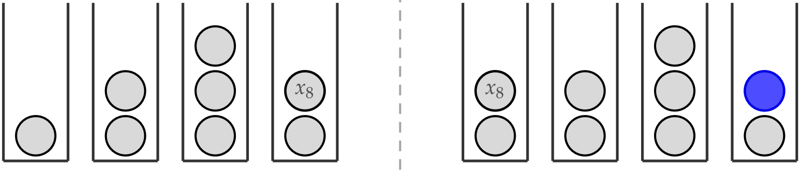
Computing A_S — Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 1



Computing A_S

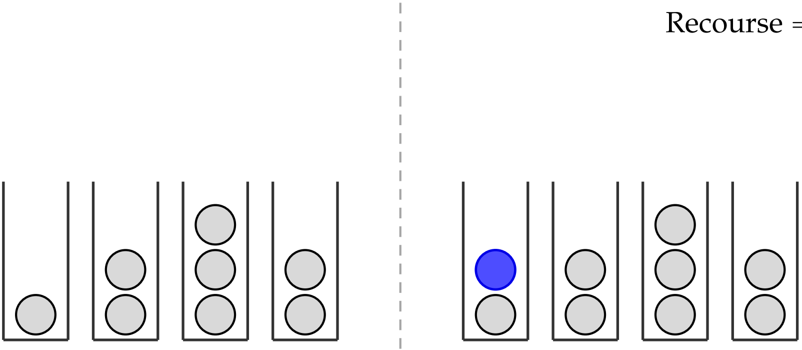
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 1



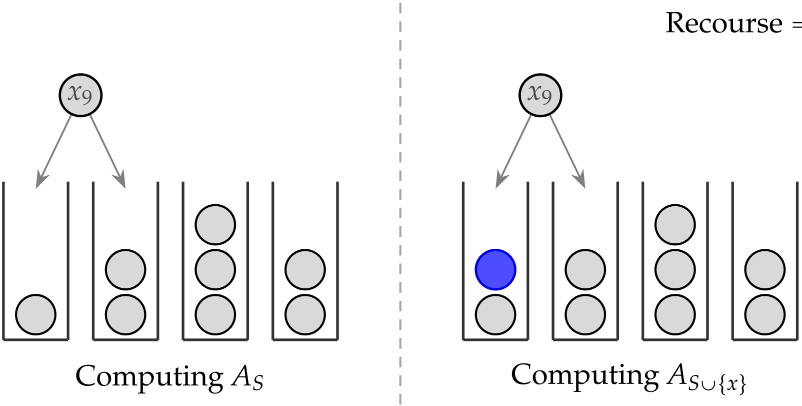
Computing A_S | Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 1

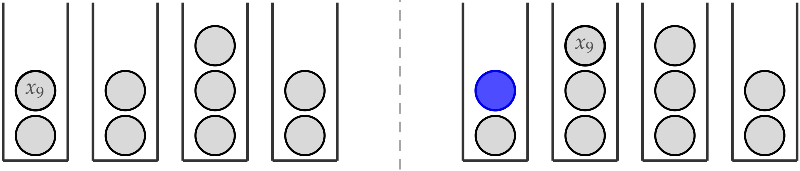


Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 2



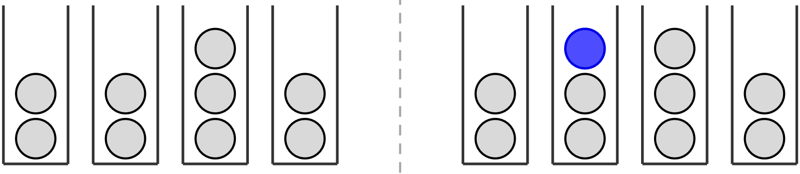
Computing A_S | Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 2



Computing A_S

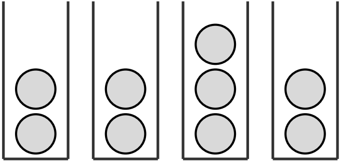
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

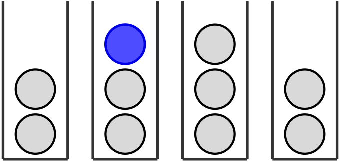
1. No recourse
2. Recourse

Analyzing the Recourse

Recourse $= 2$



Computing A_S

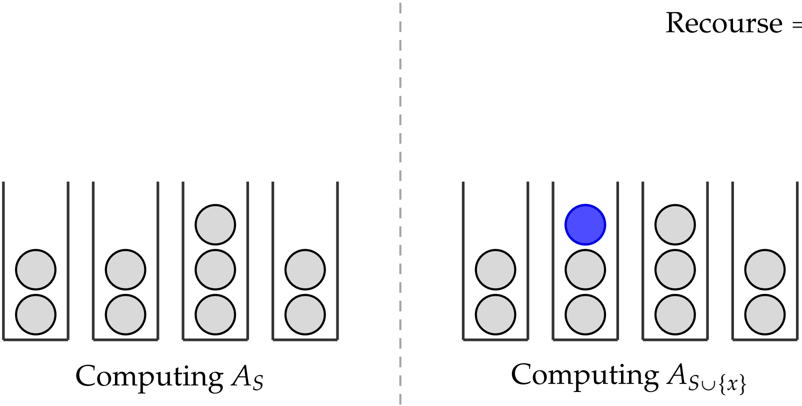


Computing $A_{S \cup \{x\}}$

Two key observations:

Analyzing the Recourse

Recourse = 2



Computing A_S

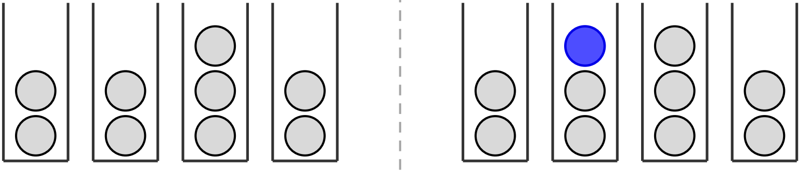
Computing $A_{S \cup \{x\}}$

Two key observations:

1. There's always one special bin with an extra ball

Analyzing the Recourse

Recourse $= 2$



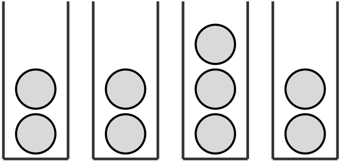
Computing A_S — Computing $A_{S\cup\{x\}}$

Two key observations:

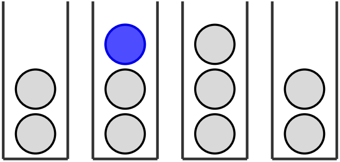
1. There's always one special bin with an extra ball
2. If a ball incurs recourse, one of its choices is the special bin

Analyzing the Recourse

Recourse = 2



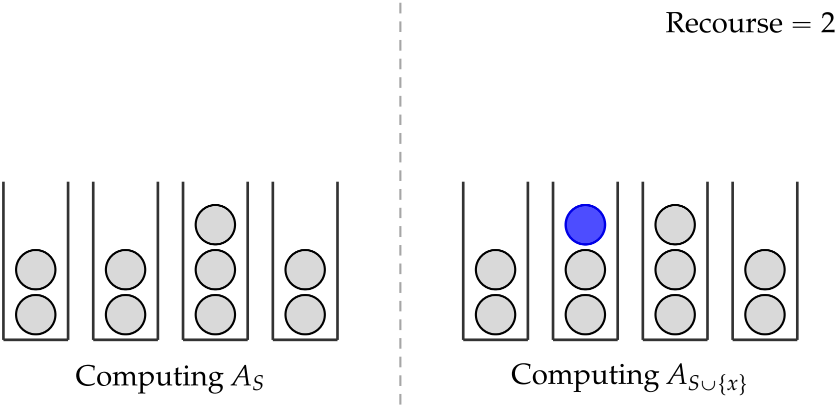
Computing A_S



Computing $A_{S \cup \{x\}}$

$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

Analyzing the Recourse



$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

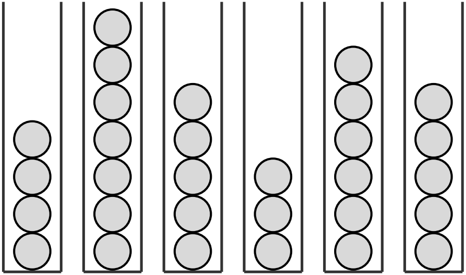
$$\implies \mathbb{E}[\text{total recourse}] = \sum_i \Pr[\text{ball } x_i \text{ incurs recourse}] = O(m/n)$$

A Simple Warmup

Theorem: There exists a history-independent solution with:

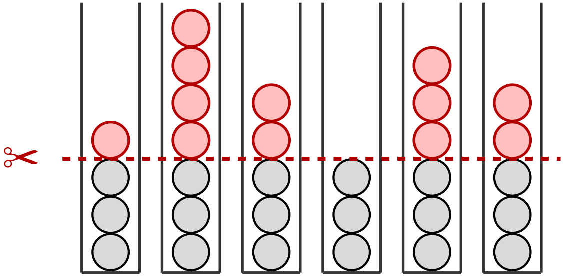
- High-probability overload $\cancel{O(1)}$ $O(\log \log n)$.
- Expected recourse $\cancel{O(\log \log (m/n))}$ $O(m/n)$.

Slice and Spread



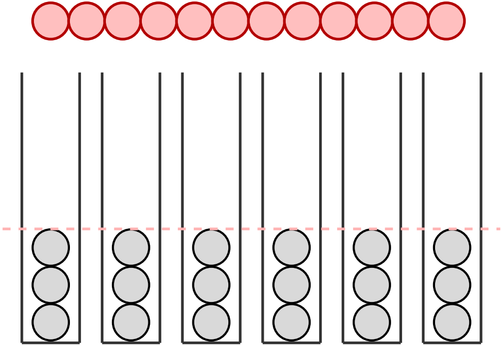
1. **Slice** off the jagged surface

Slice and Spread



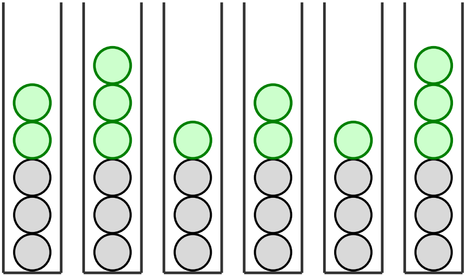
1. **Slice** off the jagged surface

Slice and Spread



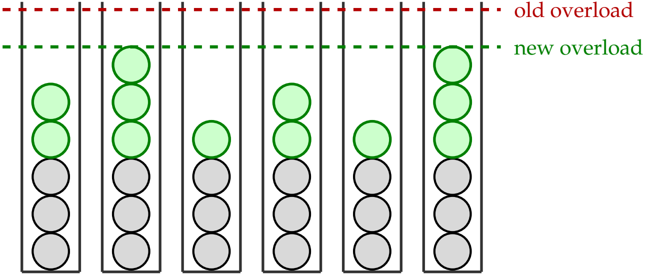
1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

Slice and Spread



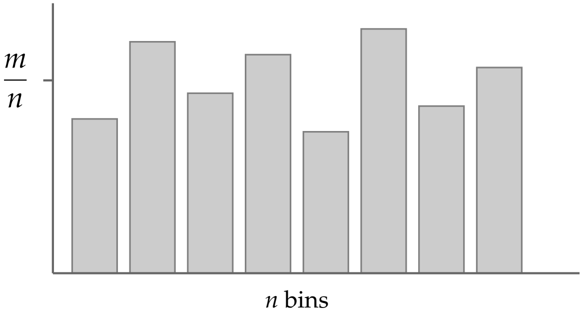
1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

Slice and Spread

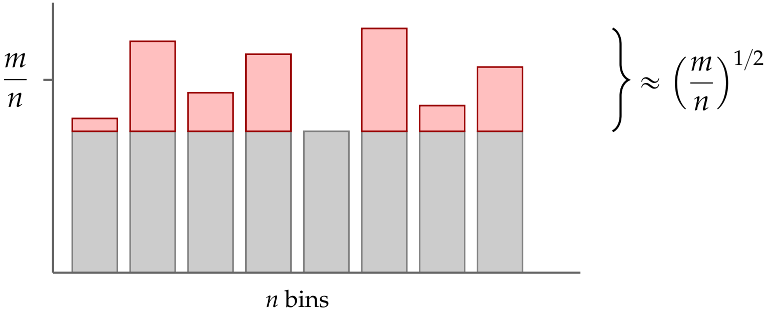


1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

SLICE AND SPREAD REDUCES OVERLOAD



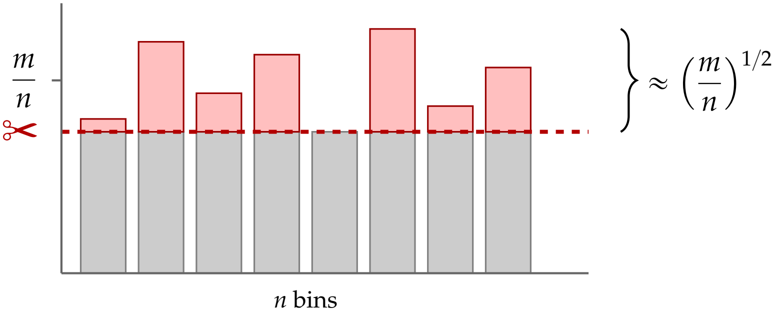
Slice and Spread Reduces Overload



Key Fact

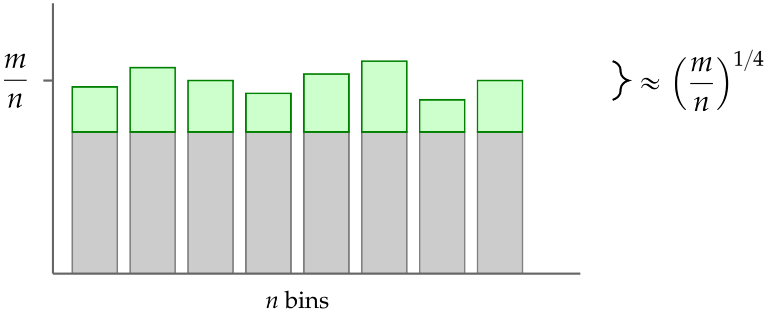
After throwing $m \gg n$ balls uniformly at random into n bins, the bin loads are within roughly $\approx \sqrt{m/n}$ with high probability in n.

Slice and Spread Reduces Overload



- Balls above dotted line $\approx (mn)^{1/2}$

Slice and Spread Reduces Overload



- Balls above dotted line $\approx (mn)^{1/2}$
- By Key Fact, bin loads within $\approx (m/n)^{1/4}$

WHAT'S THE RECOURSE?

Options:

(A) 0

(B) 1

(C) # balls being sliced off

(D) other?

Repeating Slice and Spread

Good overload, good recourse...Could we keep slicing and spreading to get better and better overload?
Proposition: Test

Algorithmic Question

Question: Which balls will we slice in each round?
Option 1: Scrape off the top **Option 2:** Maintain a priority queue in each bin **What we do:** Assign every ball a round...

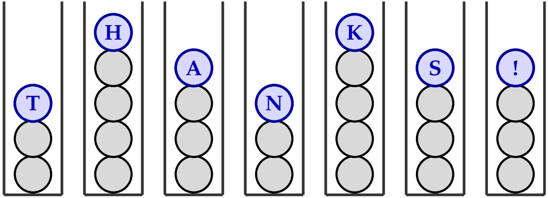
Challenge 1: Slicing Failures

Challenge: There may not be enough fresh randomness in each bin. Not enough balls assigned to that round in that bin.
Result: We won't be able to slice off enough balls, and so the jaggedness will remain.

Challenge 2: Spreading Failures

Challenge: Spreading step got unlucky and didn't spread very evenly. There's more jaggedness than we want
Result: Either we have to slice more balls than we want and may not get the overload down, or we have to slice less balls than we want and may not get the jaggedness down.

History-Independent Load Balancing



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