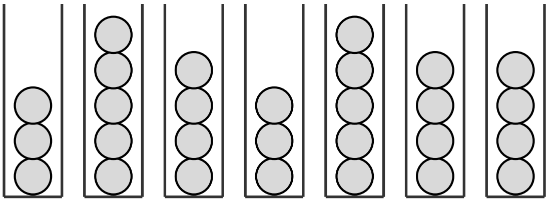
History-Independent Load Balancing



Michael A. Bender
Stony Brook University

Bill Kuszmaul
CMU

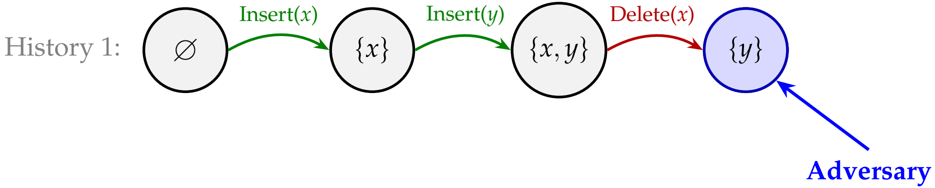
Elaine Shi
CMU

Rose Silver
CMU

History-Independent Data Structures

History 1: $\varnothing \xrightarrow{\text{Insert}(x)} \{x\} \xrightarrow{\text{Insert}(y)} \{x, y\} \xrightarrow{\text{Delete}(x)} \{y\}$

History-Independent Data Structures



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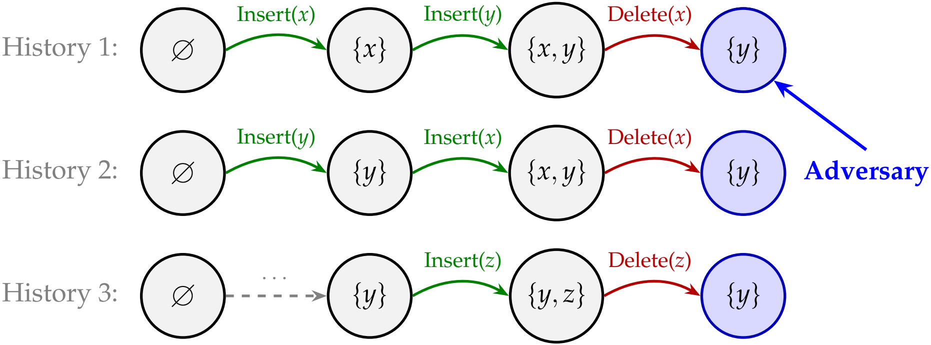

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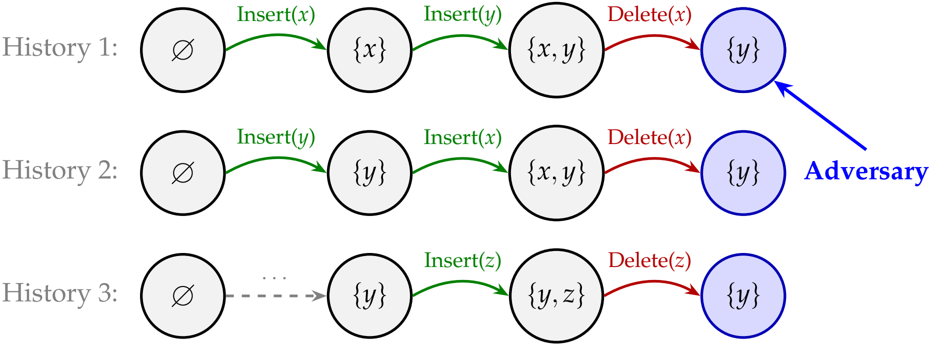

History Independence (Micciancio '97, Naor & Teague '01)

- The state reveals only the current elements—**not the history of operations.**

History Independent Data Structures

A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

Micciancio '97, Naor & Teague '01, Buchbinder & Petrank '03, Molnar et al. '06, Blelloch & Golovin '07,

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Yet fundamental questions remain open.

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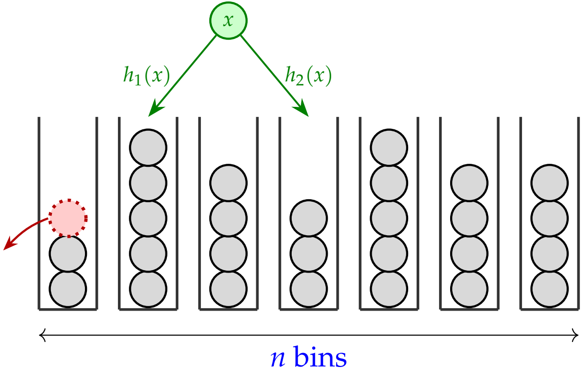
This work: History-Independent Load Balancing

Two-Choice Load Balancing



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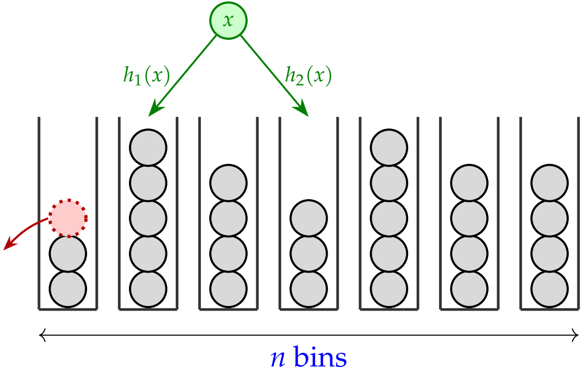

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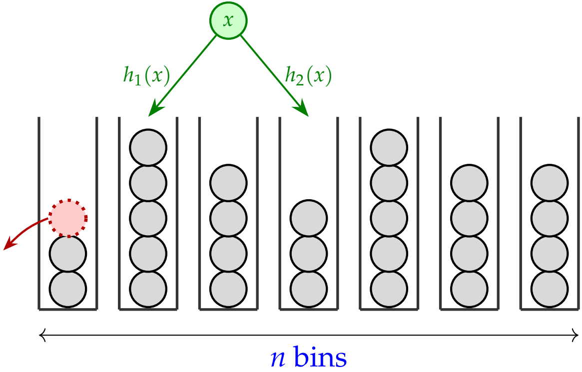

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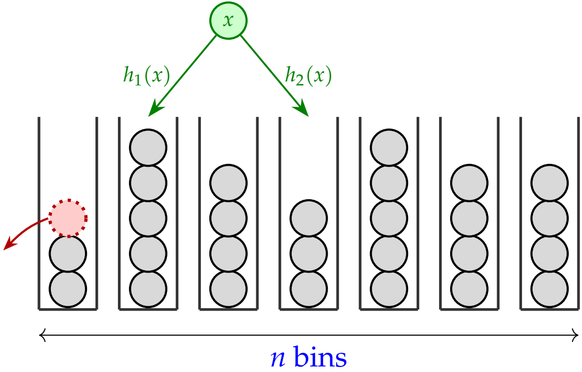
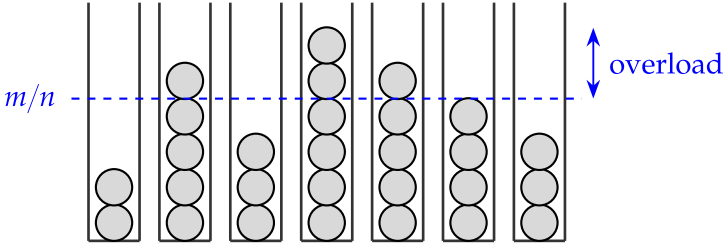


- Balls are inserted/deleted, with up to m present at a time.
- Each ball has two random bins where it can go.
- We must maintain a valid assignment of balls to bins.

Two Goals



Minimize Overload:

- i.e., the amount by which the fullest bin exceeds m/n.

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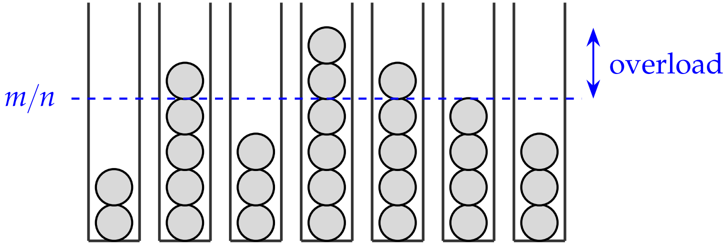

Minimize Overload:

- i.e., the amount by which the fullest bin exceeds m/n.

Minimize Recourse:

- i.e., the number of balls moved around on any given insertion/deletion.

Putting it All Together

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History-Independent Load Balancing:

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Question: Does there exist a history-independent solution with small recourse and small overload?

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History-Independent Load Balancing:

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Question: Does there exist a history-independent solution with small recourse and small overload?

Our Main Result: There exists a history-independent solution with:

- High probability overload $O(1)$
- Expected recourse $O(\log \log(m/n))$

Past Work (Not History Independent)

Overload	Recourse	Reference	Caveats
$O(\log\log n)$	0	[ABKU '94] [BCSV '00]	insertion-only
$O(1)$	$O(\log(m/n))$	[Dietzfelbinger, Weidling '07]	insertion-only
$\tilde{O}(\sqrt{m/n})$	$O(1)$	[Frieze, Petti '18]	insertion-only
$O(\log(m/n))$	0	[Bansal, Kuszmaul '22]	no reinsertions
$O(1)$	$O(m/n)$	[Dietzfelbinger, Weidling '07]	

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If we want overload $O(1)$, our result is a new state of the art!

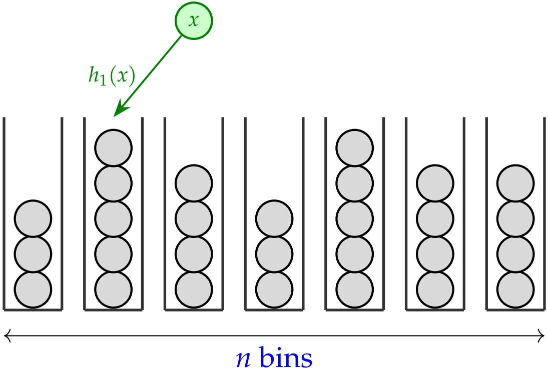
Rest of Talk: A Simple Warmup

Theorem: There exists a history-independent solution with:

- High-probability overload $\cancel{O(1)}$ $O(\log \log n)$.
- Expected recourse $\cancel{O(\log \log(m/n))}$ $O(m/n)$.

BASELINE 1: THE SINGLE-CHOICE STRATEGY

To insert a ball x, just put it in bin $h_1(x)$:



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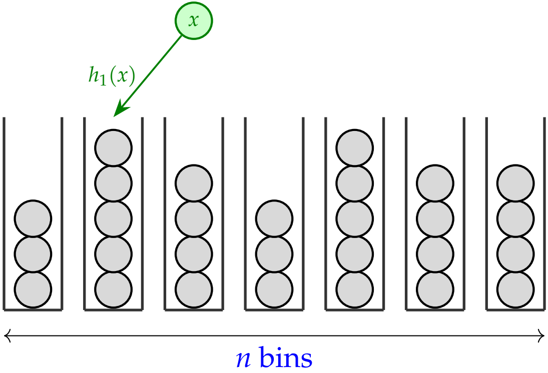

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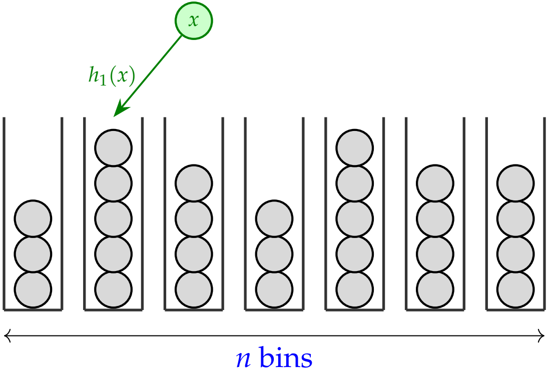

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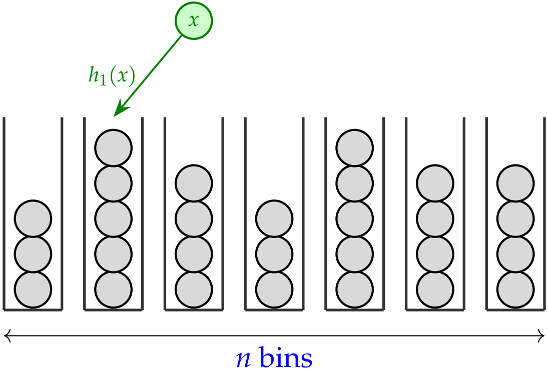
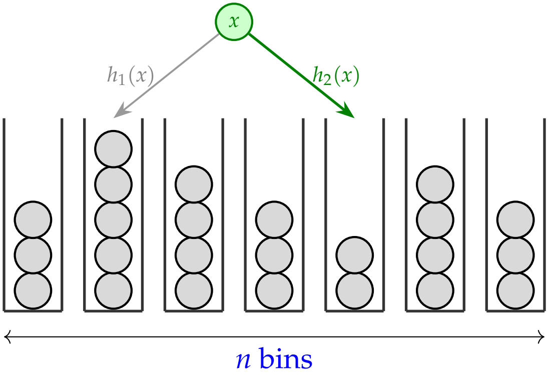


- This is history-independent ✓
- The recourse is 0 ✓
- But... the overload is huge, roughly $\sqrt{m/n}$ ✗

Baseline 2: Greedy Insertions

To insert a ball x, put it in the emptier of its choices:



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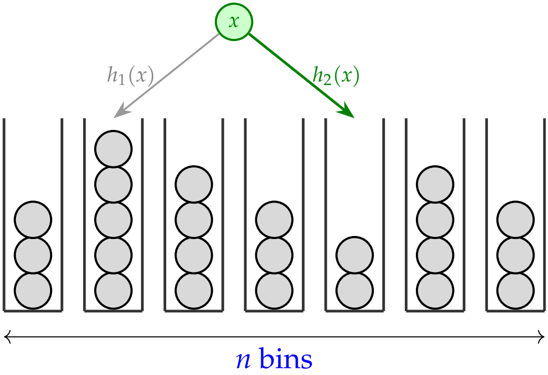

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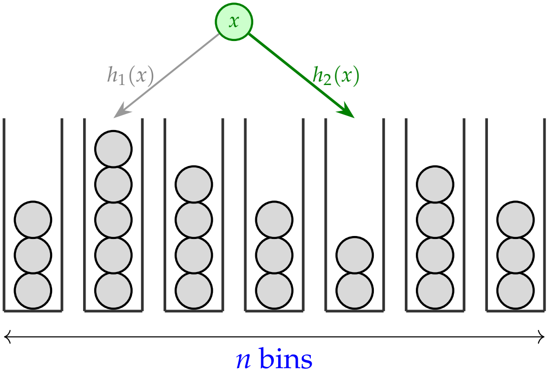

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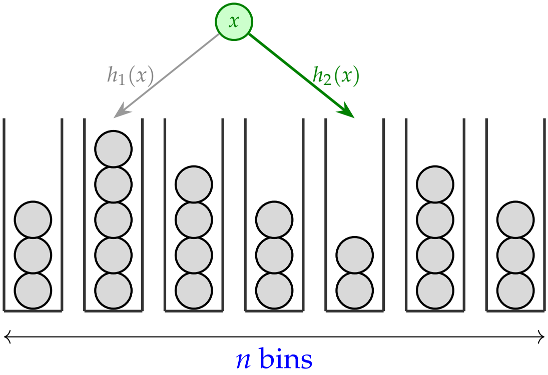
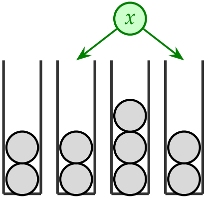


- This is **not** history-independent ✗
- The recourse is 0 ✓
- In the insertion-only case, the overload is $O(\log \log n)$ ✓

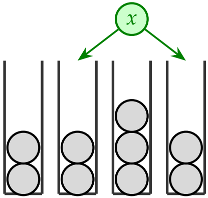
[Azar, Broder, Karlin and Upfal '94]

A Simple History-Independent Algorithm



Current state A_S

A Simple History-Independent Algorithm



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At each time step:

1. Observe current set S
2. Compute $A_{S \cup \{x\}}$
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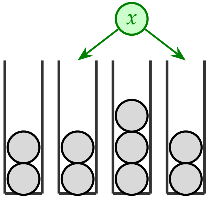

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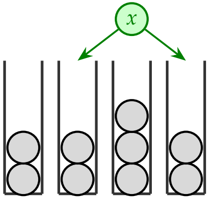

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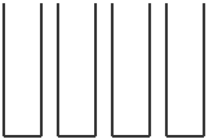

Computing $A_{S \cup \{x\}}$

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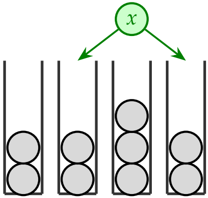

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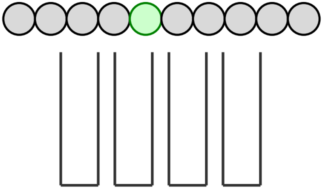

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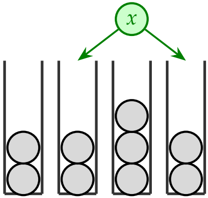

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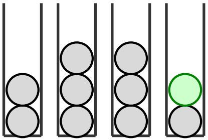

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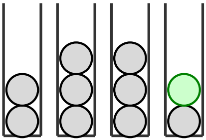
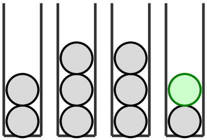


Updated state $A_{S \cup \{x\}}$



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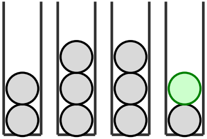
Analyzing History-Independent Greedy



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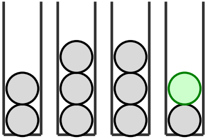
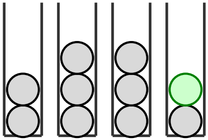


Updated state $A_{S \cup \{x\}}$

- The algorithm is history independent ✓

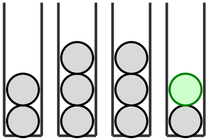
Analyzing History-Independent Greedy



Updated state $A_{S \cup \{x\}}$

- The algorithm is history independent ✓
- The overload is $O(\log \log n)$ ✓

Analyzing History-Independent Greedy

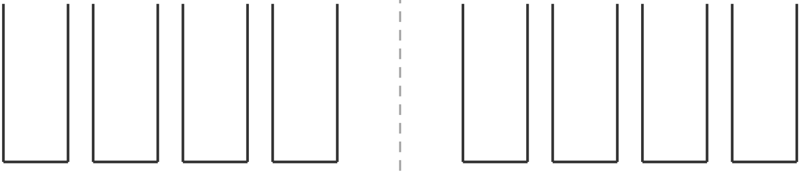


Updated state $A_{S \cup \{x\}}$

- The algorithm is history independent ✓
- The overload is $O(\log \log n)$ ✓
- What is the recourse?

Analyzing the Recourse

Recourse $= 0$



Computing A_S | Computing $A_{S \cup \{x\}}$

How many balls change assignments between A_S and $A_{S \cup \{x\}}$?

Analyzing the Recourse

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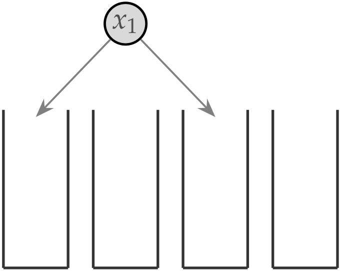

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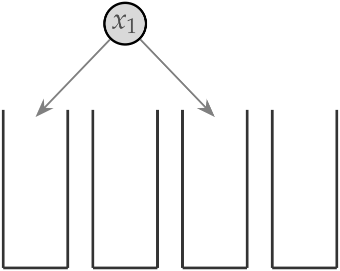

Computing $A_{S \cup \{x\}}$

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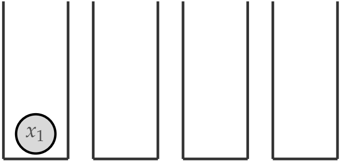

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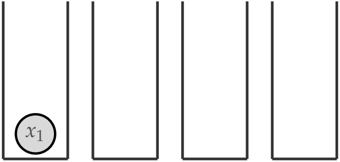

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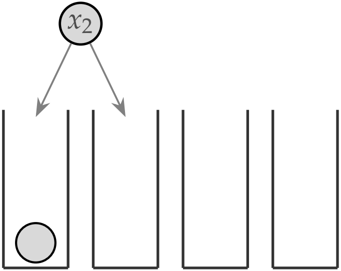

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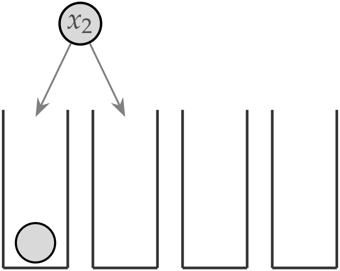

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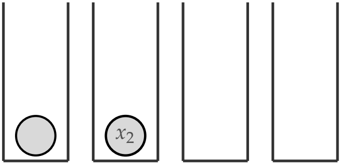

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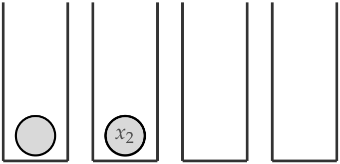

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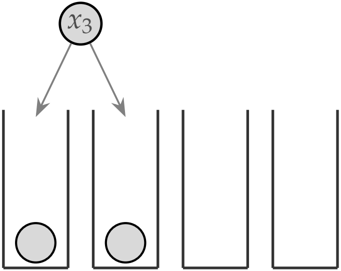

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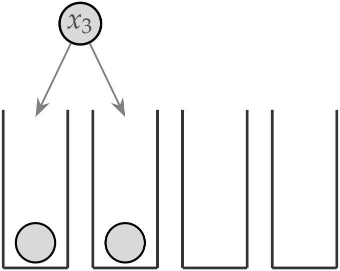

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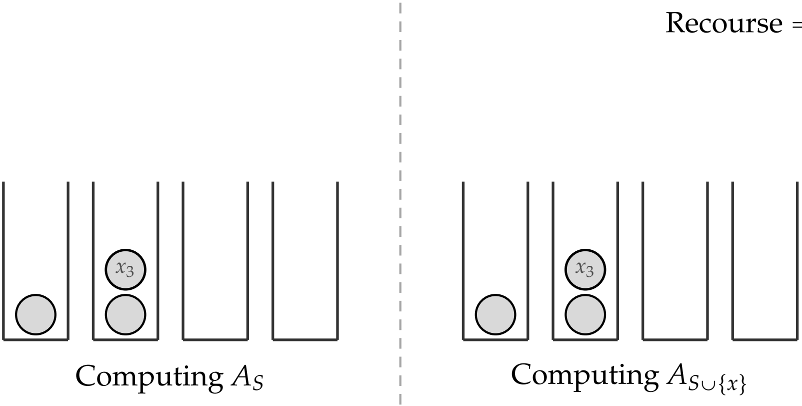

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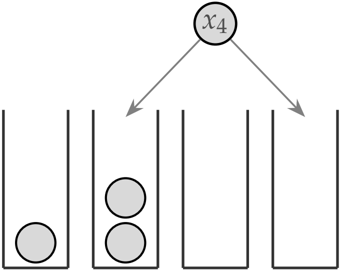
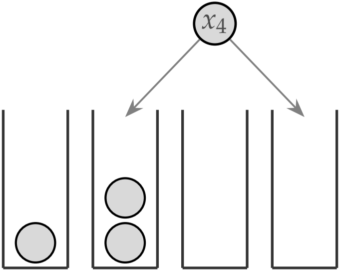


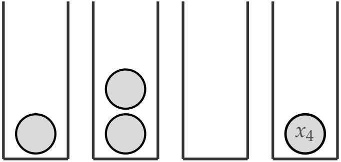
Computing A_S



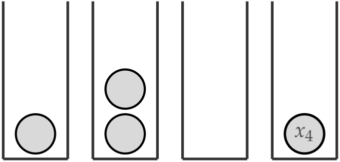
Computing $A_{S \cup \{x\}}$

Analyzing the Recourse

Recourse = 0

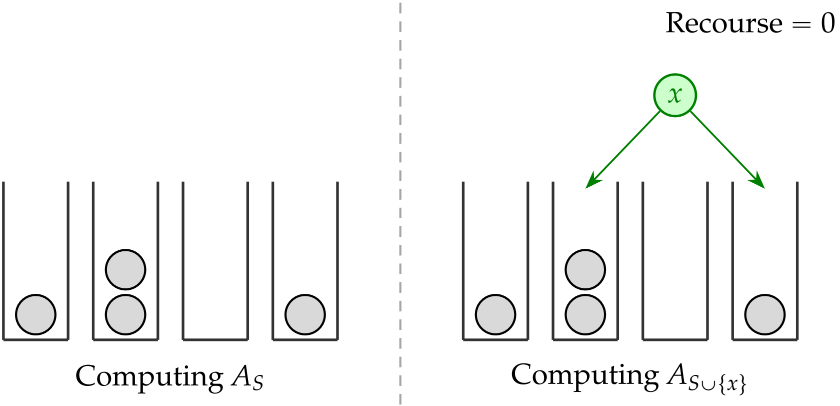


Computing A_S



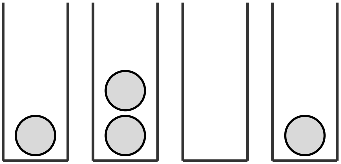
Computing $A_{S \cup \{x\}}$

Analyzing the Recourse

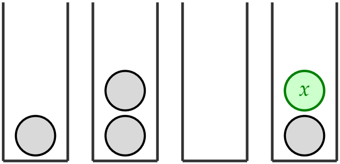


Analyzing the Recourse

Recourse = 0



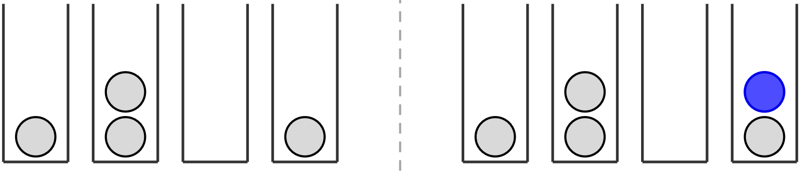
Computing A_S



Computing $A_{S \cup \{x\}}$

Analyzing the Recourse

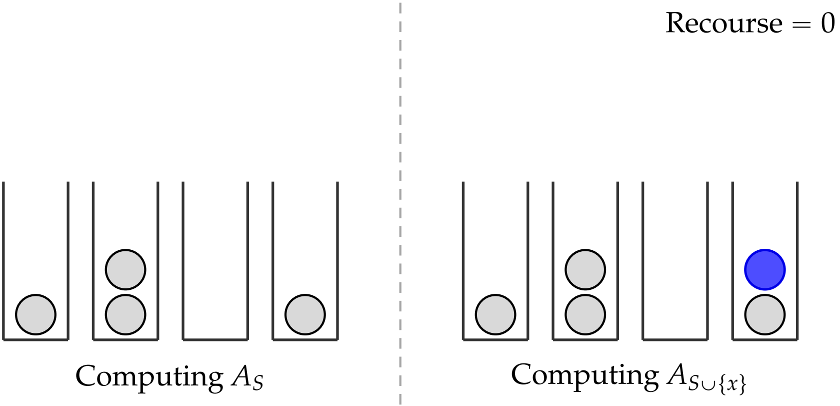
Recourse $= 0$



Computing A_S | Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

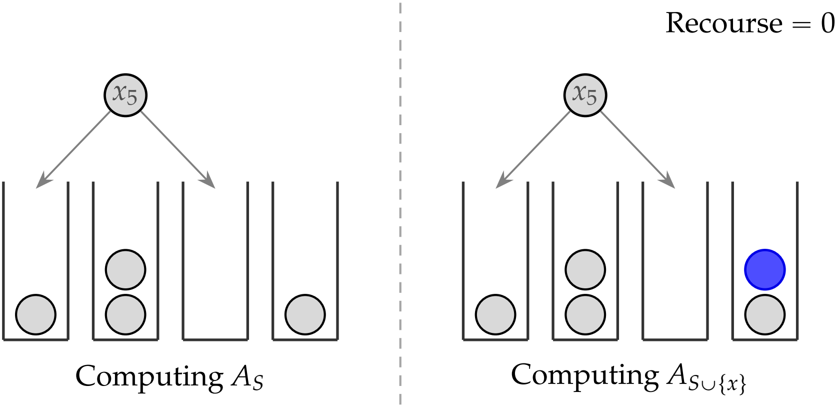
Analyzing the Recourse



Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

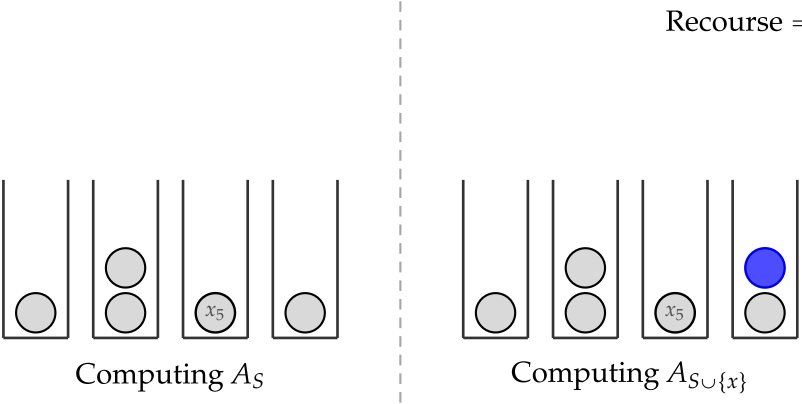


Future insertions will experience either:

1. No recourse

Analyzing the Recourse

Recourse = 0

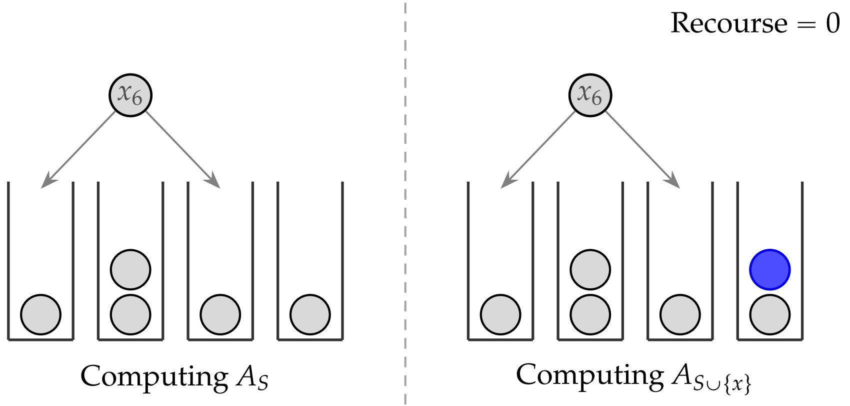


Computing A_S Computing $A_{S\cup\{x\}}$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

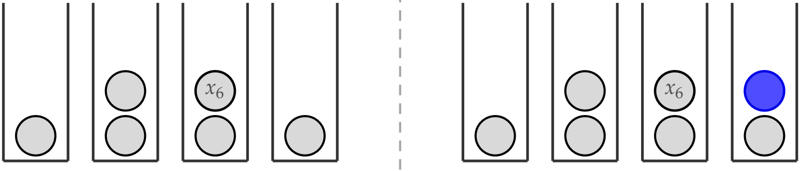


Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

Recourse = 0



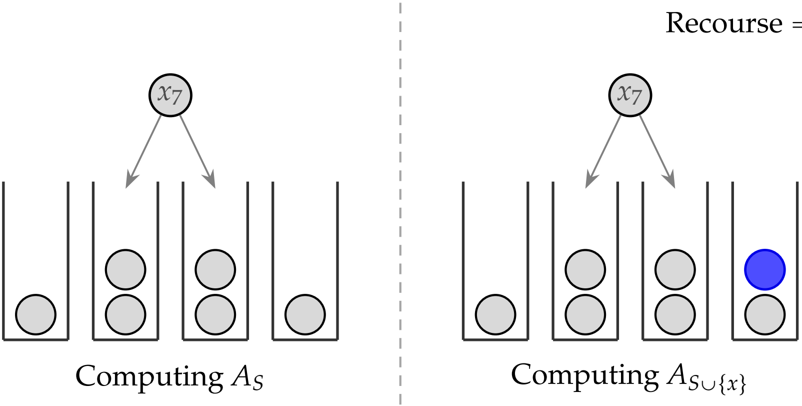
Computing A_S Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

Recourse = 0

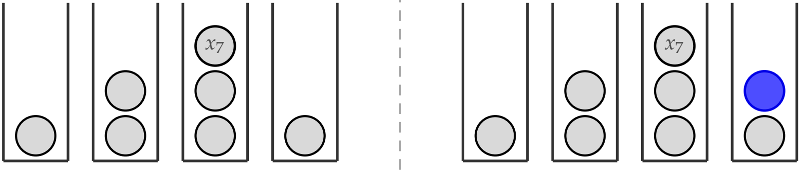


Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

Recourse $= 0$



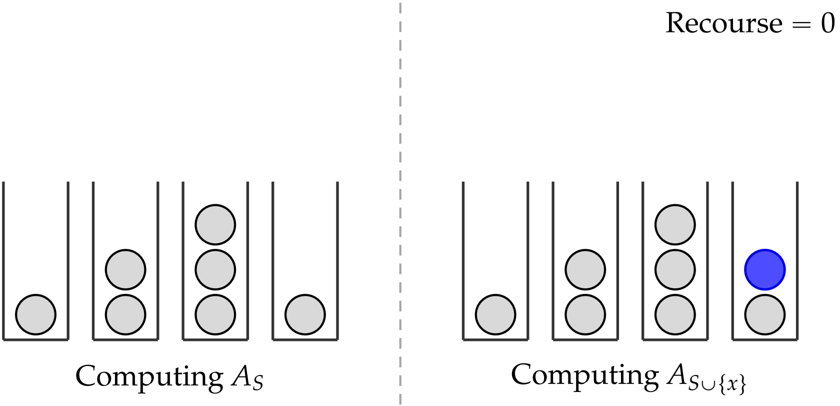
Computing A_S

Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

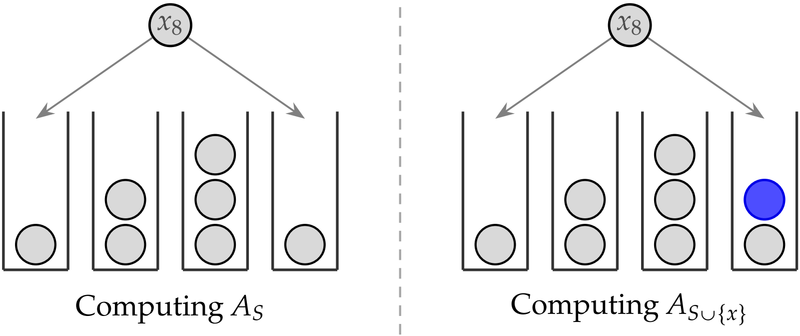


Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 0

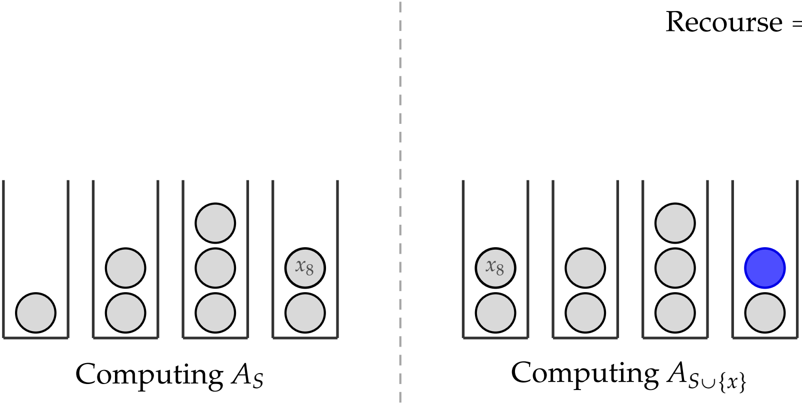


Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 1

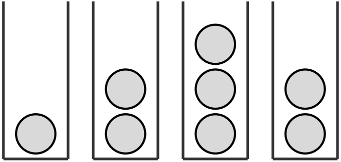


Subsequent balls will experience either:

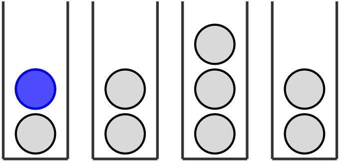
1. No recourse
2. Recourse

ANALYZING THE RECOURSE

Recourse = 1



Computing A_S

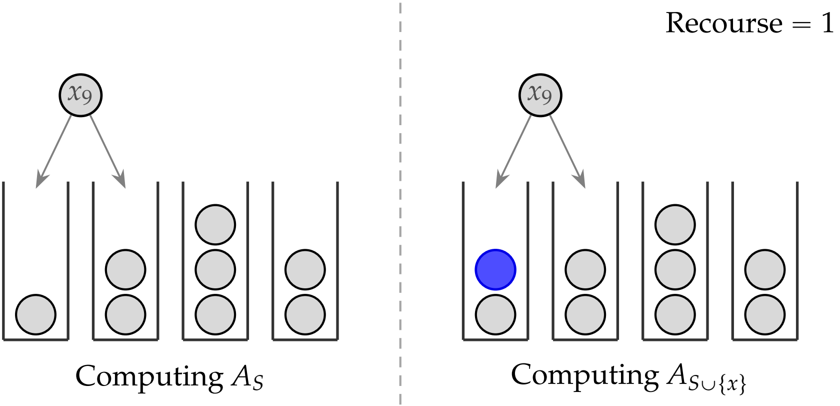


Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

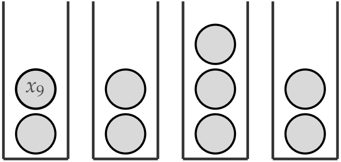


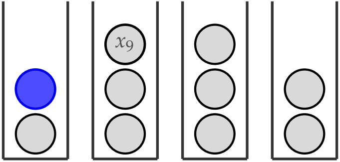
Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 2



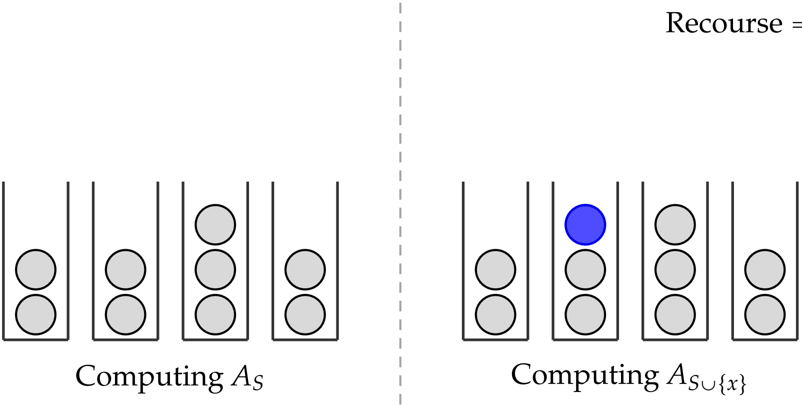


Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 2

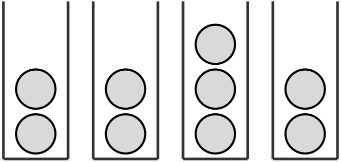


Subsequent balls will experience either:

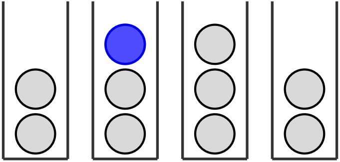
1. No recourse
2. Recourse

Analyzing the Recourse

Recourse = 2



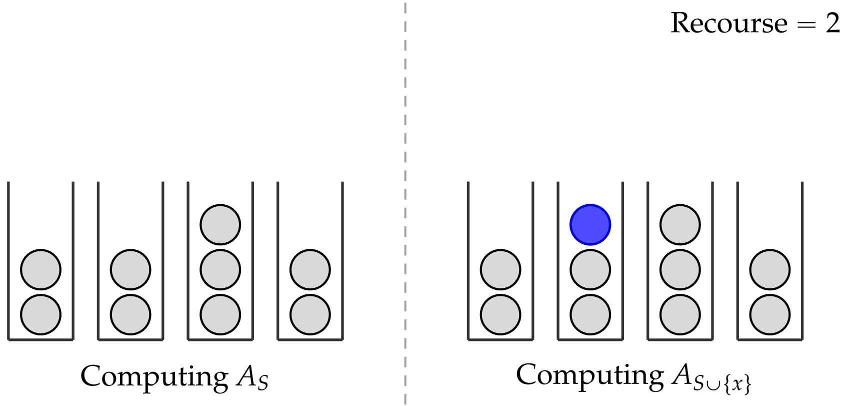
Computing A_S



Computing $A_{S \cup \{x\}}$

Two key observations:

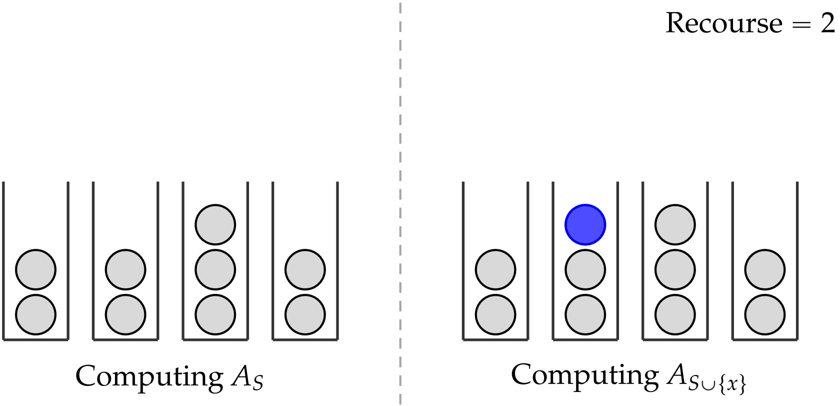
Analyzing the Recourse



Two key observations:

1. There's always one special bin with an extra ball

Analyzing the Recourse

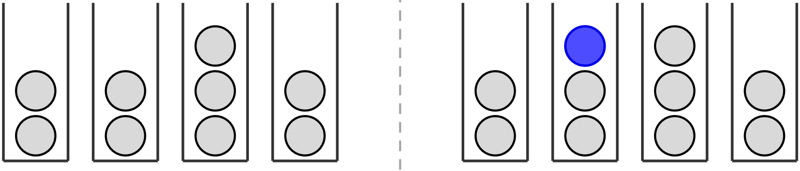


Two key observations:

1. There's always one special bin with an extra ball
2. If a ball incurs recourse, one of its choices is the special bin

Analyzing the Recourse

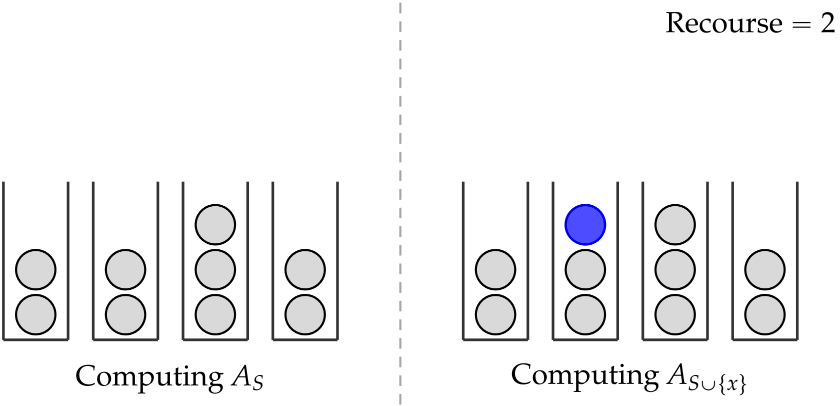
Recourse = 2



Computing A_S — Computing $A_{S \cup \{x\}}$

$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$

Analyzing the Recourse



$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

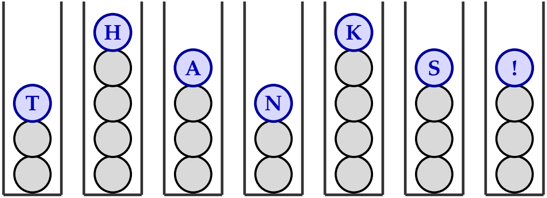
$$\implies \mathbb{E}[\text{total recourse}] = \sum_i \Pr[\text{ball } x_i \text{ incurs recourse}] = O(m/n)$$

A Simple Warmup

Theorem: There exists a history-independent solution with:

- High-probability overload $\cancel{O(1)}$ $O(\log \log n)$.
- Expected recourse $\cancel{O(\log \log(m/n))}$ $O(m/n)$.

History-Independent Load Balancing



Michael A. Bender
Stony Brook University

Bill Kuszmaul
CMU

Elaine Shi
CMU

Rose Silver
CMU