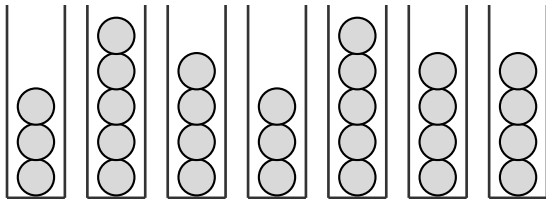


History-Independent Load Balancing



Michael A. Bender

Stony Brook University

Bill Kuszmaul

CMU

Elaine Shi

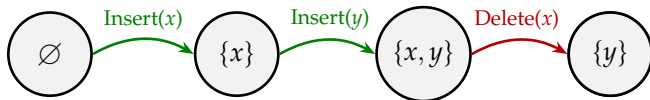
CMU

Rose Silver

CMU

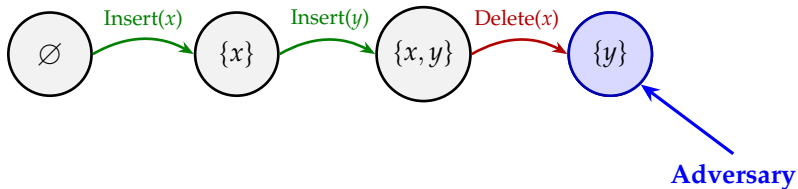
HISTORY-INDEPENDENT DATA STRUCTURES

History 1:

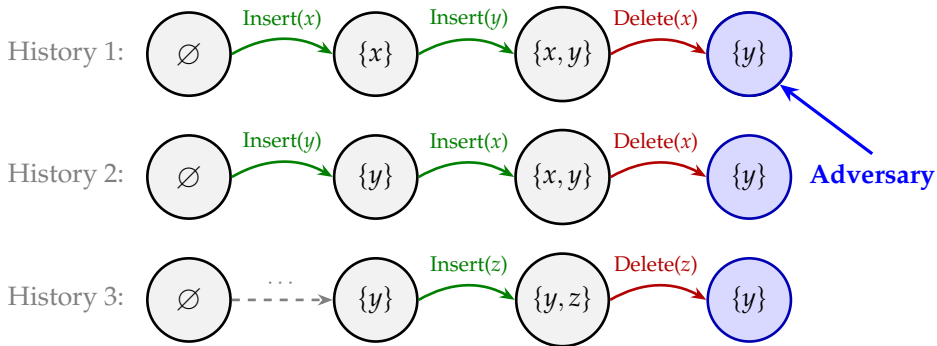


HISTORY-INDEPENDENT DATA STRUCTURES

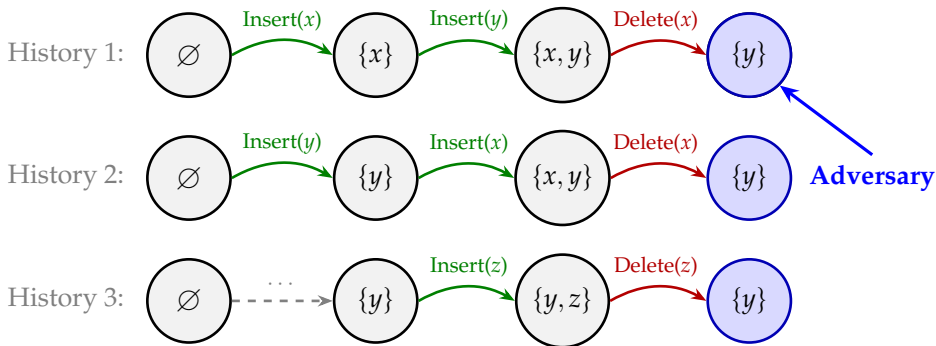
History 1:



HISTORY-INDEPENDENT DATA STRUCTURES



HISTORY-INDEPENDENT DATA STRUCTURES



History Independence (Micciancio '97, Naor & Teague '01)

- ▶ The state reveals only the current elements—**not the history of operations.**

HISTORY INDEPENDENT DATA STRUCTURES

A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

Micciancio '97, Naor & Teague '01, Buchbinder & Petrank '03, Molnar et al. '06, Blelloch & Golovin '07, Moran et al. '07, Naor et al. '08, Golovin '08–'10, Bajaj & Sion '13, Roche et al. '15, Bender et al. '16

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Yet fundamental questions remain open.

HISTORY INDEPENDENT DATA STRUCTURES

A History of Applications

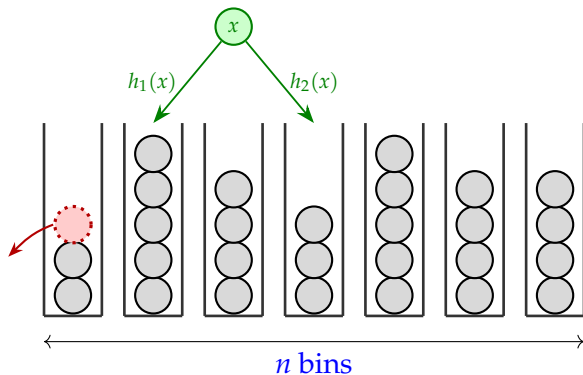
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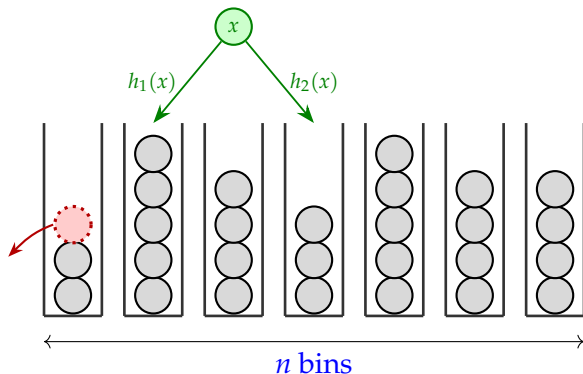
Yet fundamental questions remain open.

This work: History-Independent Load Balancing

TWO-CHOICE LOAD BALANCING

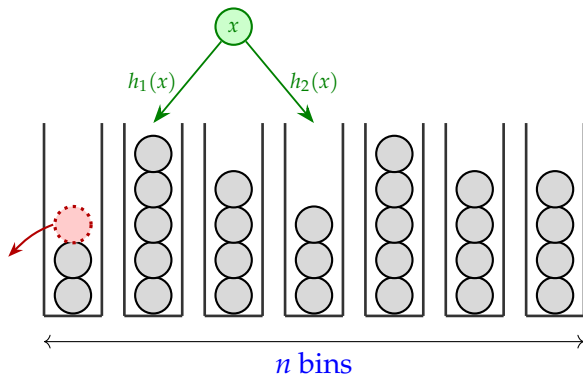


TWO-CHOICE LOAD BALANCING



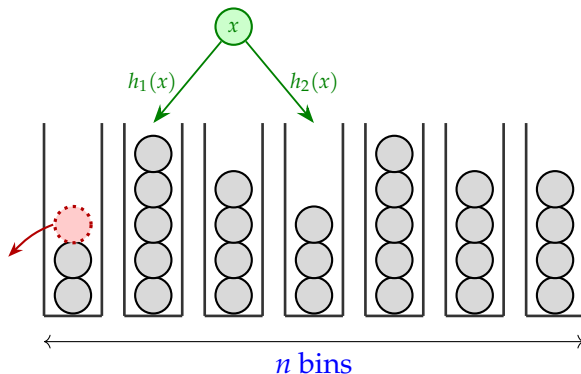
- Balls are **inserted**/**deleted**, with up to m present at a time.

TWO-CHOICE LOAD BALANCING



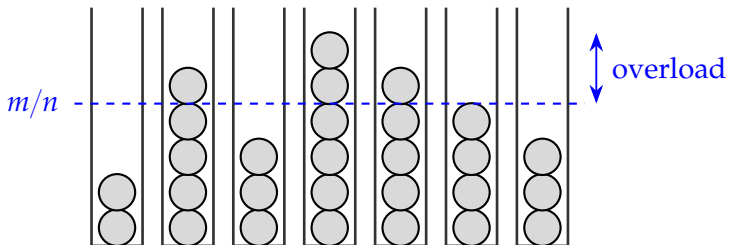
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- ▶ Each ball has two random bins where it can go.

TWO-CHOICE LOAD BALANCING



- ▶ Balls are **inserted**/**deleted**, with up to m present at a time.
- ▶ Each ball has two random bins where it can go.
- ▶ We must maintain a valid assignment of balls to bins.

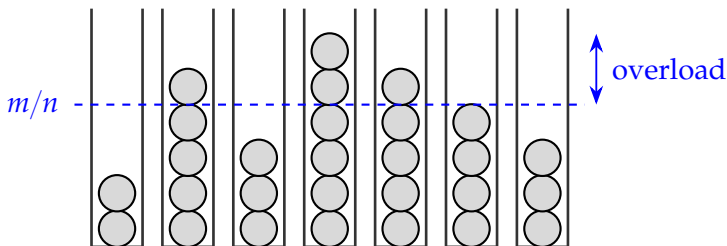
TWO GOALS



Minimize Overload:

- ▶ i.e., the amount by which the fullest bin exceeds m/n .

TWO GOALS



Minimize Overload:

- ▶ i.e., the amount by which the fullest bin exceeds m/n .

Minimize Recourse:

- ▶ i.e., the number of balls moved around on any given insertion/deletion.

PUTTING IT ALL TOGETHER

PUTTING IT ALL TOGETHER

History-Independent Load Balancing:

PUTTING IT ALL TOGETHER

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- ▶ For all sets S of balls, if the current set is S , then the assignment is always A_S .

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Question: Does there exist a **history-independent** solution with small **recourse** and small **overload**?

PUTTING IT ALL TOGETHER

History-Independent Load Balancing:

- ▶ For all sets S of balls, if the current set is S , then the assignment is always A_S .

Question: Does there exist a **history-independent** solution with small **recourse** and small **overload**?

Our Main Result: There exists a **history-independent** solution with:

- ▶ High probability **overload** $O(1)$
- ▶ Expected **recourse** $O(\log \log(m/n))$

PAST WORK (NOT HISTORY INDEPENDENT)

Overload	Recourse	Reference	Caveats
$O(\log \log n)$	0	[ABKU '94] [BCSV '00]	insertion-only
$O(1)$	$O(\log(m/n))$	[Dietzfelbinger, Weidling '07]	insertion-only
$\tilde{O}(\sqrt{m/n})$	$O(1)$	[Frieze, Petti '18]	insertion-only
$O(\log(m/n))$	0	[Bansal, Kuszmaul '22]	no reinsertions
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If we want overload $O(1)$, our result is a new state of the art!

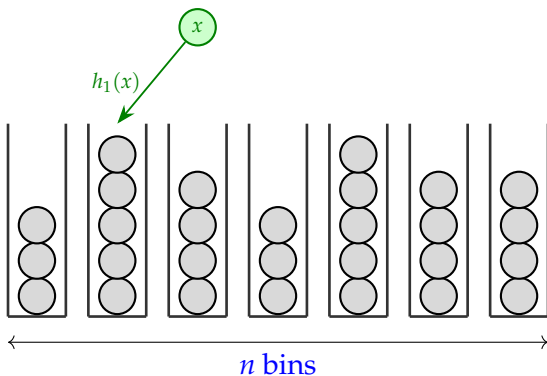
REST OF TALK: A SIMPLE WARMUP

Theorem: There exists a history-independent solution with:

- ▶ High-probability overload $\Theta(1)$ $O(\log \log n)$.
- ▶ Expected recourse $\Theta(\log \log(m/n))$ $O(m/n)$.

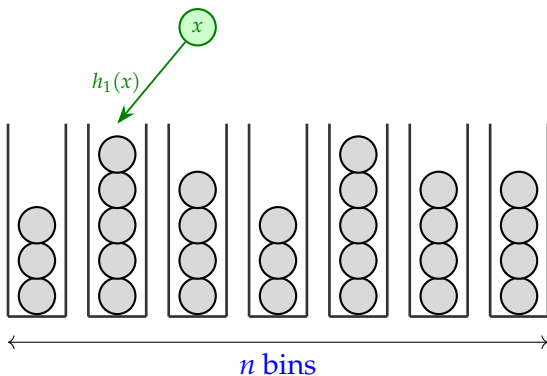
BASLINE 1: THE SINGLE-CHOICE STRATEGY

To insert a ball x , just put it in bin $h_1(x)$:



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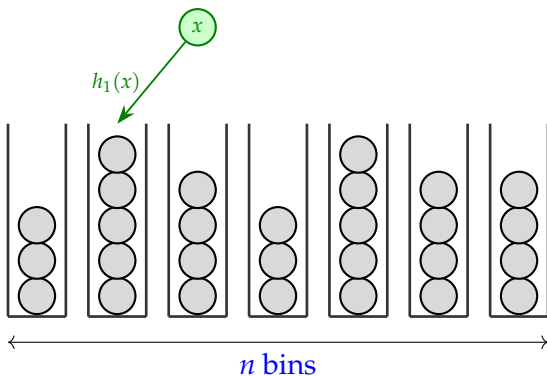
To insert a ball x , just put it in bin $h_1(x)$:



- This is history-independent ✓

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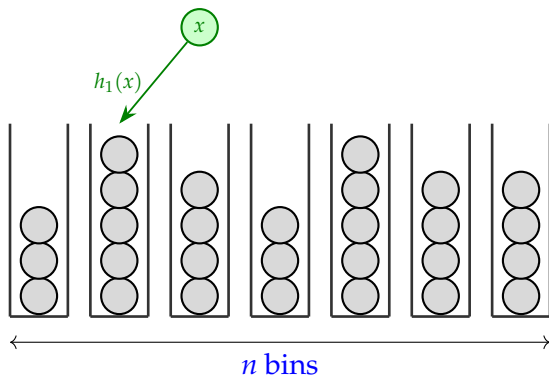
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- ▶ This is history-independent ✓
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BASLINE 1: THE SINGLE-CHOICE STRATEGY

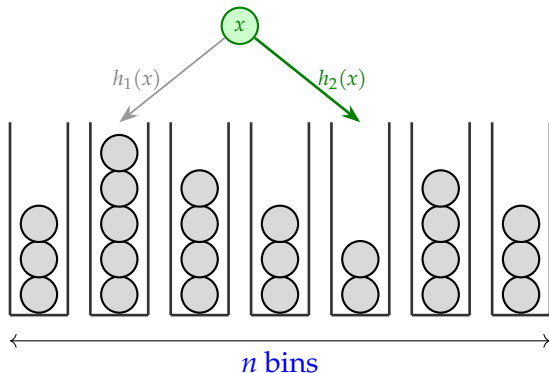
To insert a ball x , just put it in bin $h_1(x)$:



- ▶ This is history-independent ✓
- ▶ The recourse is 0 ✓
- ▶ But... the overload is huge, roughly $\sqrt{m/n}$ ✗

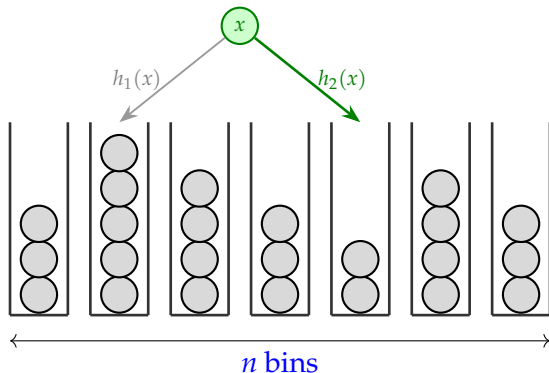
BASELINE 2: GREEDY INSERTIONS

To insert a ball x , put it in the **emptier** of its choices:



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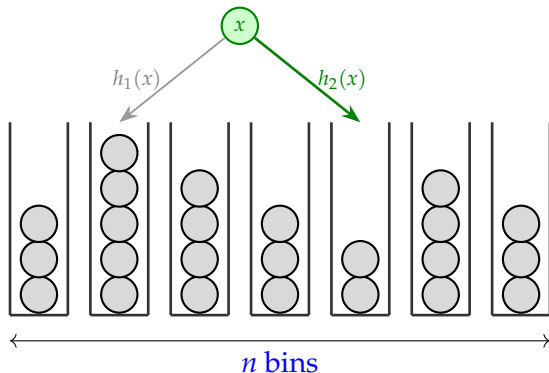
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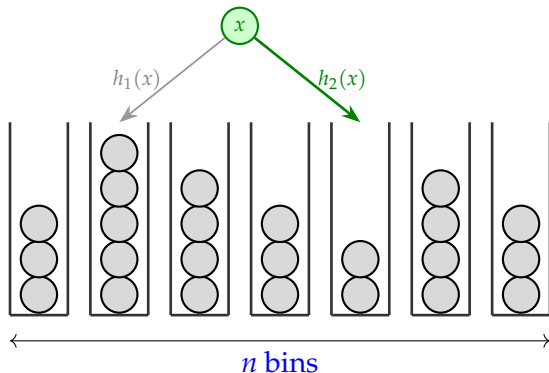
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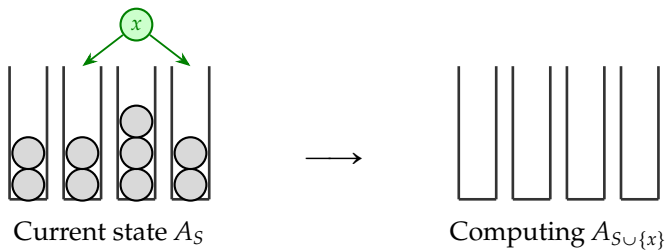
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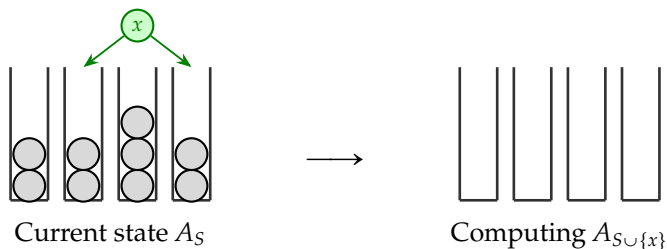
- ▶ This is **not** history-independent ✗
- ▶ The recourse is 0 ✓
- ▶ In the insertion-only case, the overload is $O(\log \log n)$ ✓

[Azar, Broder, Karlin and Upfal '94]

A SIMPLE HISTORY-INDEPENDENT ALGORITHM



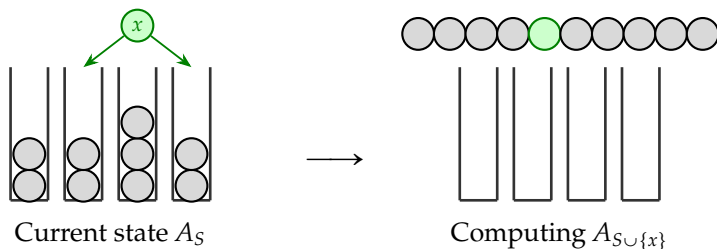
A SIMPLE HISTORY-INDEPENDENT ALGORITHM



To insert a new ball x , we compute $A_{S \cup \{x\}}$:

1. Empty out the bins.
2. Sort the balls in $S \cup \{x\}$ to get x_1, x_2, \dots .
3. Insert the balls in sorted order using greedy.

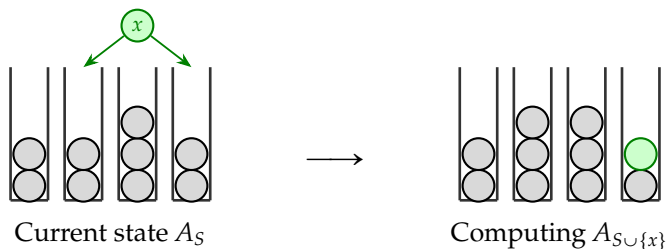
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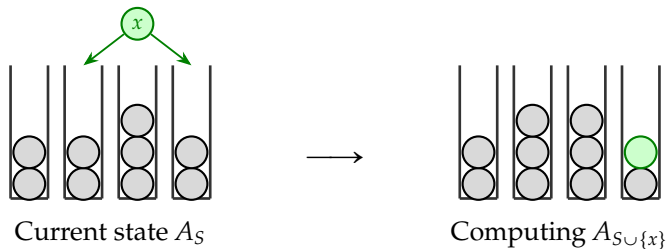
A SIMPLE HISTORY-INDEPENDENT ALGORITHM



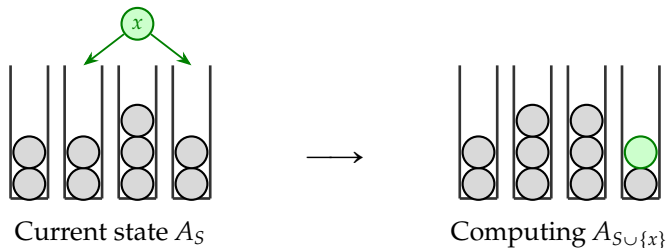
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ANALYZING HISTORY-INDEPENDENT GREEDY

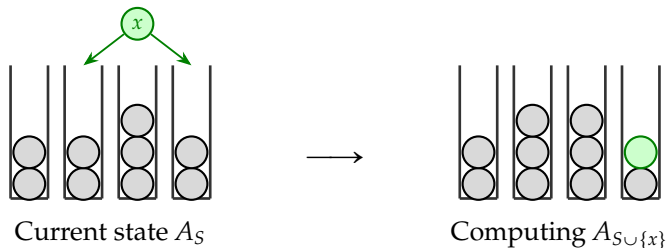


ANALYZING HISTORY-INDEPENDENT GREEDY



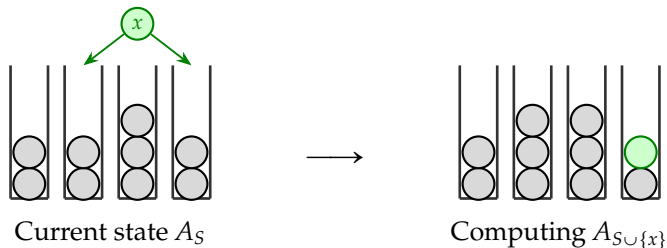
- The algorithm is history independent ✓

ANALYZING HISTORY-INDEPENDENT GREEDY



- ▶ The algorithm is history independent ✓
- ▶ The overload is $O(\log \log n)$ ✓

ANALYZING HISTORY-INDEPENDENT GREEDY



- ▶ The algorithm is history independent ✓
- ▶ The overload is $O(\log \log n)$ ✓
- ▶ What is the recourse?

ANALYZING THE RECOURSE

Recourse = 0



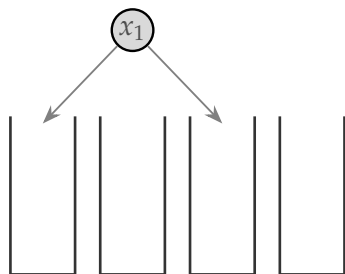
Computing A_S



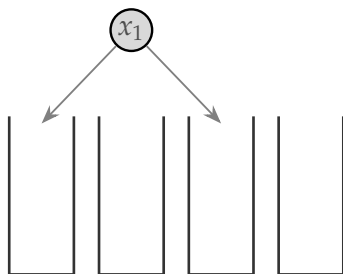
Computing $A_{S \cup \{x\}}$

How many balls change assignments between A_S and $A_{S \cup \{x\}}$?

ANALYZING THE RECOURSE



Computing A_S



Computing $A_{S \cup \{x\}}$

Recourse = 0

ANALYZING THE RECOURSE

Recourse = 0

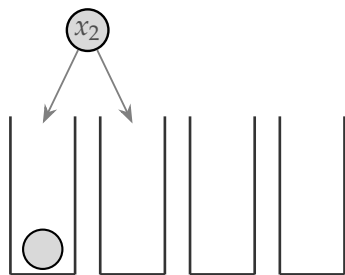


Computing A_S

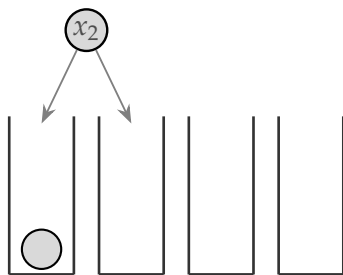


Computing $A_{S \cup \{x\}}$

ANALYZING THE RECOURSE



Computing A_S

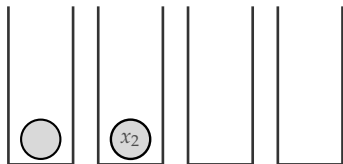


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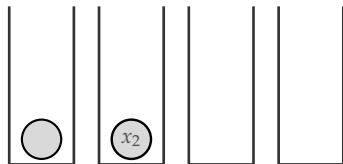
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ANALYZING THE RECOURSE

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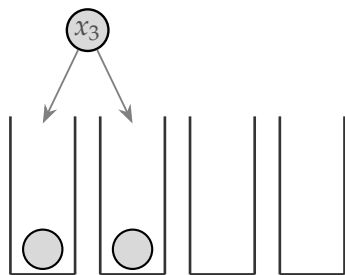


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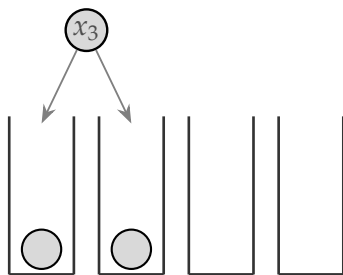


Computing $A_{S \cup \{x\}}$

ANALYZING THE RECOURSE



Computing A_S

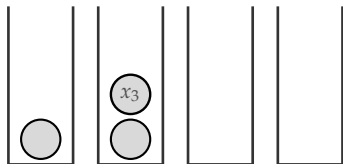


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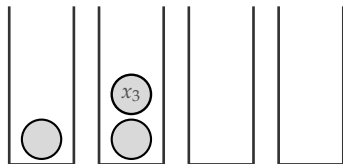
Recourse = 0

ANALYZING THE RECOURSE

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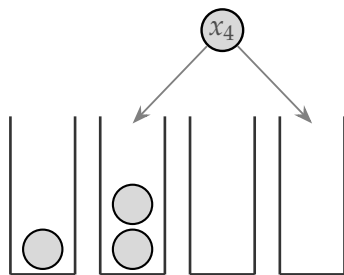


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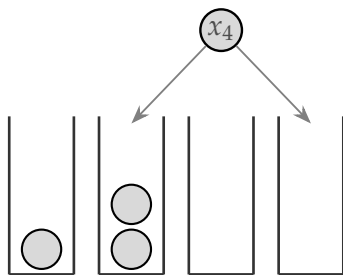


Computing $A_{S \cup \{x\}}$

ANALYZING THE RECOURSE



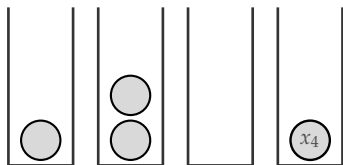
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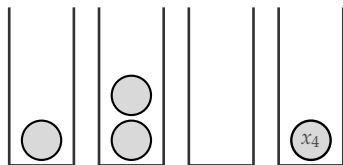
Computing $A_{S \cup \{x\}}$

ANALYZING THE RECOURSE

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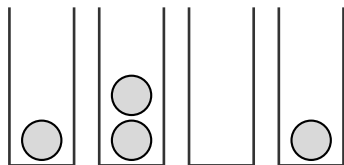


Computing A_S

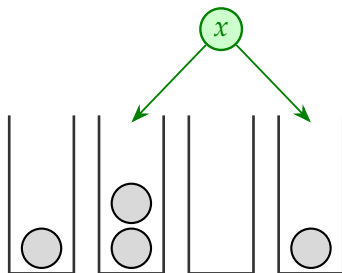


Computing $A_{S \cup \{x\}}$

ANALYZING THE RECOURSE



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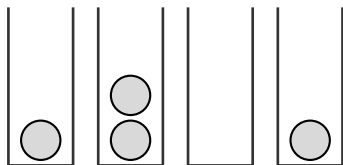


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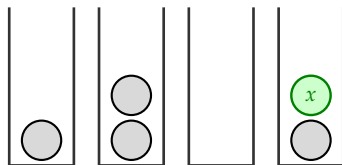
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ANALYZING THE RECOURSE

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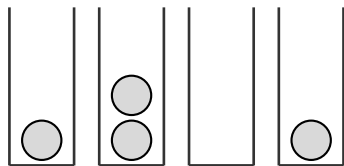
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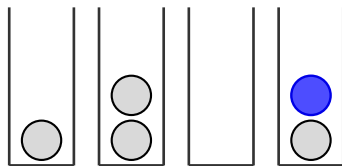
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Computing A_S

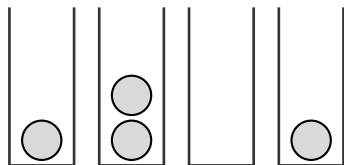


Computing $A_{S \cup \{x\}}$

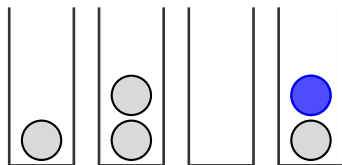
Subsequent balls will experience either:

ANALYZING THE RECOURSE

Recourse = 0



Computing A_S

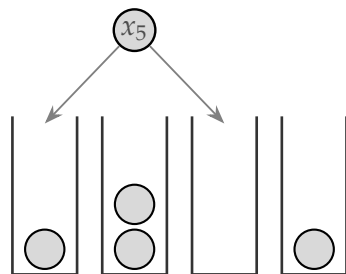


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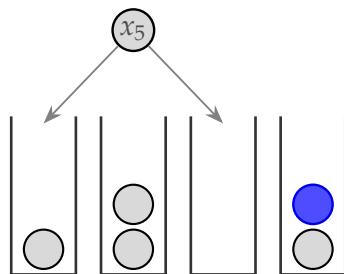
Subsequent balls will experience either:

1. No recourse

ANALYZING THE RECOURSE



Computing A_S



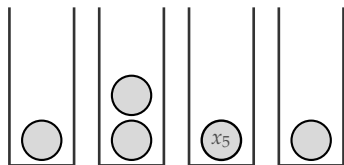
Computing $A_{S \cup \{x\}}$

Future insertions will experience either:

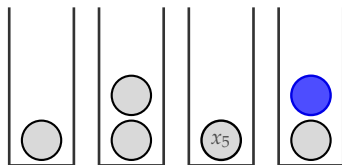
1. No recourse

ANALYZING THE RECOURSE

Recourse = 0



Computing A_S

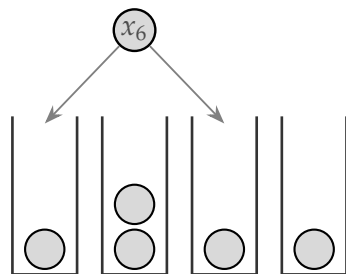


Computing $A_{S \cup \{x\}}$

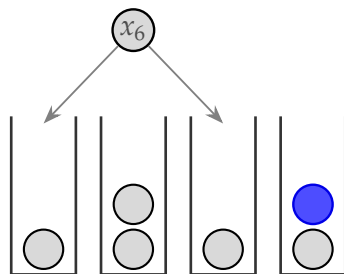
Subsequent balls will experience either:

1. No recourse

ANALYZING THE RECOURSE



Computing A_S



Recourse = 0

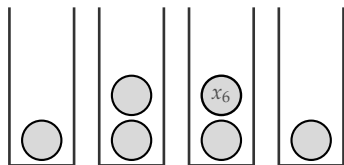
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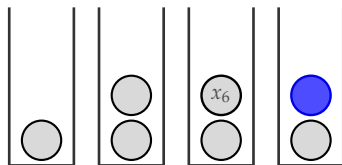
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ANALYZING THE RECOURSE

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Computing A_S

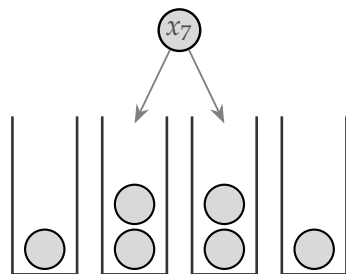


Computing $A_{S \cup \{x\}}$

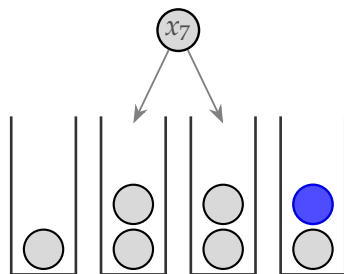
Subsequent balls will experience either:

1. No recourse

ANALYZING THE RECOURSE



Computing A_S



Recourse = 0

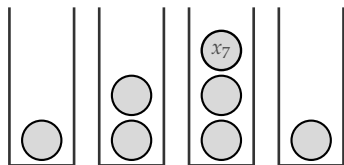
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

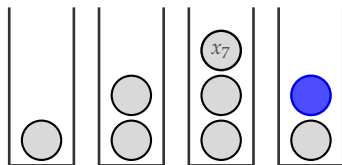
1. No recourse

ANALYZING THE RECOURSE

Recourse = 0



Computing A_S



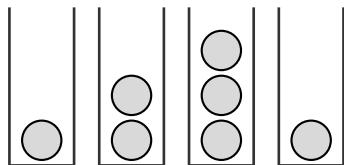
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

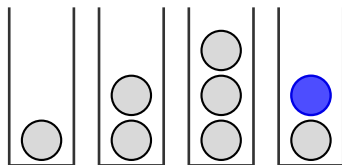
1. No recourse

ANALYZING THE RECOURSE

Recourse = 0



Computing A_S

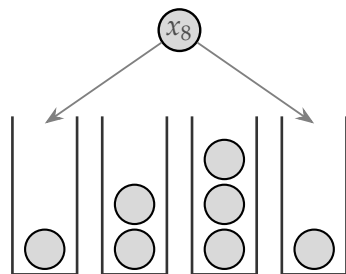


Computing $A_{S \cup \{x\}}$

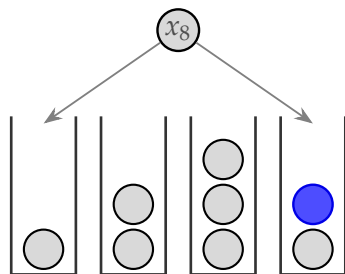
Subsequent balls will experience either:

1. No recourse
2. Recourse

ANALYZING THE RECOURSE



Computing A_S



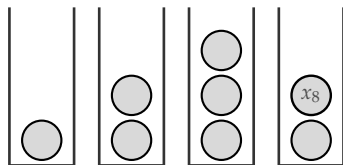
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

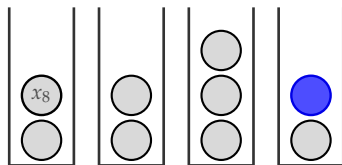
1. No recourse
2. Recourse

ANALYZING THE RECOURSE

Recourse = 1



Computing A_S



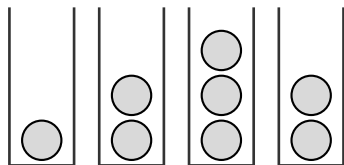
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

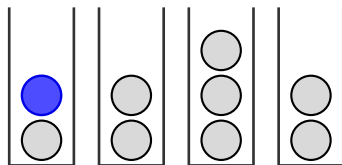
1. No recourse
2. Recourse

ANALYZING THE RECOURSE

Recourse = 1



Computing A_S

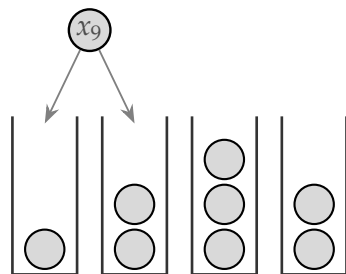


Computing $A_{S \cup \{x\}}$

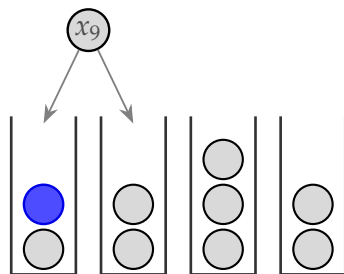
Subsequent balls will experience either:

1. No recourse
2. Recourse

ANALYZING THE RECOURSE



Computing A_S



Computing $A_{S \cup \{x\}}$

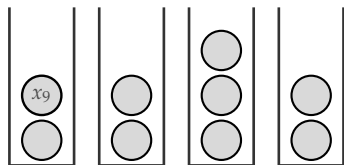
Recourse = 1

Subsequent balls will experience either:

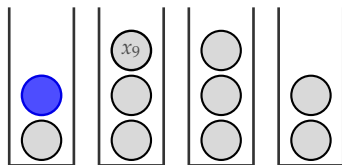
1. No recourse
2. Recourse

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S



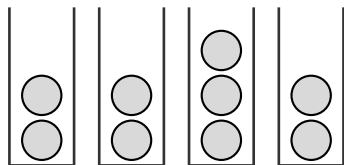
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

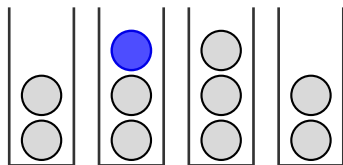
1. No recourse
2. Recourse

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S



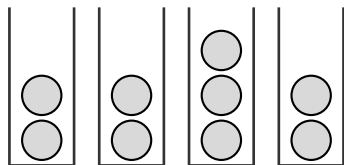
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

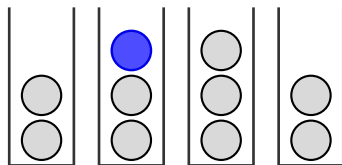
1. No recourse
2. Recourse

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S

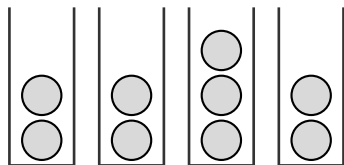


Computing $A_{S \cup \{x\}}$

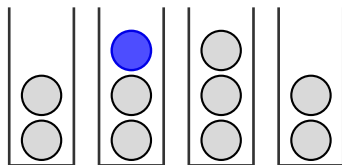
Two key observations:

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S



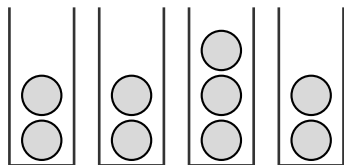
Computing $A_{S \cup \{x\}}$

Two key observations:

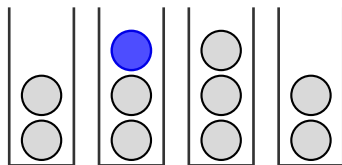
1. There's always one special bin with an extra ball

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S



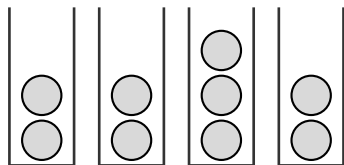
Computing $A_{S \cup \{x\}}$

Two key observations:

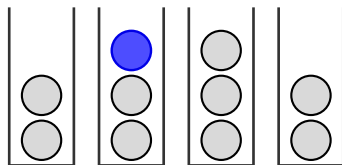
1. There's always one special bin with an extra ball
2. If a ball incurs recourse, one of its choices is the special bin

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S

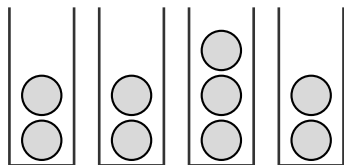


Computing $A_{S \cup \{x\}}$

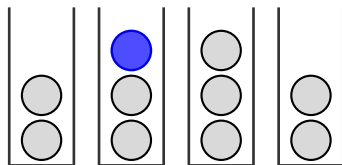
$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

ANALYZING THE RECOURSE

Recourse = 2



Computing A_S



Computing $A_{S \cup \{x\}}$

$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

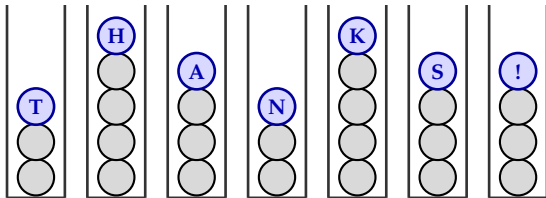
$$\implies \mathbb{E}[\text{total recourse}] = \sum_i \Pr[\text{ball } x_i \text{ incurs recourse}] = O(m/n)$$

A SIMPLE WARMUP

Theorem: There exists a history-independent solution with:

- High-probability overload $\Theta(1)$ $O(\log \log n)$.
- Expected recourse $\Theta(\log \log(m/n))$ $O(m/n)$.

History-Independent Load Balancing



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