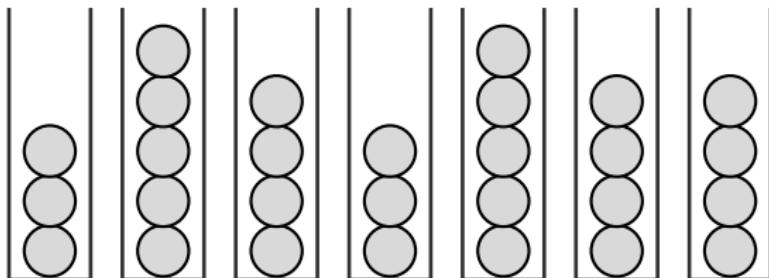


# History-Independent Load Balancing



Michael A. Bender

Stony Brook University

William Kuszmaul

CMU

Elaine Shi

CMU

Rose Silver

CMU

# NYT CENSORING CATASTROPHE

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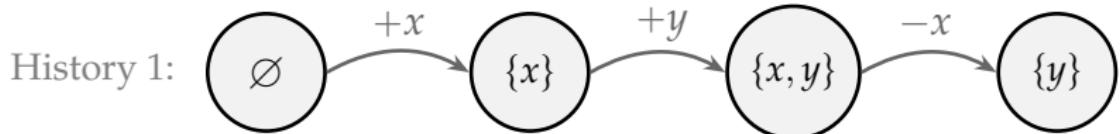
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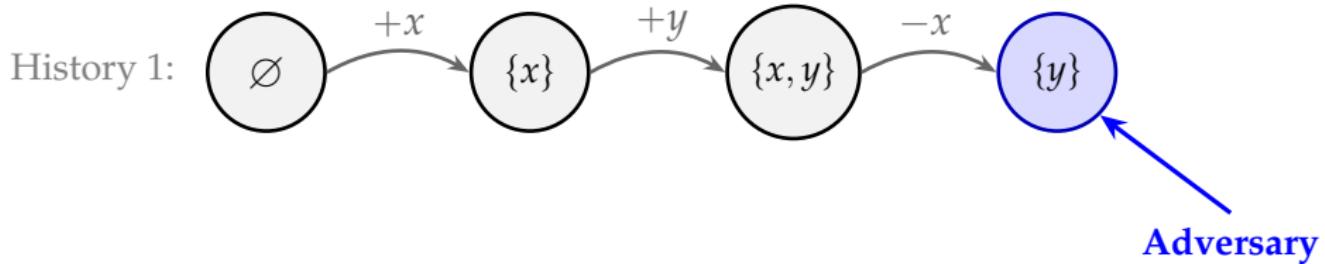
**History can be just as important as content!**

# HISTORY-INDEPENDENT DATA STRUCTURES

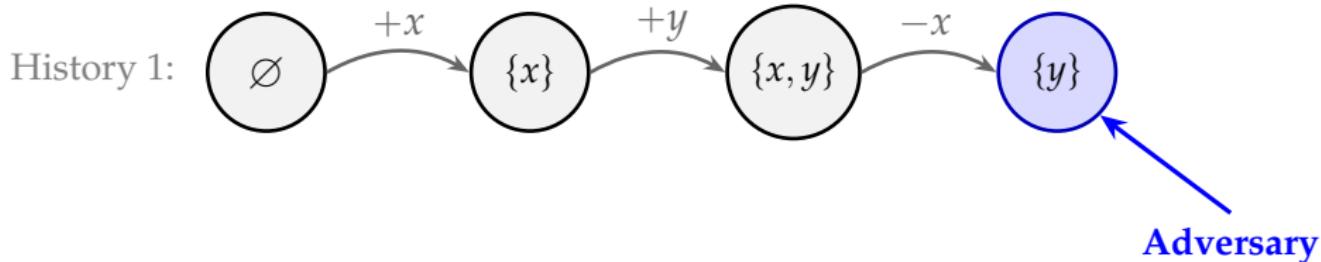
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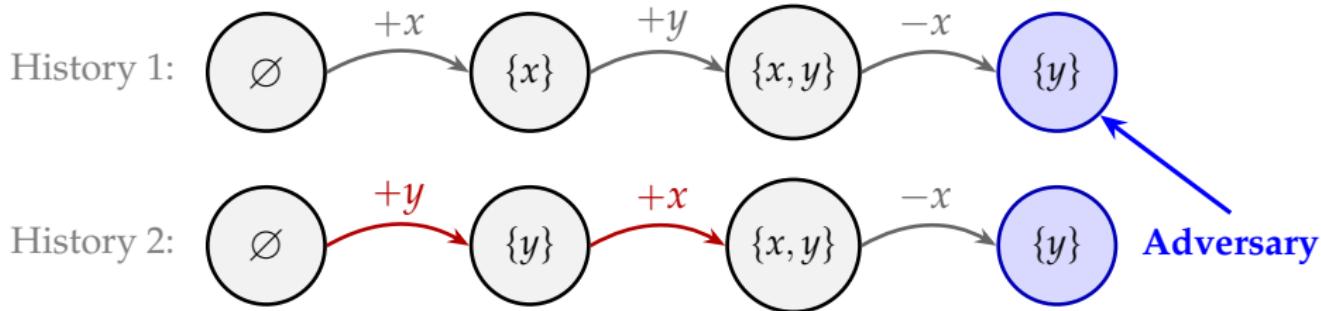
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- ▶ The state reveals only the current elements—**not the history of operations**.

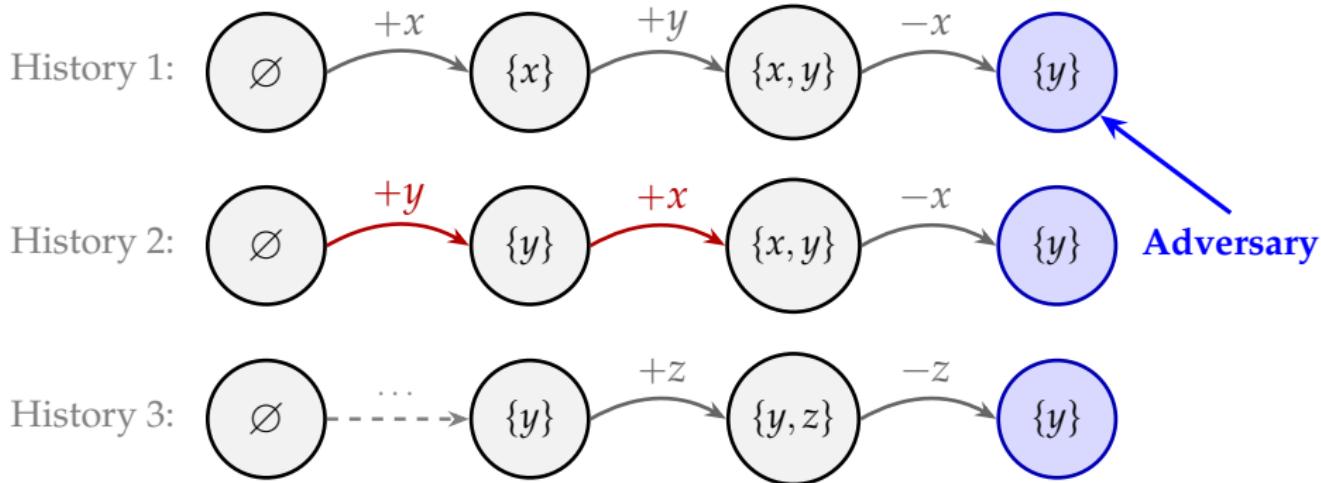
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## A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

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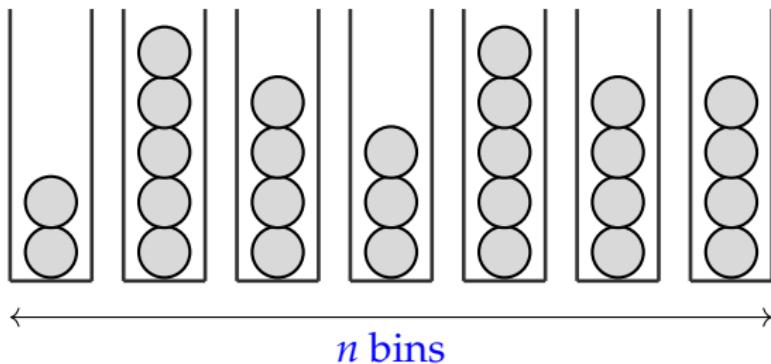
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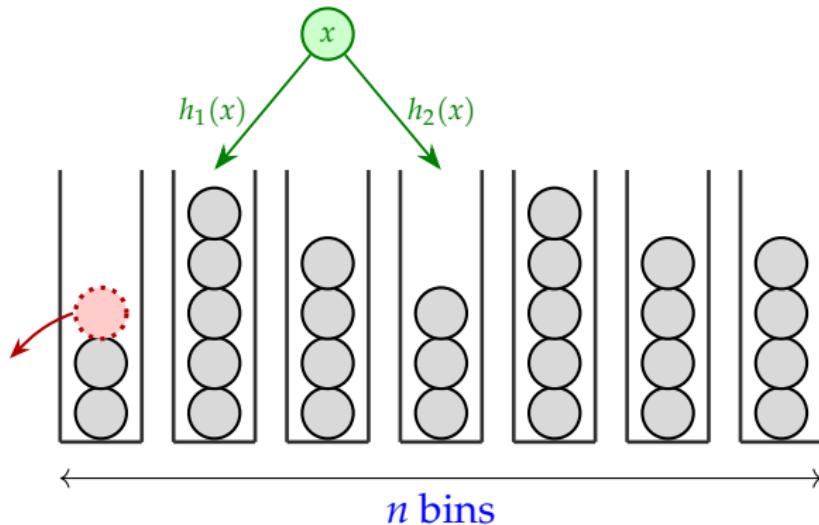
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**This work:** History-Independent Load Balancing

# TWO-CHOICE LOAD BALANCING

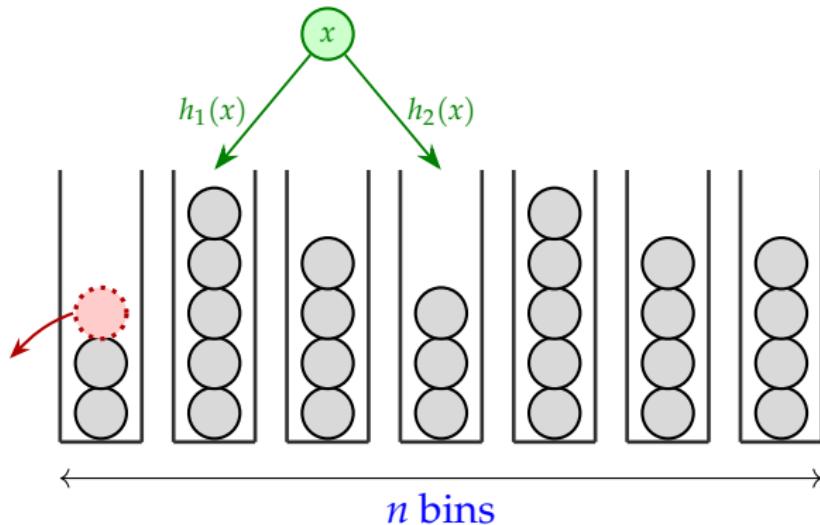


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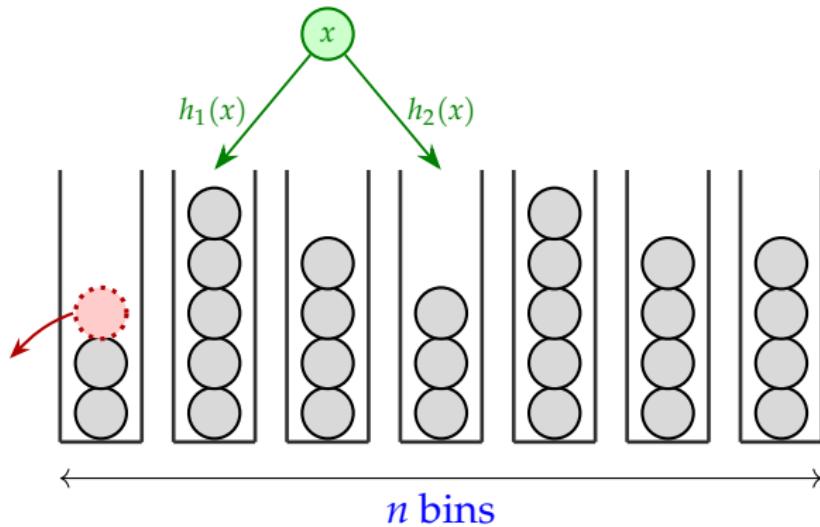
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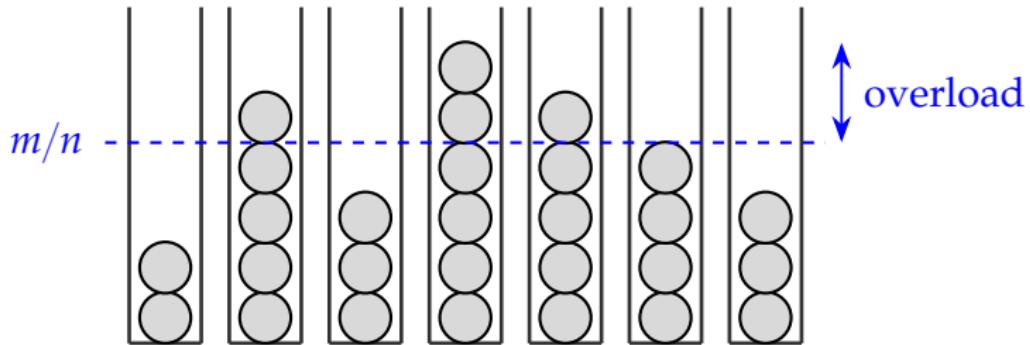
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- ▶ Balls are **inserted**/**deleted**, with up to  $m$  present at a time.
- ▶ Each ball has two random bins where it can go.
- ▶ We must maintain a valid assignment of balls to bins.

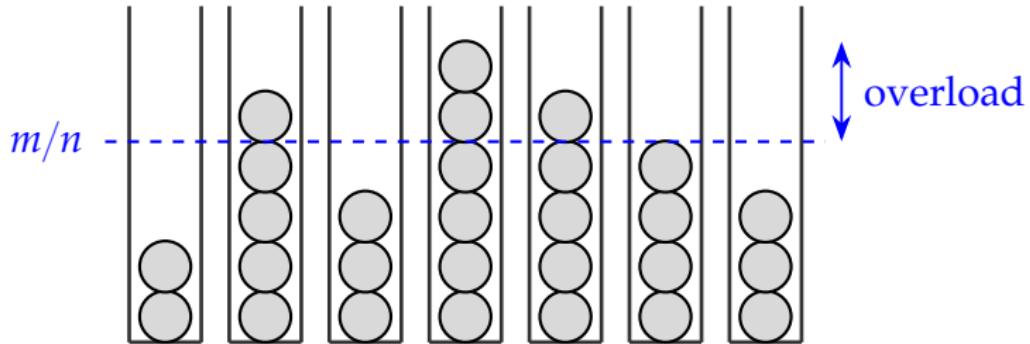
## TWO GOALS



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### Minimize Recourse:

- ▶ i.e., the number of balls moved around on any given insertion/deletion.

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**Question:** Does there exist a history-independent solution with small recourse and small overload?

**Our Main Result:** There exists a history-independent solution with:

- ▶ High probability overload  $O(1)$
- ▶ Expected recourse  $O(\log \log(m/n))$

## PAST WORK (NOT HISTORY INDEPENDENT)

---

| Overload                | Recourse       | Reference                      | Caveats         |
|-------------------------|----------------|--------------------------------|-----------------|
| $O(\log \log n)$        | 0              | [ABKU '94] [BCSV '00]          | insertion-only  |
| $O(1)$                  | $O(\log(m/n))$ | [Dietzfelbinger, Weidling '07] | insertion-only  |
| $\tilde{O}(\sqrt{m/n})$ | $O(1)$         | [Frieze, Petti '18]            | insertion-only  |
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Privacy can be leveraged as an *algorithmic tool* to outperform non-private algorithms!

## REST OF TALK

1. A Simple Warmup
2. The Full Algorithm

---

# Part 1: A Simple Warmup

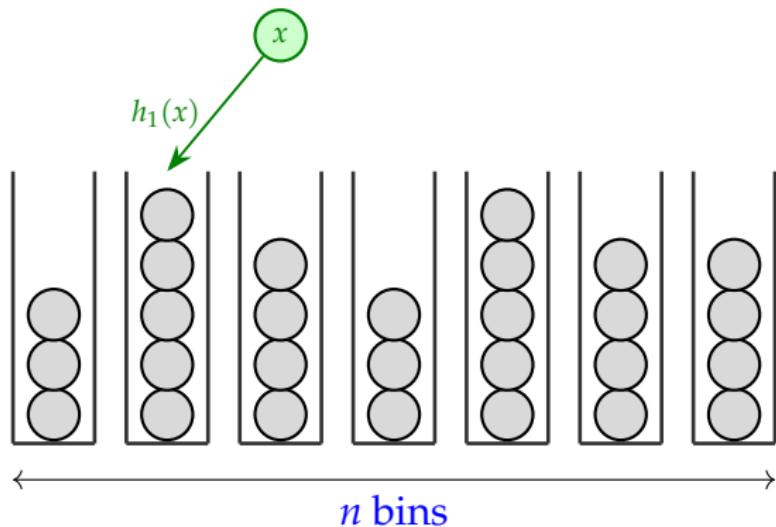
## A SIMPLE WARMUP

**Theorem:** There exists a history-independent solution with:

- ▶ High-probability overload  $\Theta(1)$   $O(\log \log n)$ .
- ▶ Expected recourse  $\Theta(\log \log(m/n))$   $O(m/n)$ .

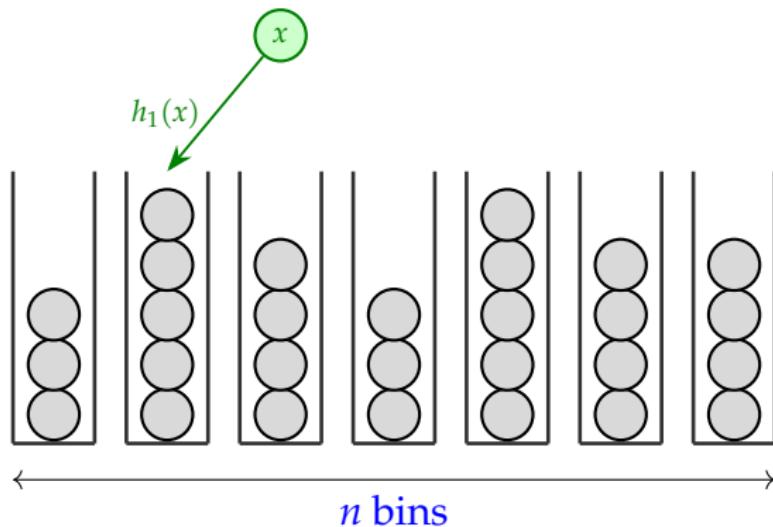
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To insert a ball  $x$ , just put it in bin  $h_1(x)$ :



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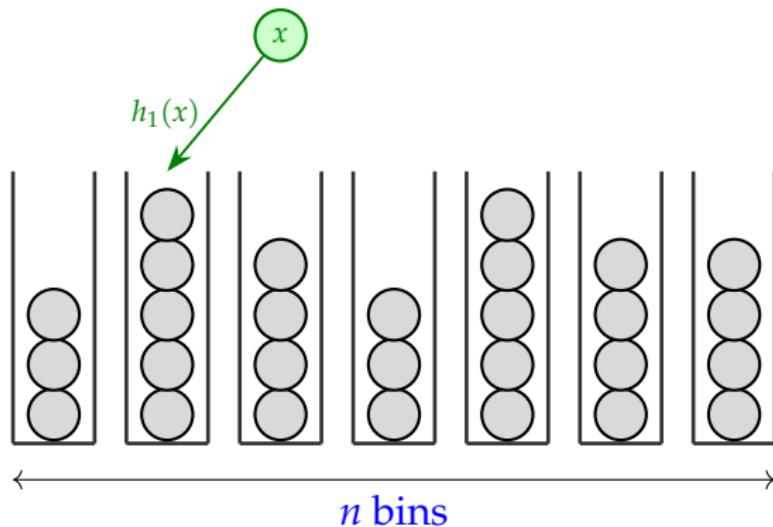
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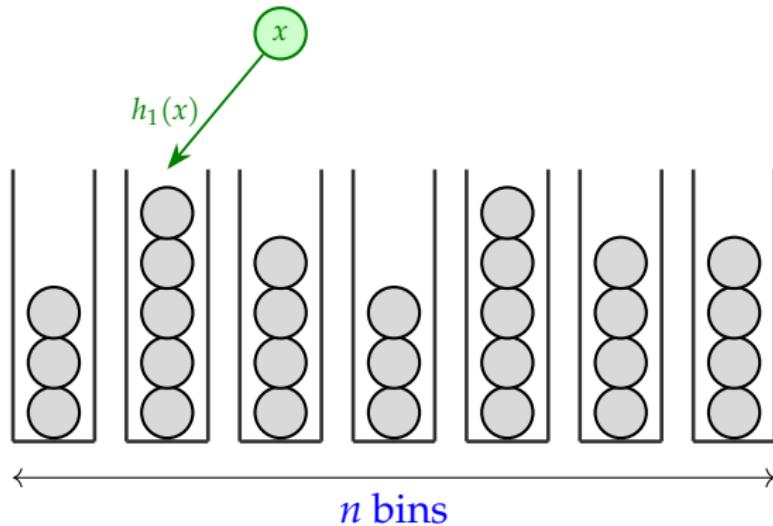
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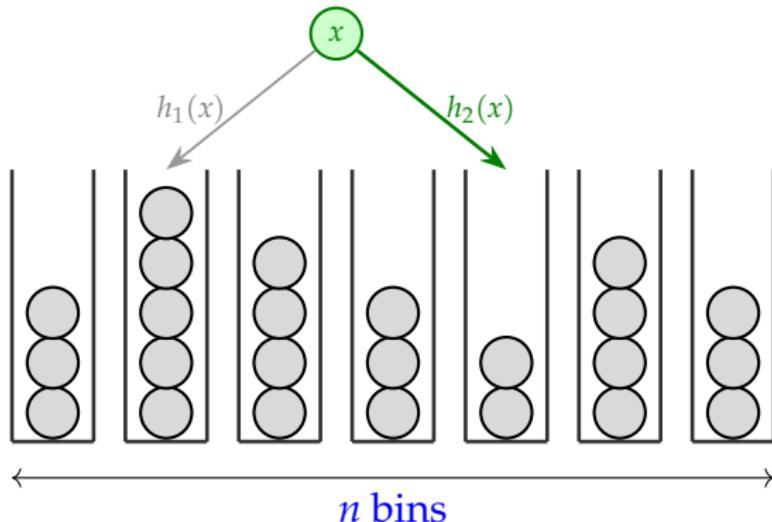
To insert a ball  $x$ , just put it in bin  $h_1(x)$ :



- ▶ This is history-independent ✓
- ▶ The recourse is 0 ✓
- ▶ But... the overload is huge, roughly  $\sqrt{m/n}$  ✗

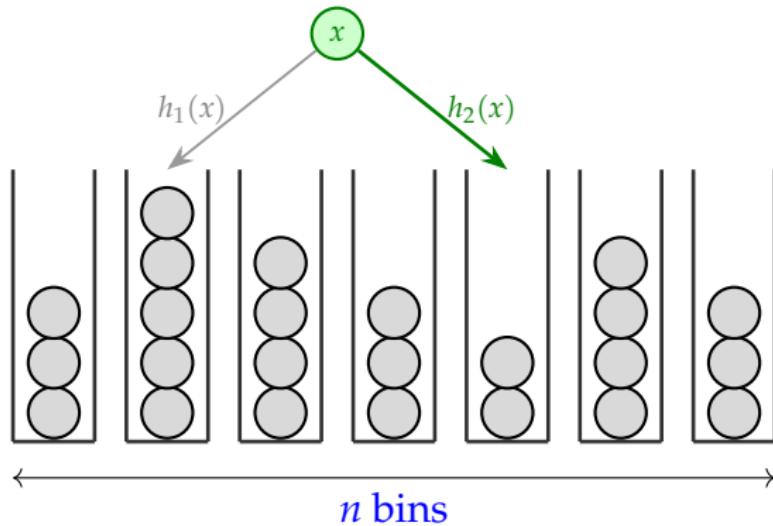
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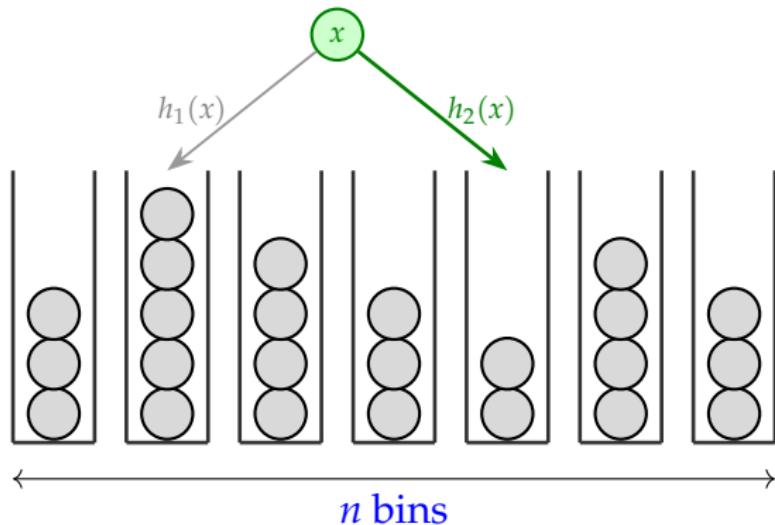
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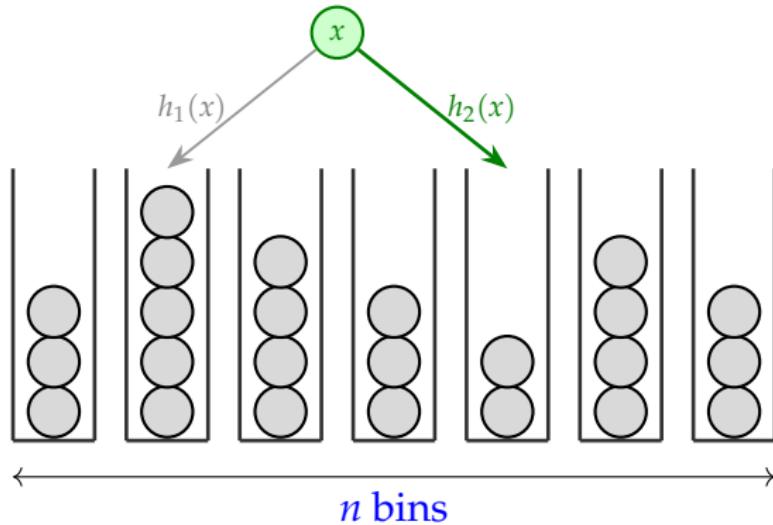
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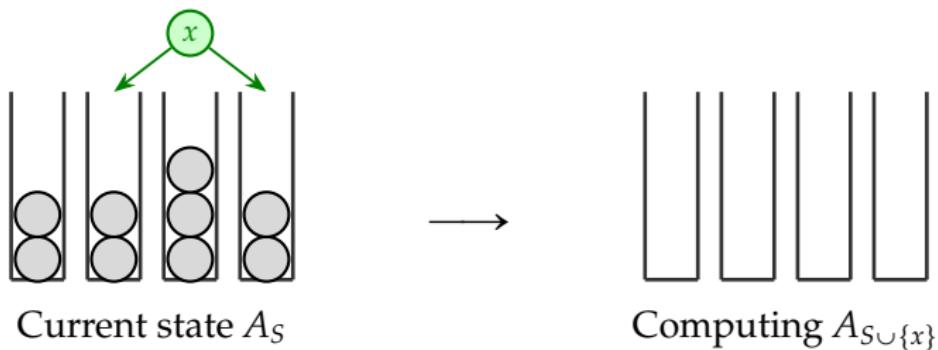
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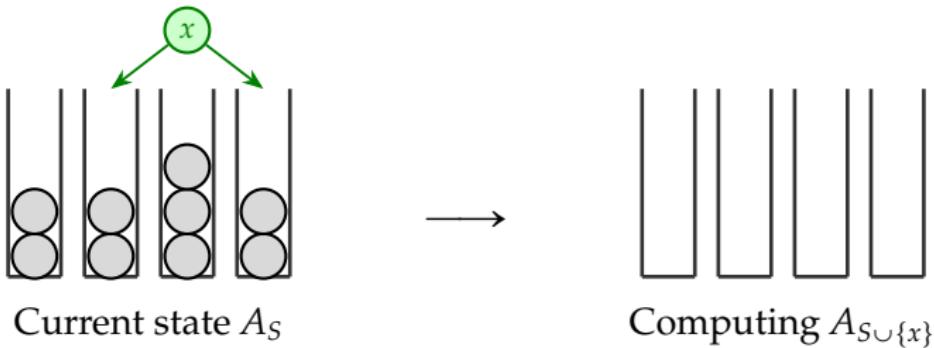
- ▶ This is **not** history-independent ✗
- ▶ The recourse is 0 ✓
- ▶ In the insertion-only case, the overload is  $O(\log \log n)$  ✓  
[Berenbrink, Czumaj, Steger, and Vöcking '00]

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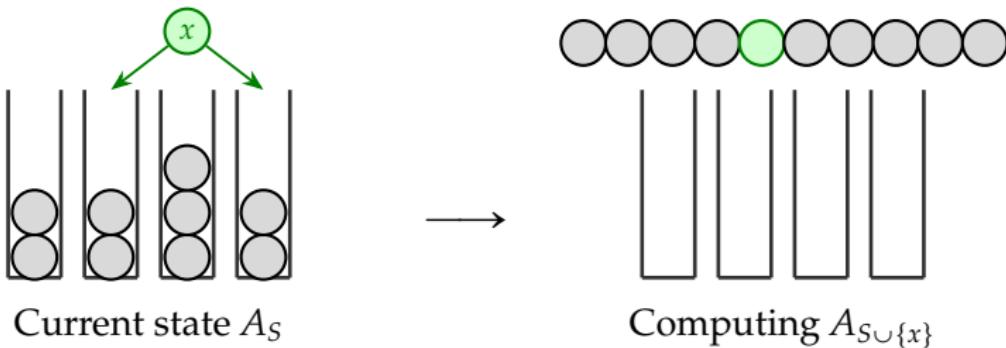
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To compute  $A_{S \cup \{x\}}$ :

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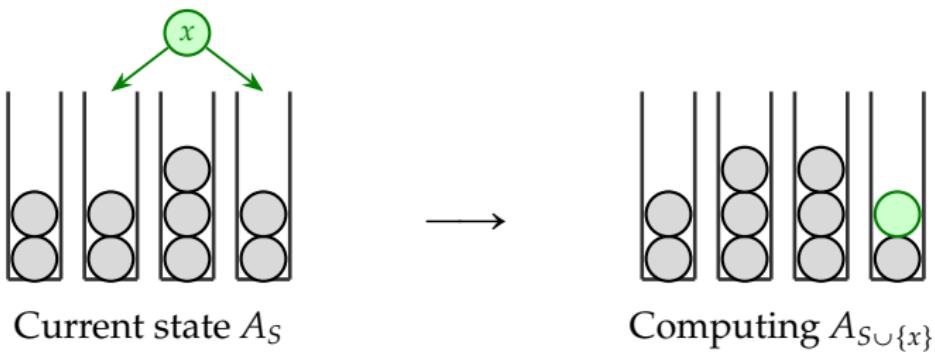
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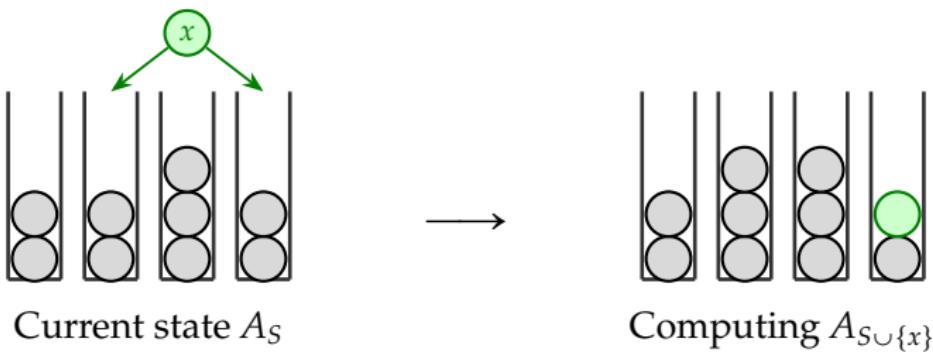
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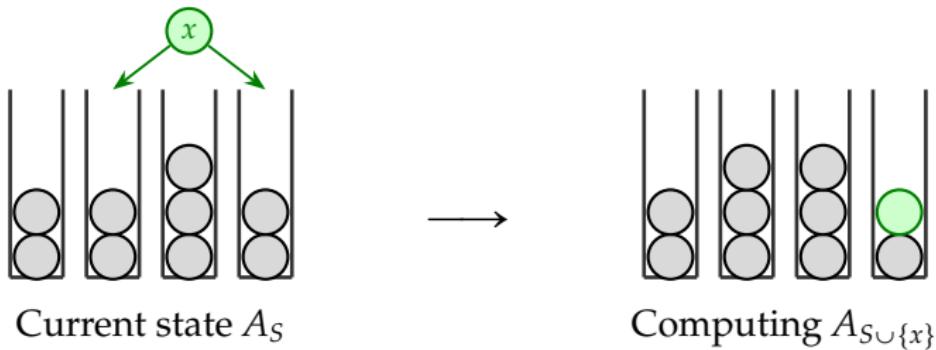


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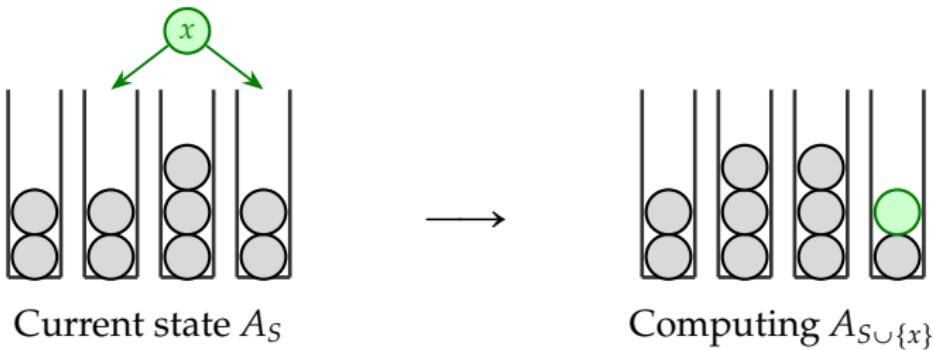
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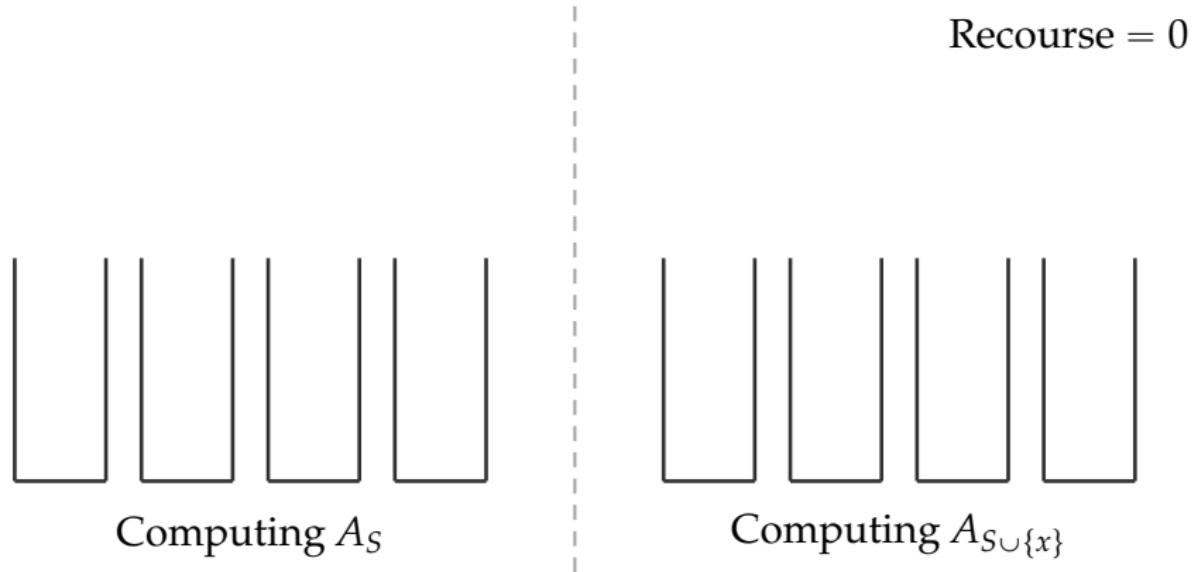
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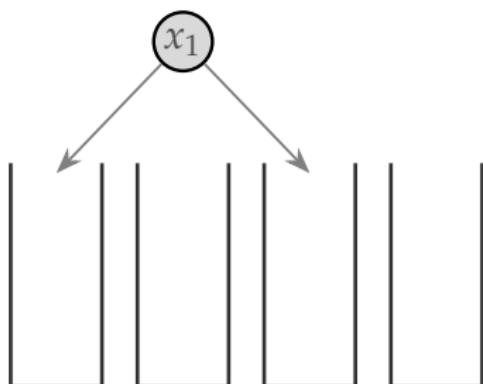
- ▶ The algorithm is history independent ✓
- ▶ The overload is  $O(\log \log n)$  ✓
- ▶ What is the recourse?

# ANALYZING THE RECOURSE

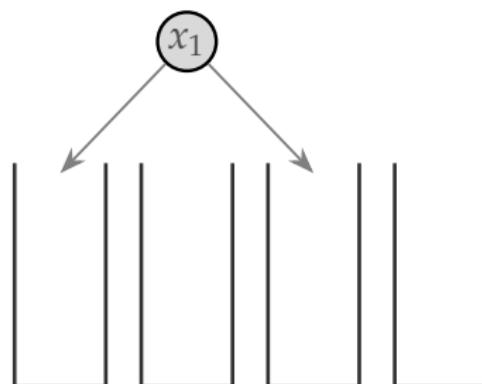


How many balls change assignments between  $A_S$  and  $A_{S \cup \{x\}}$ ?

# ANALYZING THE RECOURSE



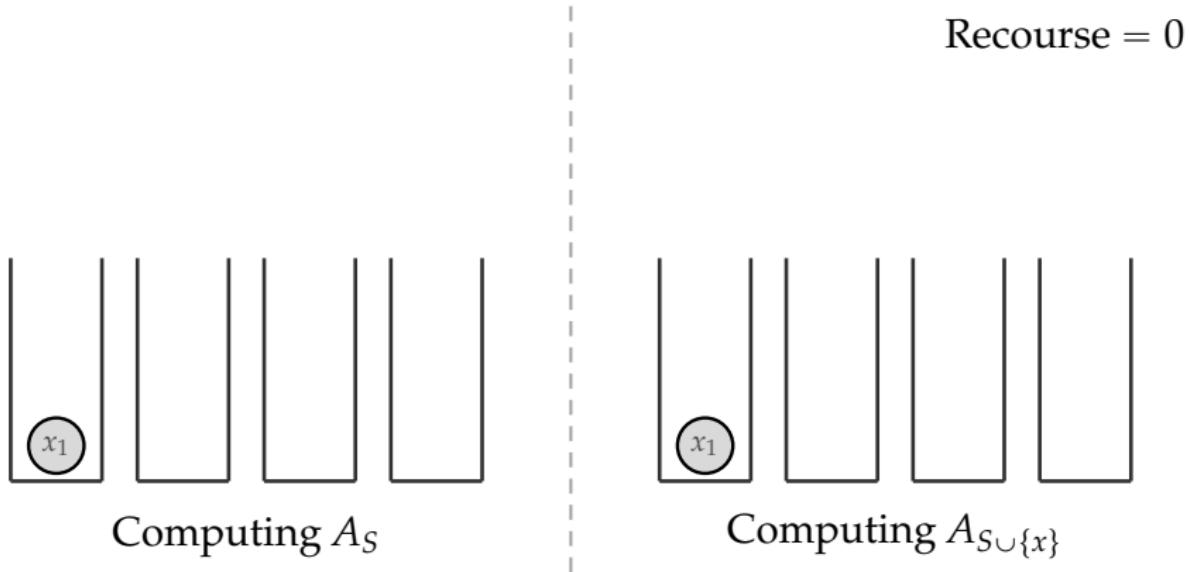
Computing  $A_S$



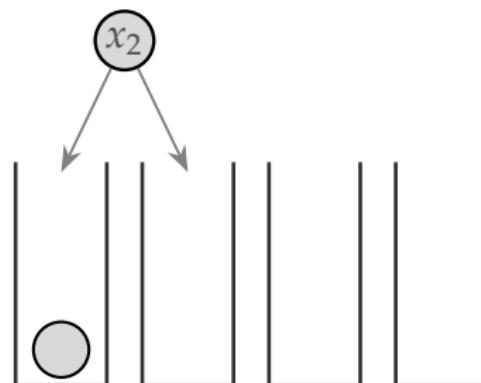
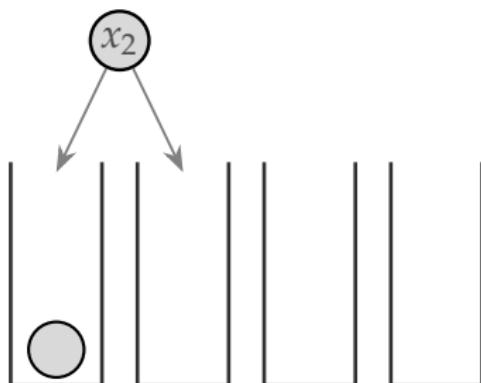
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Recourse = 0

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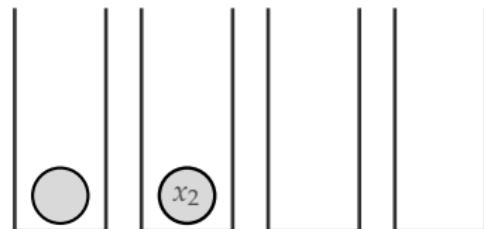


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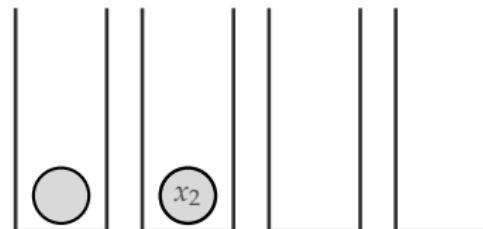


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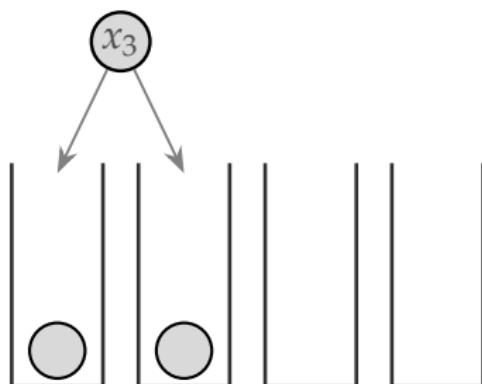
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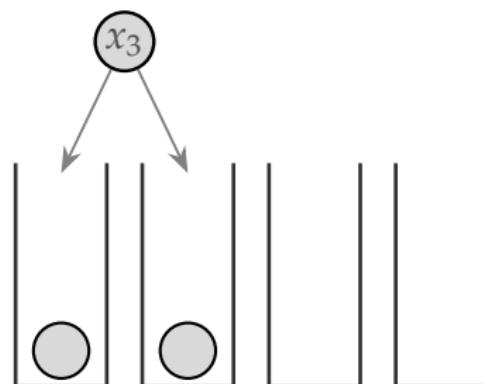
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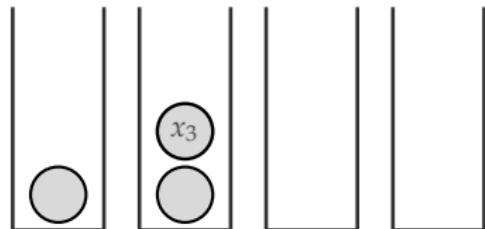
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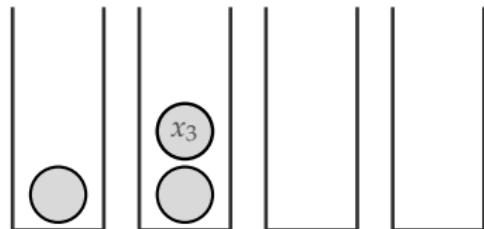
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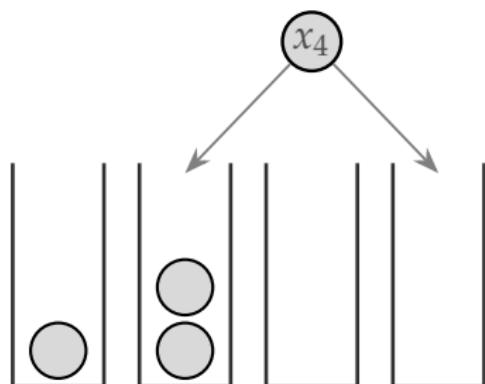
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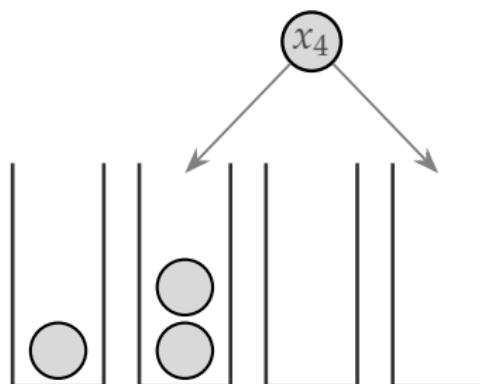
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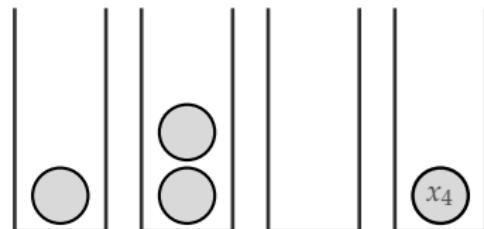
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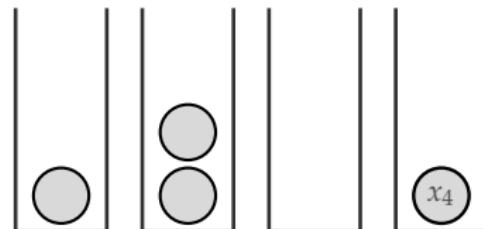
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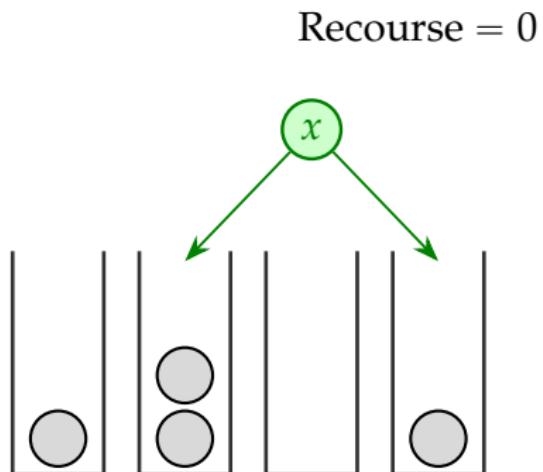
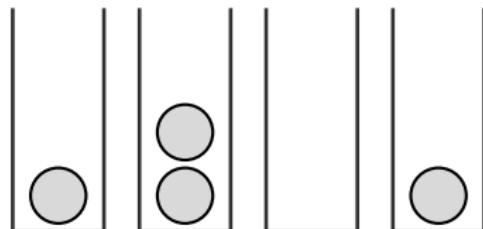
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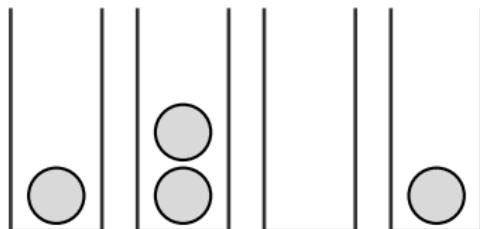
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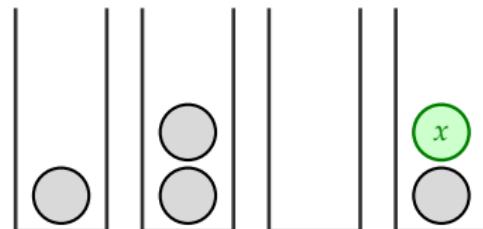
# ANALYZING THE RECOURSE



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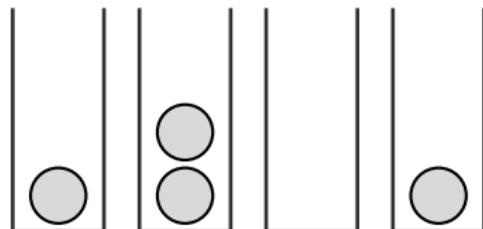
Computing  $A_S$



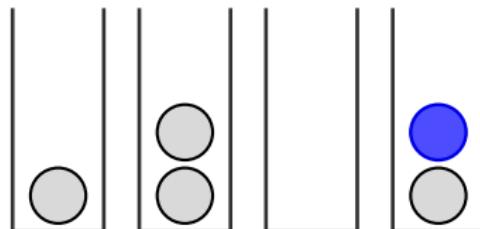
Computing  $A_{S \cup \{x\}}$

Recourse = 0

## ANALYZING THE RECOURSE



Computing  $A_S$

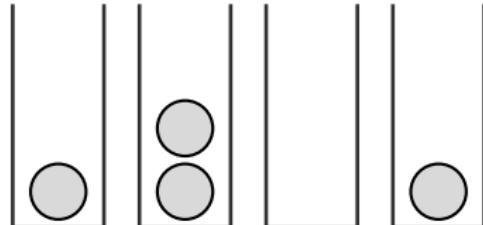


Computing  $A_{S \cup \{x\}}$

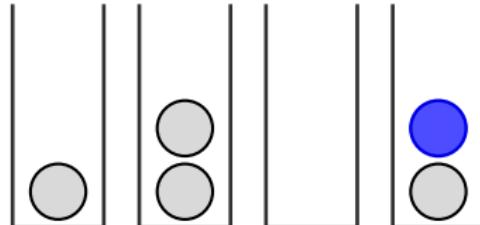
Subsequent balls will experience either:

Recourse = 0

## ANALYZING THE RECOURSE



Computing  $A_S$



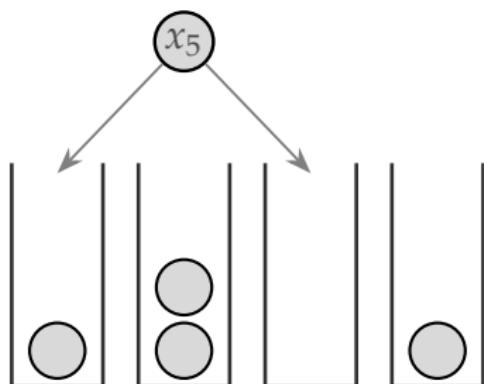
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

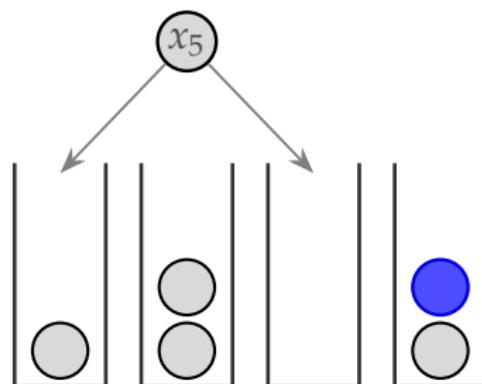
1. No recourse

$$\text{Recourse} = 0$$

## ANALYZING THE RECOURSE



Computing  $A_S$



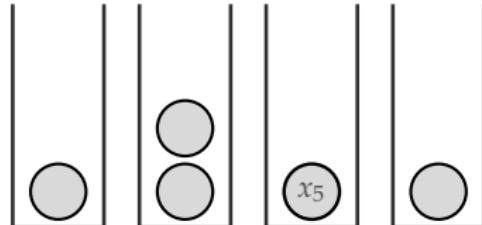
Computing  $A_{S \cup \{x\}}$

Future insertions will experience either:

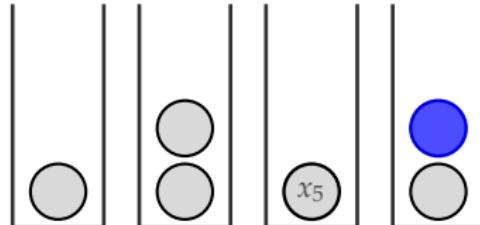
1. No recourse

Recourse = 0

# ANALYZING THE RECOURSE



Computing  $A_S$

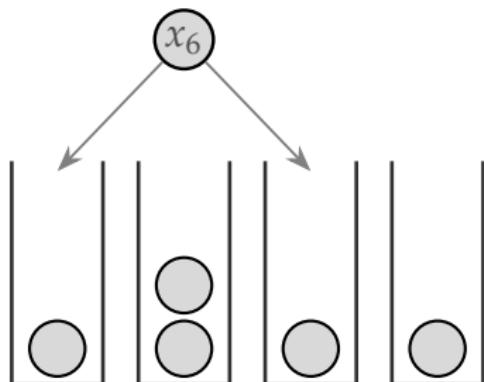


Computing  $A_{S \cup \{x\}}$

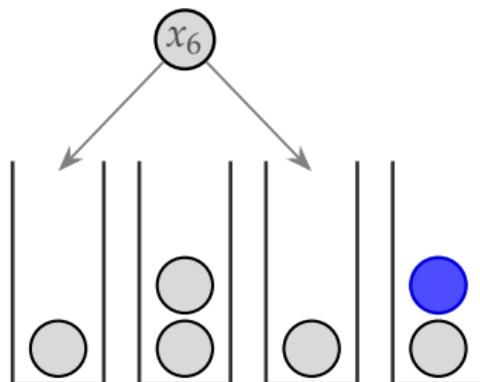
Subsequent balls will experience either:

1. No recourse

## ANALYZING THE RECOURSE



Computing  $A_S$



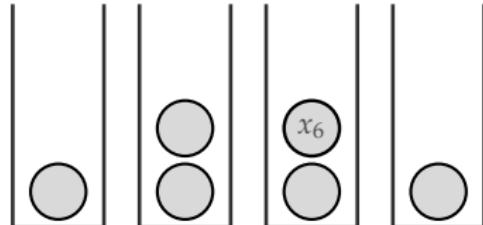
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

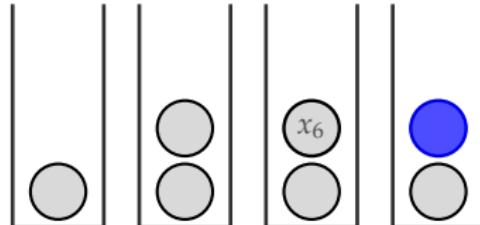
1. No recourse

Recourse = 0

# ANALYZING THE RECOURSE



Computing  $A_S$

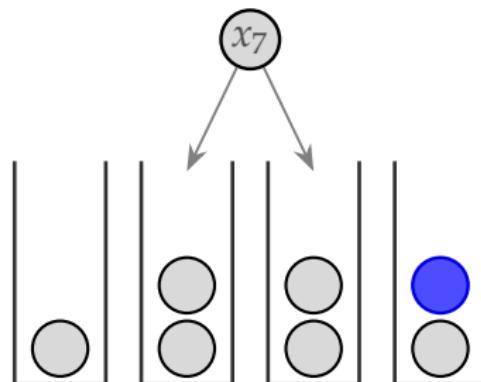
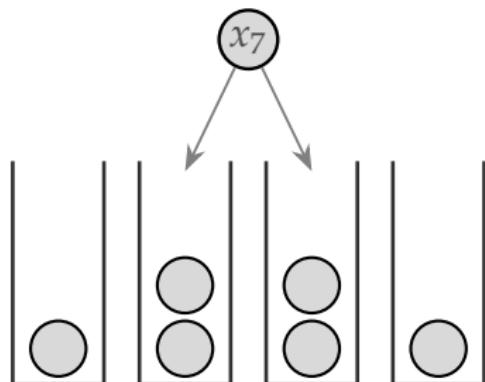


Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse

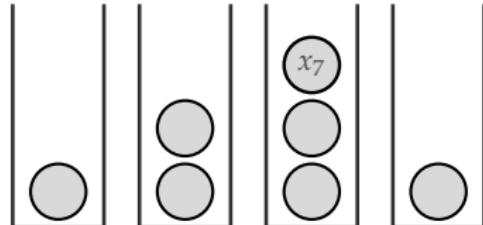
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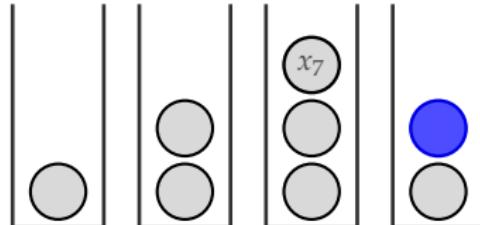
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# ANALYZING THE RECOURSE



Computing  $A_S$



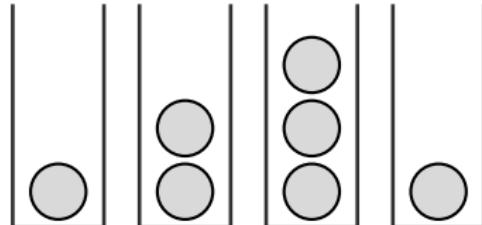
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Subsequent balls will experience either:

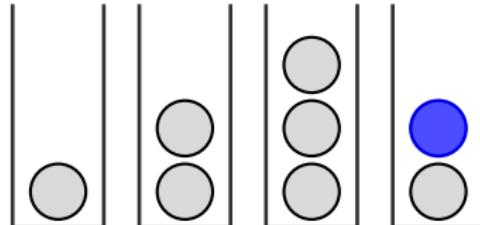
1. No recourse

$$\text{Recourse} = 0$$

## ANALYZING THE RECOURSE



Computing  $A_S$



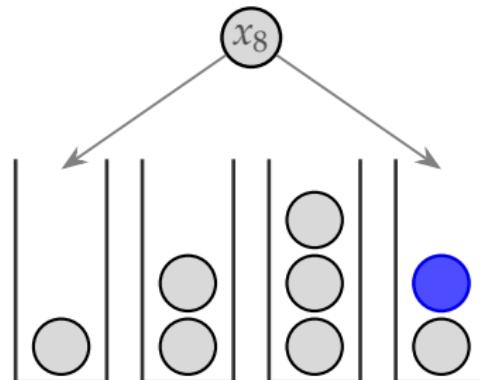
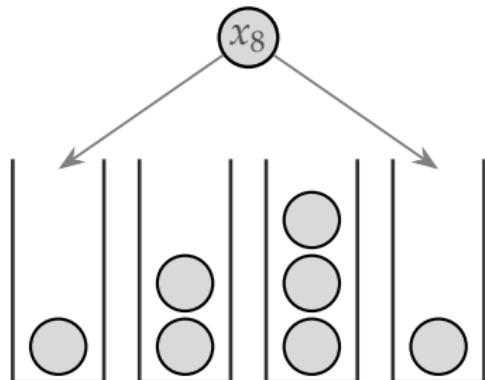
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

$$\text{Recourse} = 0$$

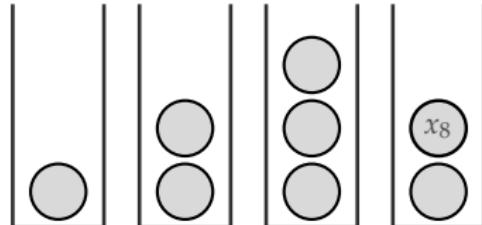
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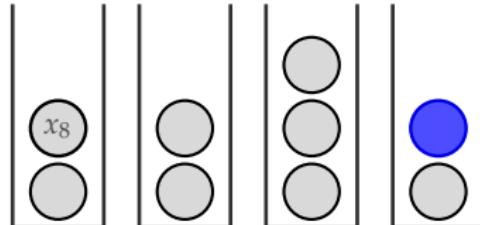
Subsequent balls will experience either:

1. No recourse
2. Recourse

# ANALYZING THE RECOURSE



Computing  $A_S$



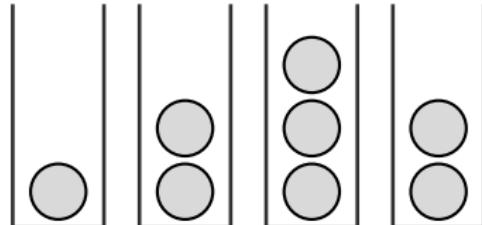
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

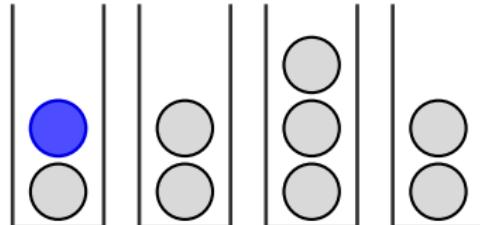
1. No recourse
2. Recourse

Recourse = 1

## ANALYZING THE RECOURSE



Computing  $A_S$



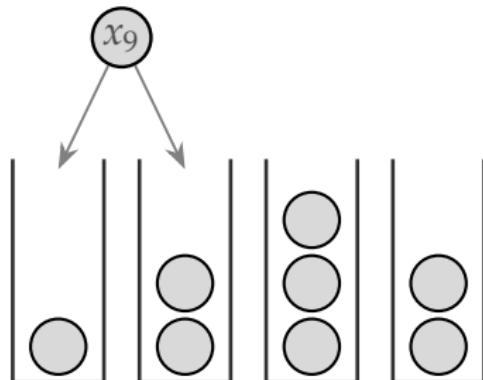
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

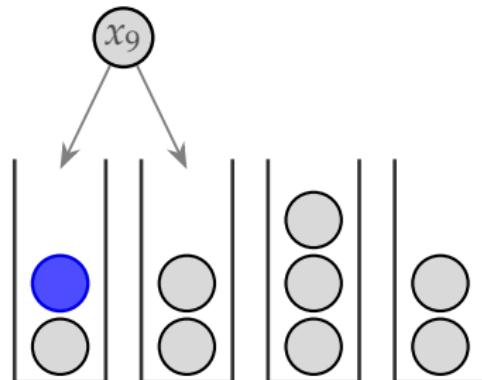
1. No recourse
2. Recourse

Recourse = 1

# ANALYZING THE RECOURSE



Computing  $A_S$



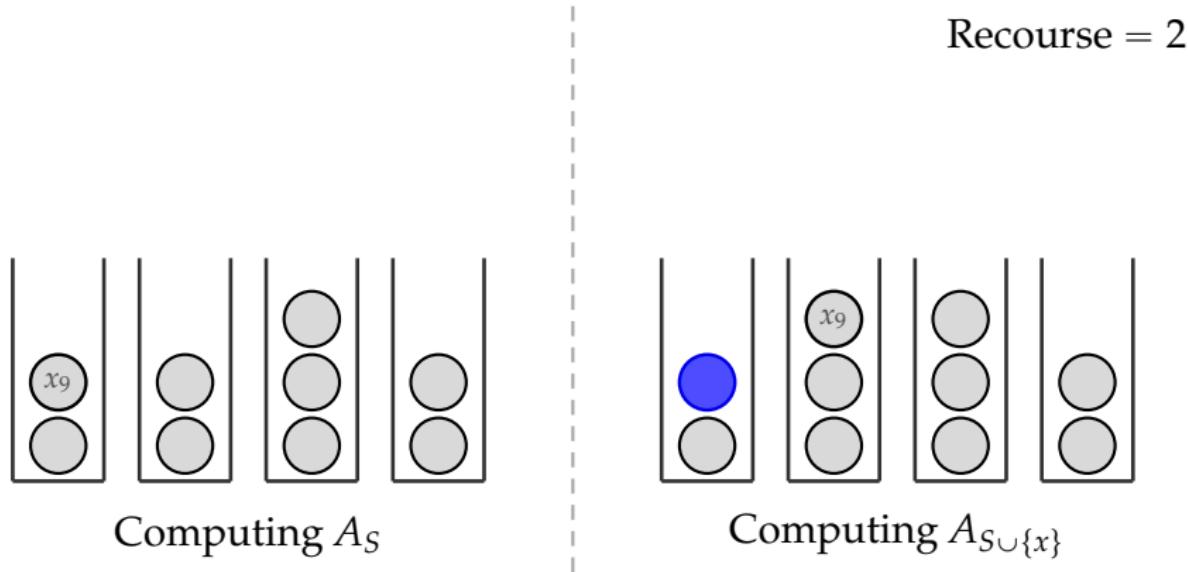
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Recourse = 1

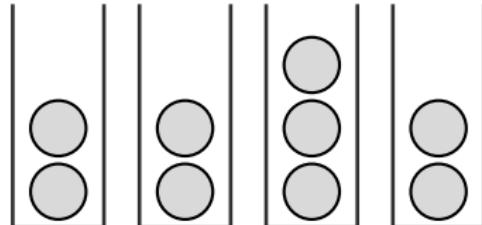
# ANALYZING THE RECOURSE



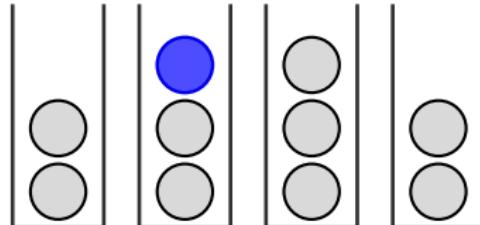
Subsequent balls will experience either:

1. No recourse
2. Recourse

# ANALYZING THE RECOURSE



Computing  $A_S$



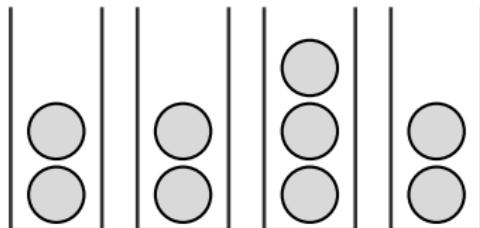
Computing  $A_{S \cup \{x\}}$

Subsequent balls will experience either:

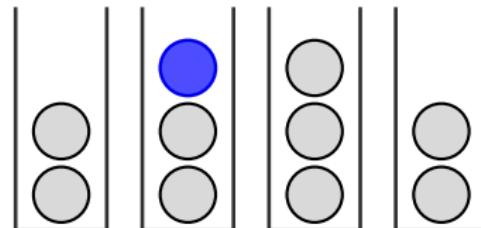
1. No recourse
2. Recourse

Recourse = 2

# ANALYZING THE RECOURSE



Computing  $A_S$

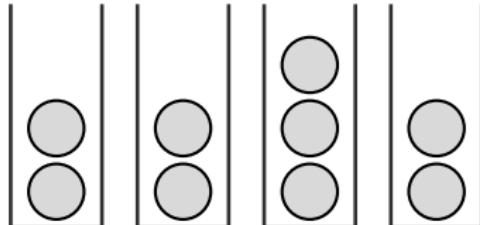


Computing  $A_{S \cup \{x\}}$

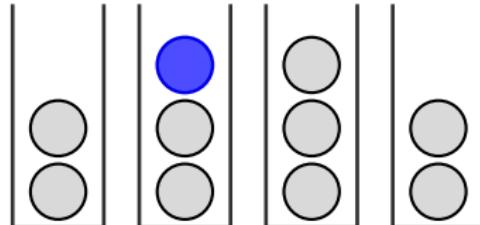
Two key observations:

Recourse = 2

# ANALYZING THE RECOURSE



Computing  $A_S$



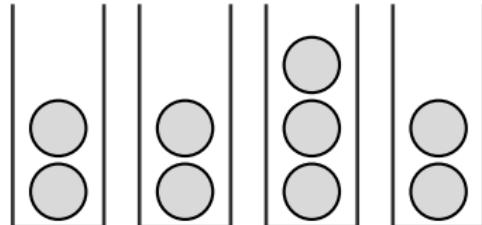
Computing  $A_{S \cup \{x\}}$

Two key observations:

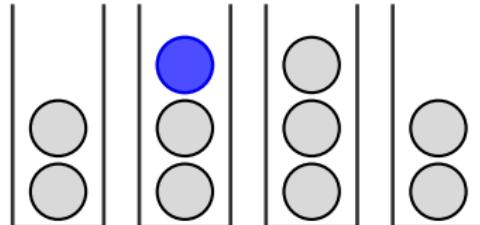
1. There's always one special bin with an extra ball

Recourse = 2

# ANALYZING THE RECOURSE



Computing  $A_S$



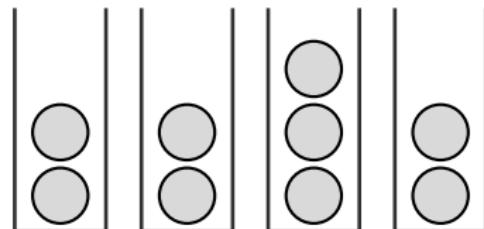
Computing  $A_{S \cup \{x\}}$

Two key observations:

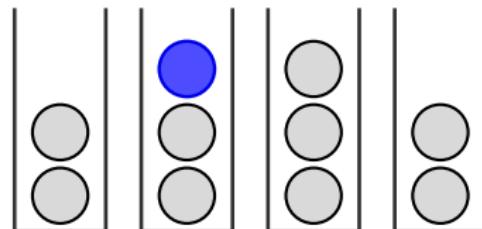
1. There's always one special bin with an extra ball
2. If a ball incurs recourse, one of its choices is the special bin

Recourse = 2

# ANALYZING THE RECOURSE



Computing  $A_S$

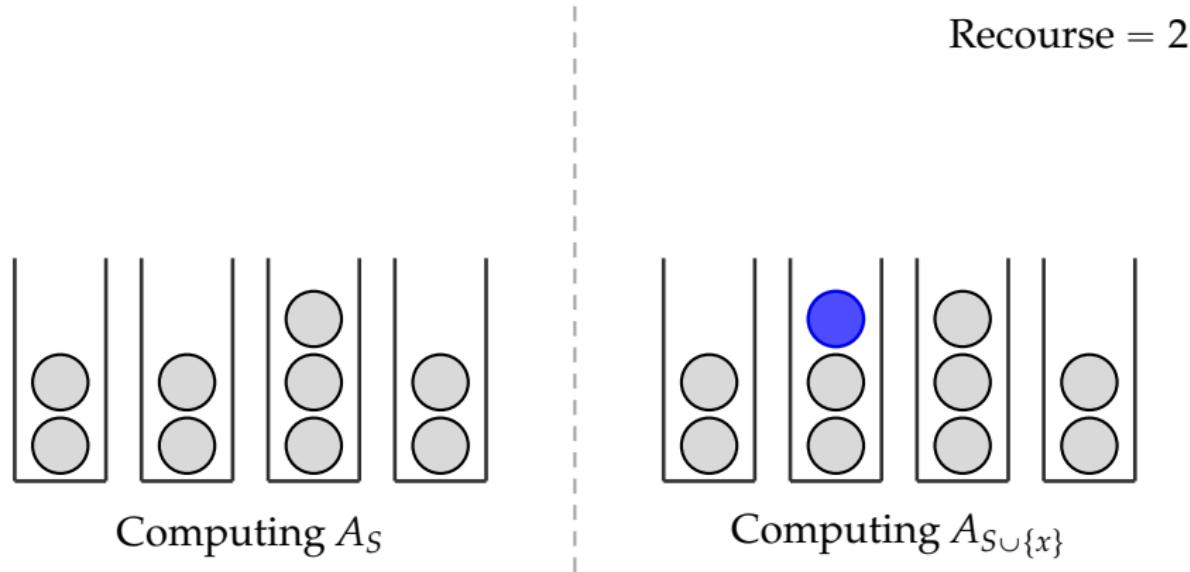


Computing  $A_{S \cup \{x\}}$

$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

Recourse = 2

# ANALYZING THE RECOURSE



$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

$$\implies \mathbb{E}[\text{total recourse}] = \sum_i \Pr[\text{ball } x_i \text{ incurs recourse}] = O(m/n)$$

## A SIMPLE WARMUP

**Theorem:** History-Independent Greedy achieves:

- ▶ High-probability overload  $\Theta(1)$   $O(\log \log n)$ .
- ▶ Expected recourse  $\Theta(\log \log(m/n))$   $O(m/n)$ .

## A SIMPLE WARMUP

**Theorem:** History-Independent Greedy achieves:

- ▶ High-probability overload  $\Theta(1) O(\log \log n)$ .
- ▶ Expected recourse  $\Theta(\log \log(m/n))$   $O(m/n)$ .



We prove a matching lower bound  
when  $m \leq n^{2-\Omega(1)}$

## REST OF TALK

1. A Simple Warmup ✓
2. The Full Algorithm

---

## Part 2: The Full Algorithm

# BAKING A CAKE



Michael



Elaine



Bill

# BAKING A CAKE

Before



Michael



Elaine



Bill

# BAKING A CAKE

Before



**Slice off the  
excess flour!**



Michael



Elaine



Bill

# BAKING A CAKE

Before



**Slice off the  
excess flour!**



Michael

**Spread it evenly!**



Elaine



Bill

# BAKING A CAKE

Before



After



**Slice off the  
excess flour!**



Michael

**Spread it evenly!**



Elaine



Bill

# BAKING A CAKE

Before



After



Slice off the  
excess flour!



Michael

Spread it evenly!



Elaine

You two rehearsed  
this, didn't you?



Bill

# BAKING A CAKE

Before



Michael



Elaine



Bill

# BAKING A CAKE

Before



**Slice off the  
excess frosting!**



Michael



Elaine



Bill

# BAKING A CAKE

Before



**Slice off the  
excess frosting!**



Michael

**Spread it smooth!**

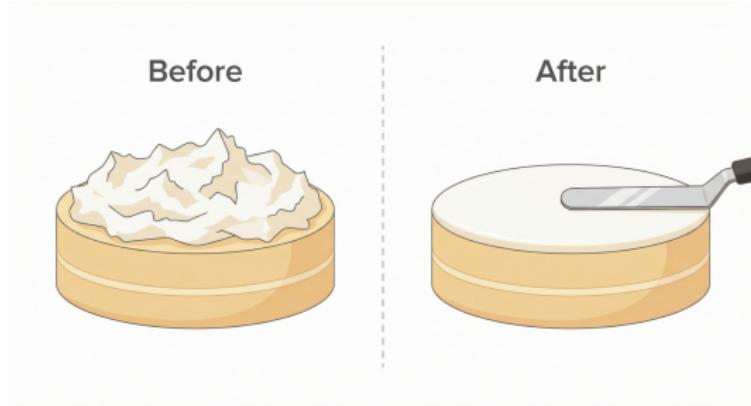


Elaine



Bill

# BAKING A CAKE



**Slice off the  
excess frosting!**



Michael

**Spread it smooth!**

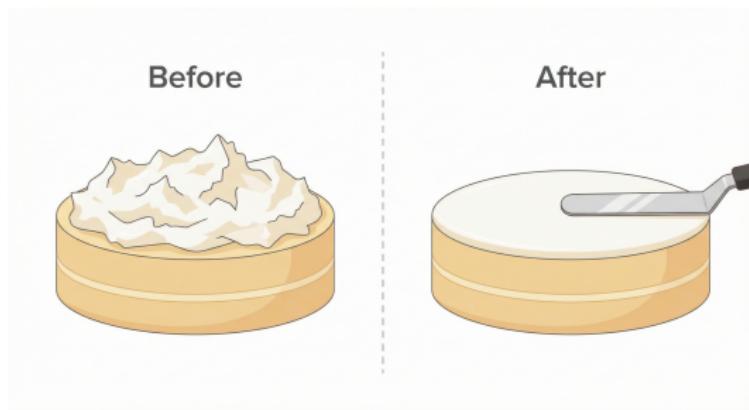


Elaine



Bill

# BAKING A CAKE



**Slice off the excess frosting!**



Michael

**Spread it smooth!**



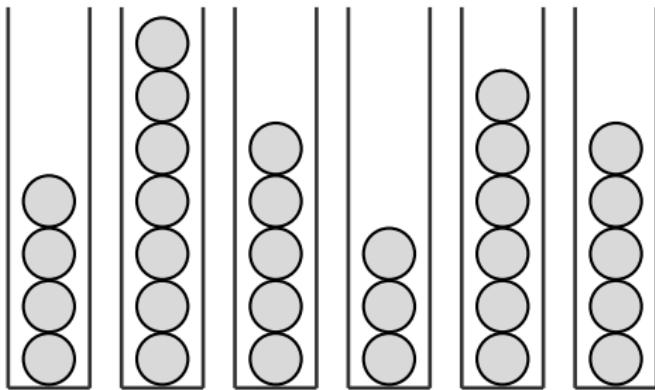
Elaine

I'm starting to think I'm the comic relief here...

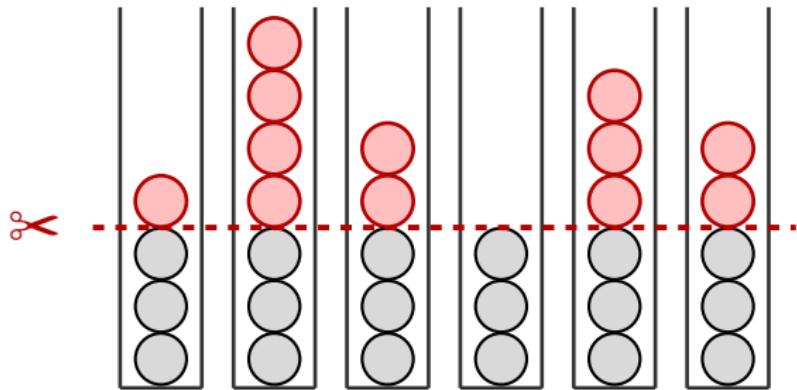


Bill

# SLICE AND SPREAD

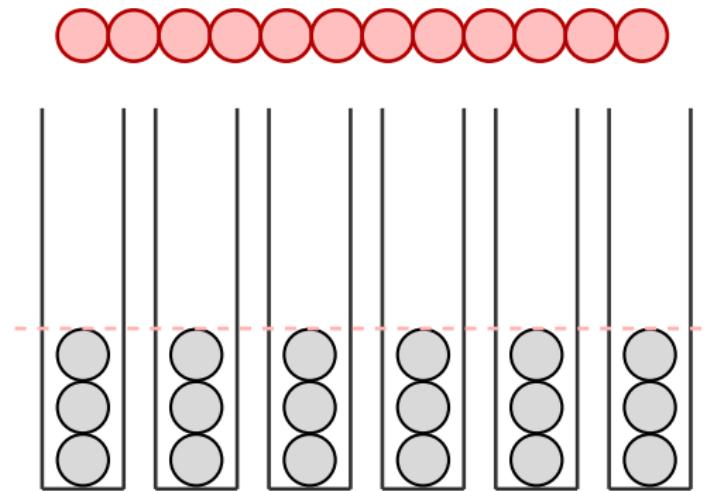


## SLICE AND SPREAD



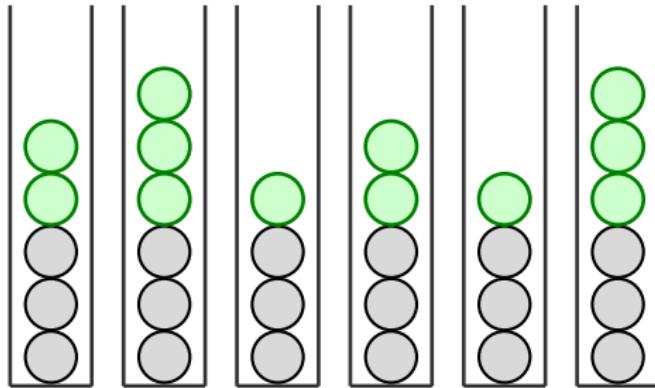
1. **Slice** off the jagged surface

## SLICE AND SPREAD



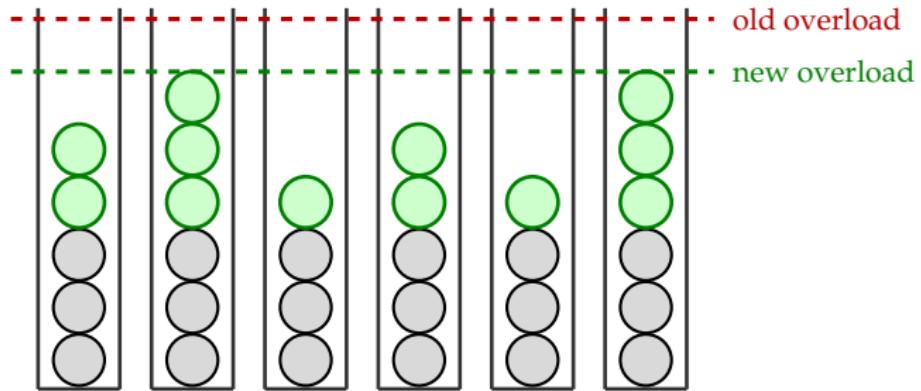
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## SLICE AND SPREAD



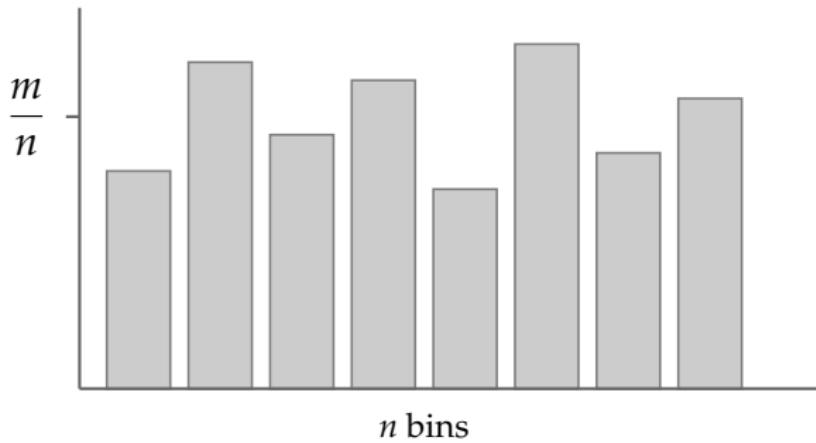
1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

# SLICE AND SPREAD

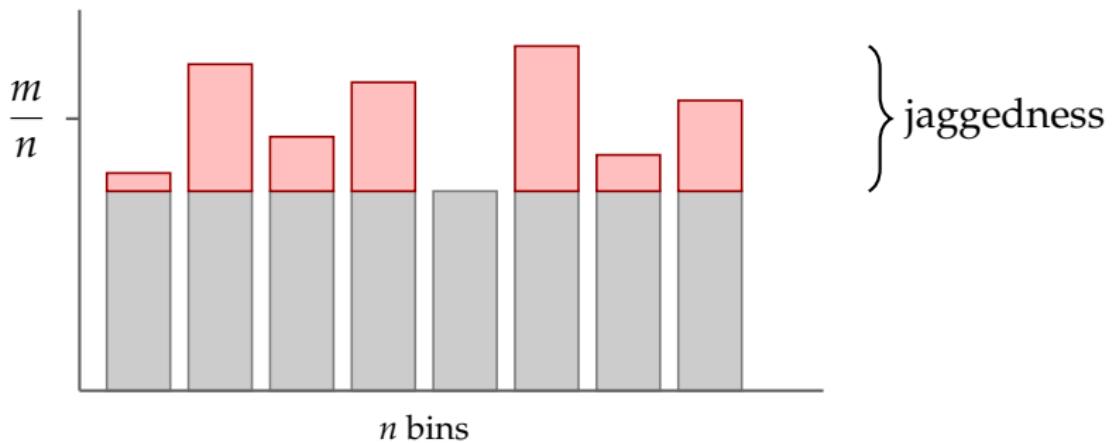


1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

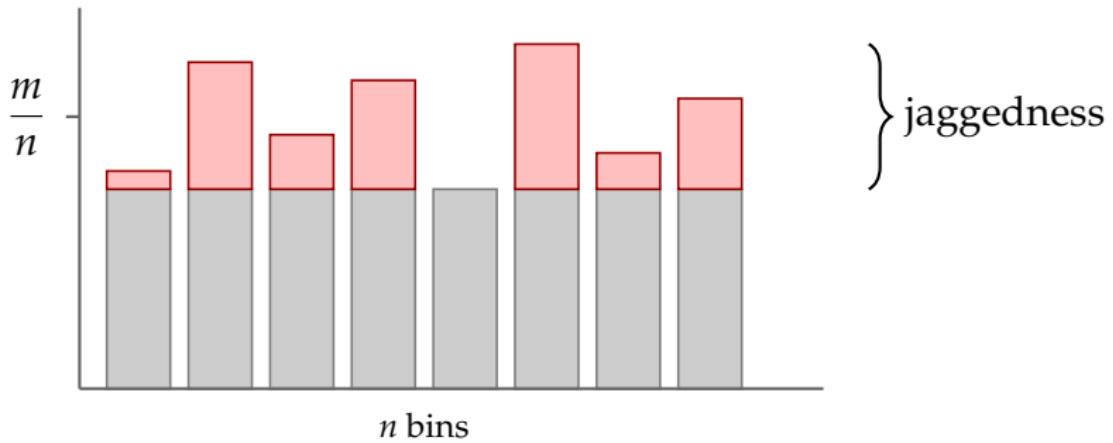
## SLICE AND SPREAD REDUCES OVERLOAD



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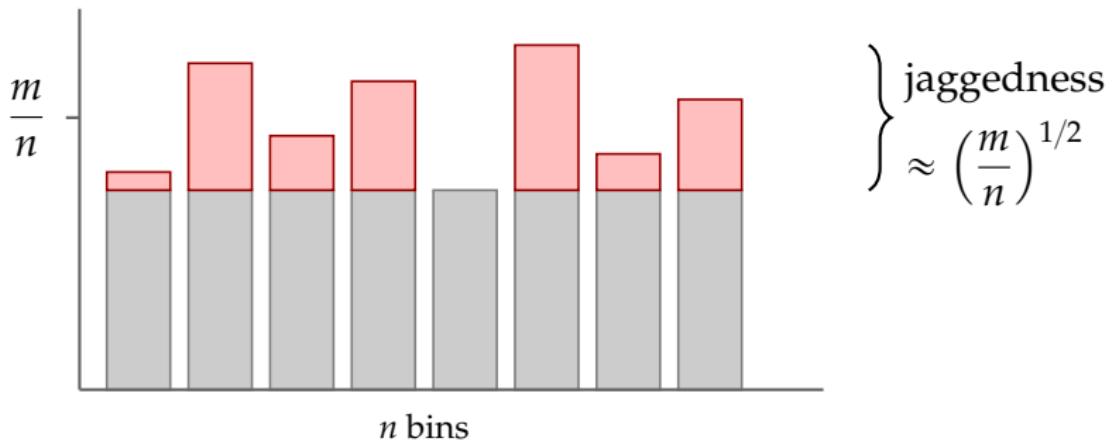


## SLICE AND SPREAD REDUCES OVERLOAD



**Q:** How much is the jaggedness?

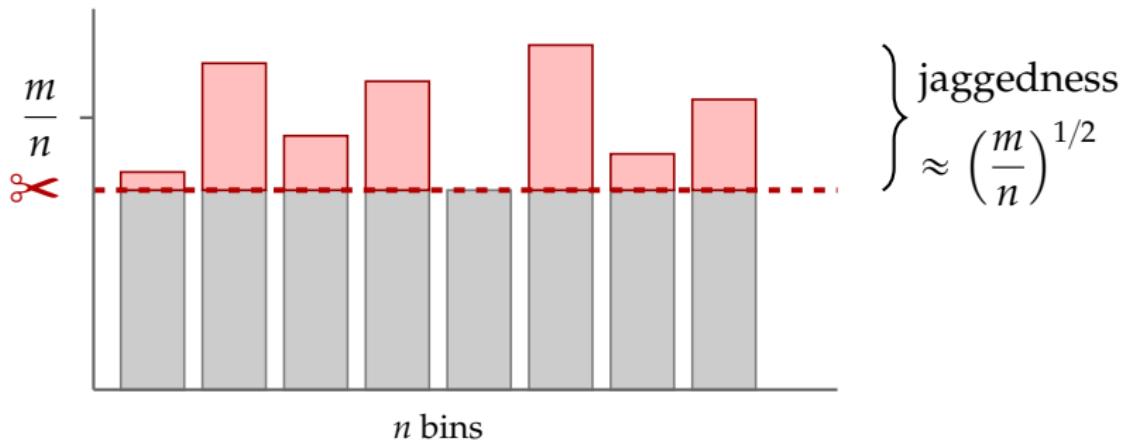
## SLICE AND SPREAD REDUCES OVERLOAD



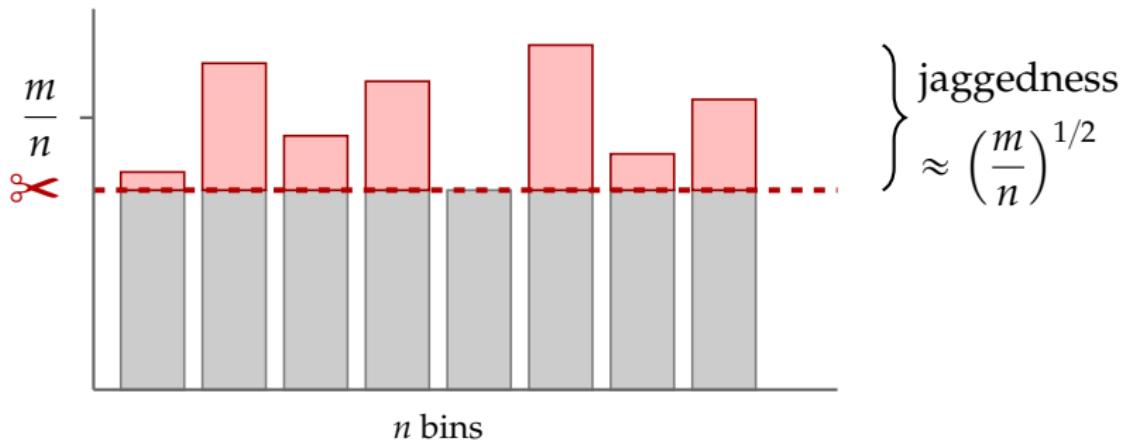
**Q:** How much is the jaggedness?

**A:** For  $m \gg n$ , the jaggedness is  $\approx (m/n)^{1/2}$  (whp in  $n$ )

## SLICE AND SPREAD REDUCES OVERLOAD

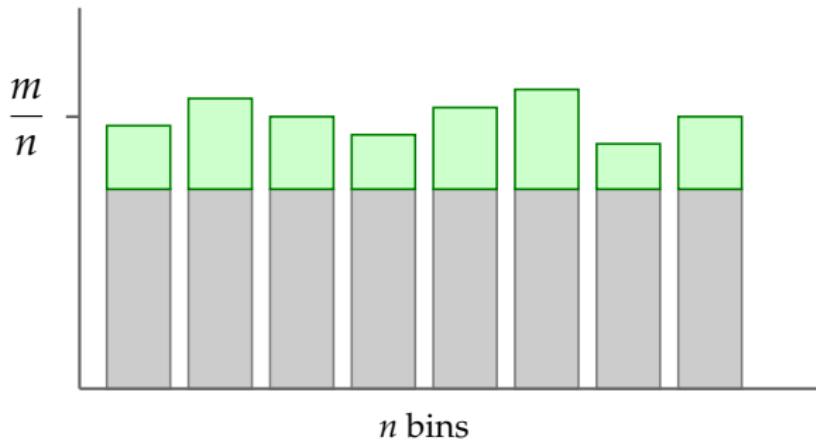


## SLICE AND SPREAD REDUCES OVERLOAD



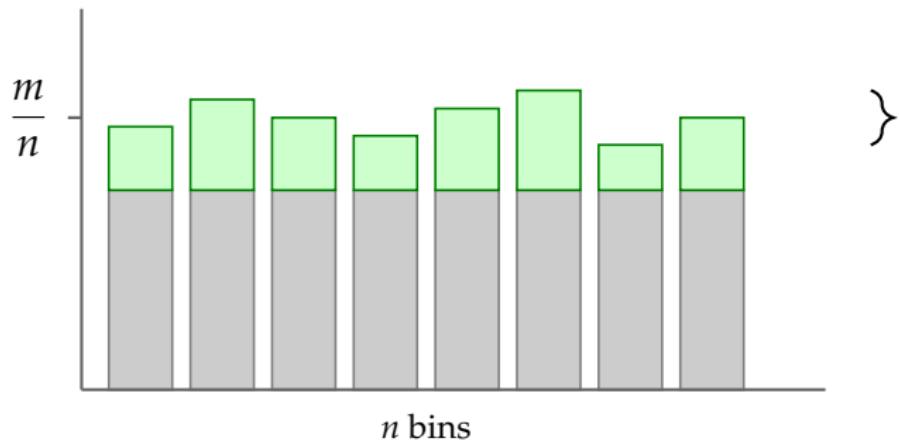
Balls sliced away:  $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

## SLICE AND SPREAD REDUCES OVERLOAD



Balls sliced away:  $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

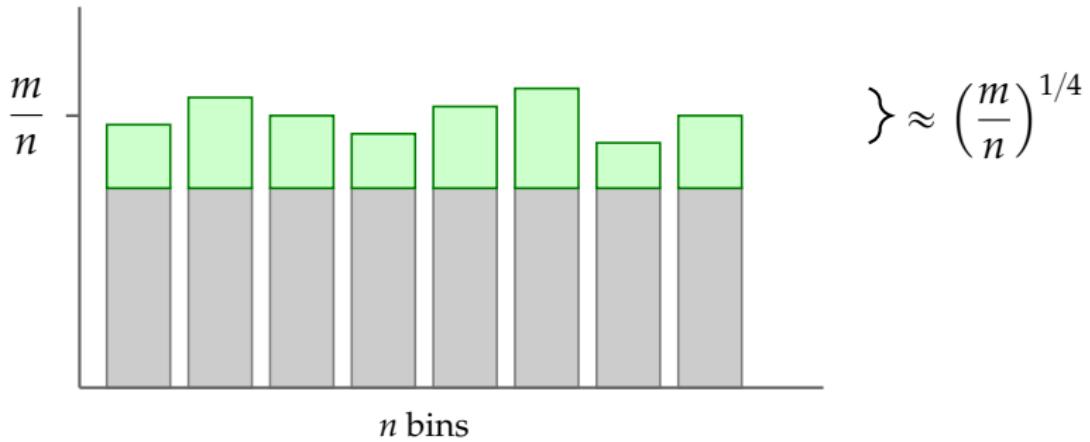
## SLICE AND SPREAD REDUCES OVERLOAD



Balls sliced away:  $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

**Q:** What is the new jaggedness?

## SLICE AND SPREAD REDUCES OVERLOAD



Balls sliced away:  $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

**Q:** What is the new jaggedness?

**A:** For  $m' \gg n$ , the new jaggedness is  $(m'/n)^{1/2} = (m/n)^{1/4}$

## SLICE AND SPREAD

- Overload:  $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$

## SLICE AND SPREAD

- ▶ Overload:  $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent?

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## SLICE AND SPREAD

- ▶ Overload:  $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse:

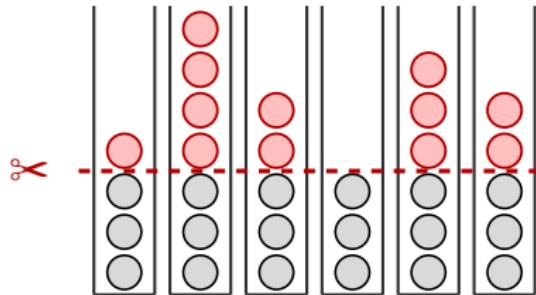
## SLICE AND SPREAD

- ▶ Overload:  $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse:  $1^*$

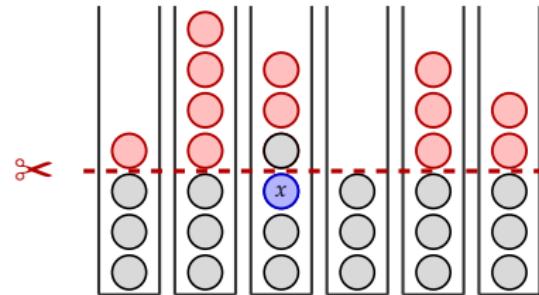
# SLICE AND SPREAD

- ▶ Overload:  $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse: 1\*

## Example



Computing  $A_S$

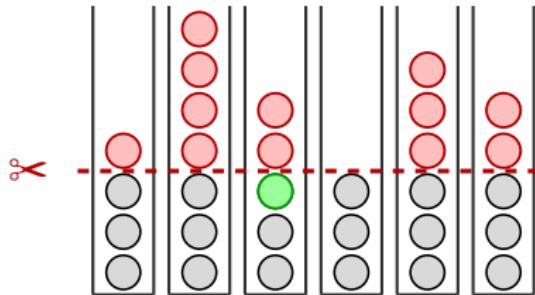


Computing  $A_{S \cup \{x\}}$

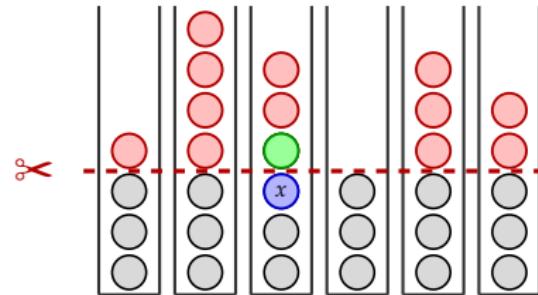
# SLICE AND SPREAD

- ▶ Overload:  $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse: 1\*

## Example

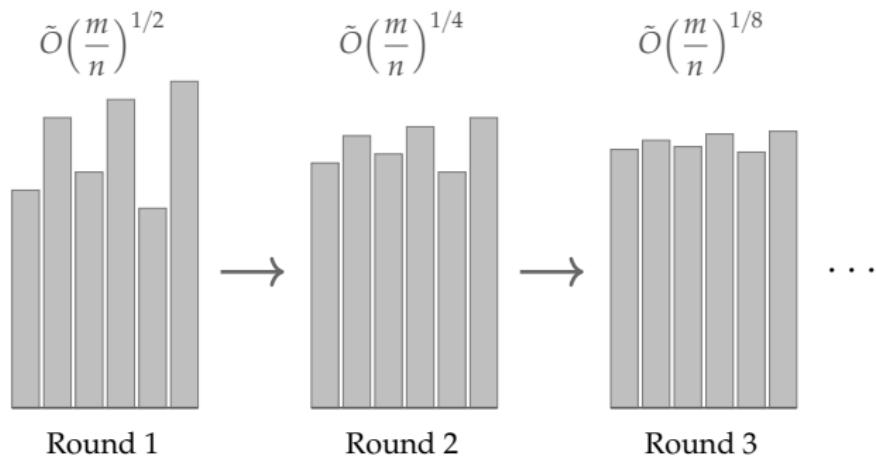


Computing  $A_S$

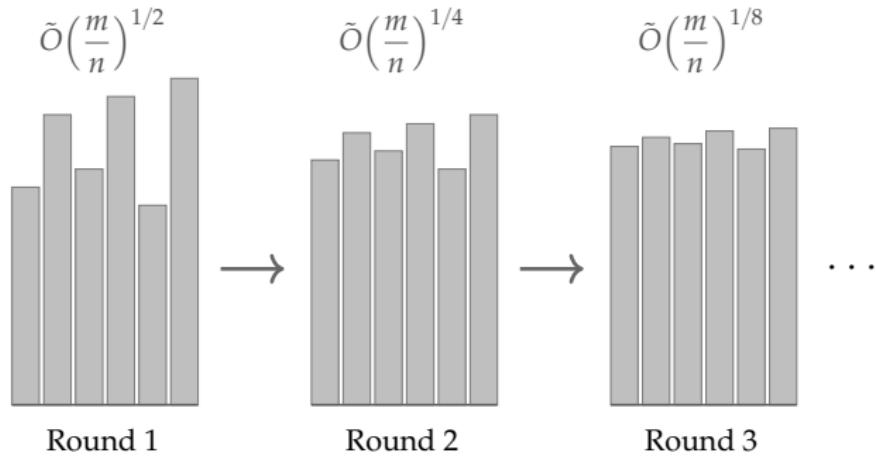


Computing  $A_{S \cup \{x\}}$

# REPEATEDLY SLICING AND SPREADING



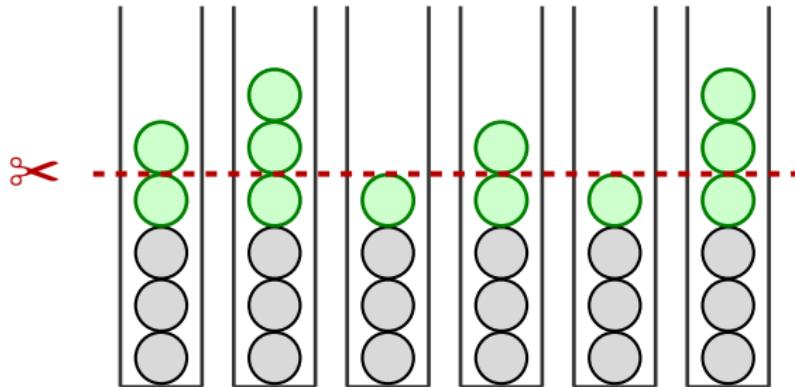
# REPEATEDLY SLICING AND SPREADING



After  $O(\log \log(m/n))$  rounds...

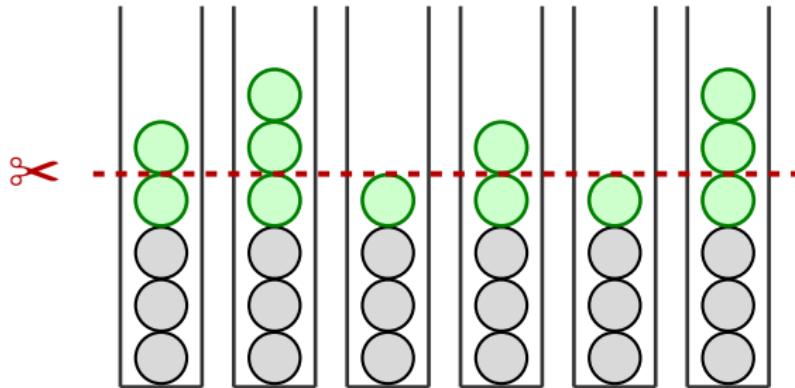
- ▶ Overload =  $O(1)$ ?
- ▶ Recourse =  $O(\log \log(m/n))$ ?

# ALGORITHMIC QUESTION



**Question:** Which balls do we slice in round  $k$ ?

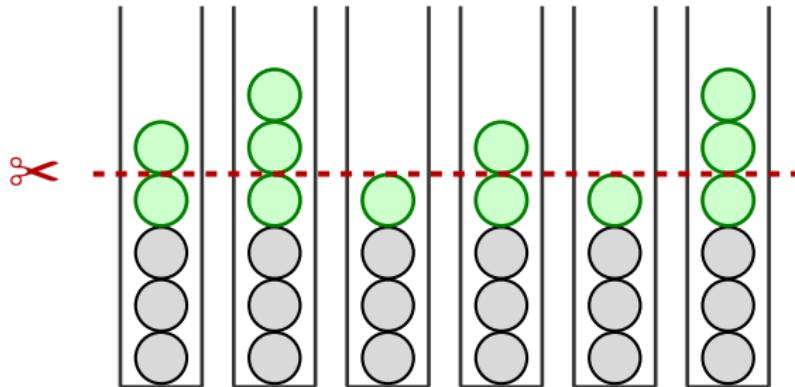
# ALGORITHMIC QUESTION



**Question:** Which balls do we slice in round  $k$ ?

- ▶ **Option 1:** Scrape off the top

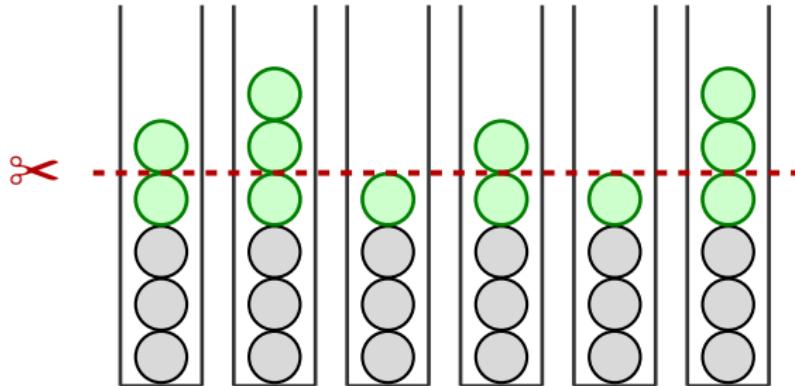
# ALGORITHMIC QUESTION



**Question:** Which balls do we slice in round  $k$ ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness

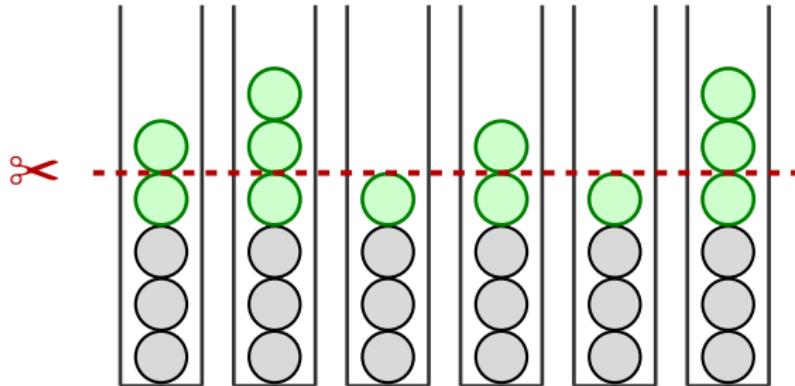
# ALGORITHMIC QUESTION



**Question:** Which balls do we slice in round  $k$ ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness
- ▶ **Option 2:** Priority queue per bin

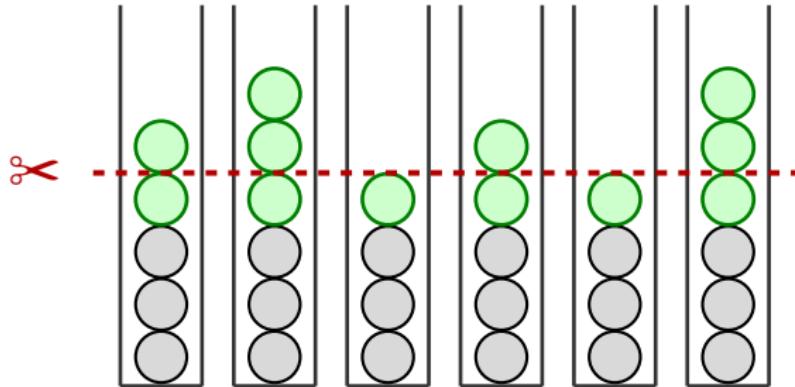
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**Question:** Which balls do we slice in round  $k$ ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness
- ▶ **Option 2:** Priority queue per bin ✗ Exploding recourse

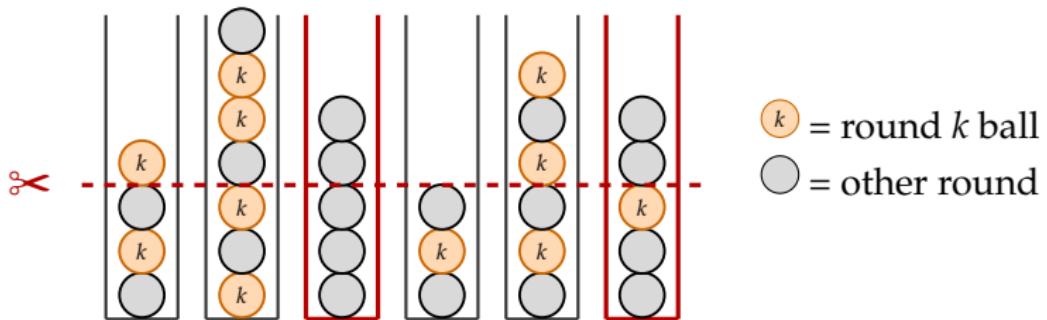
# ALGORITHMIC QUESTION



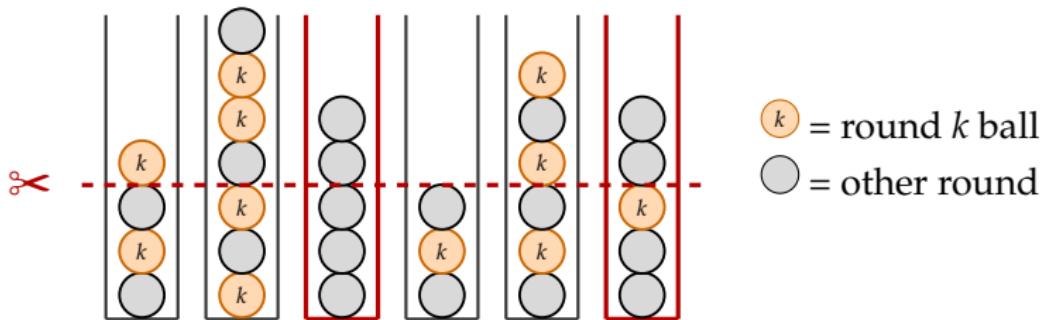
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- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness
- ▶ **Option 2:** Priority queue per bin ✗ Exploding recourse
- ▶ **Our approach:** Assign every ball a random **round number**, only choose from the balls with round number  $k$

# CHALLENGE 1: SLICING FAILURES

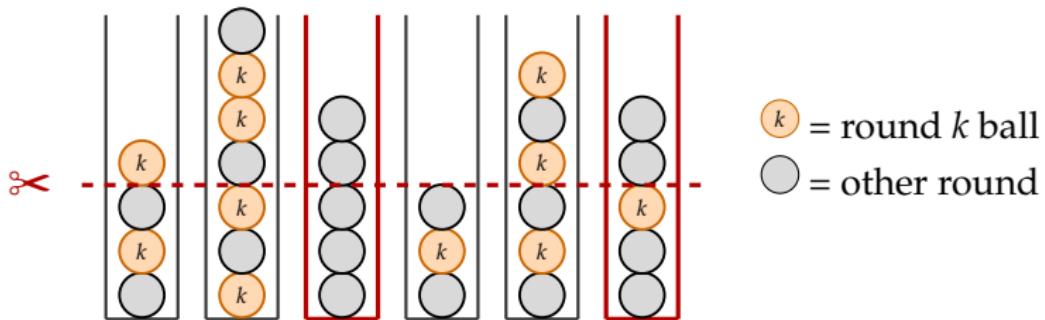


## CHALLENGE 1: SLICING FAILURES



**Challenge:** Some bins may not have enough round- $k$  balls to support slicing.

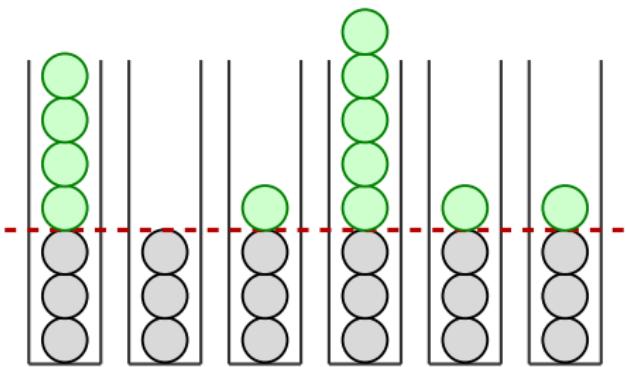
## CHALLENGE 1: SLICING FAILURES



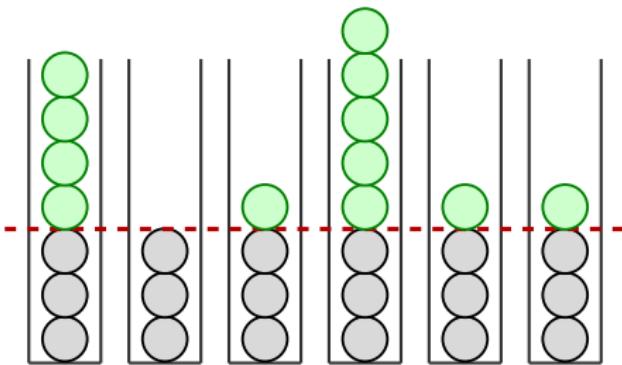
**Challenge:** Some bins may not have enough round- $k$  balls to support slicing.

**Result:** We can't slice evenly — the jaggedness remains in those bins.

## CHALLENGE 2: SPREADING FAILURES

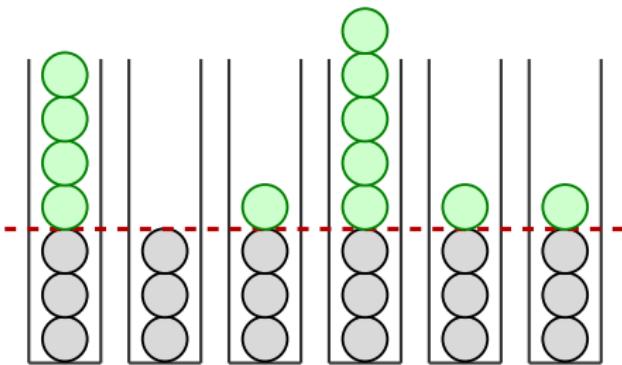


## CHALLENGE 2: SPREADING FAILURES



**Challenge:** The spreading step may distribute balls unevenly — creating new jaggedness.

## CHALLENGE 2: SPREADING FAILURES

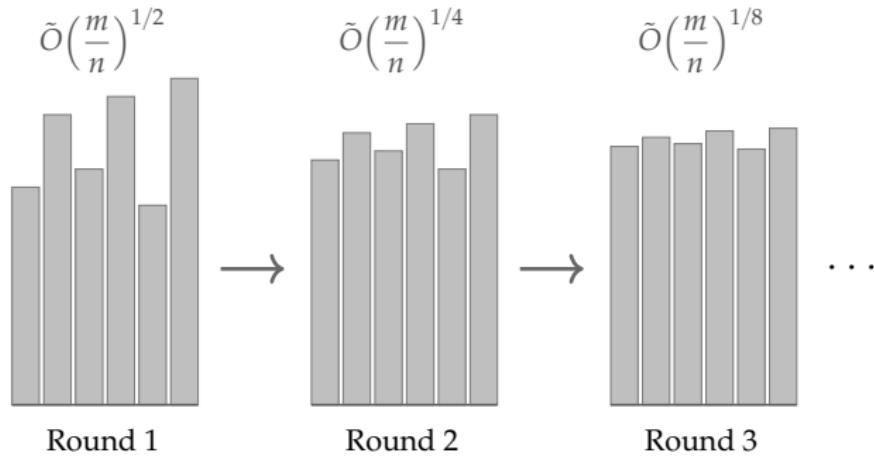


**Challenge:** The spreading step may distribute balls unevenly — creating new jaggedness.

**Dilemma:**

- ▶ Slice **more** balls  $\implies$  overload may not decrease
- ▶ Slice **fewer** balls  $\implies$  jaggedness may not decrease

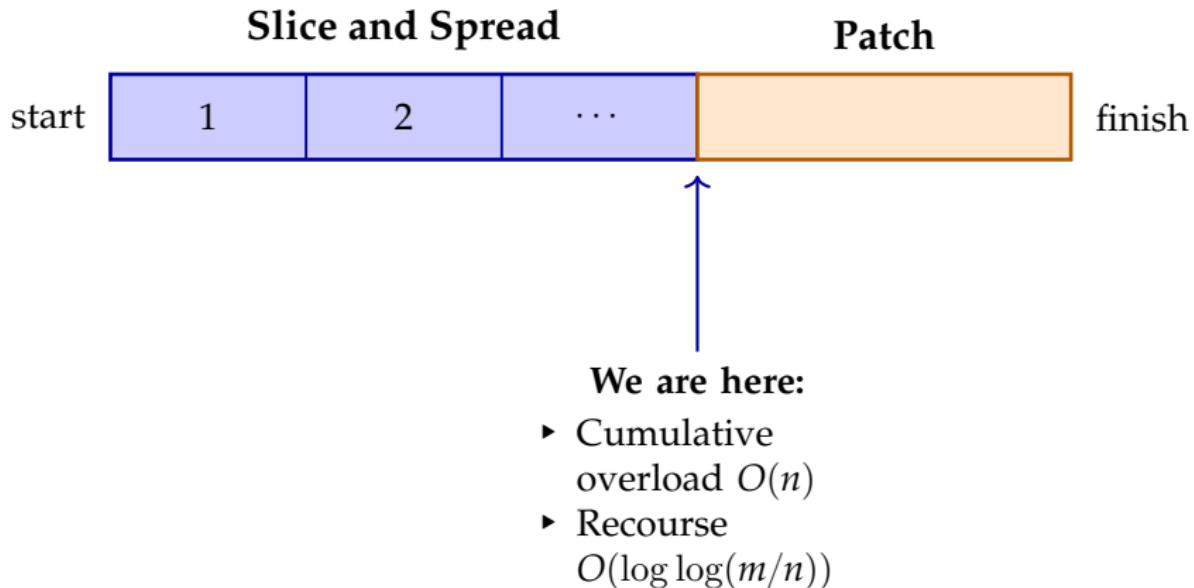
# REPEATEDLY SLICING AND SPREADING



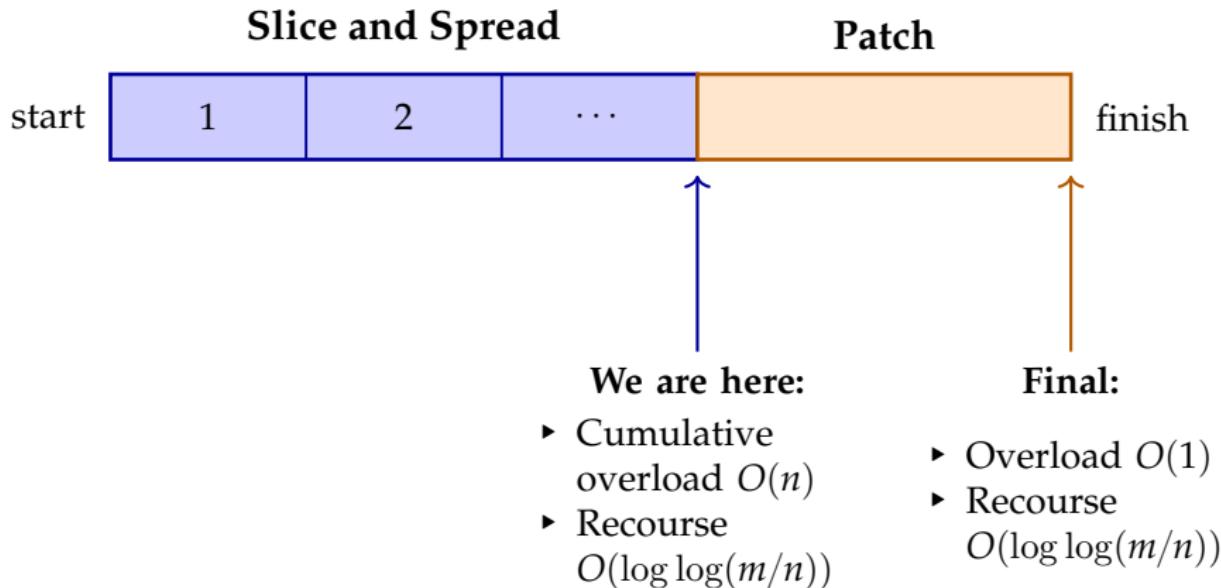
After  $O(\log \log(m/n))$  rounds...

- ▶ Overload =  $O(1)$  Cumulative overload =  $O(n)$
- ▶ Recourse =  $O(\log \log(m/n))$

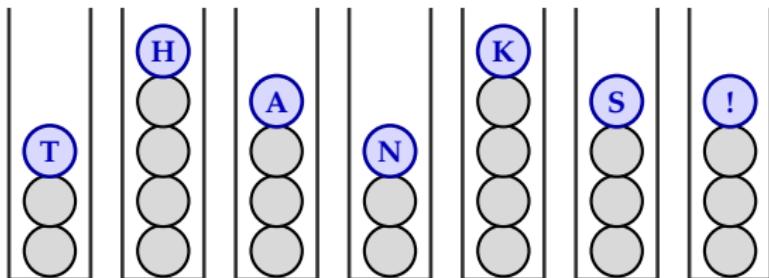
# FULL ALGORITHM



# FULL ALGORITHM



# History-Independent Load Balancing



Michael A. Bender  
Stony Brook University

William Kuszmaul  
CMU

Elaine Shi  
CMU

**Rose Silver**  
CMU