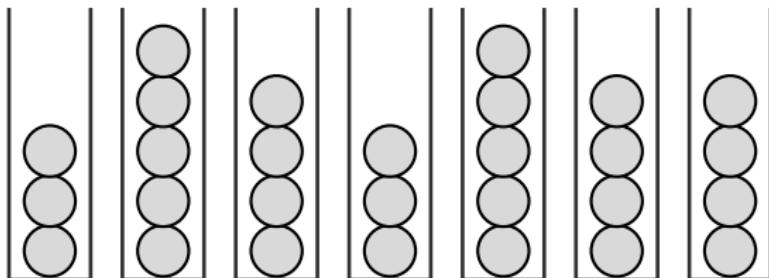


History-Independent Load Balancing



Michael A. Bender

Stony Brook University

William Kuszmaul

CMU

Elaine Shi

CMU

Rose Silver

CMU

NYT CENSORING CATASTROPHE

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Government sources differ in their accounts as to what happened when Colonel [REDACTED] tried to deliver to Mossadeq the Shah's firman dismissing him. According to General Riahi, Colonel Momtaz was on his way to the Bagh-i-Shah when he ran into Colonel [REDACTED] in the street and there-upon arrested him. According to the official communique of the Mossadeq government, [REDACTED] showed up before Mossadeq's house at 0100 hours on 16 August with four trucks full of soldiers, two jeeps, and an armored car. He claimed that

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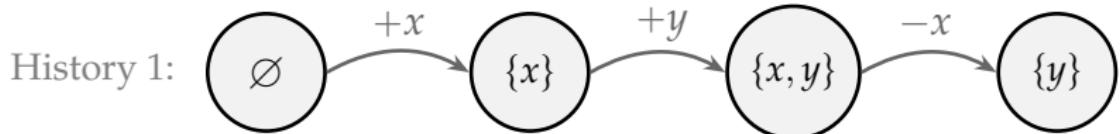
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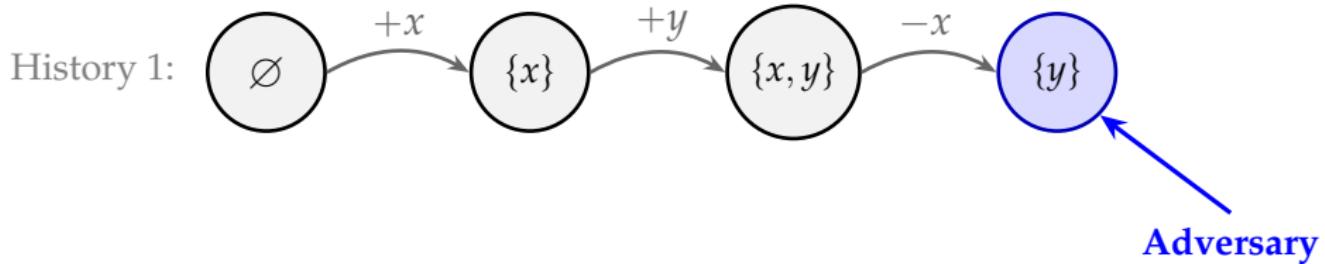
History can be just as important as content!

HISTORY-INDEPENDENT DATA STRUCTURES

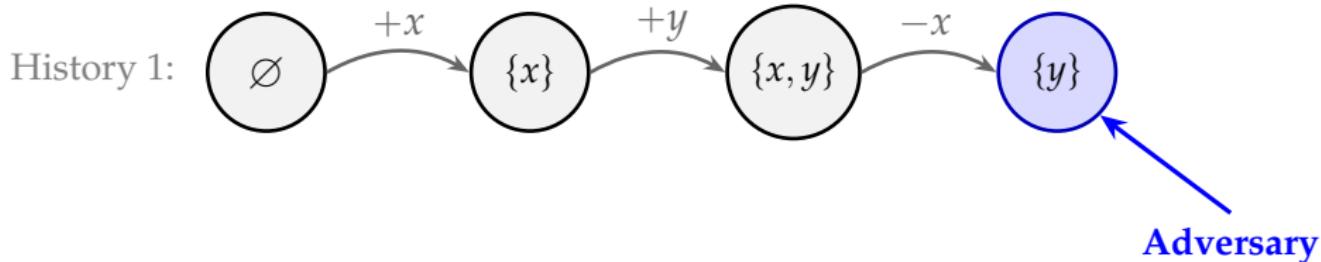
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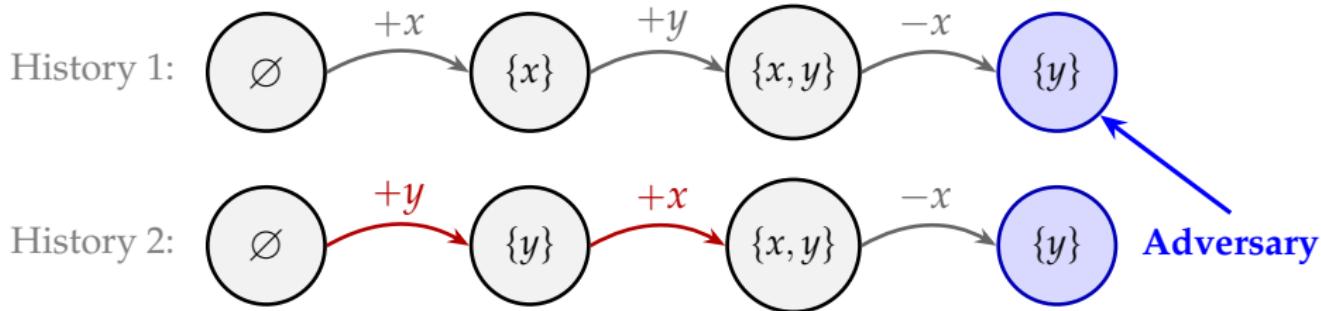
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History Independence (Micciancio '97, Naor & Teague '01)

- ▶ The state reveals only the current elements—**not the history of operations**.

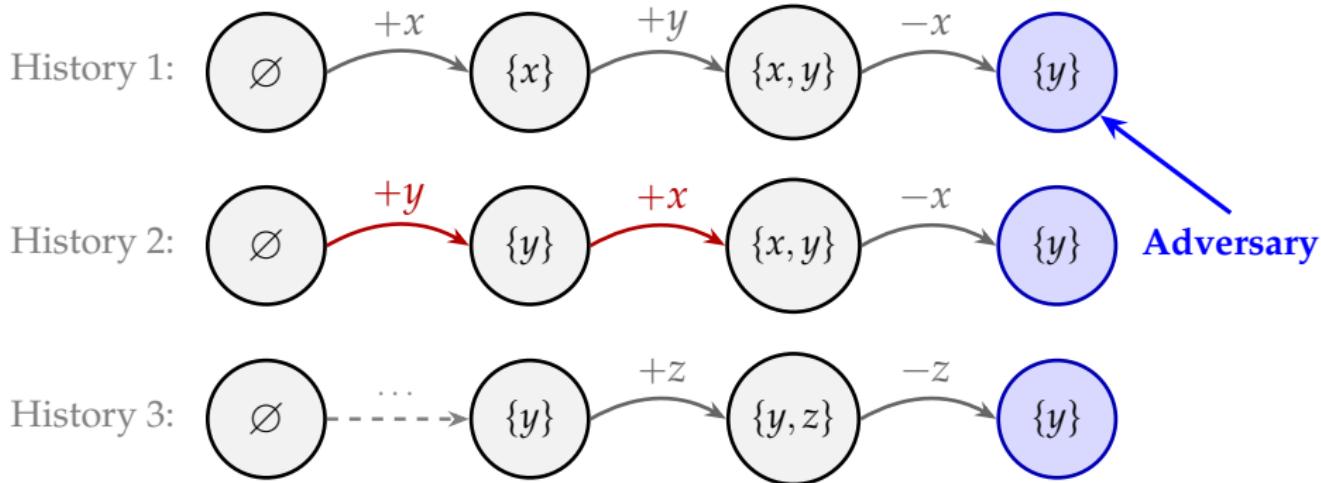
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A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

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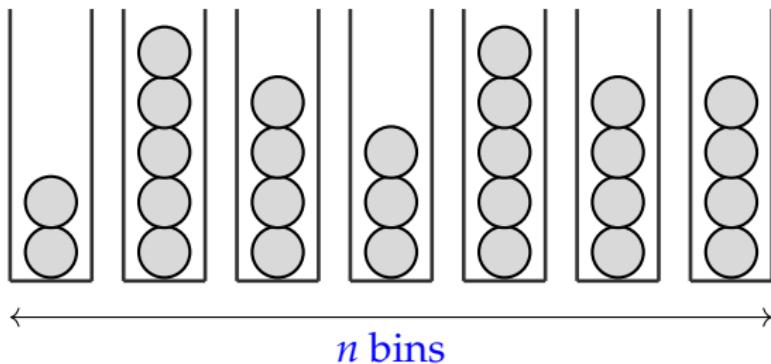
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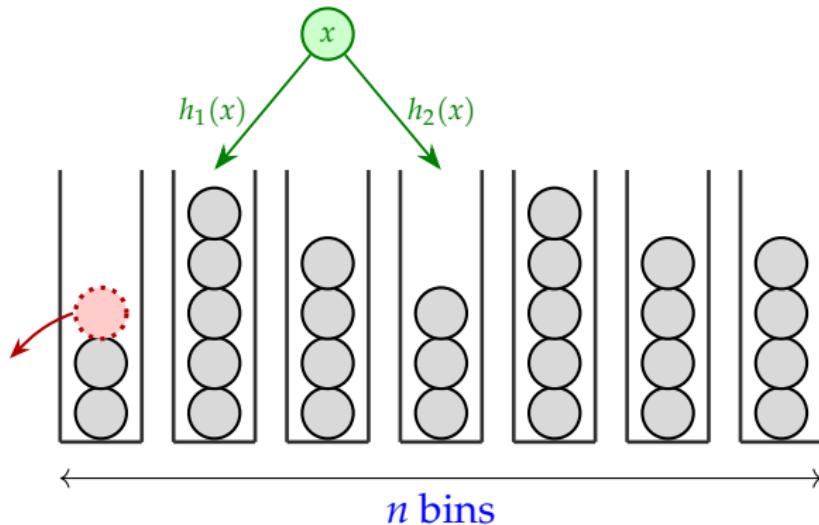
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This work: History-Independent Load Balancing

TWO-CHOICE LOAD BALANCING

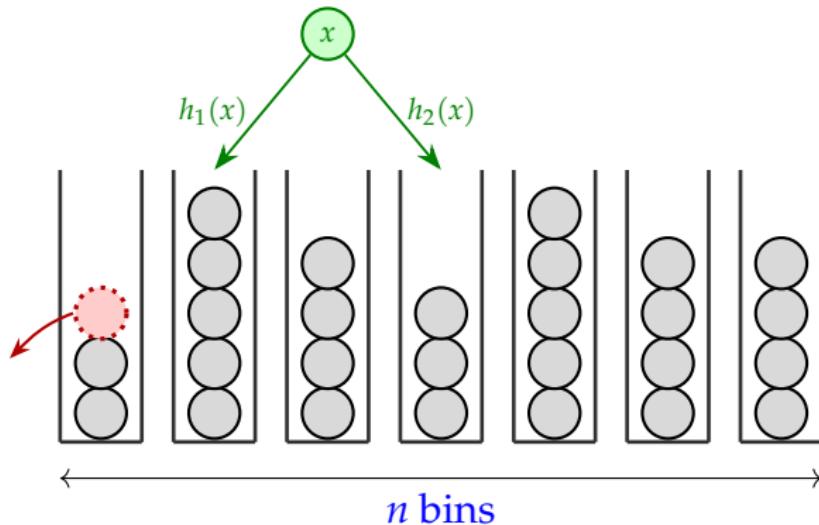


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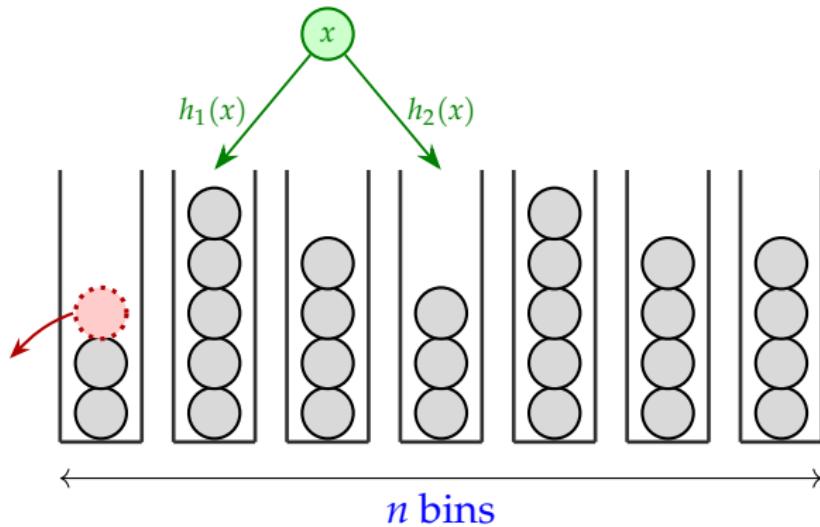
- ▶ Balls are **inserted**/**deleted**, with up to m present at a time.

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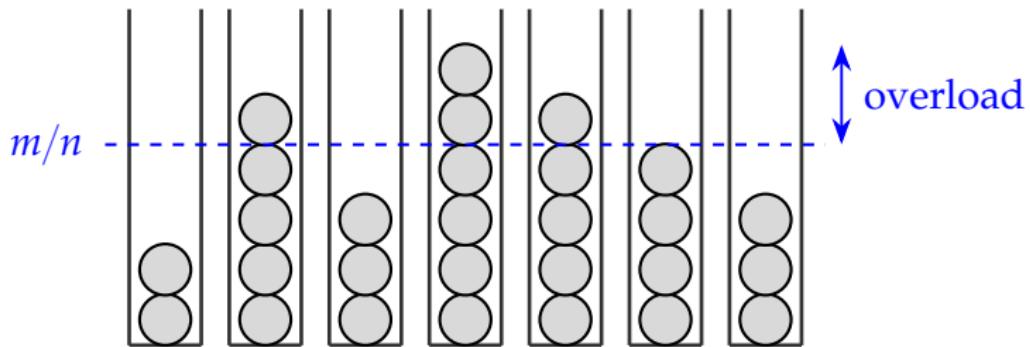
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TWO-CHOICE LOAD BALANCING



- ▶ Balls are **inserted**/**deleted**, with up to m present at a time.
- ▶ Each ball has two random bins where it can go.
- ▶ We must maintain a valid assignment of balls to bins.

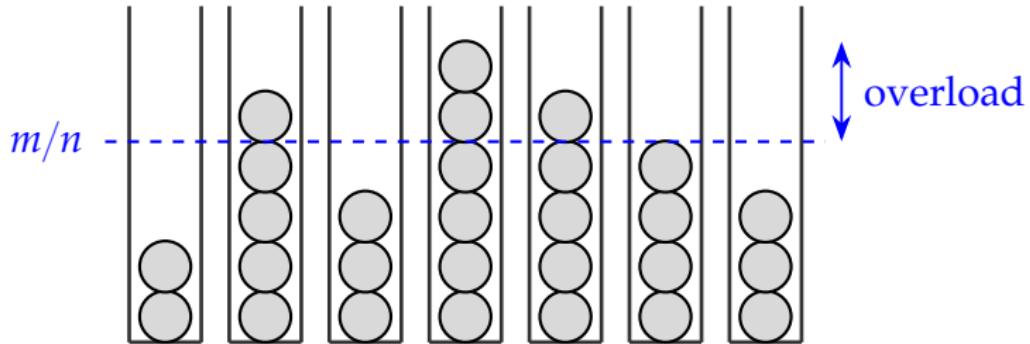
TWO GOALS



Minimize Overload:

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TWO GOALS



Minimize Overload:

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Minimize Recourse:

- ▶ i.e., the number of balls moved around on any given insertion/deletion.

PUTTING IT ALL TOGETHER

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History-Independent Load Balancing:

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- ▶ We call this assignment A_S .

Question: Does there exist a history-independent solution with small recourse and small overload?

Our Main Result: There exists a history-independent solution with:

- ▶ High probability overload $O(1)$
- ▶ Expected recourse $O(\log \log(m/n))$

PAST WORK (NOT HISTORY INDEPENDENT)

Overload	Recourse	Reference	Caveats
$O(\log \log n)$	0	[ABKU '94] [BCSV '00]	insertion-only
$O(1)$	$O(\log(m/n))$	[Dietzfelbinger, Weidling '07]	insertion-only
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Privacy can be leveraged as an *algorithmic tool* to outperform non-private algorithms!

REST OF TALK

1. A Simple Warmup
2. The Full Algorithm

Part 1: A Simple Warmup

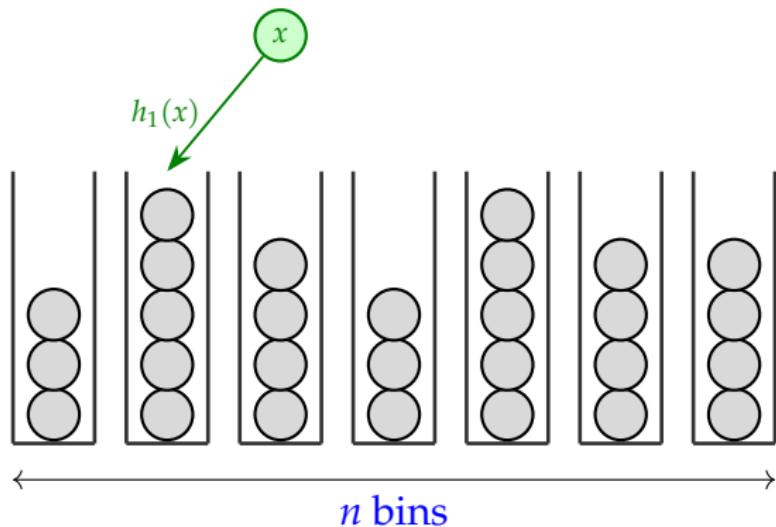
A SIMPLE WARMUP

Theorem: There exists a history-independent solution with:

- ▶ High-probability overload $\Theta(1)$ $O(\log \log n)$.
- ▶ Expected recourse $\Theta(\log \log(m/n))$ $O(m/n)$.

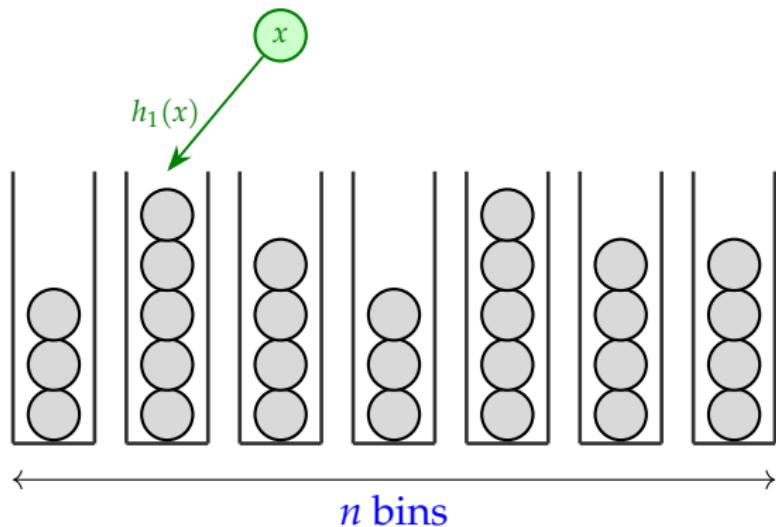
BASELINE 1: THE SINGLE-CHOICE STRATEGY

To insert a ball x , just put it in bin $h_1(x)$:



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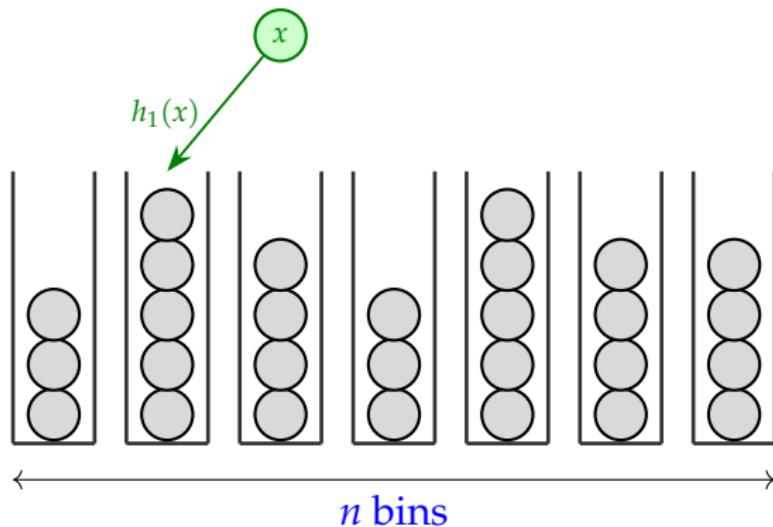
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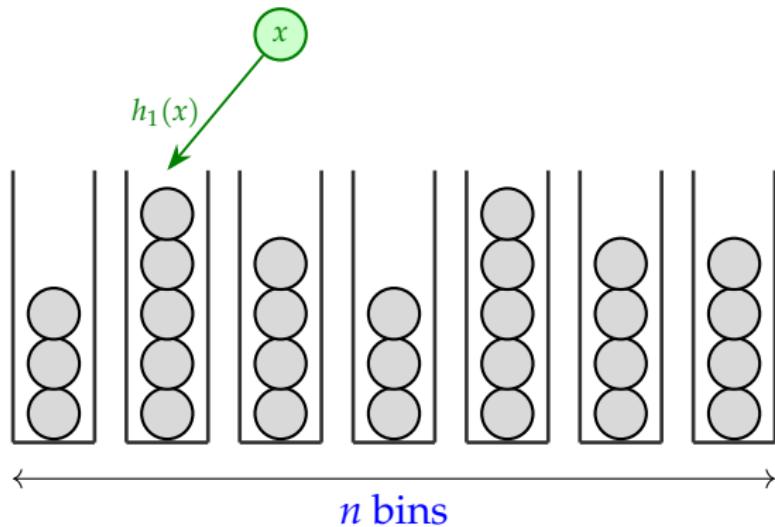
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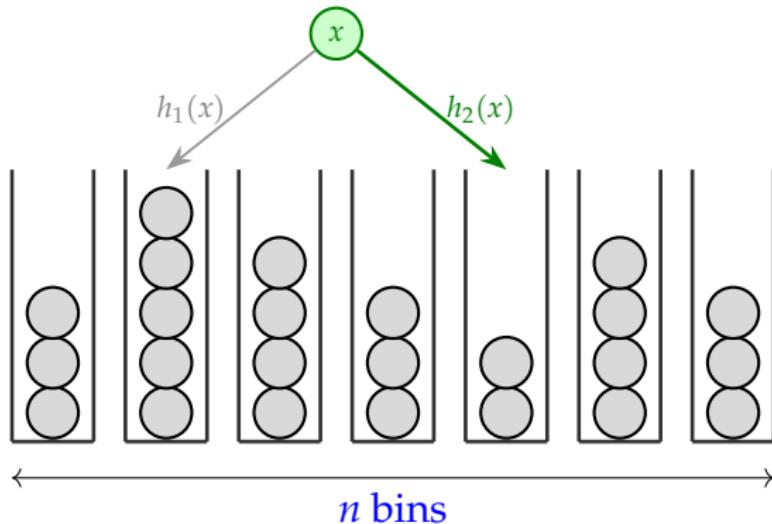
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- ▶ This is history-independent ✓
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- ▶ But... the overload is huge, roughly $\sqrt{m/n}$ ✗

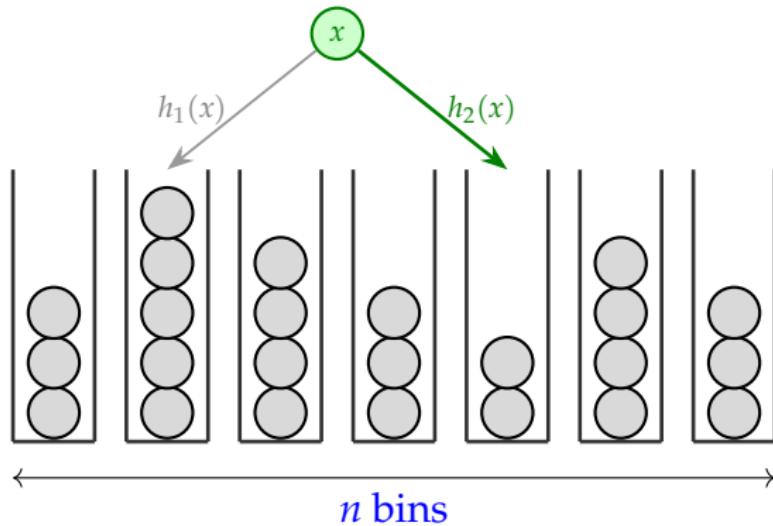
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To insert a ball x , put it in the **emptier** of its choices:



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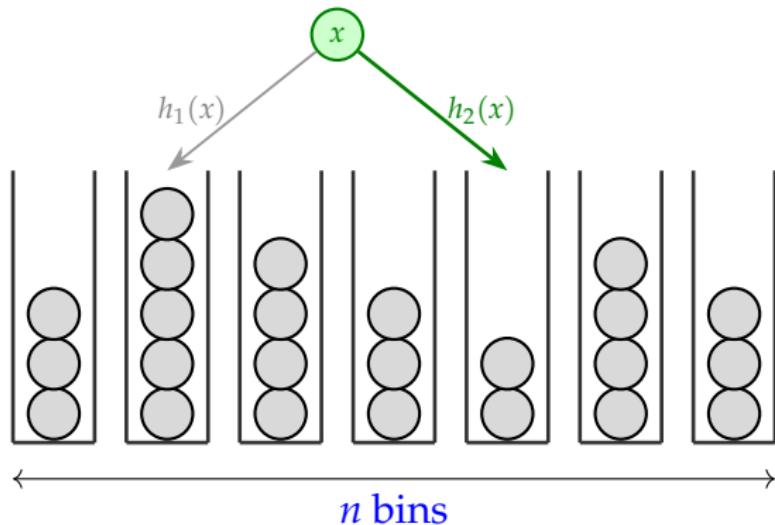
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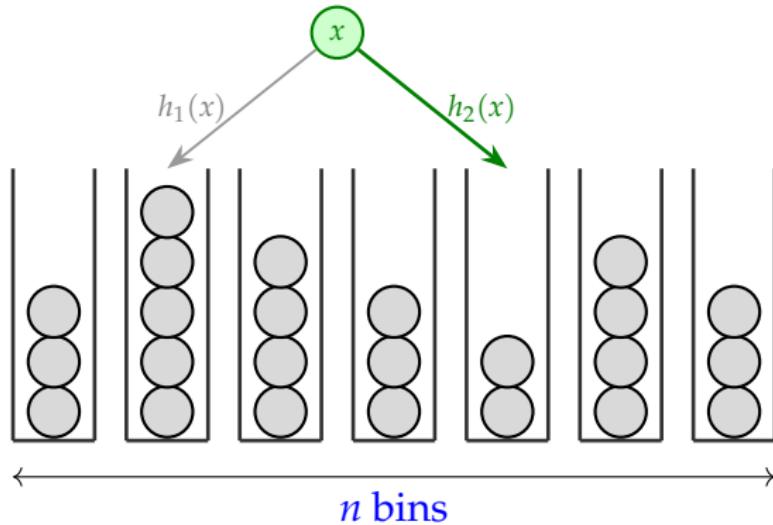
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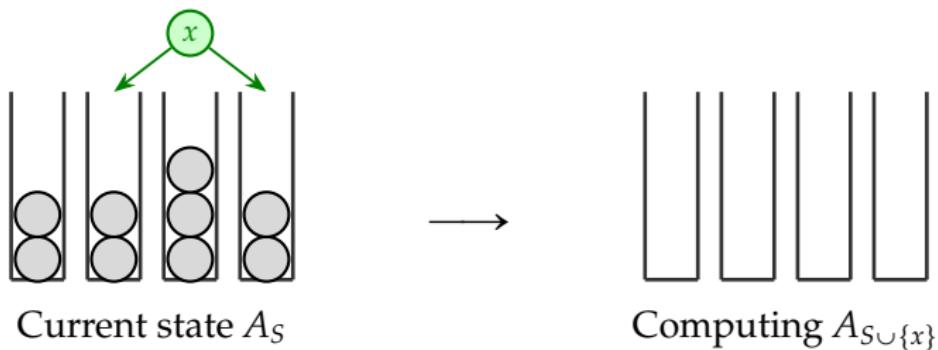
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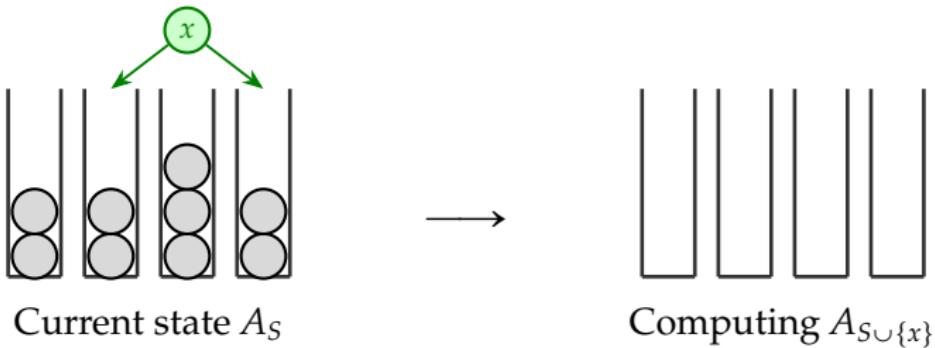
- ▶ This is **not** history-independent ✗
- ▶ The recourse is 0 ✓
- ▶ In the insertion-only case, the overload is $O(\log \log n)$ ✓
[Berenbrink, Czumaj, Steger, and Vöcking '00]

WARMUP: HISTORY-INDEPENDENT GREEDY

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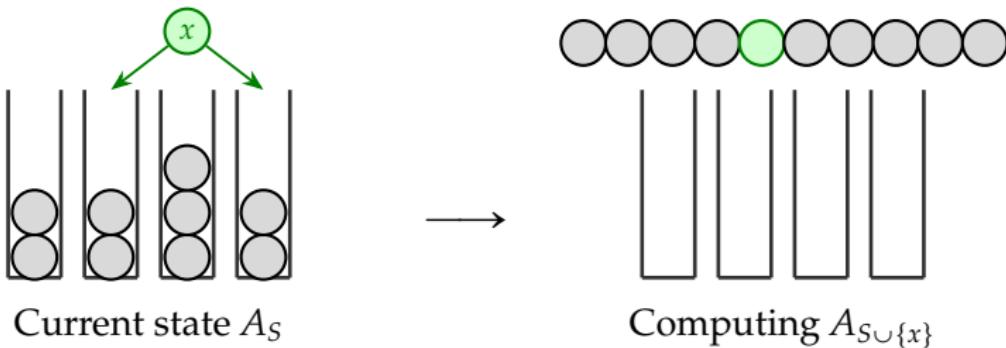
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1. Empty out the bins.
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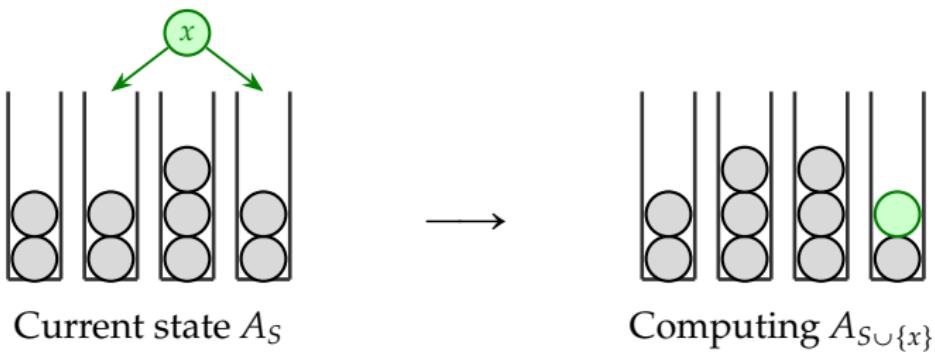
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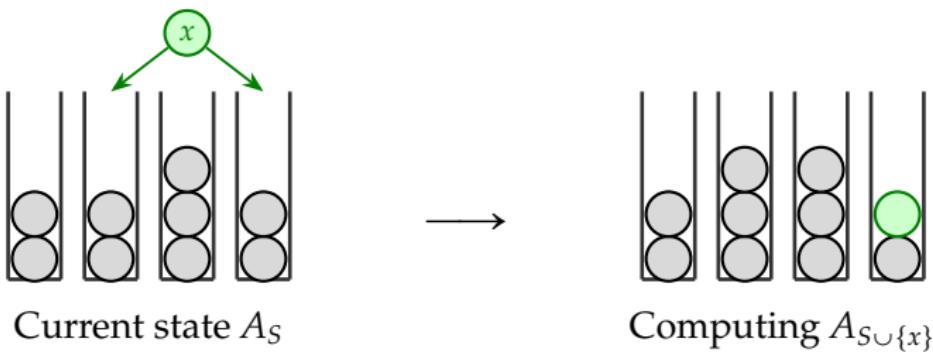
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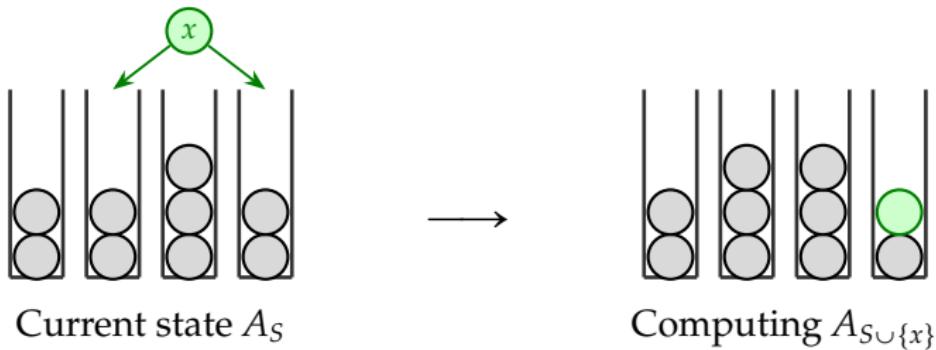


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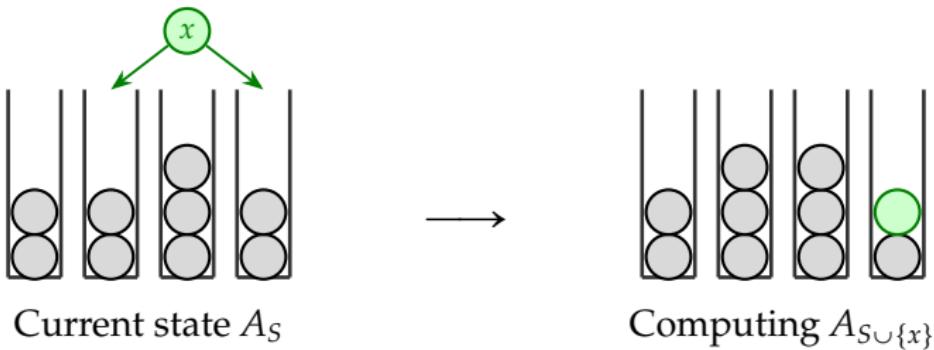
- ▶ The algorithm is history independent ✓

ANALYZING HISTORY-INDEPENDENT GREEDY



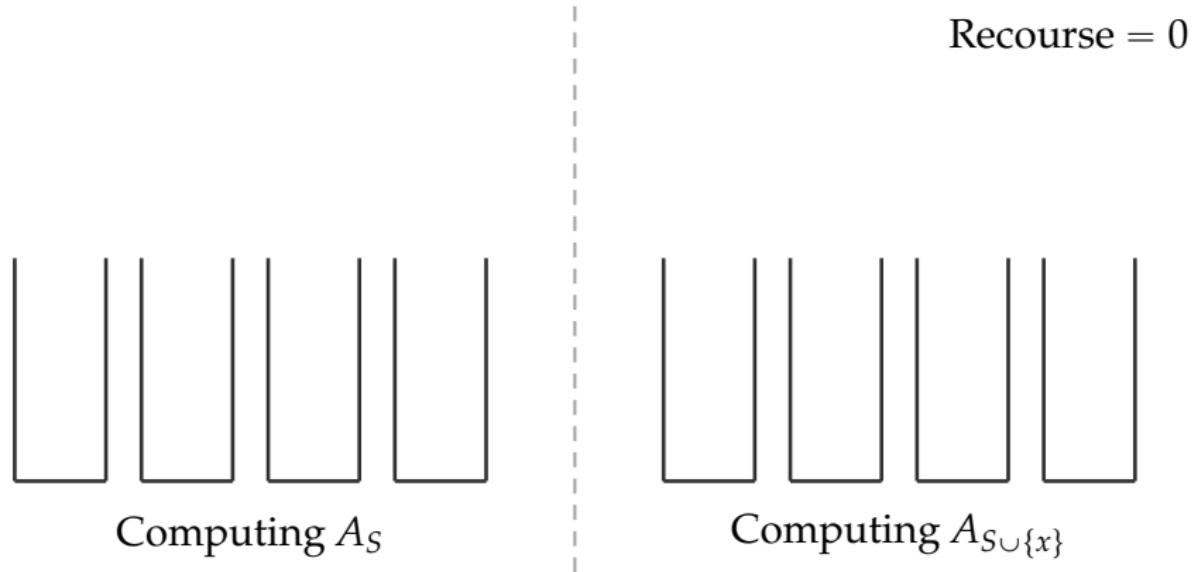
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ANALYZING HISTORY-INDEPENDENT GREEDY



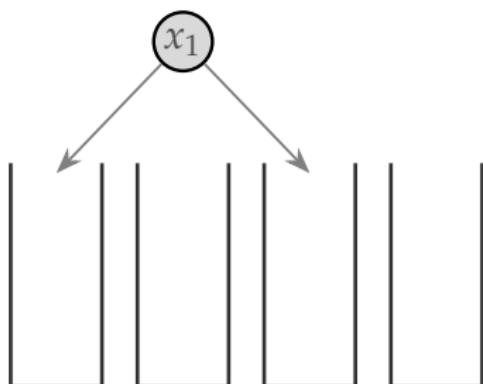
- ▶ The algorithm is history independent ✓
- ▶ The overload is $O(\log \log n)$ ✓
- ▶ What is the recourse?

ANALYZING THE RECOURSE

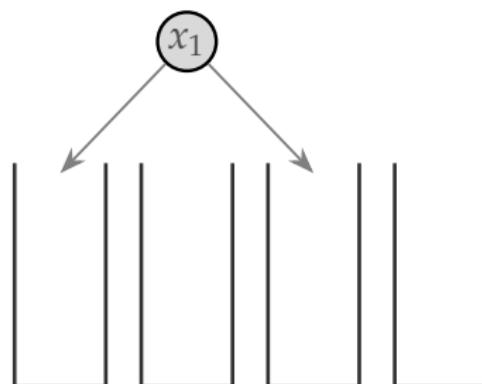


How many balls change assignments between A_S and $A_{S \cup \{x\}}$?

ANALYZING THE RECOURSE



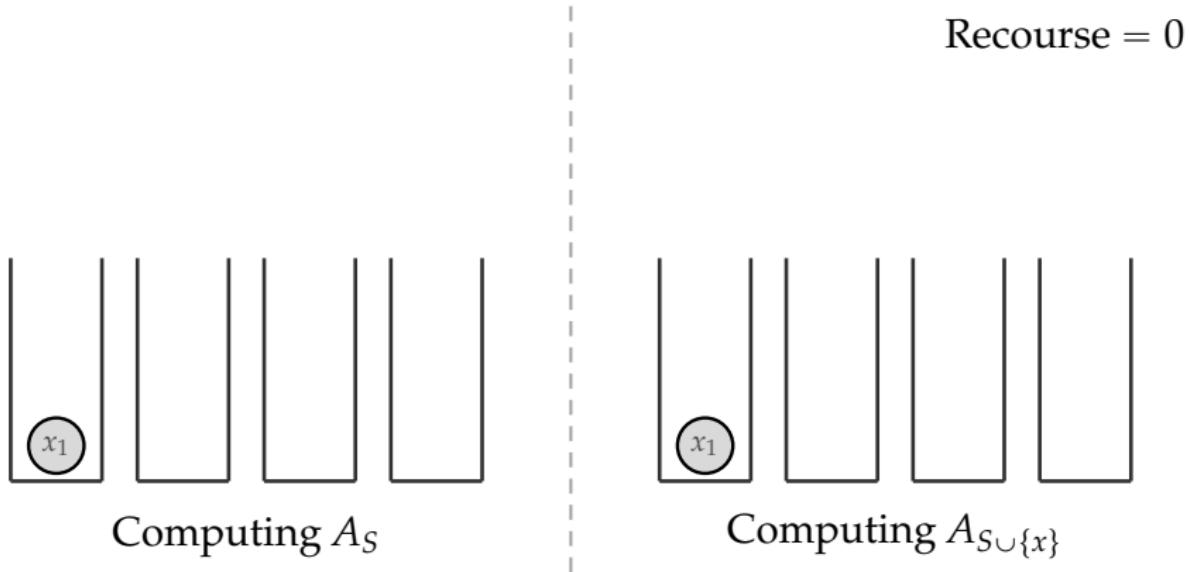
Computing A_S



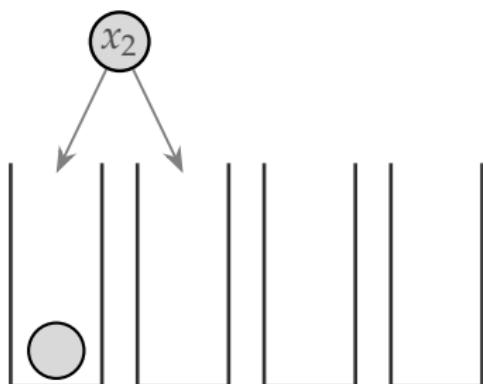
Computing $A_{S \cup \{x\}}$

Recourse = 0

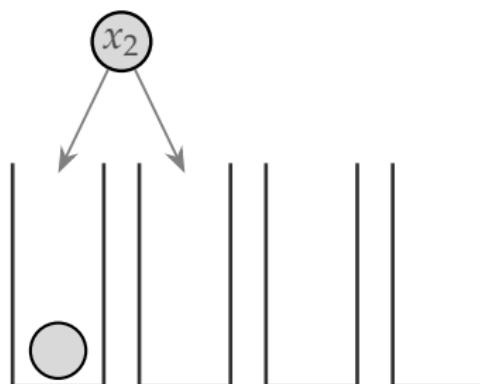
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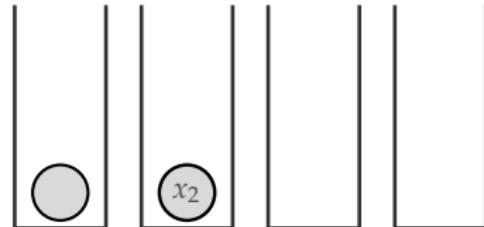
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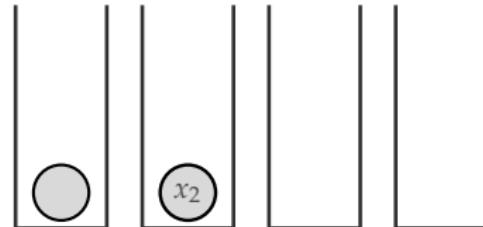
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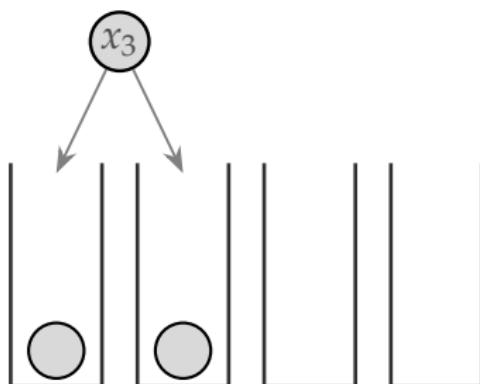
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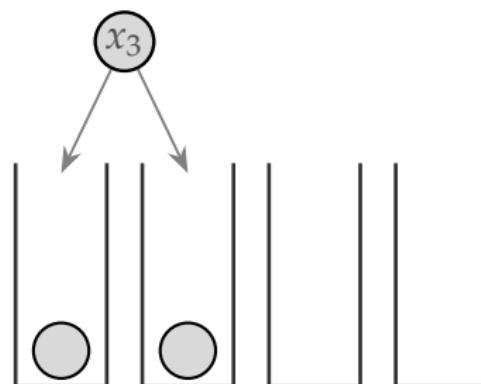
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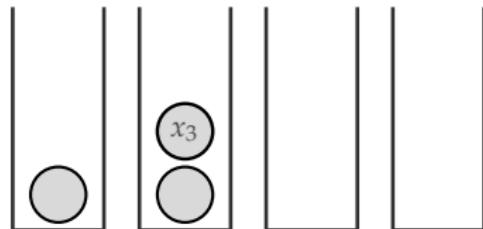
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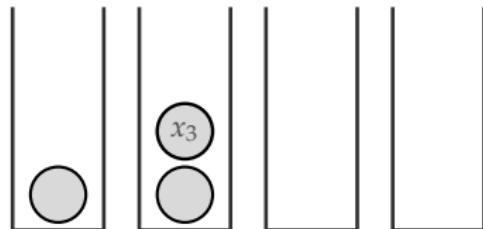
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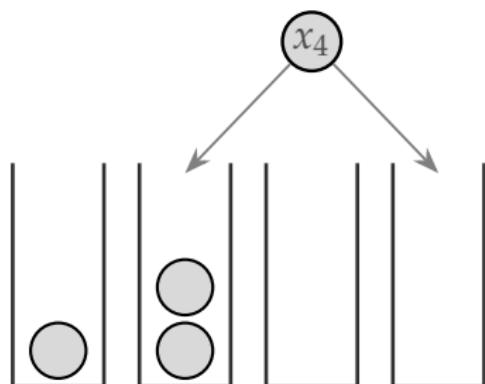
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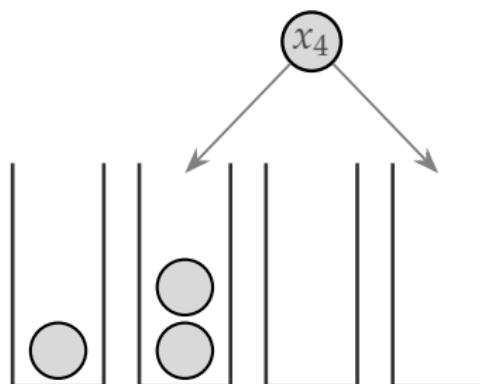
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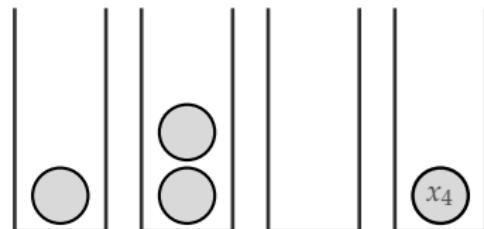
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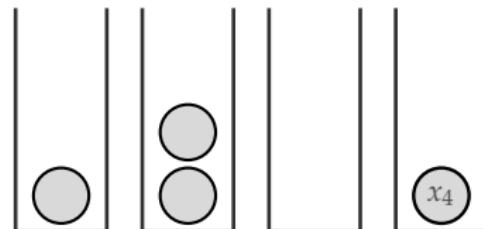
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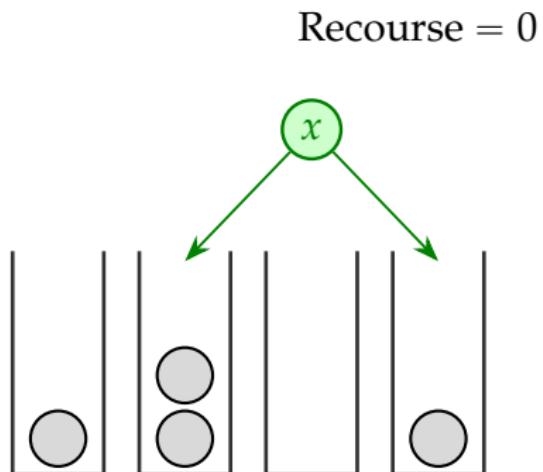
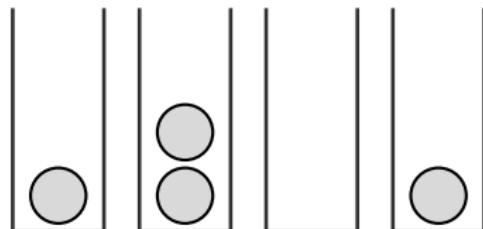
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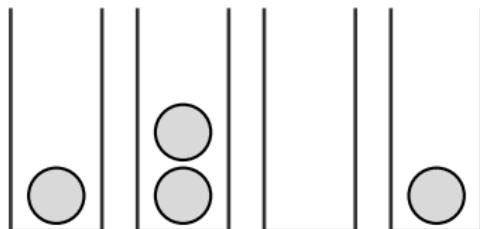
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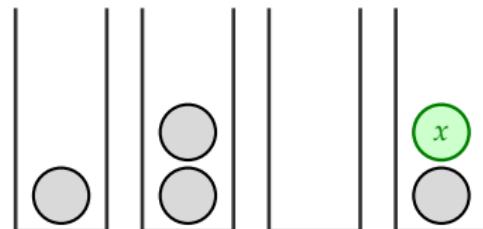
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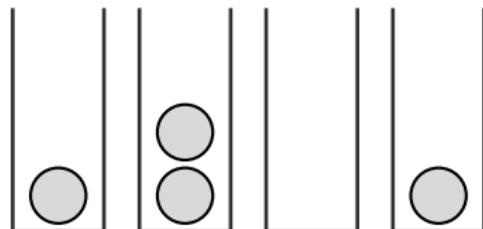
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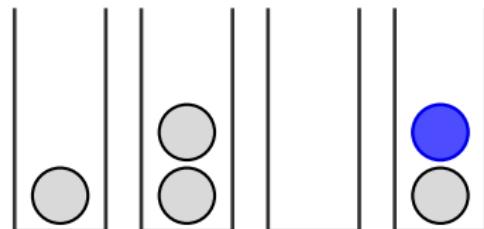
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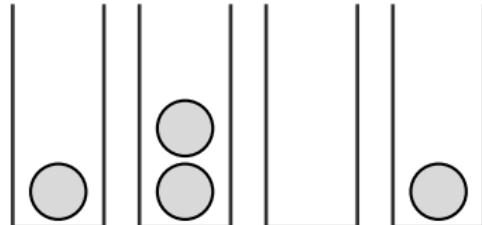


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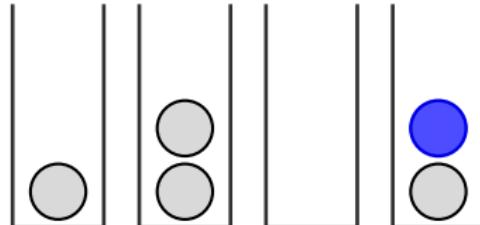
Subsequent balls will experience either:

Recourse = 0

ANALYZING THE RECOURSE



Computing A_S

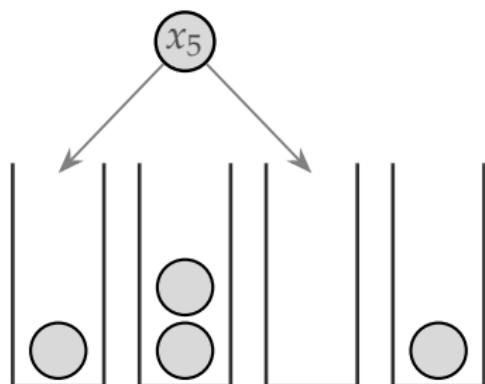


Computing $A_{S \cup \{x\}}$

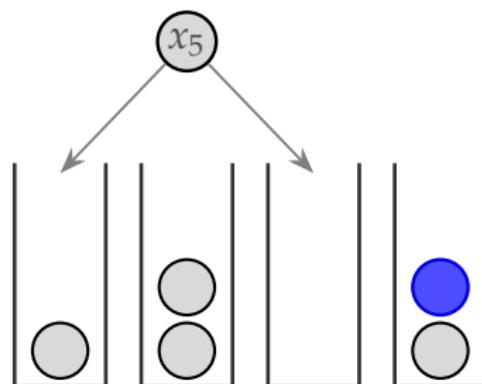
Subsequent balls will experience either:

1. No recourse

ANALYZING THE RECOURSE



Computing A_S



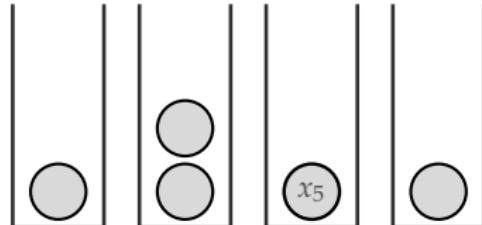
Computing $A_{S \cup \{x\}}$

Future insertions will experience either:

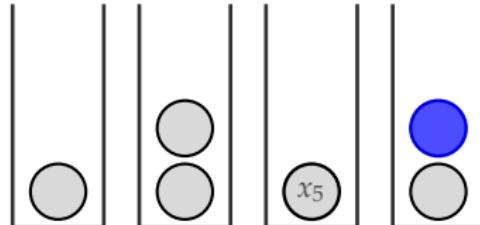
1. No recourse

Recourse = 0

ANALYZING THE RECOURSE



Computing A_S

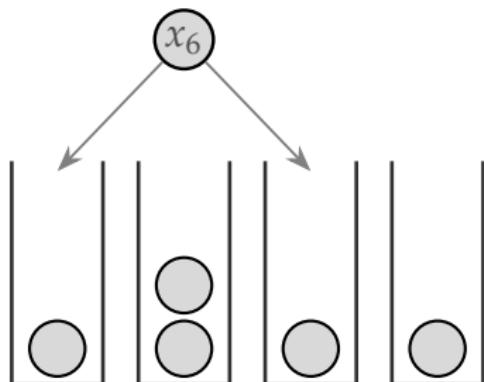


Computing $A_{S \cup \{x\}}$

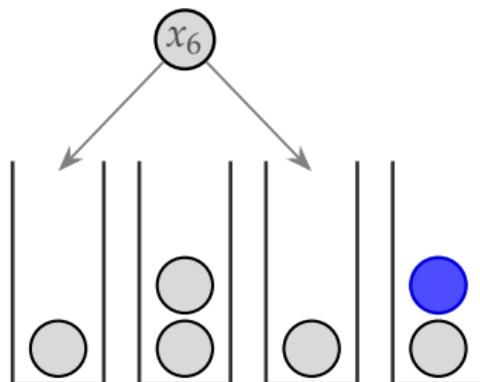
Subsequent balls will experience either:

1. No recourse

ANALYZING THE RECOURSE



Computing A_S



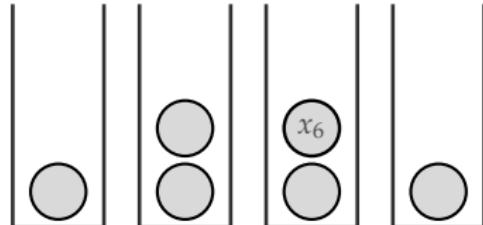
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

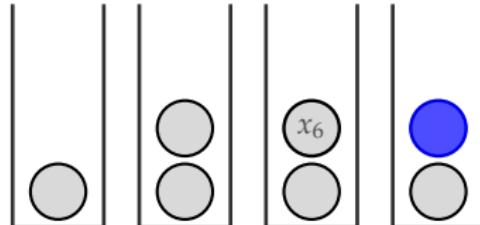
1. No recourse

Recourse = 0

ANALYZING THE RECOURSE



Computing A_S



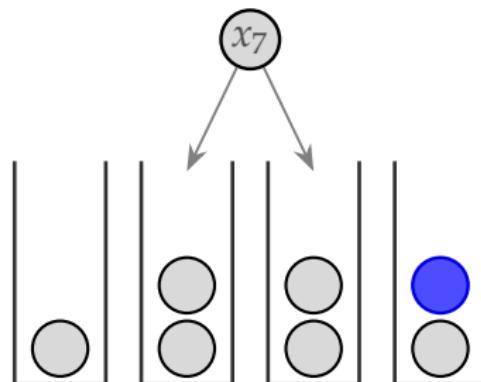
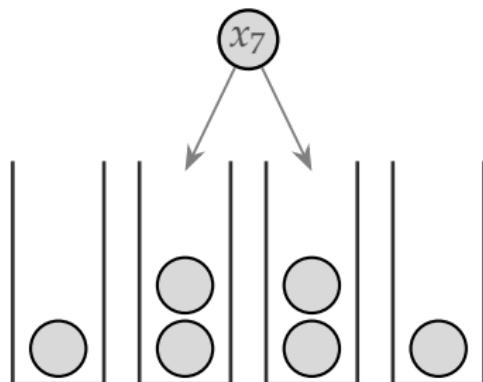
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse

Recourse = 0

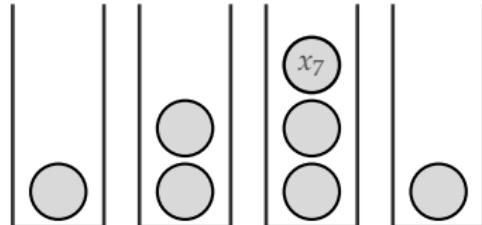
ANALYZING THE RECOURSE



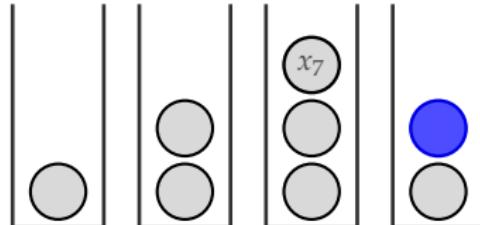
Subsequent balls will experience either:

1. No recourse

ANALYZING THE RECOURSE



Computing A_S



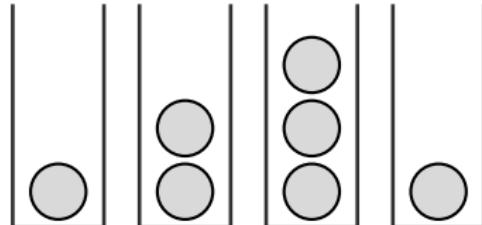
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

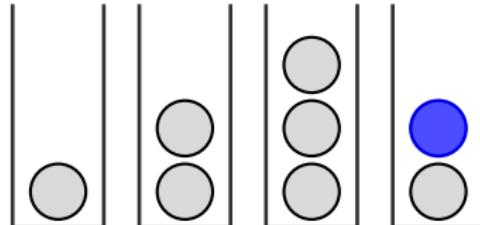
1. No recourse

$$\text{Recourse} = 0$$

ANALYZING THE RECOURSE



Computing A_S



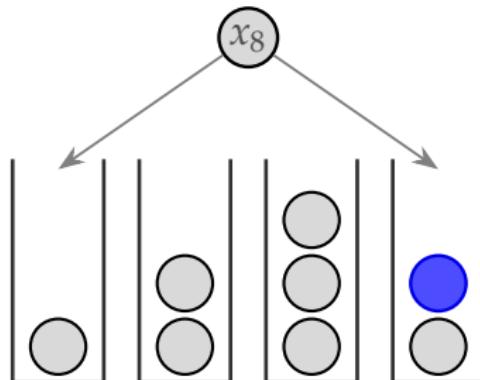
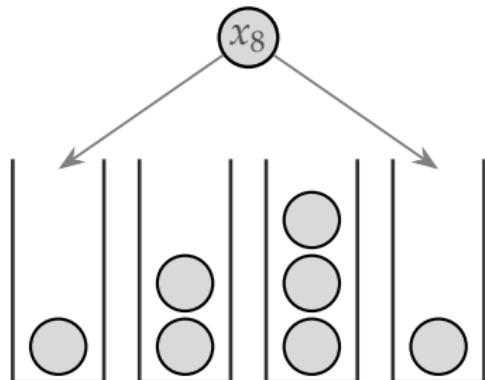
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

1. No recourse
2. Recourse

$$\text{Recourse} = 0$$

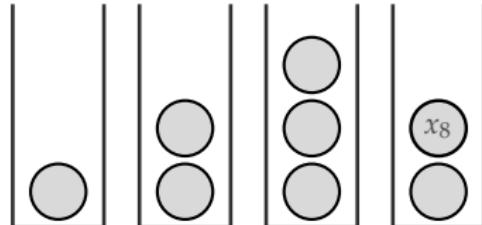
ANALYZING THE RECOURSE



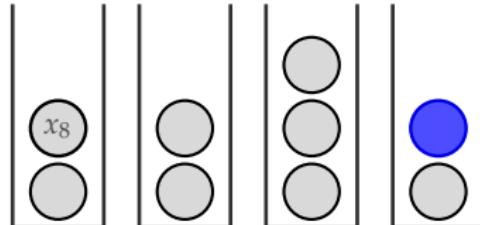
Subsequent balls will experience either:

1. No recourse
2. Recourse

ANALYZING THE RECOURSE



Computing A_S



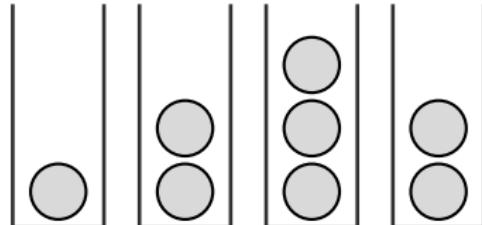
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

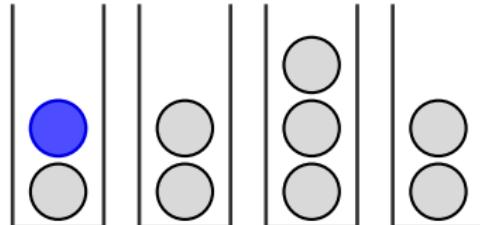
1. No recourse
2. Recourse

Recourse = 1

ANALYZING THE RECOURSE



Computing A_S



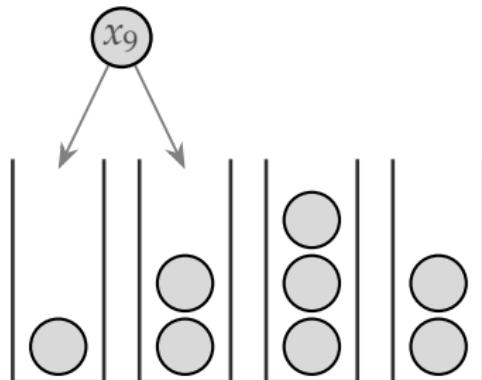
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

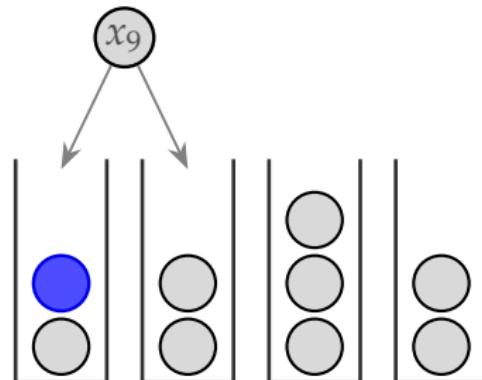
1. No recourse
2. Recourse

Recourse = 1

ANALYZING THE RECOURSE



Computing A_S



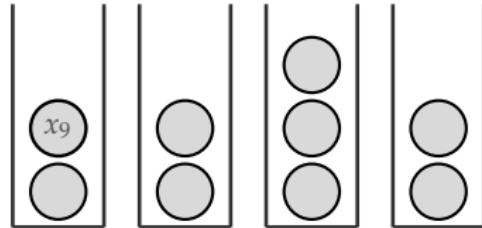
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

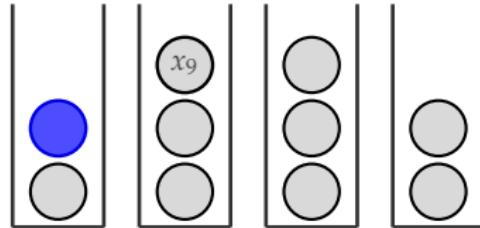
1. No recourse
2. Recourse

Recourse = 1

ANALYZING THE RECOURSE



Computing A_S



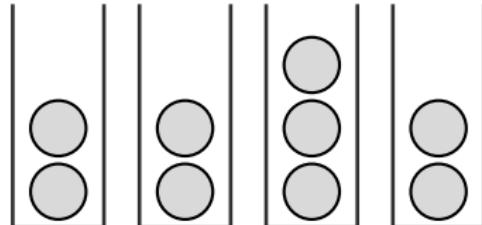
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

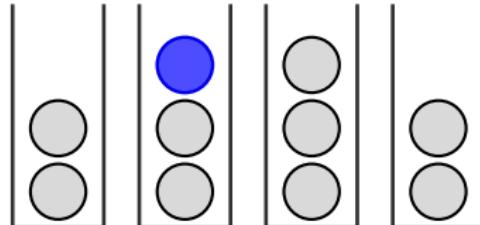
1. No recourse
2. Recourse

Recourse = 2

ANALYZING THE RECOURSE



Computing A_S



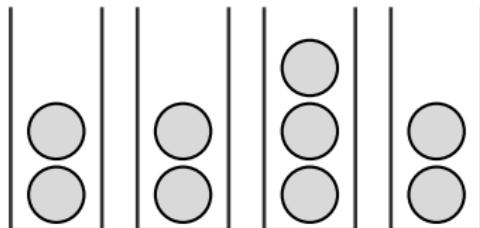
Computing $A_{S \cup \{x\}}$

Subsequent balls will experience either:

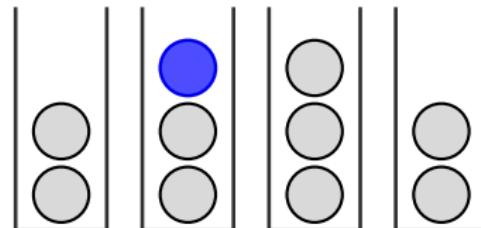
1. No recourse
2. Recourse

Recourse = 2

ANALYZING THE RECOURSE



Computing A_S

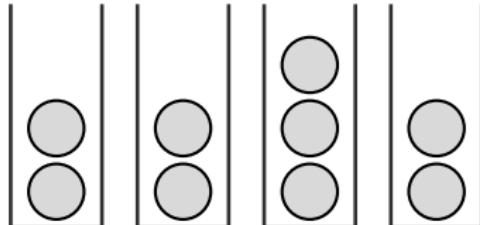


Computing $A_{S \cup \{x\}}$

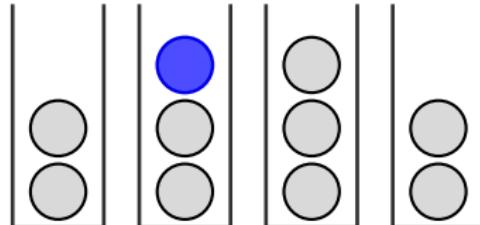
Two key observations:

Recourse = 2

ANALYZING THE RECOURSE



Computing A_S



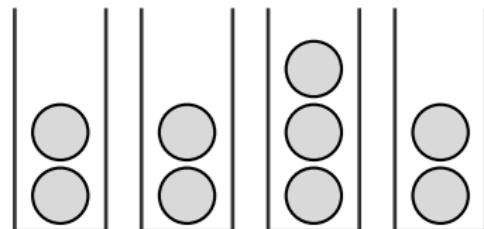
Computing $A_{S \cup \{x\}}$

Two key observations:

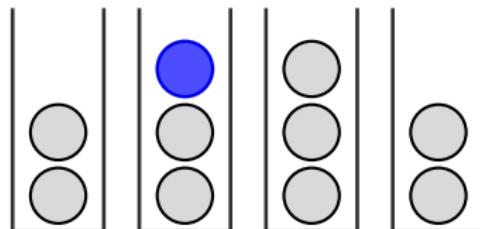
1. There's always one special bin with an extra ball

Recourse = 2

ANALYZING THE RECOURSE



Computing A_S



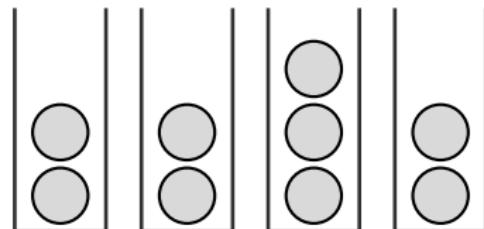
Computing $A_{S \cup \{x\}}$

Two key observations:

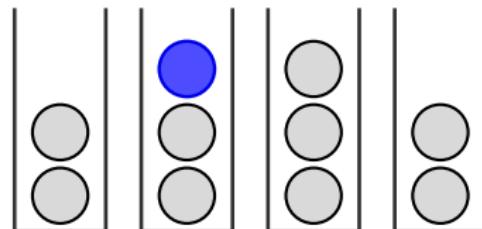
1. There's always one special bin with an extra ball
2. If a ball incurs recourse, one of its choices is the special bin

Recourse = 2

ANALYZING THE RECOURSE



Computing A_S

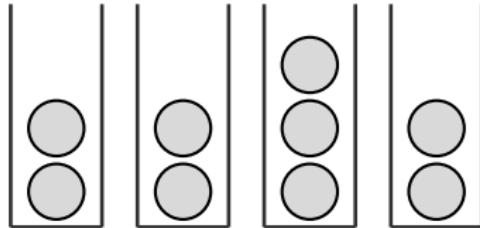


Computing $A_{S \cup \{x\}}$

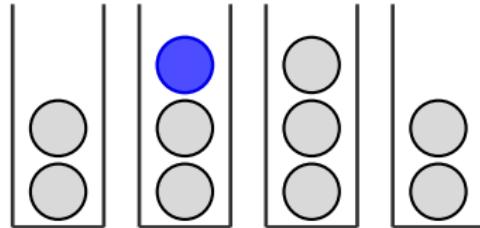
$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

Recourse = 2

ANALYZING THE RECOURSE



Computing A_S



Computing $A_{S \cup \{x\}}$

$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

$$\implies \mathbb{E}[\text{total recourse}] = \sum_i \Pr[\text{ball } x_i \text{ incurs recourse}] = O(m/n)$$

Recourse = 2

A SIMPLE WARMUP

Theorem: History-Independent Greedy achieves:

- ▶ High-probability overload $\Theta(1)$ $O(\log \log n)$.
- ▶ Expected recourse $\Theta(\log \log(m/n))$ $O(m/n)$.

A SIMPLE WARMUP

Theorem: History-Independent Greedy achieves:

- ▶ High-probability overload $\Theta(1)$ $O(\log \log n)$.
- ▶ Expected recourse $\Theta(\log \log(m/n))$ $O(m/n)$.



We prove a matching lower bound
when $m \leq n^{2-\Omega(1)}$

REST OF TALK

1. A Simple Warmup ✓
2. The Full Algorithm

Part 2: The Full Algorithm

BAKING A CAKE



Michael



Elaine



Bill

BAKING A CAKE

Before



Michael



Elaine



Bill

BAKING A CAKE

Before



**Slice off the
excess flour!**



Michael



Elaine



Bill

BAKING A CAKE

Before



**Slice off the
excess flour!**



Michael

Spread it evenly!



Elaine



Bill

BAKING A CAKE

Before



After



**Slice off the
excess flour!**



Michael

Spread it evenly!



Elaine



Bill

BAKING A CAKE

Before



After



Slice off the
excess flour!



Michael

Spread it evenly!



Elaine

You two rehearsed
this, didn't you?



Bill

BAKING A CAKE

Before



Michael



Elaine



Bill

BAKING A CAKE

Before



**Slice off the
excess frosting!**



Michael



Elaine



Bill

BAKING A CAKE

Before



**Slice off the
excess frosting!**



Michael

Spread it smooth!

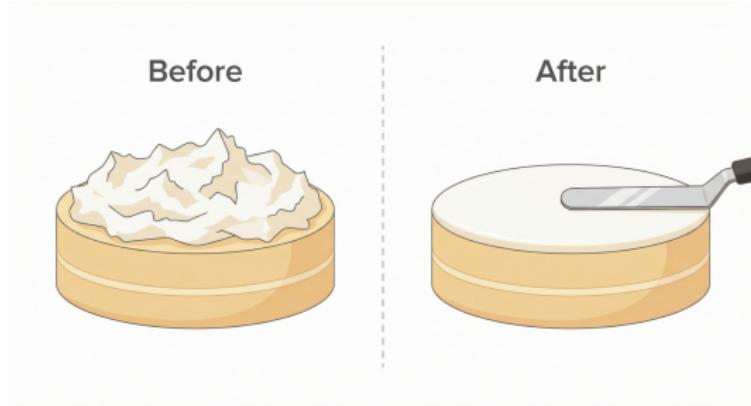


Elaine



Bill

BAKING A CAKE



**Slice off the
excess frosting!**



Michael

Spread it smooth!

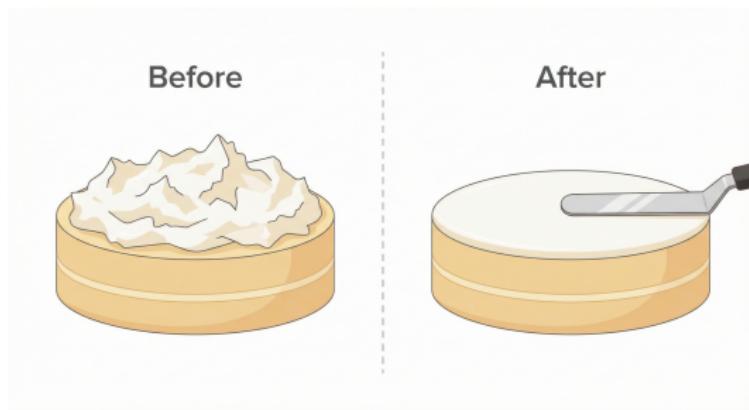


Elaine



Bill

BAKING A CAKE



Slice off the excess frosting!



Michael

Spread it smooth!



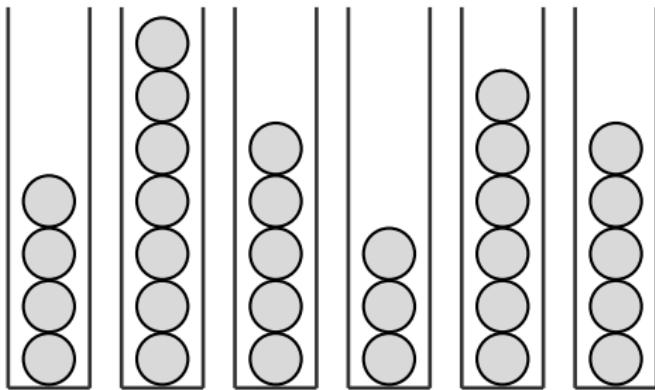
Elaine

I'm starting to think I'm the comic relief here...

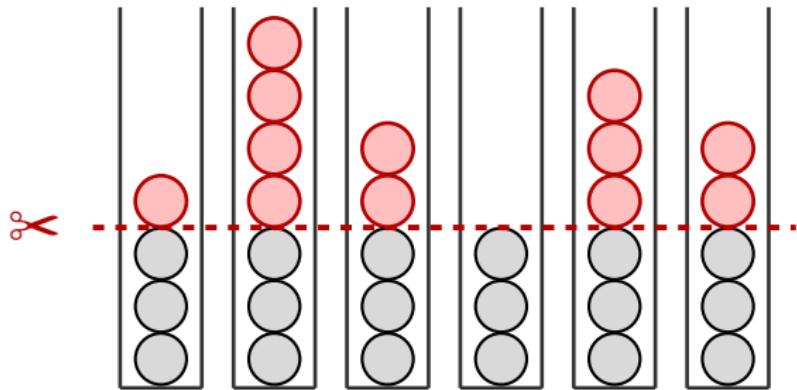


Bill

SLICE AND SPREAD

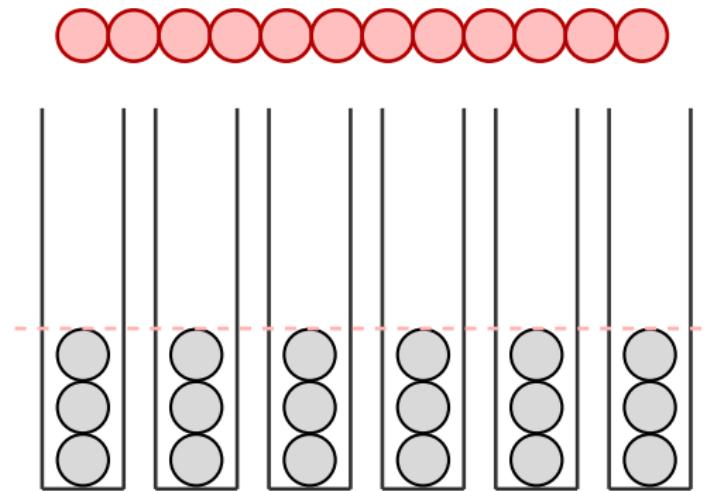


SLICE AND SPREAD



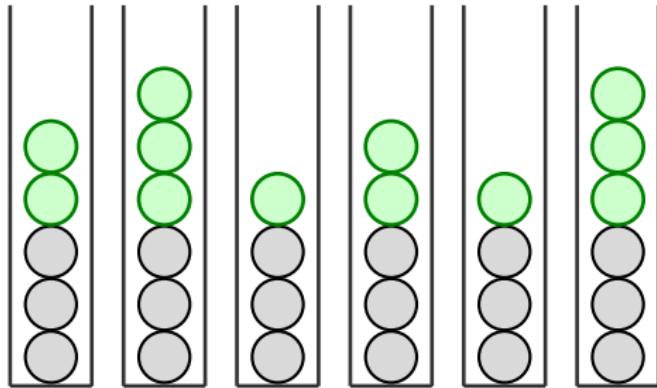
1. **Slice** off the jagged surface

SLICE AND SPREAD



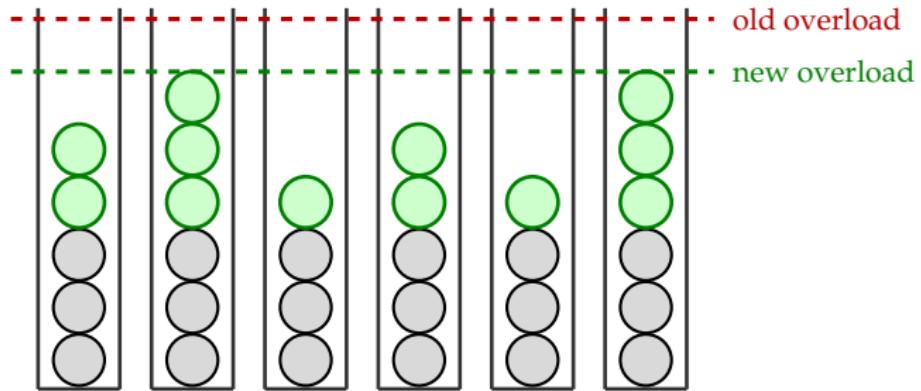
1. **Slice** off the jagged surface

SLICE AND SPREAD



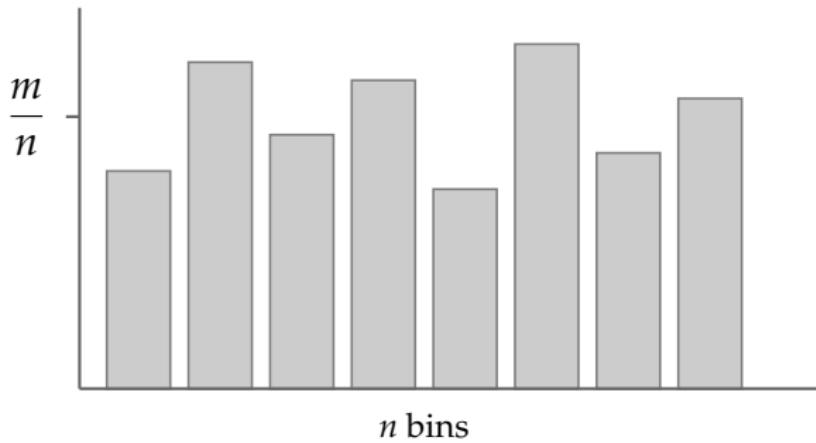
1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

SLICE AND SPREAD

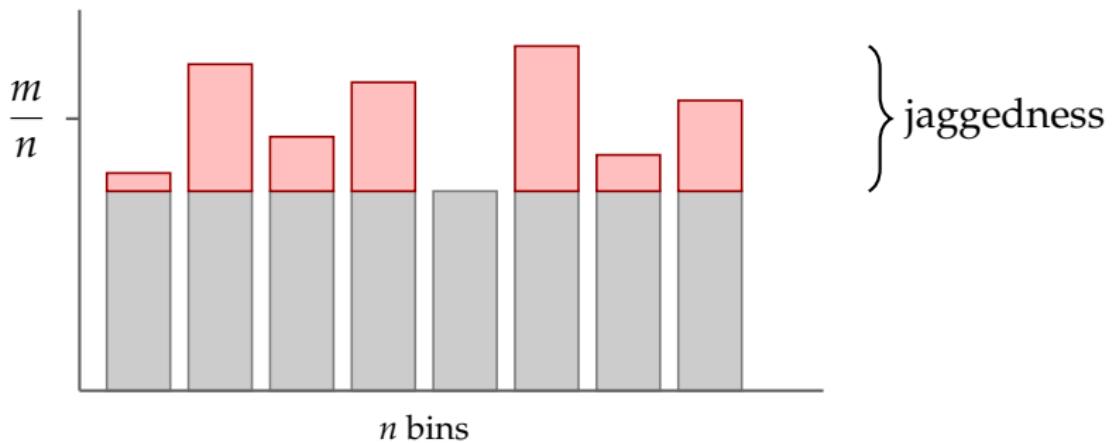


1. **Slice** off the jagged surface
2. **Spread** balls to their second-choice bins

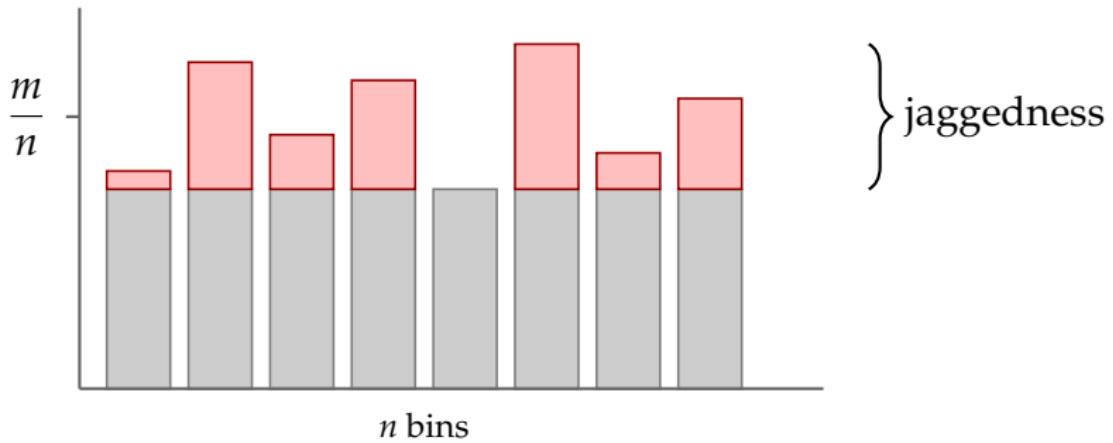
SLICE AND SPREAD REDUCES OVERLOAD



SLICE AND SPREAD REDUCES OVERLOAD

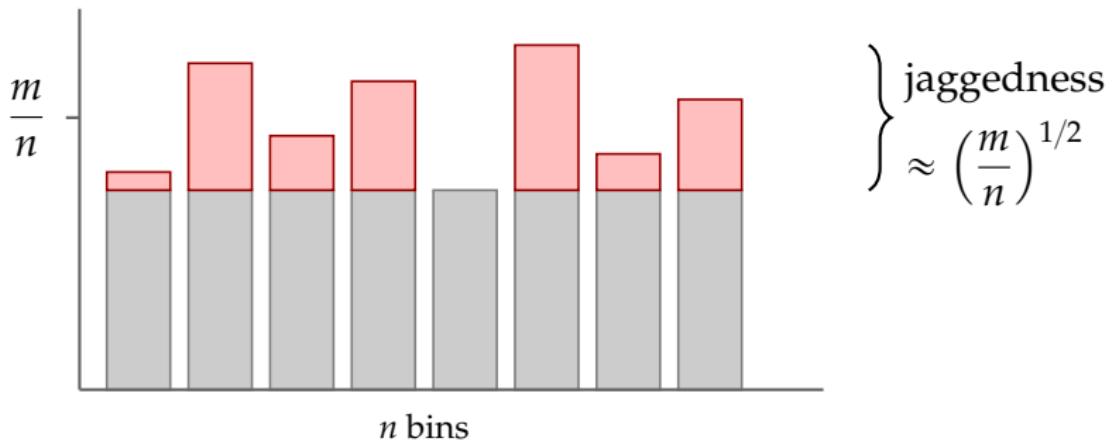


SLICE AND SPREAD REDUCES OVERLOAD



Q: How much is the jaggedness?

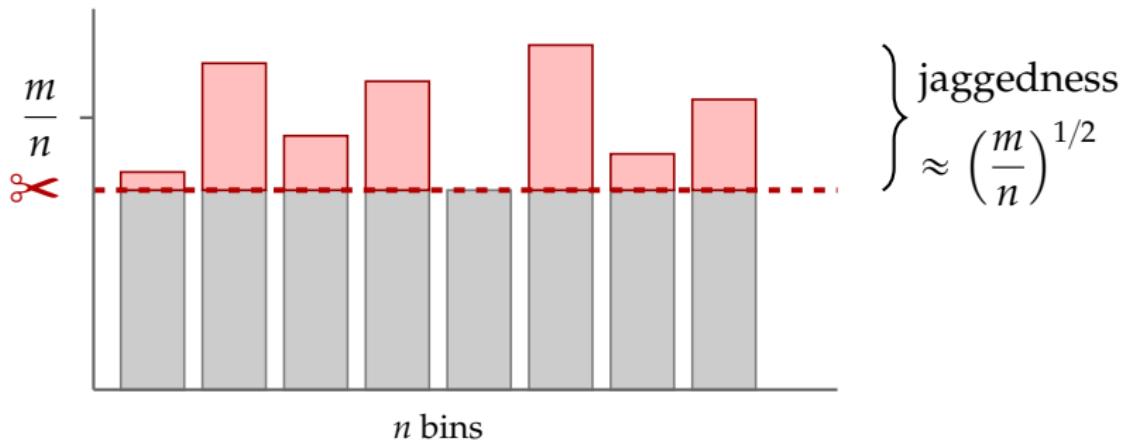
SLICE AND SPREAD REDUCES OVERLOAD



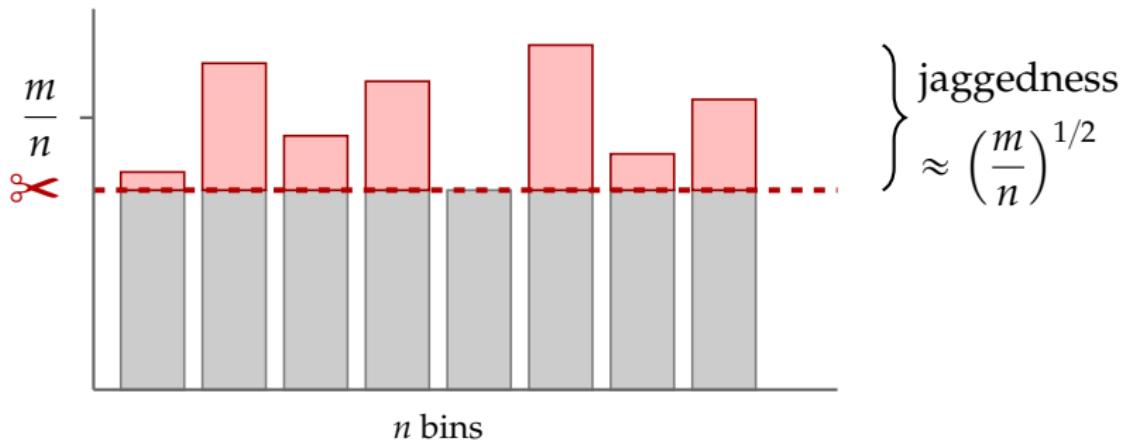
Q: How much is the jaggedness?

A: For $m \gg n$, the jaggedness is $\approx (m/n)^{1/2}$ (whp in n)

SLICE AND SPREAD REDUCES OVERLOAD

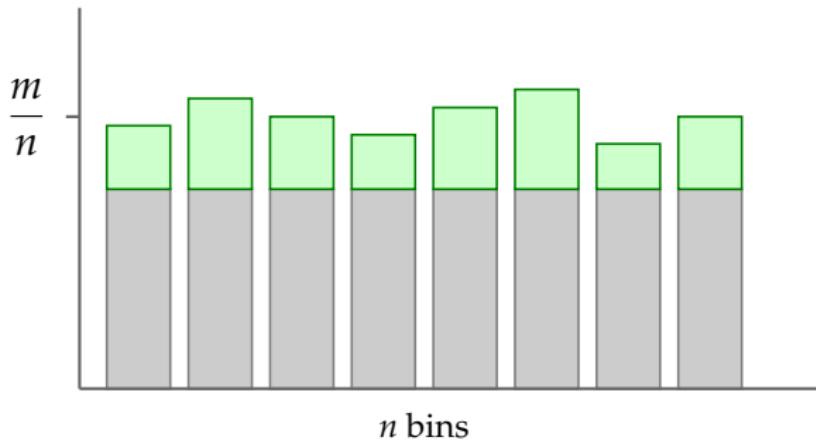


SLICE AND SPREAD REDUCES OVERLOAD



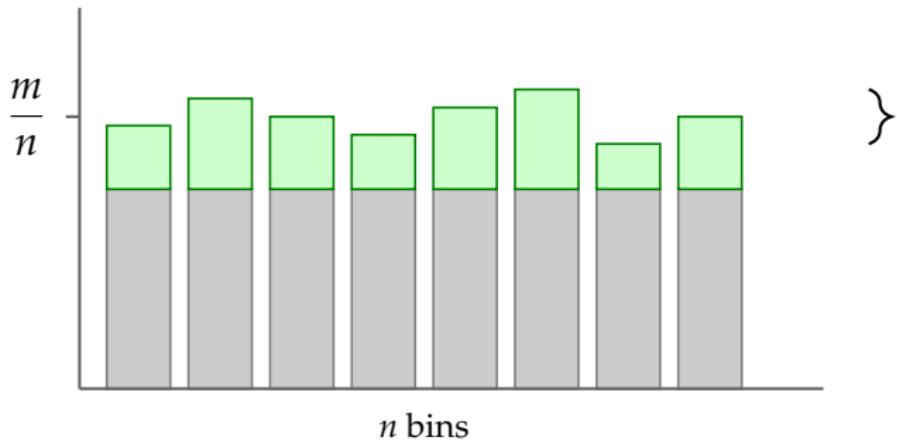
Balls sliced away: $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

SLICE AND SPREAD REDUCES OVERLOAD



Balls sliced away: $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

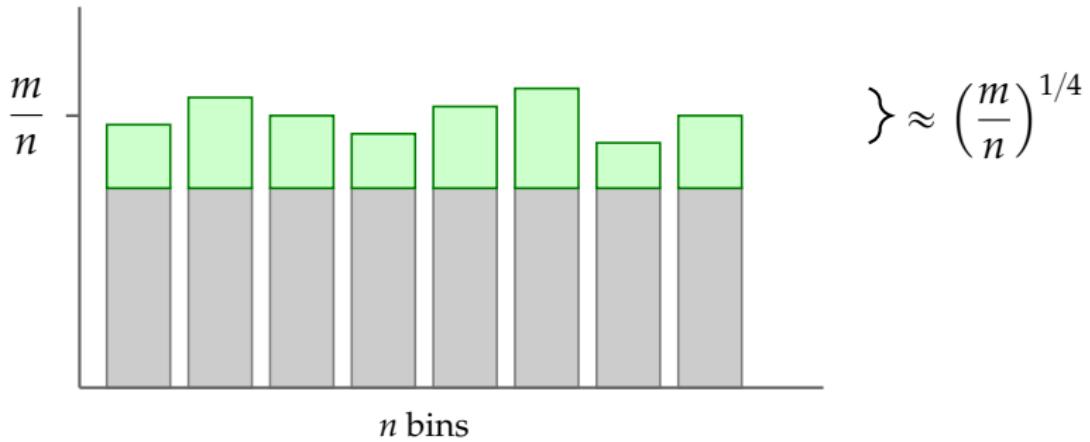
SLICE AND SPREAD REDUCES OVERLOAD



Balls sliced away: $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

Q: What is the new jaggedness?

SLICE AND SPREAD REDUCES OVERLOAD



Balls sliced away: $m' \approx n \cdot (m/n)^{1/2} = (mn)^{1/2}$

Q: What is the new jaggedness?

A: For $m' \gg n$, the new jaggedness is $(m'/n)^{1/2} = (m/n)^{1/4}$

SLICE AND SPREAD

- Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$

SLICE AND SPREAD

- ▶ Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent?

SLICE AND SPREAD

- ▶ Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!

SLICE AND SPREAD

- ▶ Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse:

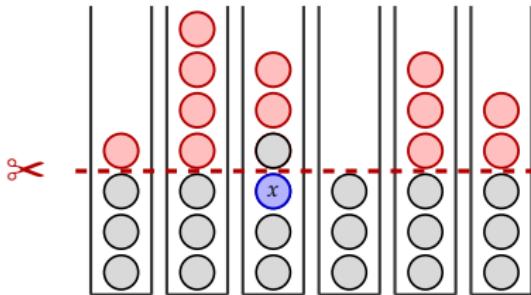
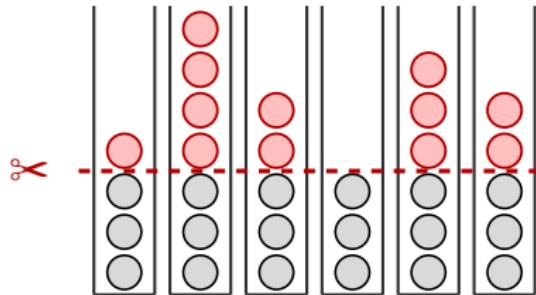
SLICE AND SPREAD

- ▶ Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse: 1

SLICE AND SPREAD

- ▶ Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse: 1

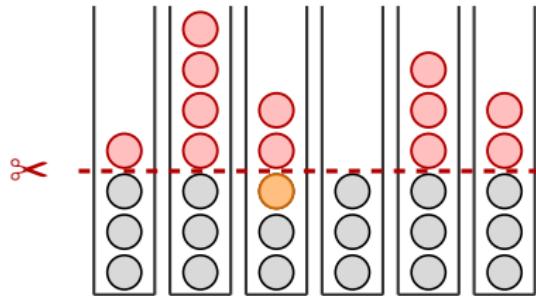
Example



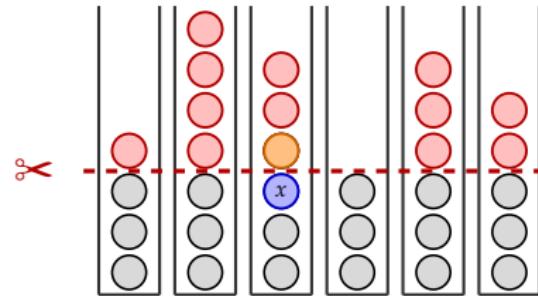
SLICE AND SPREAD

- ▶ Overload: $(m/n)^{1/2} \rightarrow (m/n)^{1/4}$
- ▶ History independent? Yes!
- ▶ Recourse: 1

Example

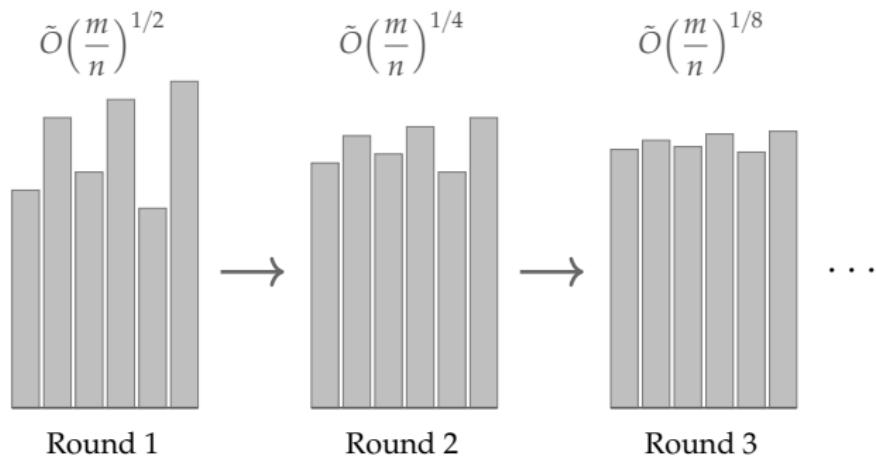


Computing A_S

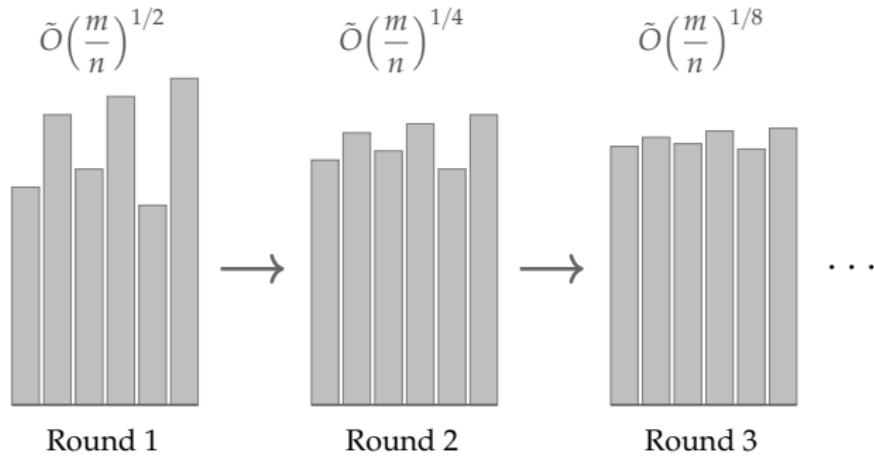


Computing $A_{S \cup \{x\}}$

REPEATEDLY SLICING AND SPREADING



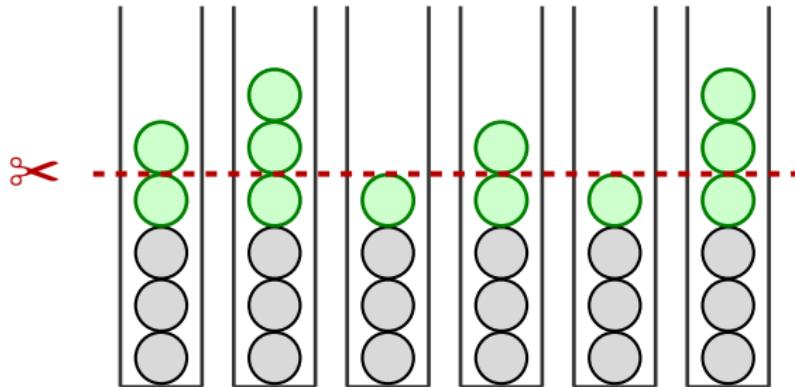
REPEATEDLY SLICING AND SPREADING



After $O(\log \log(m/n))$ rounds...

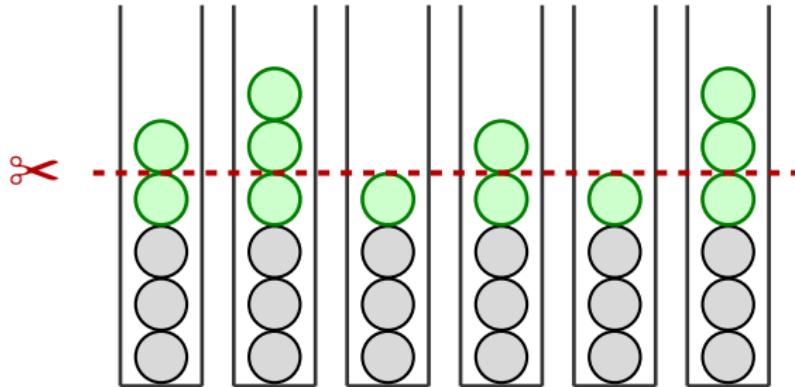
- ▶ Overload = $O(1)$?
- ▶ Recourse = $O(\log \log(m/n))$?

ALGORITHMIC QUESTION



Question: Which balls do we slice in round k ?

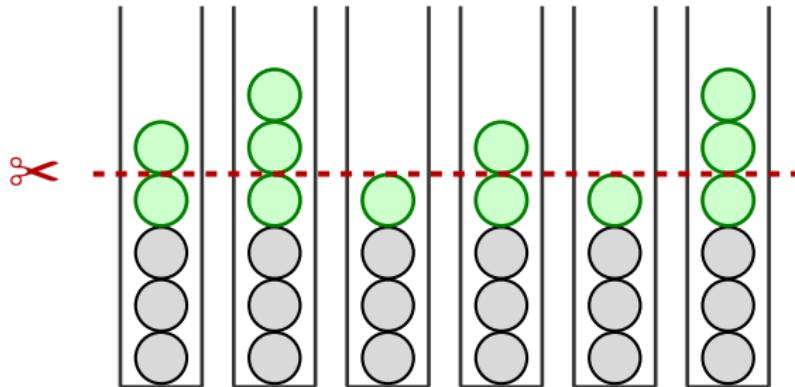
ALGORITHMIC QUESTION



Question: Which balls do we slice in round k ?

- ▶ **Option 1:** Scrape off the top

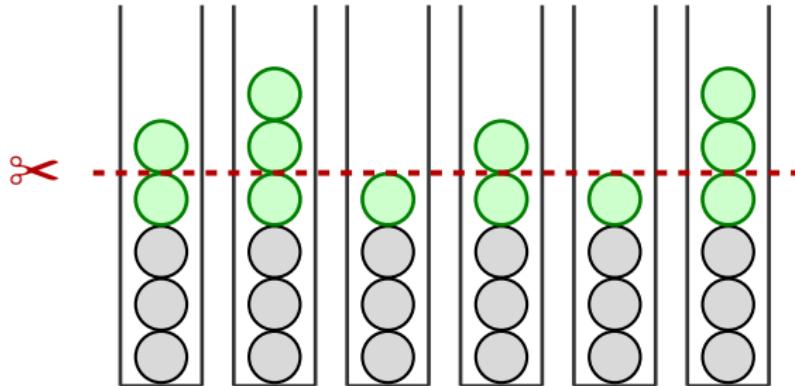
ALGORITHMIC QUESTION



Question: Which balls do we slice in round k ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness

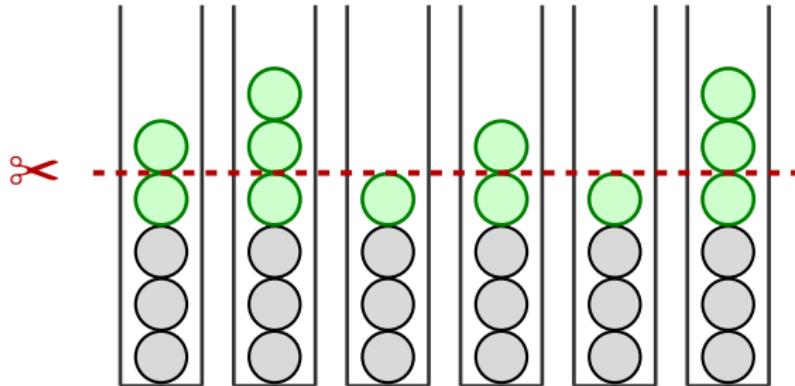
ALGORITHMIC QUESTION



Question: Which balls do we slice in round k ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness
- ▶ **Option 2:** Priority queue per bin

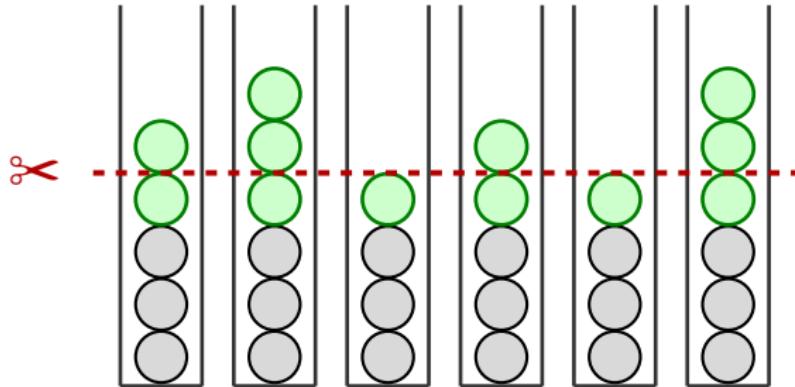
ALGORITHMIC QUESTION



Question: Which balls do we slice in round k ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness
- ▶ **Option 2:** Priority queue per bin ✗ Exploding recourse

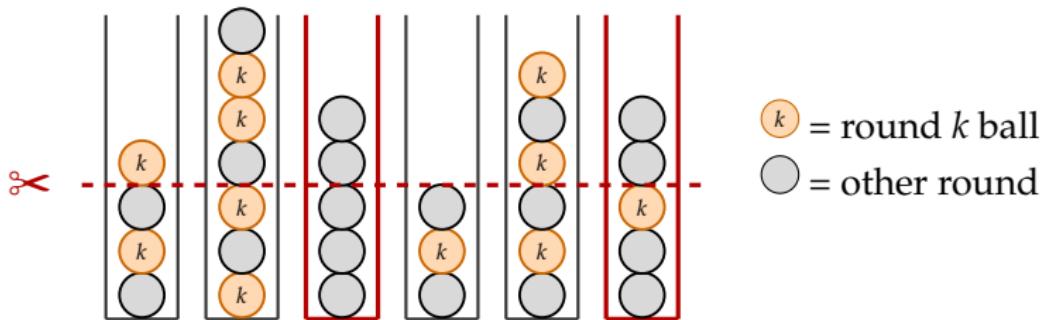
ALGORITHMIC QUESTION



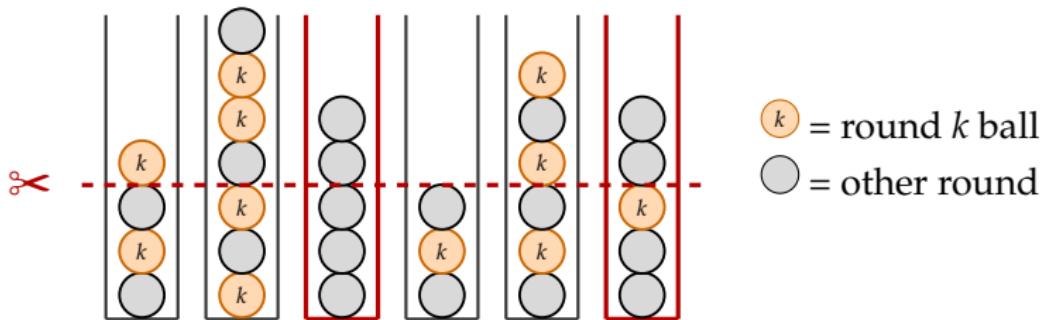
Question: Which balls do we slice in round k ?

- ▶ **Option 1:** Scrape off the top ✗ Reuses stale randomness
- ▶ **Option 2:** Priority queue per bin ✗ Exploding recourse
- ▶ **Our approach:** Assign every ball a random **round number**, only choose from the balls with round number k

CHALLENGE 1: SLICING FAILURES

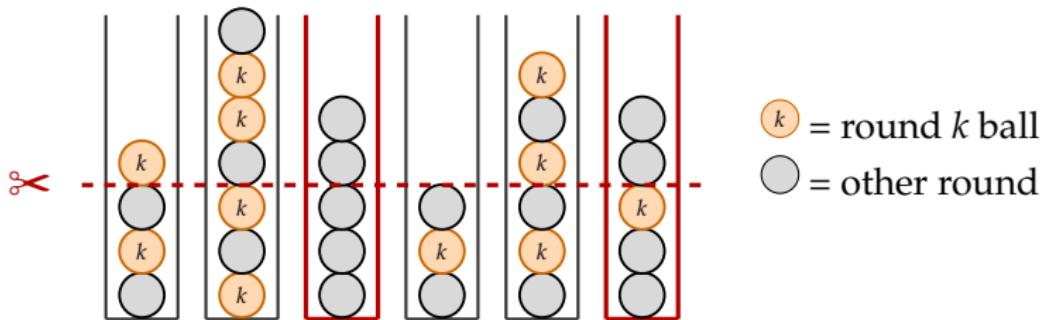


CHALLENGE 1: SLICING FAILURES



Challenge: Some bins may not have enough round- k balls to support slicing.

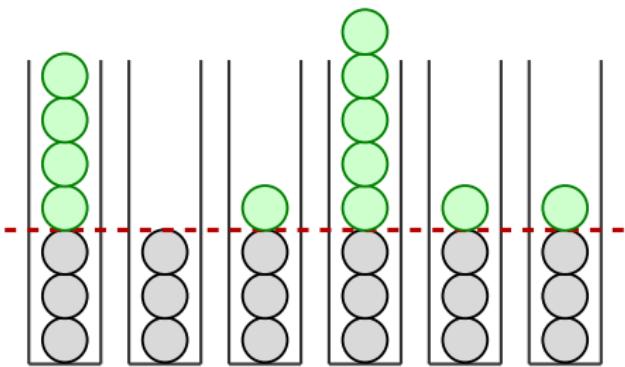
CHALLENGE 1: SLICING FAILURES



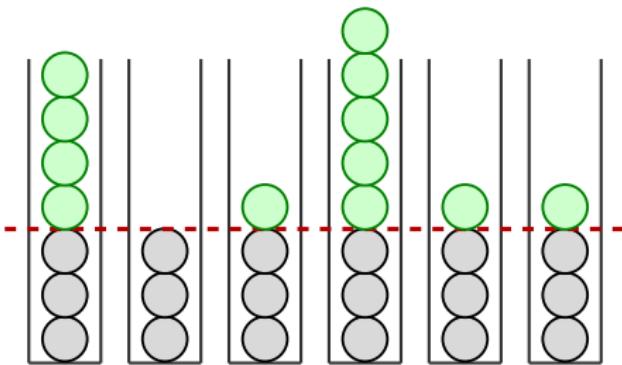
Challenge: Some bins may not have enough round- k balls to support slicing.

Result: We can't slice evenly — the jaggedness remains in those bins.

CHALLENGE 2: SPREADING FAILURES

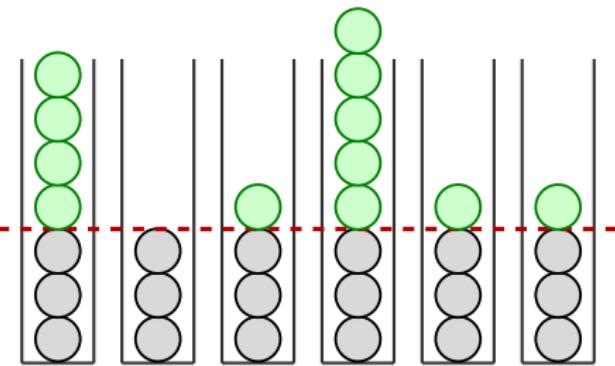


CHALLENGE 2: SPREADING FAILURES



Challenge: The spreading step may distribute balls unevenly — creating new jaggedness.

CHALLENGE 2: SPREADING FAILURES

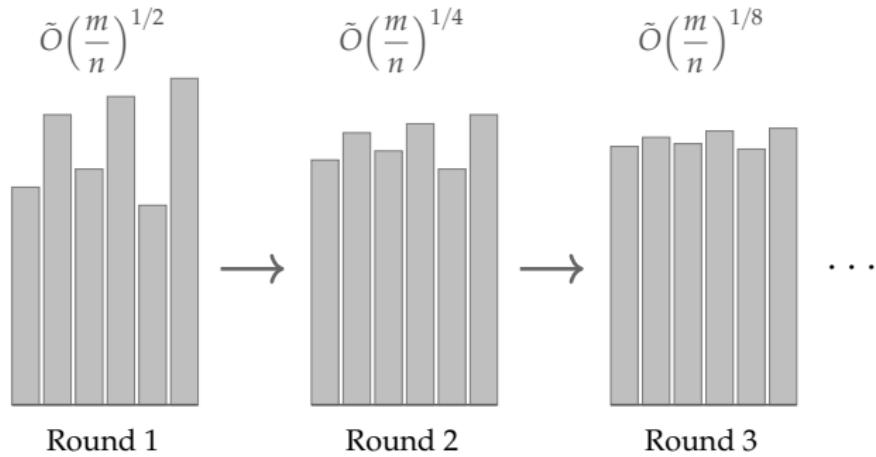


Challenge: The spreading step may distribute balls unevenly — creating new jaggedness.

Dilemma:

- ▶ Slice **more** balls \implies overload may not decrease
- ▶ Slice **fewer** balls \implies jaggedness may not decrease

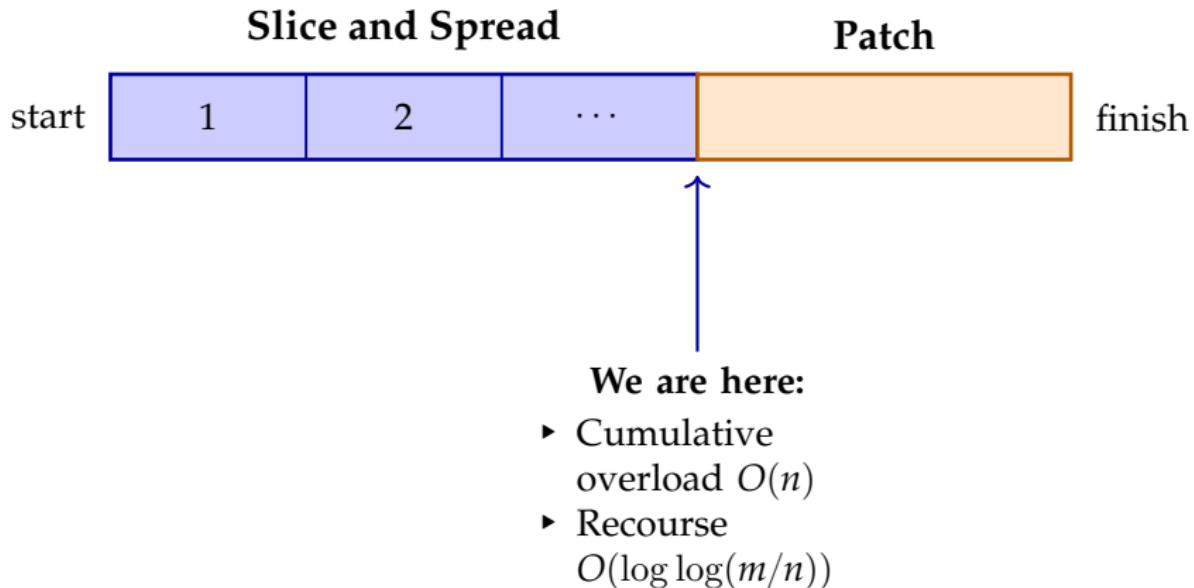
REPEATEDLY SLICING AND SPREADING



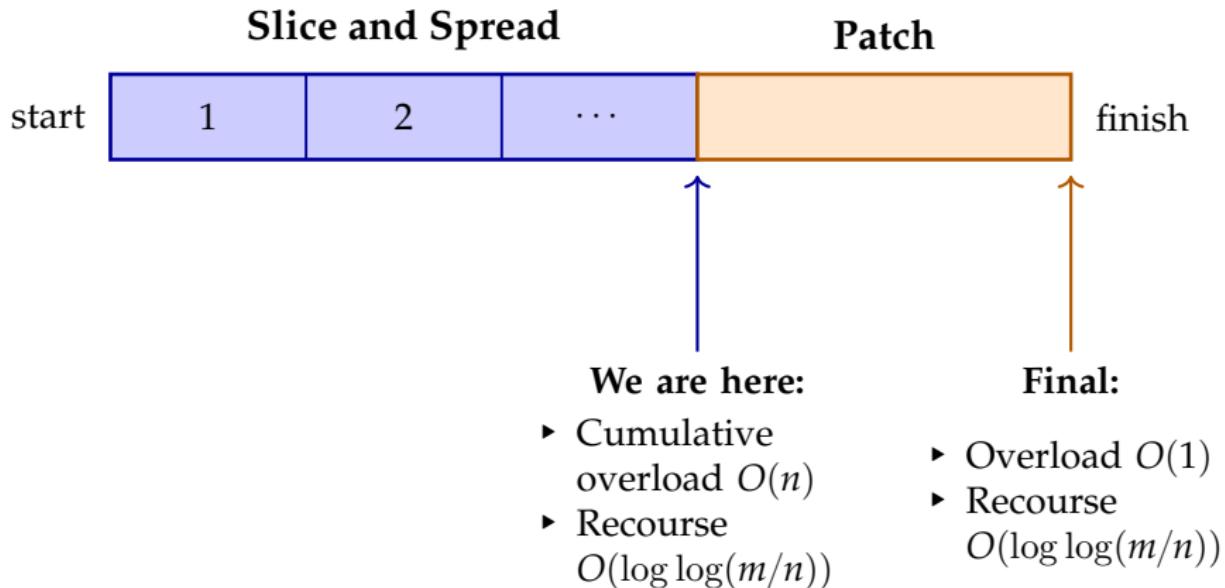
After $O(\log \log(m/n))$ rounds...

- ▶ Overload = $O(1)$ Cumulative overload = $O(n)$
- ▶ Recourse = $O(\log \log(m/n))$

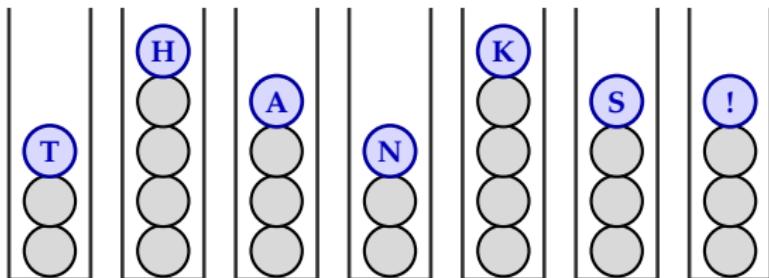
FULL ALGORITHM



FULL ALGORITHM



History-Independent Load Balancing



Michael A. Bender
Stony Brook University

William Kuszmaul
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Elaine Shi
CMU

Rose Silver
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