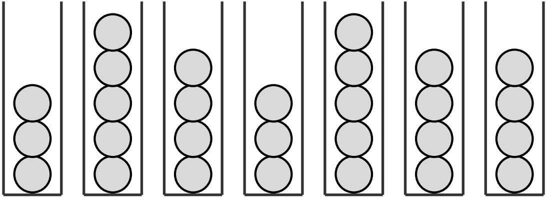
History-Independent Load Balancing



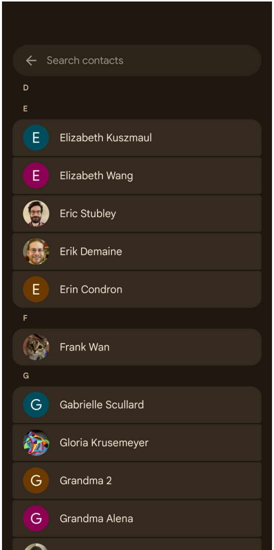
Michael A. Bender — Stony Brook University
Bill Kuszmaul — CMU
Elaine Shi — CMU
Rose Silver — CMU

History Independent Data Structures

History Independence (Micciancio '97, Naor & Teague '01)

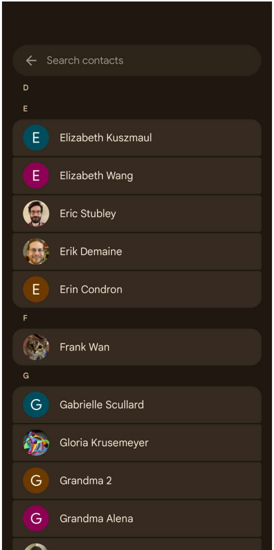
> *"If an adversary were to see the state of the data structure, they would learn only the current set of elements, and nothing else about the history of past operations."*

History vs Content



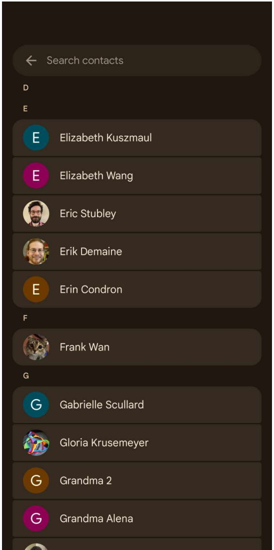
- If someone hacks my phone, they can learn my contacts list.

History vs Content



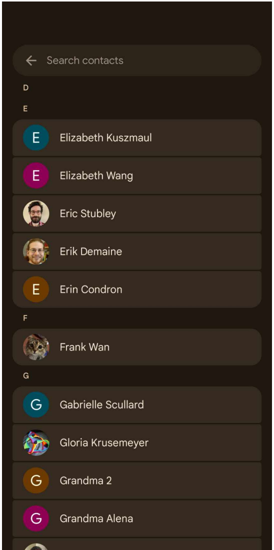
- If someone hacks my phone, they can learn my contacts list.
- But can they learn who my contacts were in the past?

History vs Content



- If someone hacks my phone, they can learn my contacts list.
- But can they learn who my contacts were in the past?
- What about the order in which contacts were added?

History vs Content



- If someone hacks my phone, they can learn my contacts list.
- But can they learn who my contacts were in the past?
- What about the order in which contacts were added?
- A history independent data structure protects this kind of information.

History Independence is a Security Guarantee

History Independence (Micciancio '97, Naor & Teague '01)

> *"If an adversary were to see the state of the data structure, they would learn only the current set of elements, and nothing else about the history of past operations."*

History Independence is a Security Guarantee

History Independence (Micciancio '97, Naor & Teague '01)

"If an adversary were to see the state of the data structure, they would learn only the current set of elements, and nothing else about the history of past operations."

A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

Micciancio '97, Naor & Teague '01, Buchbinder & Petrank '03, Molnar et al. '06, Blelloch & Golovin '07, Moran et al. '07, Naor et al. '08, Golovin '08–'10, Bajaj & Sion '13, Roche et al. '15, Bender et al. '16

History Independence is a Security Guarantee

History Independence (Micciancio '97, Naor & Teague '01)

"If an adversary were to see the state of the data structure, they would learn only the current set of elements, and nothing else about the history of past operations."

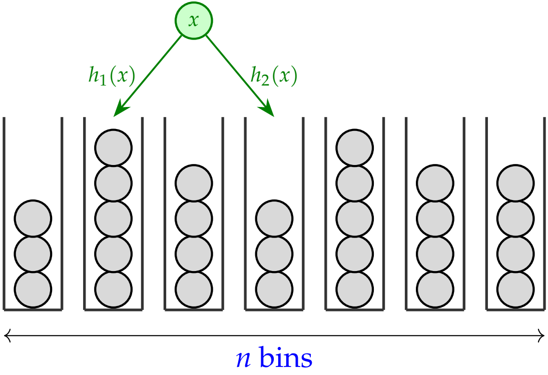
A History of Applications

Hash tables, trees, memory allocation, PMAs, graph algorithms, cache-oblivious data structures, and more.

Micciancio '97, Naor & Teague '01, Buchbinder & Petrank '03, Molnar et al. '06, Blelloch & Golovin '07, Moran et al. '07, Naor et al. '08, Golovin '08–'10, Bajaj & Sion '13, Roche et al. '15, Bender et al. '16

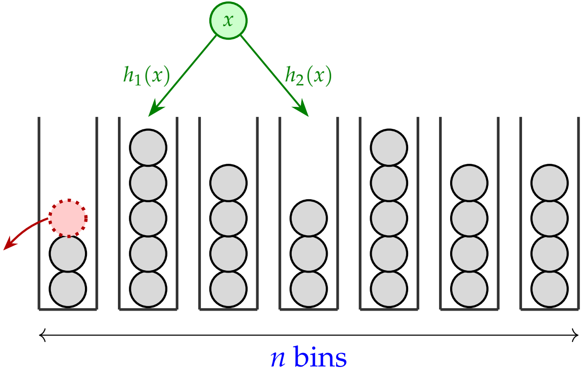
Yet fundamental questions remain open.

Two-Choice Load Balancing



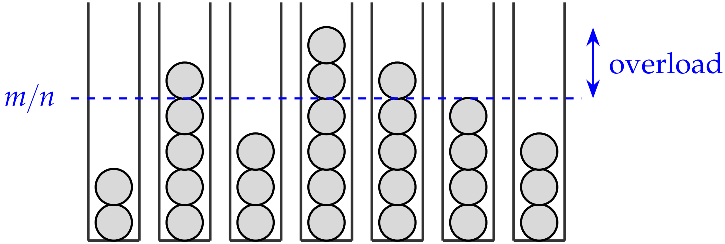
- Balls are inserted/deleted, with up to m present at a time.
- Each ball has two random bins where it can go.
- We must maintain a valid assignment of balls to bins.

Two-Choice Load Balancing



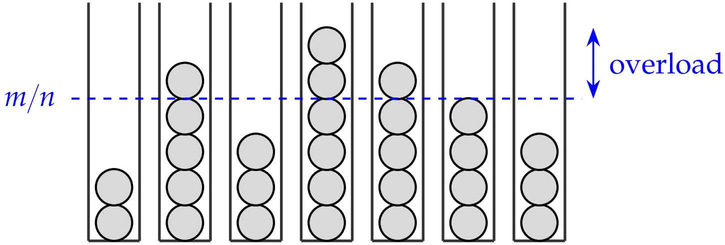
- Balls are inserted/deleted, with up to m present at a time.
- Each ball has two random bins where it can go.
- We must maintain a valid assignment of balls to bins.

Two Goals



Minimize Overload:
The amount by which the fullest bin exceeds m/n is small.

Two Goals



Minimize Overload:
The amount by which the fullest bin exceeds m/n is small.

Minimize Recourse:
On any given insertion/deletion, the number of balls moved around is small.

This Paper

Question: Does there exist a history-independent solution with small recourse and overload?

This Paper

Question: Does there exist a history-independent solution with small recourse and overload?

Theorem: There exists a history-independent solution with:

- Overload $O(1)$, with high probability.
- Expected recourse $O(\log \log(m/n))$.

What About Non-History-Independent Solutions?

What About Non-History-Independent Solutions?

Lots of work on the insertion-only case.
[Azar, Broder, Karlin and Upfal '94] [Berenbrink, Czumaj, Steger, and Vöcking '00][Dietzfelbinger and Weidling '07] [Frieze and Petti '18] . . .

What About Non-History-Independent Solutions?

Lots of work on the insertion-only case.
[Azar, Broder, Karlin and Upfal '94] [Berenbrink, Czumaj, Steger, and Vöcking '00][Dietzfelbinger and Weidling '07] [Frieze and Petti '18] ...

But the fully dynamic case has remained largely open.
[Vöcking '99] [Dietzfelbinger and Weidling '07] [Bender, Conway, Farach-Colton, Kuszmaul, Tagliavini '21] [Bansal, Kuszmaul '22] ...

What About Non-History-Independent Solutions?

Lots of work on the insertion-only case.
[Azar, Broder, Karlin and Upfal '94] [Berenbrink, Czumaj, Steger, and Vöcking '00][Dietzfelbinger and Weidling '07] [Frieze and Petti '18] . . .

But the fully dynamic case has remained largely open.
[Vöcking '99] [Dietzfelbinger and Weidling '07] [Bender, Conway, Farach-Colton, Kuszmaul, Tagliavini '21] [Bansal, Kuszmaul '22] . . .

Open Question:
Is there a fully dynamic solution with recourse $o(m/n)$ and overload $O(\log \log n)$?

What About Non-History-Independent Solutions?

Lots of work on the insertion-only case.
[Azar, Broder, Karlin and Upfal '94] [Berenbrink, Czumaj, Steger, and Vöcking '00][Dietzfelbinger and Weidling '07] [Frieze and Petti '18] ...

But the fully dynamic case has remained largely open.
[Vöcking '99] [Dietzfelbinger and Weidling '07] [Bender, Conway, Farach-Colton, Kuszmaul, Tagliavini '21] [Bansal, Kuszmaul '22] ...

Open Question:
Is there a fully dynamic solution with recourse $o(m/n)$ and overload $O(\log \log n)$?

Answer:
Yes! We get recourse $O(\log \log (m/n))$ and overload $O(1)$.

This Paper

Question: Does there exist a history-independent solution with small recourse and overload?

Theorem: There exists a history-independent solution with:

- Overload $O(1)$, with high probability.
- Expected recourse $O(\log \log(m/n))$.

This Paper

Question: Does there exist a history-independent solution with small recourse and overload?

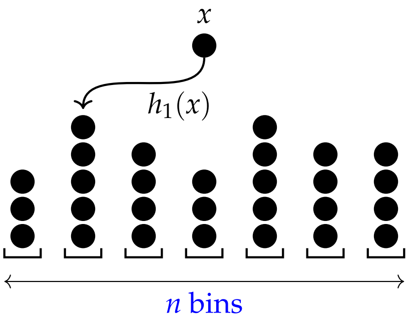
Theorem: There exists a history-independent solution with:

- Overload $O(1)$, with high probability.
- Expected recourse $O(\log\log(m/n))$.

Rest of Talk: A simple history-independent algorithm with overload $O(\log\log n)$ and expected recourse $O(m/n)$.

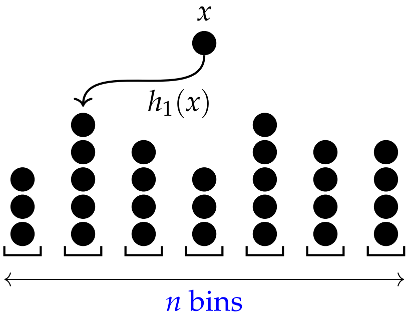
Warmup 1: The Single-Choice Strategy

To insert a ball x, just put it in bin $h_1(x)$:



WARMUP 1: THE SINGLE-CHOICE STRATEGY

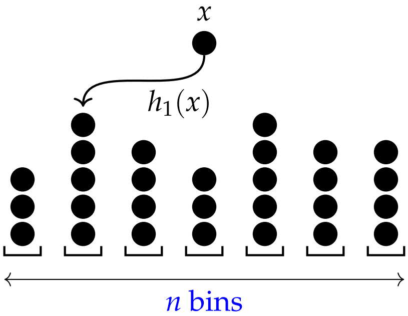
To insert a ball x, just put it in bin $h_1(x)$:



- This is history-independent ✓

Warmup 1: The Single-Choice Strategy

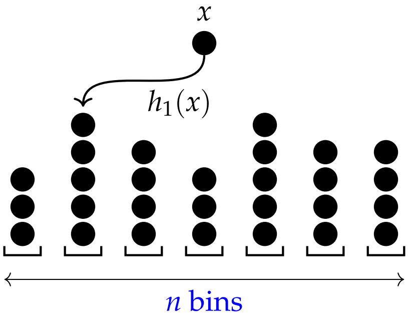
To insert a ball x, just put it in bin $h_1(x)$:



- This is history-independent ✓
- The recourse is 0 ✓

Warmup 1: The Single-Choice Strategy

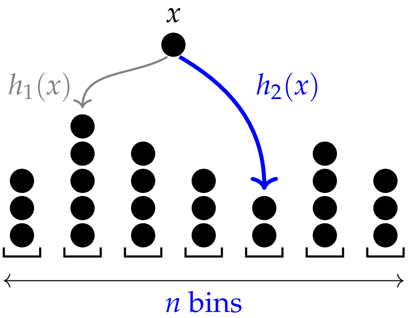
To insert a ball x, just put it in bin $h_1(x)$:



- This is history-independent ✓
- The recourse is 0 ✓
- But... the overload is huge, roughly $\sqrt{m/n}$ ✗

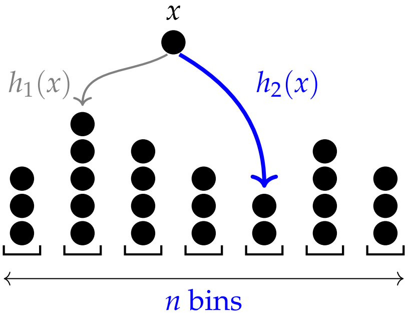
Warmup 2: Greedy Insertions

To insert a ball x, put it in the emptier of its choices:



Warmup 2: Greedy Insertions

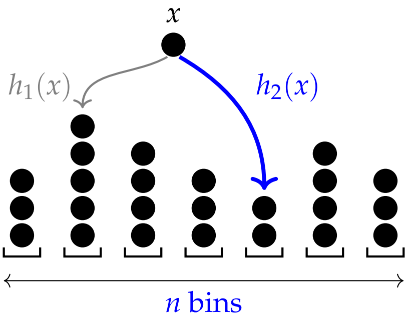
To insert a ball x, put it in the emptier of its choices:



- This is **not** history-independent ✗

WARMUP 2: GREEDY INSERTIONS

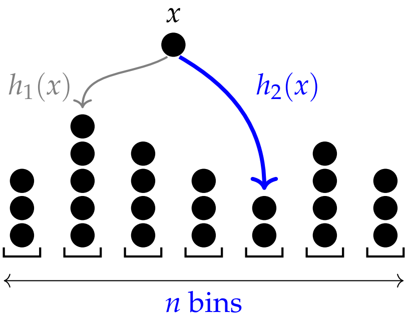
To insert a ball x, put it in the emptier of its choices:



- This is **not** history-independent ✗
- The recourse is 0 ✓

Warmup 2: Greedy Insertions

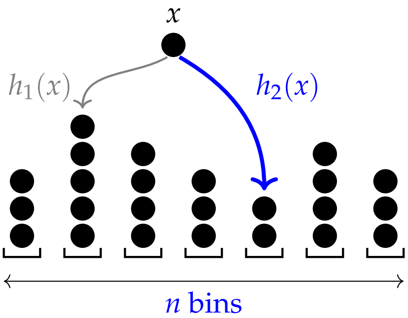
To insert a ball x, put it in the emptier of its choices:



- This is **not** history-independent ✗
- The recourse is 0 ✓
- In the insertion-only case, the overload is $O(\log \log n)$ ✓

[Azar, Broder, Karlin and Upfal '94]

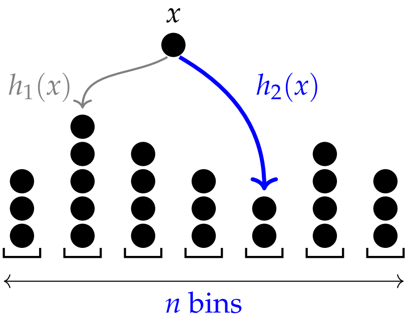
A Simple History-Independent Algorithm



Given a set S of balls, define `Greedy`(S) as:

- Start with empty bins.
- Sort the balls in S to get a sequence $x_1, x_2, \ldots$.
- Insert $x_1, x_2, \ldots$ using the greedy algorithm.

A Simple History-Independent Algorithm

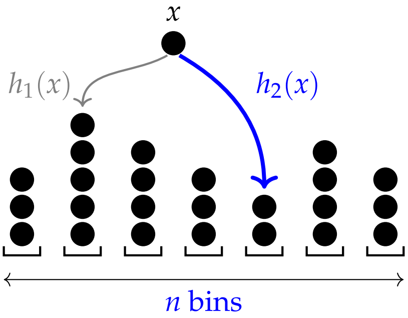


Given a set S of balls, define $\texttt{Greedy}(S)$ as:

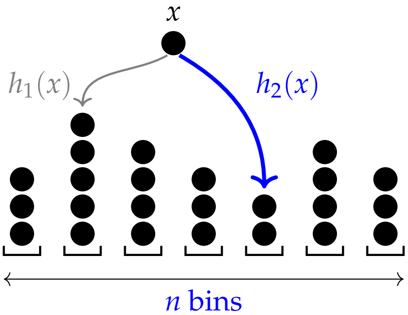
- Start with empty bins.
- Sort the balls in S to get a sequence $x_1, x_2, \ldots$.
- Insert $x_1, x_2, \ldots$ using the greedy algorithm.

A History-Independent Algorithm: At any given moment, if S is the current set of balls, use allocation $\texttt{Greedy}(S)$.

Analyzing History-Independent Greedy

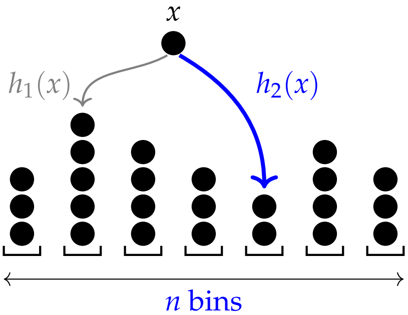


Analyzing History-Independent Greedy



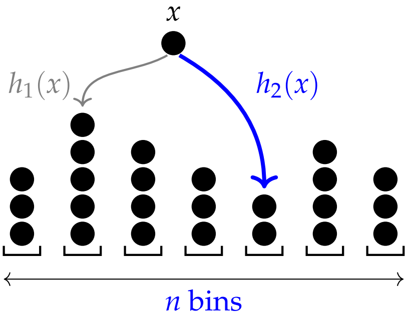
- The algorithm is history independent ✓

Analyzing History-Independent Greedy



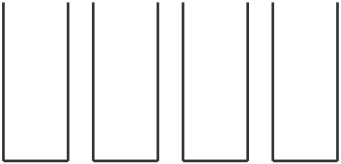
- The algorithm is history independent ✓
- The overload is $O(\log \log n)$ ✓

Analyzing History-Independent Greedy

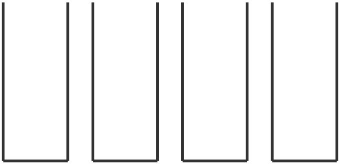


- The algorithm is history independent ✓
- The overload is $O(\log \log n)$ ✓
- But what about the recourse?

ANALYZING THE RECOURSE



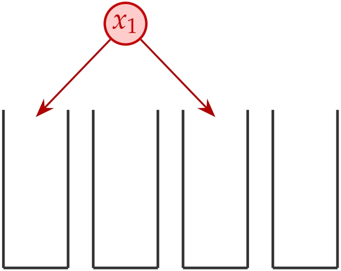
Computing `Greedy`(S)



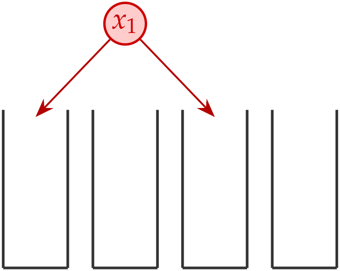
Computing `Greedy`$(S \cup \{x^*\})$

How does `Greedy`(S) change if we add a ball $x^\star$?

Analyzing the Recourse

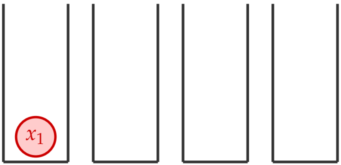


Computing $\texttt{\textbf{Greedy}}(S)$

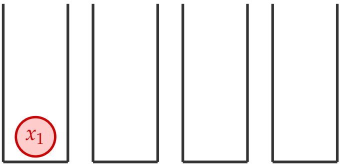


Computing $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

Analyzing the Recourse

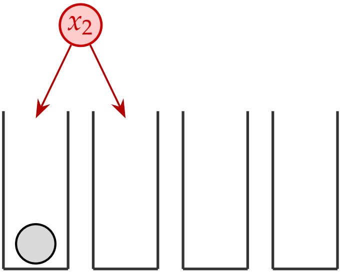


Computing **`Greedy`**(S)

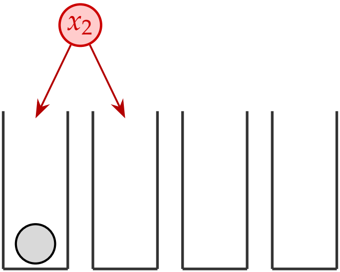


Computing **`Greedy`**$(S \cup \{x^*\})$

Analyzing the Recourse

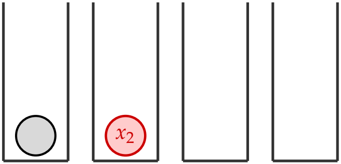


Computing `Greedy`(S)

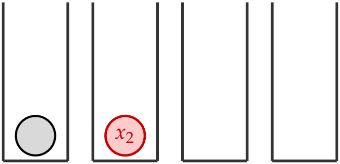


Computing `Greedy`$(S \cup \{x^*\})$

Analyzing the Recourse

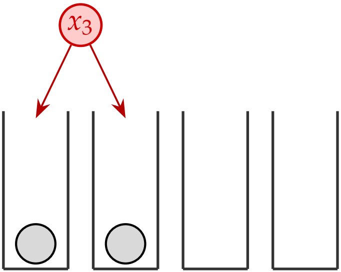


Computing `Greedy`(S)

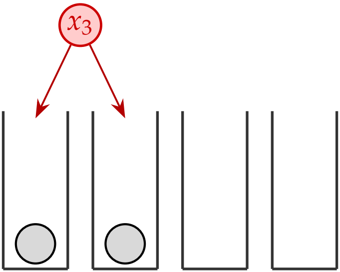


Computing `Greedy`$(S \cup \{x^*\})$

Analyzing the Recourse

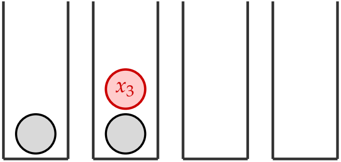


Computing $\texttt{Greedy}(S)$

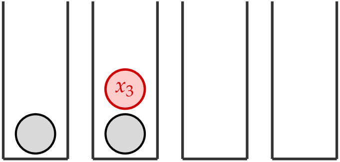


Computing $\texttt{Greedy}(S \cup \{x^*\})$

Analyzing the Recourse

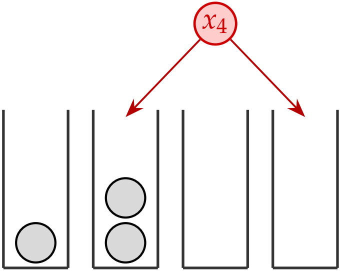


Computing $\texttt{\textbf{Greedy}}(S)$

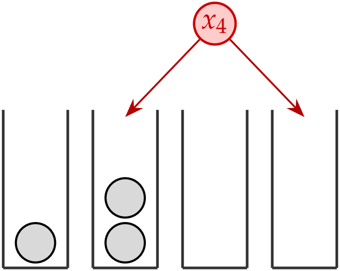


Computing $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

Analyzing the Recourse

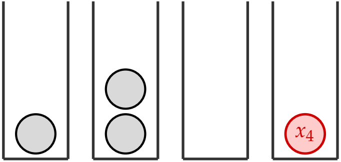


Computing $\texttt{\textbf{Greedy}}(S)$

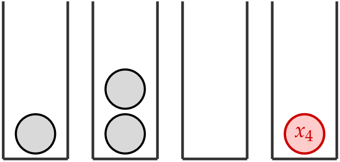


Computing $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

Analyzing the Recourse

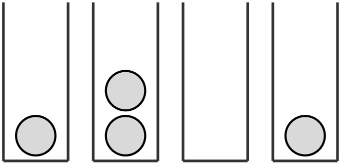


Computing $\texttt{\textbf{Greedy}}(S)$

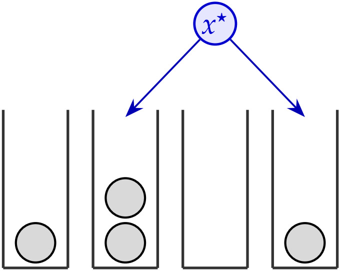


Computing $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

ANALYZING THE RECOURSE

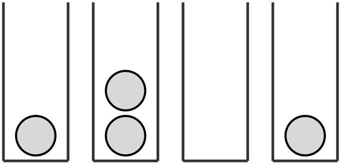


Computing `Greedy`(S)

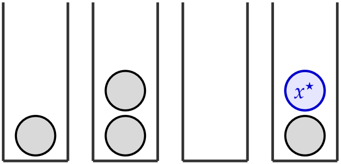


Computing `Greedy`$(S \cup \{x^*\})$

Analyzing the Recourse

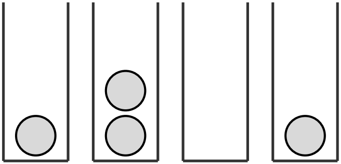


Computing `Greedy`(S)

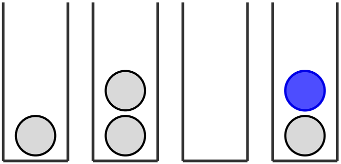


Computing `Greedy`$(S \cup \{x^*\})$

Analyzing the Recourse

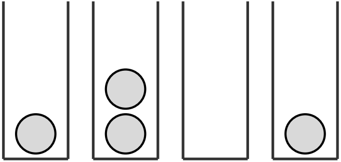


Computing **`Greedy`**(S)

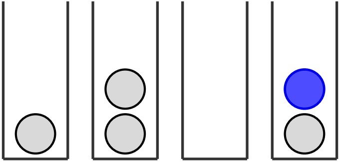


Computing **`Greedy`**$(S \cup \{x^*\})$

Analyzing the Recourse



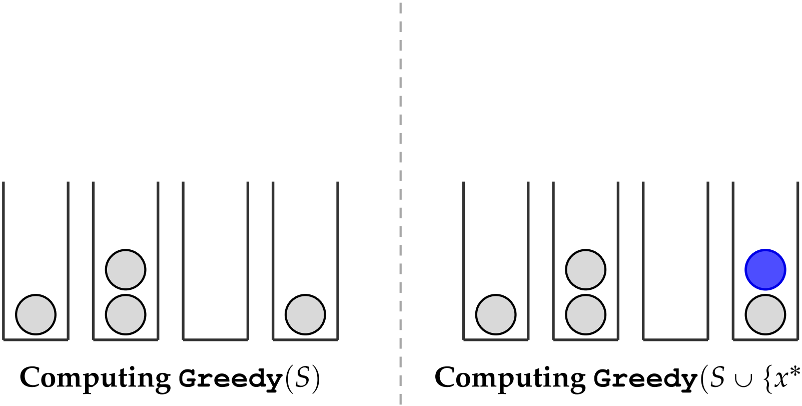
Computing $\texttt{Greedy}(S)$



Computing $\texttt{Greedy}(S \cup \{x^*\})$

Subsequent balls will experience either:

ANALYZING THE RECOURSE

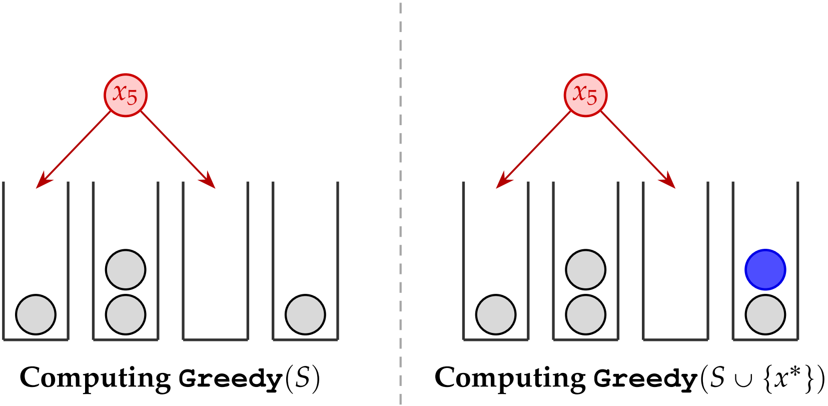


Computing `Greedy`(S) — **Computing `Greedy`**$(S \cup \{x^*\})$

Subsequent balls will experience either:

1. No recourse

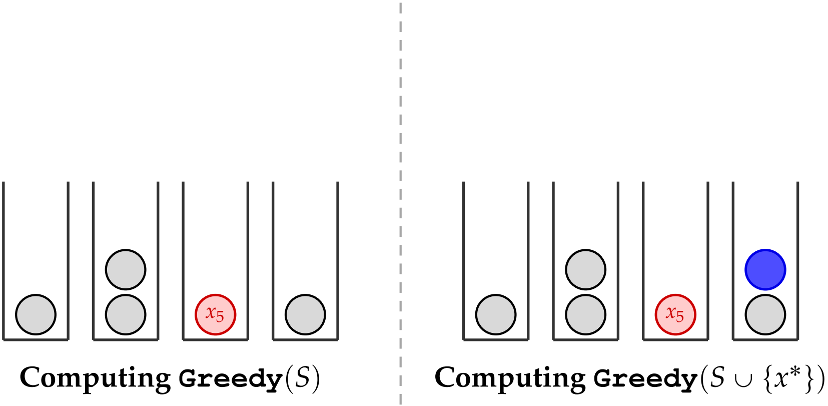
Analyzing the Recourse



Future insertions will experience either:

1. No recourse

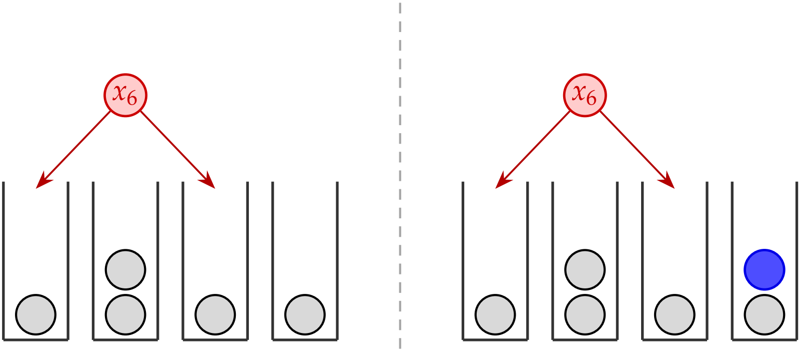
Analyzing the Recourse



Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

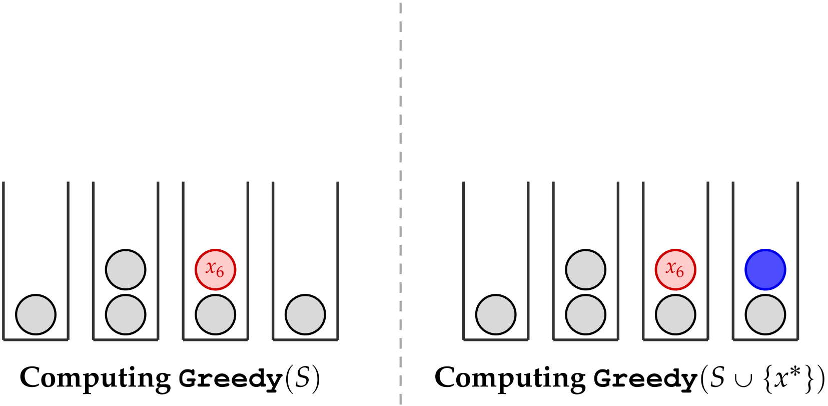


Computing $\texttt{Greedy}(S)$ **Computing** $\texttt{Greedy}(S \cup \{x^*\})$

Subsequent balls will experience either:

1. No recourse

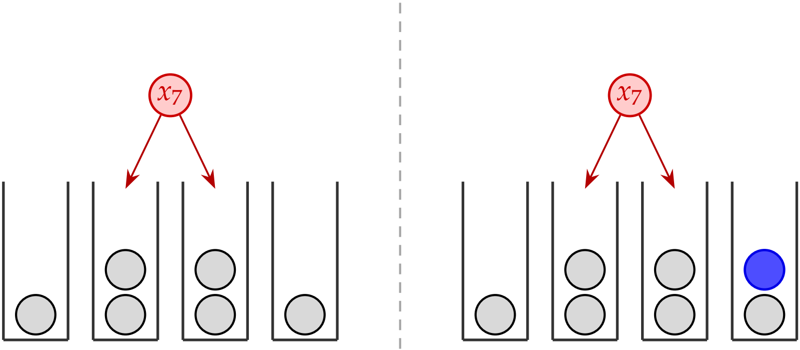
Analyzing the Recourse



Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

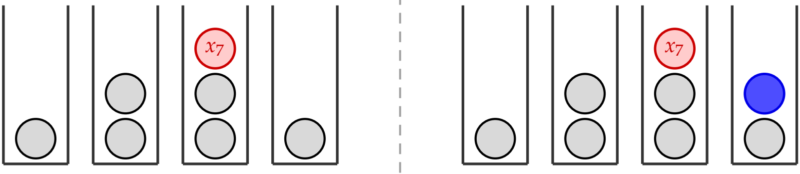


Computing $\texttt{\textbf{Greedy}}(S)$ **Computing** $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse

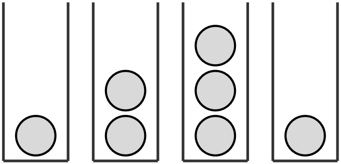


Computing $\texttt{\textbf{Greedy}}(S)$ | **Computing** $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

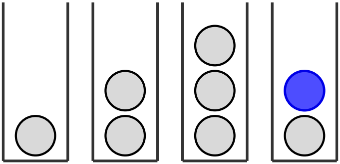
Subsequent balls will experience either:

1. No recourse

Analyzing the Recourse



Computing `Greedy`(S)

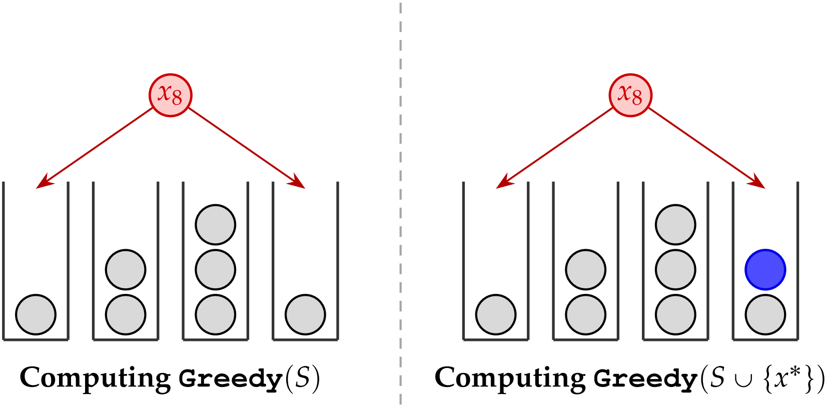


Computing `Greedy`$(S \cup \{x^*\})$

Subsequent balls will experience either:

1. No recourse
2. Recourse

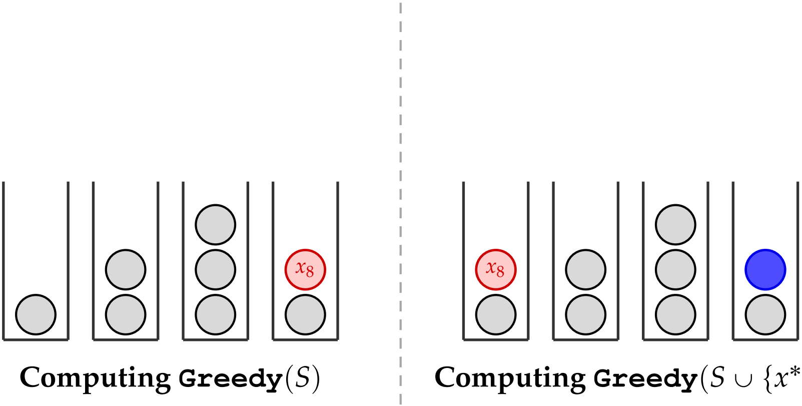
ANALYZING THE RECOURSE



Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

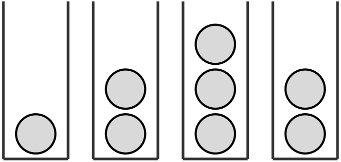


Computing $\texttt{Greedy}(S)$ | **Computing** $\texttt{Greedy}(S \cup \{x^*\})$

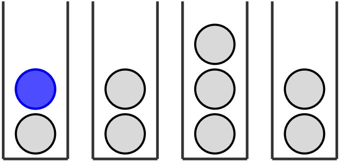
Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse



Computing `Greedy`(S)

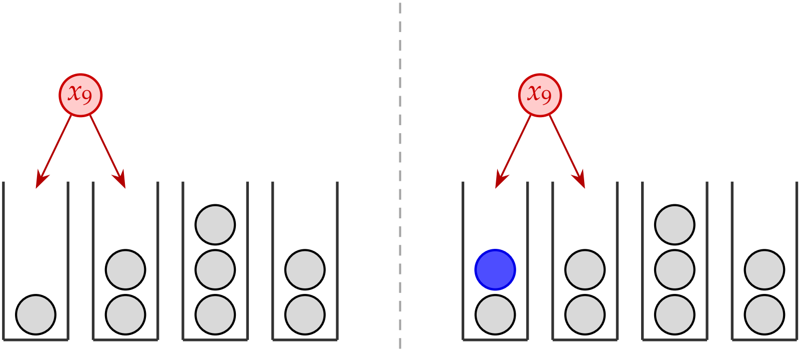


Computing `Greedy`$(S \cup \{x^*\})$

Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse

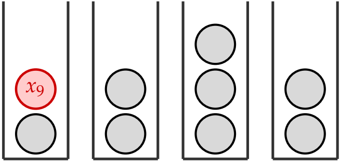


Computing $\texttt{Greedy}(S)$ **Computing** $\texttt{Greedy}(S \cup \{x^*\})$

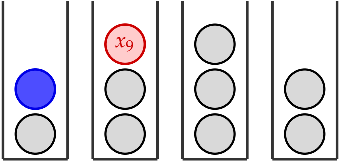
Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse



Computing $\texttt{Greedy}(S)$

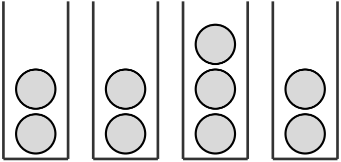


Computing $\texttt{Greedy}(S \cup \{x^*\})$

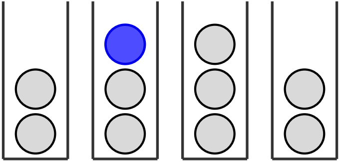
Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse



Computing $\texttt{Greedy}(S)$

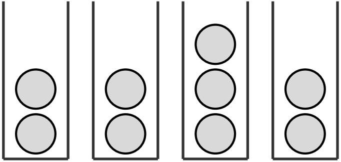


Computing $\texttt{Greedy}(S \cup \{x^*\})$

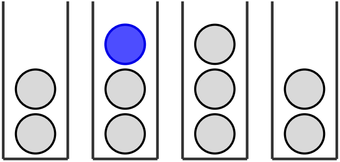
Subsequent balls will experience either:

1. No recourse
2. Recourse

Analyzing the Recourse



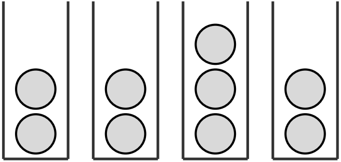
Computing `Greedy`(S)



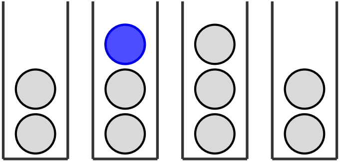
Computing `Greedy`$(S \cup \{x^*\})$

Two key observations:

Analyzing the Recourse



Computing `Greedy`(S)

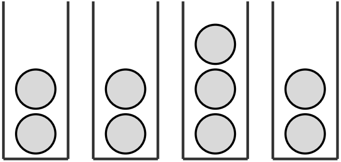


Computing `Greedy`$(S \cup \{x^*\})$

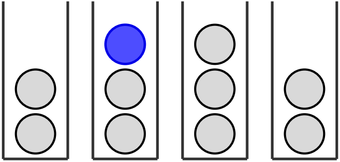
Two key observations:

1. There's always one special bin with an extra ball

Analyzing the Recourse



Computing $\texttt{Greedy}(S)$

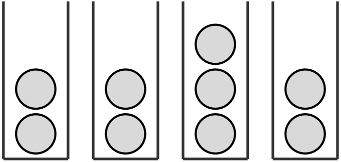


Computing $\texttt{Greedy}(S \cup \{x^*\})$

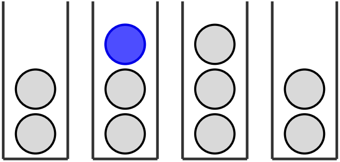
Two key observations:

1. There's always one special bin with an extra ball
2. If a ball incurs recourse, one of its choices is the special bin

Analyzing the Recourse



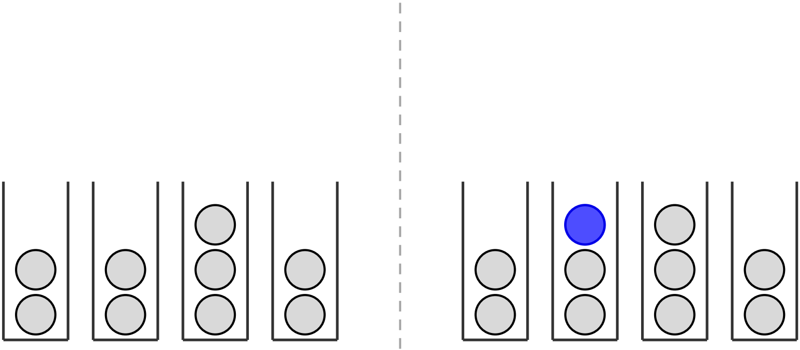
Computing `Greedy`(S)



Computing `Greedy`$(S \cup \{x^*\})$

$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

Analyzing the Recourse



Computing $\texttt{\textbf{Greedy}}(S)$ **Computing** $\texttt{\textbf{Greedy}}(S \cup \{x^*\})$

$$\Pr[\text{ball } x_i \text{ incurs recourse}] = O(1/n)$$

$$\implies \mathbb{E}[\text{total recourse}] = \sum_i \Pr[\text{ball } x_i \text{ incurs recourse}] = O(m/n)$$

This Paper

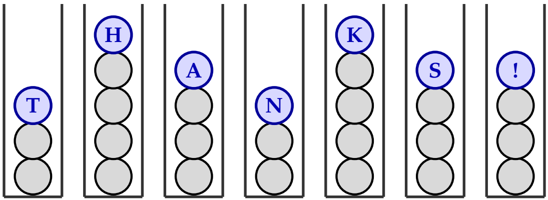
Question: Does there exist a history-independent solution with small recourse and overload?

Theorem: There exists a history-independent solution with:

- Overload $O(1)$, with high probability.
- Expected recourse $O(\log \log (m/n))$.

Rest of Talk: A simple history-independent algorithm with overload $O(\log \log n)$ and expected recourse $O(m/n)$. ✓

History-Independent Load Balancing



Michael A. Bender
Stony Brook University

Bill Kuszmaul
CMU

Elaine Shi
CMU

Rose Silver
CMU