Regulated Tempered Stable Processes with Stochastic Volatility: A Crypto Option Pricing Perspective

Hengxu Li, Junyan Ye

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Abstract

We propose a model for the price dynamics of Bitcoin and Ethereum that combines tempered stable subordinators with Heston's stochastic volatility model. By introducing the stochastic nature of volatility into the tempered stable model, we are able to capture the high variability of the BTC and ETH price process. We consider the use of Gaussian mixture with risk-neutral correction, resulting in a log-price process that consists of a risk-neutral Heston's stochastic volatility price process and a Gaussian mixture of the tempered stable subordinator and standard Brownian motion. We show that this model is able to accurately capture the stylized facts of the BTC and ETH price dynamics, as evidenced by the good fit of the model to the empirical data. Our results suggest that the combination of tempered stable subordinators and Heston's model is a promising approach for modeling the price dynamics of Bitcoin and Ethereum.

1 Introduction

The cryptocurrency market is characterized by high levels of volatility due to fluctuating supply and demand, shifting investor attention, and emotional market dynamics. One approach to modeling the dynamics of financial assets is the Black-Scholes model, but this model does not account for the fluctuating volatility of Bitcoin (BTC) and Ethereum (ETH) prices, which is a significant factor in the cryptocurrency market. As noted in [DB19], Heston's stochastic volatility model can capture the fluctuations in BTC and ETH price volatility. However, this model does not consider the presence of jumps in the BTC and ETH price process, which occur frequently in the market. To address this issue, [Xia21] proposes the use of a tempered stable subordinator, which is a Lévy process that can incorporate jumps into the model. Additionally, [ZF22] suggests that the combination of a tempered stable subordinator with a stochastic clock can provide a better fit for the BTC price process, with some modification of the Laplace transform. Finally, in an effort to consider both the high fluctuations in volatility and jumps, we propose a combined model that combines Heston's stochastic volatility process with the tempered stable model through independent addition, which can be seen as the convolution of two independent random variables. The Laplace transform of this model can be derived using the formula for the Laplace transform of the convolution of independent variables.

The remainder of this paper is organized as follows: in **Section 2**, we present the data we collected for the purpose of calibrating several models and deriving our combined Heston-tempered stable model for BTC and ETH prices. In **Section 3**, we describe each of the models we consider, including the Black-Scholes model, Heston's stochastic volatility model, and tempered stable subordinators with averaged versions and stochastic clocks. We also analyze the statistical properties and average relative price errors (ARPE) of these models when calibrated using our collected data, and present our final combined Heston-tempered stable model. Finally, in **Section 4**, we draw conclusions based on the calibration results of these models and their comparison in terms of statistical properties and ARPE, and discuss the effectiveness of these models for BTC and ETH price modeling.

2 Data

The data used in this study consists of European-style options on BTC and ETH price indices, collected from the leading BTC-USD exchange Deribit. These options are cash-settled in BTC, with USD serving as the numeraire. The data was extracted on December 31, 2022. The options data cover the period from January 13, 2023 to December 29, 2023, with expiries ranging from 13 days to 363 days. Options with longer maturities are not frequently traded on the exchange and were therefore excluded from the analysis.

It is worth noting that the options prices in our data set are relatively expensive due to the high volatility of BTC and ETH. Additionally, the strikes for short-term options (up to 3 months) tend to range from approximately 85% to 125% of the initial values of the underlying BTC prices, as opposed to higher maturities, which typically have strikes ranging from 100% to 500%.

3 Calibration

In this study, we introduce several stochastic models and evaluate their ability to fit the option dynamics of BTC and ETH. To do this, we calibrate the models using the average relative pricing error (**ARPE**) as an error function, in order to determine the optimal parameters for each model. The models we consider include the Black-Scholes model, the Heston stochastic volatility model, and several versions of the tempered stable subordinator. For the latter models, we use a Gaussian mixture technique to construct the price process. We then use the Laplace transform and its inverse formula to compute option prices, which serve as model prices.

The formula for **ARPE** is

$$ARPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|marketprice_i - modelprice_i|}{marketprice_i} \tag{1}$$

After calibrating the models, we compare their skewness and kurtosis and present a final comparison of these statistical measures along with the values of **ARPE**. The length and frequency of the time series are chosen arbitrarily.

3.1 Gaussian mixture and option pricing

We use $\Lambda = (\Lambda_t)_{t\geq 0}$ to denotes time-change process, which can be considered a "clock". Then we let W be a standard Brownian motion independent of Λ, and define the following process

$$H_t = \kappa t + \mu\Lambda_t + \sigma W_{\Lambda_t}, \quad t \geq 0, \tag{2}$$

where $\kappa \in \mathcal{R}$ is drift parameter, $\mu \in \mathcal{R}$ is a Brownian location parameter and $\sigma > 0$ the Brownian scale parameter. By applying the tower property of conditional expectations, the Laplace Transform of H_t for fixed $t > 0$ is

$$\psi_{H_t}(u) := \mathbf{E}[e^{-uH_t}] = e^{-\kappa t u}\psi_{\Lambda_t}(\mu u - \frac{1}{2}\sigma^2 u^2) \tag{3}$$

We use the above process $(H_t)_{t\geq 0}$ to capture jumps for risky BTC(ETH) asset prices, then we consider the following risk-neutralized log-price

$$\log S_t = \log\frac{S_0}{\psi_{H_t}(-1)} + H_t, \quad t \geq 0, \tag{4}$$

where $\psi_{H_t}(-1)$ is equal to $\mathbf{E}[e^{H_t}]$. The price of a European-style call option written on this with strike price K and maturity $T > 0$ can be calculated as

$$\Gamma = \mathbf{E}[e^{rT}(S_T - K)^+] = S_0 e^{-qT}\hat{p} - Ke^{-rT}p^*, \tag{5}$$

where the in-the money probabilities are given as follows:

$$p^* = \int_{\log(K\psi_{H_T}(-1)/S_0)}^{\infty} f_{H_T}(x)dx \tag{6}$$

and

$$\hat{p} = \int_{\log(K\psi_{H_T}(-1)/S_0)}^{\infty} x f_{H_T}(x) dx \tag{7}$$

f_{H_T} is the probability density of H_T. According to [Xia21], the in-the-money probabilities can be expressed in Fourier inverse formula

$$p^* = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\frac{(S_0/(K\psi_{H_T}(-1)))^{iu}\psi_{H_T}(-iu)}{iu} du \tag{8}$$

and

$$\hat{p} = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\frac{(S_0/(K\psi_{H_T}(-1)))^{iu}\psi_{H_T}(-iu-1)}{iu\psi_{H_T}(-1)} du \tag{9}$$

3.2 Moments calculation

Assume that we have obtained the Laplace transform of a random varible X as $\psi_X(u)$, the 1 to 4 degree moments are derives by the following formula:

$$\begin{aligned}
\mathbf{E}[X] &= -\psi_X^{'}(u), \\
\mathbf{E}[X^2] &= \psi_X^{''}(u), \\
\mathbf{E}[X^3] &= -\psi_X^{'''}(u), \\
\mathbf{E}[X^4] &= \psi_X^{(4)}(u)
\end{aligned} \tag{10}$$

3.3 The Black-Scholes model

Let $(S_t)_{t\geq 0}$ be the BTC (ETH) price process. Assuming a zero interest rate, the price process is given by

$$S_t = S_0 \exp\left(\frac{-\sigma^2 t}{2} + \sigma W_t\right), \tag{11}$$

where $(W_t)t \geq 0$ is a standard Brownian motion. S_0 is the initial value of the BTC (ETH) price and σ is the volatility parameter. The term $\frac{-\sigma^2 t}{2}$ ensures that $(S_t)t \geq 0$ is a martingale.

The Laplace transform $\psi_t(u)$ of the Black-Scholes log-price process $\log S_t$ is given by

$$\psi_t(u) = S_0^{-u} \exp\left(\frac{1}{2}\sigma^2 t(u+u^2)\right) \tag{12}$$

The volatility parameter σ has been optimized, resulting in a value of 59.73% for BTC (with ARPE = 0.3904) and 69.05% for ETH (with ARPE = 0.3461). It is evident that the optimal Black-Scholes BTC (ETH) price volatility is relatively high compared to normal assets, such as stocks in the S&P 500.

Skewness and Kurtosis are easily obtained from moments by formula (10), and density function can be also derived by inverse formula of Laplace transform.

3.4 Heston's stochastic volatility model

To account for the high variability of the BTC and ETH price process volatility, we consider Heston's stochastic volatility model [Hes93]. Heston's model is characterized by a set of parameters that describe the behavior of the squared volatility process $(\nu_t)_{t\geq 0}$. These parameters are:

- κ: This is the rate at which the volatility mean reverts back to the long-term mean η. A high value of κ indicates that the volatility mean reverts quickly, while a low value indicates slow mean reversion.

- ρ: This is the correlation between the two Brownian motions $(W_t)t \geq 0$ and $(\widetilde{W_t})t \geq 0$ in the squared volatility process. A positive value of ρ indicates that the two Brownian motions are positively correlated, while a negative value indicates negative correlation.

	$\hat{\kappa}$	$\hat{\rho}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\nu_0}$	ARPE (%)
BTC	0.4661	-0.07886	10.16	-0.121	0.0236	10.45
ETH	0.55	-0.1085	13.46	-0.1373	0.1086	13.73

Figure 1: Heston calibration

- θ: This is the long-term mean of the squared volatility process.
- η: This is the long-term variance of the volatility process.
- ν_0: This is the initial value of the squared volatility process at time $t = 0$.

The risk-neutral BTC and ETH price process $(S_t)_{t\geq 0}$ is constructed as follows:

$$d \log S_t = \sqrt{\nu_t} dW_t, \tag{13}$$

where $(\nu_t)_{t\geq 0}$ denotes the squared volatility process, which is given by

$$d\nu_t = \kappa(\eta - \nu_t)dt + \theta\sqrt{\nu_t} d\widetilde{W_t} \tag{14}$$

with $(W_t)t \geq 0$ and $(\widetilde{W_t})t \geq 0$ being two correlated standard Brownian motions, and the correlation is given by

$$Cov(dW_t, d\widetilde{W_t}) = \rho dt. \tag{15}$$

The Laplace Transform $\psi_t(u)$ of $\log S_t$ is given by

$$\begin{aligned} \psi_t(u) = \exp(u \log S_0) \times \exp[\eta\kappa\theta^{-2}[(\kappa + \rho\theta u - d)t - 2 \log \frac{1 - g \exp(-td)}{1 - g}]] \\ \times \exp(\nu_0\theta^{-2} \frac{(\kappa + \rho\theta u - d)(1 - \exp(-td)}{1 - g \exp(-td)}) \end{aligned} \tag{16}$$

where

$$d = [(\kappa + \rho\theta u)^2 - \theta^2(u + u^2)]^{\frac{1}{2}}$$

and

$$g = \frac{\kappa + \rho\theta u - d}{\kappa + \rho\theta u + d}$$

Based on the given parameter estimates in figure 1, we can make the following observations: Heston's stochastic volatility model was able to capture some of the dynamics of the BTC and ETH price processes, particularly the mean reversion rates of the volatility and the correlations between the squared volatility processes. Our calibration results showed that the mean reversion rate of the BTC volatility was lower than that of ETH, and that the two squared volatility processes were negatively correlated. The long-term mean of the squared volatility process was higher for ETH compared to BTC, while the long-term variance was higher for BTC. These findings suggest that the BTC and ETH price processes may exhibit different patterns of volatility behavior, and that a model that can more accurately capture these differences may be needed to improve the fit of the option pricing model.

3.5 Tempered stable subordinators

We first introduce tempered stable subordinators, then construct the average style and stochastic clocks upon them.

The tempered stable subordinator is a Lévy process, which is a type of stochastic process that exhibits independent and stationary increments and has càdlàg (right-continuous with left-hand limits) sample paths. The one-sided tempered stable distribution is a family of distributions characterized by the parameters $a > 0$, $b > 0$ and $c \in (0, 1)$, denoted by $TS(a, b; c)$:

- a: This is the stability parameter, which determines the tail behavior of the distribution. A value of $a > 0$ corresponds to a distribution with heavy tails, with larger values of a corresponding to heavier tails.

- b: This is the tempering parameter, which controls the degree of heavy-tailed behavior. A value of $b > 0$ corresponds to a tempered stable distribution, which exhibits heavy tails but is more peaked and has a finite mean compared to the stable distribution ($b = 0$).

- c: This is the skewness parameter, which determines the skewness of the distribution. A value of $c \in (0, 1)$ corresponds to a left-skewed distribution, while a value of $c > 1$ corresponds to a right-skewed distribution.

Let $(\Omega, \mathbf{F}, \mathbf{P})$ be a complete probability space, with respect to which all stochastic are considered. Let $X \equiv (X_t)_{t\geq 0}$ denotes a tempered stable subordinator, a Lévy process with a marginal one-sided tempered stable distribution. It starts from 0 $\mathbf{P}$-a.s., has independent and stationary increments and has $\mathbf{P}$-a.s. càdlàg sample paths.

Its Laplace transform is given by Equation (17). This Laplace transform is used to compute the probability that the subordinator does not exceed a certain threshold within a given time period.

$$\begin{aligned}\psi_{X_t}(u) \equiv \psi_{X_t}(u|a, b; c) := \mathbf{E}[e^{uX_t}] = \exp(at\Gamma(-c)((b + u)^c - b^c)),\\ u \in \mathbf{C} - (-\infty, -b], \quad t \geq 0\end{aligned} \tag{17}$$

which satisfies $\psi_{X_t}(u) = (\psi_{X_1}(u))^t$ for every $t \geq 0$.

3.5.1 Average-tempered stable subordinators

We now study the running average of the tempered stable subordinator given by the time-scaled integral functional of X,

$$\widetilde{X_t} := \frac{1}{t}\int_0^t X_s ds, \quad t > 0, \tag{18}$$

with $\widetilde{X_0} = 0, \mathbf{P}$-a.s. Then according to [Xia21], we derive the Laplace transform

$$\begin{aligned}\psi_{\widetilde{X_t}}(u) \equiv \psi_{\widetilde{X_t}}(u|a, b; c) := \mathbf{E}[e^{-u\widetilde{X_t}}] = \exp \frac{at\Gamma(-c)((b + u)^{c+1} - b^c(b + (c + 1)u))}{(c + 1)u},\\ u \in \mathbf{C} - (-\infty, -b], \quad t \geq 0.\end{aligned} \tag{19}$$

In the following discussion, the average tempered stable subordinator determined by $\psi_{\widetilde{X_t}}(u|a, b; c)$ is denoted by $ATS(a, b; c)$.

Based on [Xia21], we can obtain the Gamma process $G(a, b) \equiv G_t$ and average Gamma process $AG(a, b) \equiv AG_t$ by taking the limitation of c to 0. To be specific, $TS(a, b; c) \longrightarrow G(a, b)$ and $ATS(a, b; c) \longrightarrow AG(a, b)$ as $c \longrightarrow 0$. Then we derive the following Laplace transform of

$$\psi_{G_t}(u) := \mathbf{E}[e^{-uG_t}] = (1 + \frac{u}{b})^{-at}, \quad \psi_{AG_t}(u) = e^a(1 + \frac{u}{b})^{-a(1+b/u)}, \quad Re(u) > 0, \tag{20}$$

From [Xia21], we learn that average tempered stable subordinators have the following moments formula, which lead to our calculation of the log-price process's moments,

$$M_{\widetilde{X_t}}(n) = \begin{cases} 1 & n = 0 \\ at\sum_{k=0}^{n} \begin{pmatrix} n \\ k \end{pmatrix} \frac{\Gamma(k - c + 1)}{(k + 2)b^{k-c+1}} M_{\widetilde{X_t}}(n - k) & n \in \mathbf{N}_{++} \end{cases} \quad t \geq 0 \tag{21}$$

according to relationship of tempered stable subordinator and average tempered stable subordinator, we can easily derive the following moments(statistical magnitude) transformation formula:

$$
\begin{aligned}
\mathbf{E}[\widetilde{X_t}] &= \frac{at\Gamma(1-c)}{2b^{1-c}} = \frac{1}{2}\mathbf{E}[X_t],\\
\mathbf{var}[\widetilde{X_t}] &= \frac{at\Gamma(2-c)}{3b^{2-c}} = \frac{1}{3}\mathbf{var}[X_t],\\
\mathbf{skew}[\widetilde{X_t}] &= \frac{3\sqrt{3}(2-c)\Gamma(3-c)}{4\sqrt{atb^c(\Gamma(2-c))^3}} = \frac{3\sqrt{3}}{4}\mathbf{skew}[X_t],\\
\mathbf{kurt}[\widetilde{X_t}] &= \frac{9\Gamma(4-c)}{5atb^c(\Gamma(2-c))^2} = \frac{9}{5}\mathbf{kurt}[X_t]
\end{aligned}
$$

Then we have been get statistical magnitude for both average recipe and non-average recipe.

3.5.2 Regulating stochastic clocks

We first let $X \equiv (X_t)_{t\geq 0}$ be a Lévy processes. From [ZF22], let $\widetilde{X_t^{(n)}}$ denotes the third recipe of stochastic clocks, defined by $\widetilde{X^{(0)}} := X$ with n-dimension averaging formula

$$
\widetilde{X_0^{(n)}} = 0; \quad \widetilde{X_t^{(n)}} := \frac{1}{t^n}\int \cdots \int_0^t X_s ds \cdots ds, \quad t>0, n \in \mathbf{N}_{++} \tag{22}
$$

then we have that for fixed t $>$ 0,

$$
\psi_{\widetilde{X_t^{(n)}}|c=0}(u) = \left(1+\frac{u}{b\Gamma(n+1)}\right)^{-at} \exp\left(\frac{atnu}{b\Gamma(n+2)}\,{}_2F_1\left(1,\frac{1}{n}+1,\frac{1}{n}+2,-\frac{u}{b\Gamma(n+1)}\right)\right) \tag{23}
$$

$$
\psi_{\widetilde{X_t^{(n)}}|c>0}(u) = \left(\exp(atb^c\Gamma(-c)\left({}_2F_1(-c,\frac{1}{n},\frac{1}{n}+1,-\frac{u}{b\Gamma(n+1)}-1\right)\right) \tag{24}
$$

In the special case n = 1, equation (24) reduces into equation (19), corresponding to the Lévy density derived in section 3.4.1. The calibration results are summarized in Figure 2.

3.6 Tempered stable subordinators combined with stochastic volatility

In this section, we present a combination of tempered stable subordinators and Heston's stochastic volatility model in order to introduce the stochastic nature of volatility into our tempered stable model. We continue to consider the use of Gaussian mixture with risk-neutral correction, resulting in the combined log-price process of BTC(ETH) given by equation (1). This process consists of a risk-neutral Heston's stochastic volatility price process, denoted as $S_t^{'}$, and a Gaussian mixture of the tempered stable subordinator and standard Brownian motion, denoted as H_t. The correction term $-\log\psi_{H_t}(-1)$ is included for risk-neutralization, ensuring that the expected value of $\log S_t$ is equal to S_0.

$$
\log S_t = -\log\psi_{H_t}(-1) + \log S_t^{'} + H_t, \tag{25}
$$

The Laplace transform of this log-price process is given by

$$
\psi_{\log S_t}(u) = e^{u\log\psi_{H_t}(-1)}\psi_{\log S_t^{'}}(u)\psi_{H_t}(u), \tag{26}
$$

where $\psi_{\log S_t}(u)$ is the Laplace transform of the log-price process, $\psi_{\log S_t^{'}}(u)$ is the Laplace transform of the risk-neutral Heston's stochastic volatility price process, and $\psi_{H_t}(u)$ is the Laplace transform of the Gaussian mixture of the tempered stable subordinator and standard Brownian motion.

The calibration results are summarized in Figure 3. We also identify the best fit model, whose implied prices are further visually compared with the market prices in Figure 4 and 5.

	n	$\hat{a}_n$	$\hat{b}_n$	$\hat{\mu}_n$	ARPE
c = 0 (VG)	1	25.219657	35.467720	-3.462143	0.336007
	2	21.542141	10.389041	-3.779917	0.329210
	3	46.521129	5.457749	-2.918681	0.342083
	4	73.155257	1.687171	-3.357152	0.345759
	5	85.242394	0.321350	-3.517543	0.348744
	n	$\hat{a}_n$	$\hat{b}_n$	$\hat{\mu}_n$	ARPE
c = 0.25	1	3.270138	10.521099	-3.331862	0.319821
	2	3.558529	2.717845	-3.194711	0.321597
	3	7.865521	1.206716	-3.144380	0.319196
	4	9.372745	0.177949	-3.050799	0.322878
	5	12.059098	0.019795	-3.135383	0.337863
	n	$\hat{a}_n$	$\hat{b}_n$	$\hat{\mu}_n$	ARPE
c = 0.5 (NIG)	1	0.877162	4.900884	-3.069026	0.318634
	2	0.884716	0.347913	-2.478224	0.335839
	3	3.061162	0.372710	-2.857201	0.322375
	4	11.429698	0.231097	-3.140824	0.320780
	5	14.116161	0.006605	-2.437168	0.331545
	n	$\hat{a}_n$	$\hat{b}_n$	$\hat{\mu}_n$	ARPE
c = 0.75	1	0.218787	1.394313	-2.866514	0.318236
	2	0.509664	0.463359	-2.756164	0.317647
	3	1.687974	0.244725	-2.870747	0.319524
	4	6.683835	0.077320	-2.758321	0.327094
	5	21.650225	0.007520	-2.740293	0.317439

Figure 2: Tempered stable model with stochastic clocks calibration for BTC. VG: variance gamma. NIG: normal inverse Gaussian

	c	$\hat{a}$	$\hat{b}$	$\hat{\mu}$	$\hat{\kappa}$	$\hat{\rho}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{v_0}$	ARPE (%)
BTC	0	0.05062	0.3216	-1.74	16.15	-0.1164	10.85	0.4549	0.02372	10.45
	0.25	0.0324	0.3402	-3.526	14.51	-0.06457	9.453	0.4479	0.02501	10.93
	0.5	0.05227	0.3428	-4.843	12.65	-0.05454	9.257	0.4025	0.05086	9.791
	0.75	0.05697	0.8074	-4.743	15.02	-0.1082	14.54	0.461	0.03593	10.31
	c	$\hat{a}$	$\hat{b}$	$\hat{\mu}$	$\hat{\kappa}$	$\hat{\rho}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{v_0}$	ARPE (%)
ETH	0	0.2638	0.5705	-3.409	15.45	-0.1173	14.22	0.4659	0.04532	11.14
	0.25	0.1299	0.2815	-4.183	17.1	-0.02875	13.09	0.4176	0.06036	10.78
	0.5	0.1726	0.8116	-3.445	12.38	-0.1012	13.65	0.4331	0.09563	10.08
	0.75	0.05697	0.5184	-4.743	15.02	-0.1082	14.54	0.461	0.03593	9.898

Figure 3: Tempered stable subordinators combined with stochastic volatility calibration

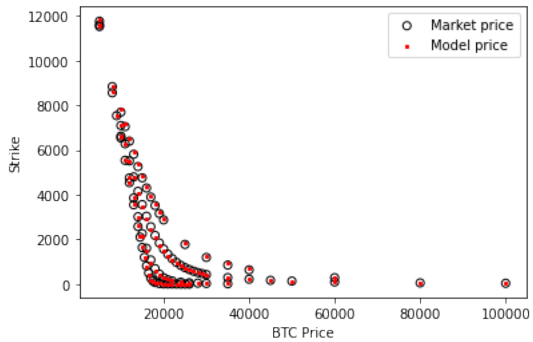

Figure 4: Best model for BTC,
T = 13, 55, 181, 363 days

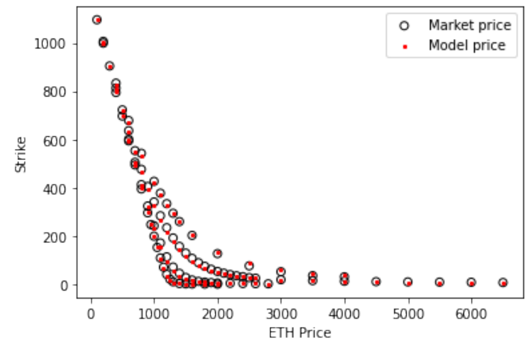

Figure 5: Best model for ETH,
T = 13, 55, 181, 363 days

4 Conclusion

In this paper, we have introduced a hybrid model for the price evolution of BTC and ETH, combining the features of Heston's stochastic volatility model with those of tempered stable subordinators. By introducing the notion of a Gaussian mixture, we have been able to incorporate the heavy-tailed behavior of these cryptocurrencies while still maintaining a risk-neutral framework. Our combined Heston-tempered stable model provided the best overall fit to the BTC and ETH options data, with relatively low ARPE values and more balanced statistical measures. These results suggest that the combination of a stochastic volatility process with a tempered stable subordinator may be a useful approach for modeling the dynamics of cryptocurrency prices. Overall, this work provides a promising approach for modeling the complex dynamics of cryptocurrencies and can potentially be extended to other financial assets exhibiting heavy-tailed behavior.

The high volatility of BTC and ETH prices is a key factor that affects the options prices in our data set, which are relatively expensive compared to options on more stable assets. Additionally, our analysis is limited to options on BTC and ETH price indices from the Deribit exchange, and only includes options with maturities ranging from 13 days to 363 days. This may introduce some biases into our results, as options with longer maturities or options from other exchanges may behave differently. Future research could investigate the performance of our model on a wider range of options or on other cryptocurrency exchanges to further assess its generalizability.

Appendix A Option data from Deribit

https://github.com/williamli-15/Regulated-tempered-stable-processes-with-stochastic-volatility/blob/main/cleaned_df.csv

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