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2 Every Flare, Everywhere: Untriggered Searches for Astrophysical Neutrino Transients Using Data

3 From the IceCube Detector

4 *by*

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21 Abstract

22 Recent results from the IceCube collaboration regarding the blazar TXS 0506+056 suggest
23 the presence of neutrino flares that are not temporally coincident with a significant corre-
24 sponding gamma ray flare. Such flares are particularly difficult to identify, as their presence
25 must be inferred from the temporal distribution of neutrino data alone. While previously
26 existing statistical methods fit for only the largest neutrino flare candidate at a particular
27 location, this work introduces an extension to fit for the contributions from multiple flare
28 candidates at once, thereby increasing the sensitivity to sub-threshold neutrino flares. The
29 need for a multiple flare fit is further motivated by the increasing duration of the IceCube
30 data available for analysis. In this work, this "multi-flare" method is applied to data from
31 the IceCube Neutrino Observatory using two different source catalogs to explore emission
32 from candidates similar to TXS 0506+056. This method is additionally used to create a
33 "multi-flare" skymap, describing the temporal variability of IceCube data at every point in
34 the neutrino sky. These results can be used to place further constraints on potential pop-
35 ulations of transient neutrino sources, serving as a complement to existing time-integrated
36 and time-dependent methods.

37 Acknowledgements

38 I'd like to thank the academy...

39 **Contents**

40	Abstract	i
41	Acknowledgements	ii
42	1 Introduction	1
43	2 Astrophysical Messengers	4
44	2.1 Cosmic Rays	4
45	2.1.1 Energy Spectrum	4
46	2.1.2 Acceleration Mechanism	5
47	2.2 Photons	7
48	2.2.1 Leptonic Radiative Processes	8
49	2.2.2 Hadronic Radiative Processes	9
50	2.2.3 Gamma Rays as an Astrophysical Messenger	10
51	2.3 Neutrinos	11
52	2.3.1 Production Mechanisms	11
53	2.3.2 Observed Neutrino Spectrum	13
54	2.3.3 Neutrinos as Astrophysical Messengers	13
55	2.3.4 AGN as Neutrino Source Candidates	15
56	2.3.5 Galactic Neutrino Sources	18
57	2.3.6 Constraints on Neutrino Source Populations from Diffuse Measurements	19
58	3 The IceCube Detector	22

59	3.1 Detection Mechanism	22
60	3.2 The Physical Components of the IceCube Detector	25
61	3.3 Event Types	28
62	3.3.1 Tracks	30
63	3.3.2 Cascades	32
64	3.3.3 Double Cascades	33
65	3.4 Triggering	33
66	3.5 Directional Reconstruction	34
67	3.5.1 LineFit	34
68	3.5.2 SplineMPE	35
69	3.5.3 Paraboloid	35
70	3.6 Energy Reconstruction	36
71	3.7 IceCube Event Samples Overview	38
72	3.7.1 PointSourceTracks v002p03	39
73	3.7.2 PointSourceTracks v003p02	39
74	3.7.3 NorthernTracks v002p06	41
75	4 Statistical Methods in Neutrino Astronomy	43
76	4.1 Clustering Analysis Outline for a Generalized TS	43
77	4.2 Unbinned Time Integrated Methods	46
78	4.3 Unbinned Time-Dependent Methods	49
79	4.3.1 Optimizing for a Single Flare	50
80	4.3.2 Fitting Ensembles of Multiple Flares	52
81	5 Applications of the Multi-flare Algorithm to Source Catalogs	57
82	5.1 Self-Triggered Catalog	58
83	5.2 Fermi 3LAC Blazars	61
84	6 An All-Sky Multi-flare Analysis	73

85	6.1 Motivation	73
86	6.2 Analysis Construction	74
87	6.3 Results	76
88	6.3.1 Hotspots and Populations Analysis	76
89	6.3.2 Locations of External Interest	79
90	7 Outlook	89
91	8 Appendix: Statistics Background	91
92	8.1 Estimators	91
93	8.2 Likelihoods	92
94	8.3 Confidence Intervals	93
95	8.4 Hypothesis Testing	94
96	8.4.1 State relevant null and alternative hypotheses	94
97	8.4.2 State a test statistic	95
98	8.4.3 Select a significance level at which the null hypothesis will be rejected	97
99	8.4.4 Compute the TS from observed data	99
100	8.4.5 Decide whether to accept or reject the null hypothesis	99
101	8.5 Trial Factors	100
102	Bibliography	102

¹⁰³ **Chapter 1**

¹⁰⁴ **Introduction**

¹⁰⁵ Though cosmic rays were first discovered in 1912, their origins have to date remained a
¹⁰⁶ mystery. Since cosmic rays are charged particles and their paths through the universe are
¹⁰⁷ bent by magnetic fields, identifying the sources of cosmic rays is not as simple as tracing
¹⁰⁸ back their arrival directions at Earth. The discovery of the astrophysical neutrino flux in
¹⁰⁹ 2013 [1] provides a new avenue for investigating this problem, as the sources of astrophysical
¹¹⁰ neutrinos are suspected to be the same as high energy cosmic rays, but neutrinos are not
¹¹¹ charged, and thus their arrival directions point directly back to their source. Identifying
¹¹² the sources of astrophysical neutrinos (and consequently cosmic rays) would provide an
¹¹³ entirely new view of the universe, potentially allowing us to examine regions opaque to more
¹¹⁴ traditional astrophysical messengers such as photons.

¹¹⁵ With the IceCube Neutrino Observatory having collected over 10 years of data, the
¹¹⁶ first hints of astrophysical neutrino sources are beginning to be visible: a high energy alert
¹¹⁷ event from the direction of the blazar TXS 0506+056 sparked multi-messenger followup that
¹¹⁸ suggests neutrino emission from that location [2][3], and recent studies of time integrated
¹¹⁹ neutrino emission from candidate blazars reveal a 3.5σ excess of astrophysical neutrino events
¹²⁰ from the direction of the Seyfert II galaxy NGC 1068 [4].

¹²¹ These first hints of astrophysical neutrino sources raise several further questions: Could
¹²² there be more sources like TXS 0506+056 or NGC 1068? How many? Is neutrino emission

123 time-dependent, or steady? Perhaps most importantly, if there are more sources, how can
124 we identify them? Though neutrinos are an excellent astrophysical messenger due to the
125 correlation of their arrival directions with their source of origin, neutrino astronomy is made
126 significantly more difficult by the existence of the atmospheric neutrino flux, which produces
127 a large, irreducible background for searches of astrophysical neutrino sources. Because of
128 this, high energy neutrino astronomy relies either on triggering off a multi-messenger signal
129 (such as a blazar that was also detected in gamma rays), or leveraging knowledge about the
130 pattern of neutrino emission (for example, the arrival directions of astrophysical neutrinos
131 should be clustered near their sources of origin, while the atmospheric background should
132 be isotropically distributed).

133 This work will focus specifically on exploring the possibility of time-dependent neutrino
134 emission, motivated in part by the archival followup that was performed by IceCube at the
135 location of the blazar TXS 0506+056 [3]. A novel method for fitting decorrelated ensembles of
136 neutrino flare candidates is introduced and subsequently applied to source catalogs assembled
137 according properties of the TXS 0506+056 result that could potentially define a class of
138 astrophysical neutrino sources. This new method of fitting ensembles of neutrino flares is
139 intended to fill a methodological gap that exists in the field, as it provides a framework for
140 combining ("stacking") neutrino flare candidates.

141 This method is additionally applied to the entire neutrino sky, allowing for a more gen-
142 eral examination of potential clustering of neutrino data in both space and time. If neutrino
143 data is clustered beyond what is expected from the background hypothesis (isotropic arrival
144 directions distributed uniformly in time), then that would be evidence of a population of
145 astrophysical neutrino sources. Even in the case of a non-discovery, the method introduced
146 here produces neutrino "light-curves" (here referred to as "flare curves") which are of po-
147 tential use for future multi-messenger searches that may wish to incorporate information on
148 historical neutrino emission from a particular source candidate.

.	Time-Integrated	Untriggered, Time-Dependent
Single Source	[4]	[3]
Source Stacking	[5]	This work!

Table 1.1: Examples of some of the types of astrophysical neutrino source searches that have been performed using IceCube data. Time-integrated analyses search for an excess of events over the entire data sample livetime, ignoring any information about potential temporal clustering. By contrast, untriggered, time-dependent searches attempt to fit for temporal neutrino clusters ("flares") without using a multi-messenger lightcurve as a template. Single source searches report the most significant result from a small number of source candidate locations, corrected by a trial factor, while source stacking analyses combine information from many source locations under the assumption that each source candidate is a weak emitter, thereby increasing the search sensitivity to low individual source flux.

¹⁴⁹ Chapter 2

¹⁵⁰ Astrophysical Messengers

¹⁵¹ This work will focus primarily on cosmic rays, photons, and neutrinos, however other mes-
¹⁵² sengers, such as gravitational waves, can provide unique information as well.

¹⁵³ 2.1 Cosmic Rays

¹⁵⁴ 2.1.1 Energy Spectrum

¹⁵⁵ Cosmic rays are high energy protons and atomic nuclei, originating from outside the Earth's
¹⁵⁶ atmosphere. It is difficult to do traditional astronomy using cosmic rays (in the sense of using
¹⁵⁷ their arrival directions to identify sources) due to the fact that cosmic rays are charged, and
¹⁵⁸ consequently their paths are bent by magnetic fields in the universe. However, the cosmic
¹⁵⁹ ray spectrum is well studied, with many measurements having been made over the past
¹⁶⁰ century since their initial discovery. In general, the spectrum can be modeled as a power
¹⁶¹ law (eq. 2.1):

$$N(E)dE \propto E^{-x}dE \quad (2.1)$$

¹⁶² where N describes the number of cosmic ray particles, E is their energy, and x is an energy-
¹⁶³ dependent spectral index. The spectrum has several notable features, shown in figure 2.1.

164 At lower energies (below 10^{16} eV), the spectrum is well described by an $E^{-2.7}$ flux. At 10^{16}
 165 eV, the spectrum softens to $E^{-3.1}$, commonly referred to as the "knee". At even higher
 166 energies (around $10^{18.5}$ eV), the spectrum hardens again. This region is referred to as the
 167 "ankle", and is thought to correspond to the point where extragalactic cosmic rays begin to
 168 dominate the flux.

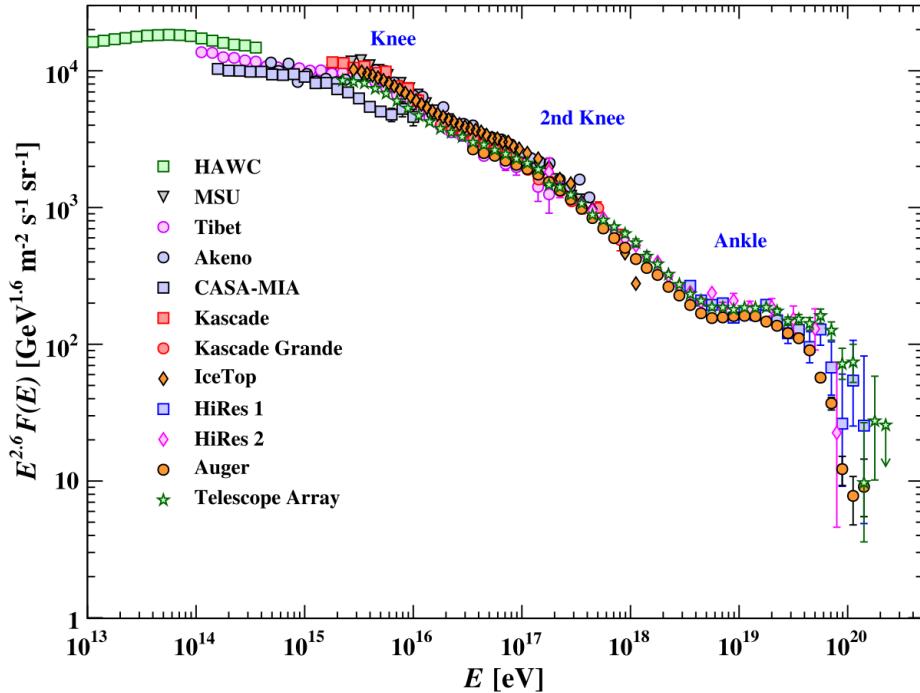


Figure 2.1: A plot of the cosmic ray spectrum assembled from observations from a variety of experiments. Major features visible are the knee ($\approx 10^{16}$ eV), and the ankle ($\approx 10^{19}$ eV). [6]

169 2.1.2 Acceleration Mechanism

170 Though the exact origin of cosmic rays is still unknown, we can infer how these particles are
 171 being accelerated based on our knowledge of how charged particles behave.

172 Suppose a particle encounters a moving cloud of plasma. If it is large enough, the
 173 cloud will act as a massive scatterer. The overall collision is the result of a large number
 174 of individual scatterings inside the cloud, and the outgoing angle of the original particle is

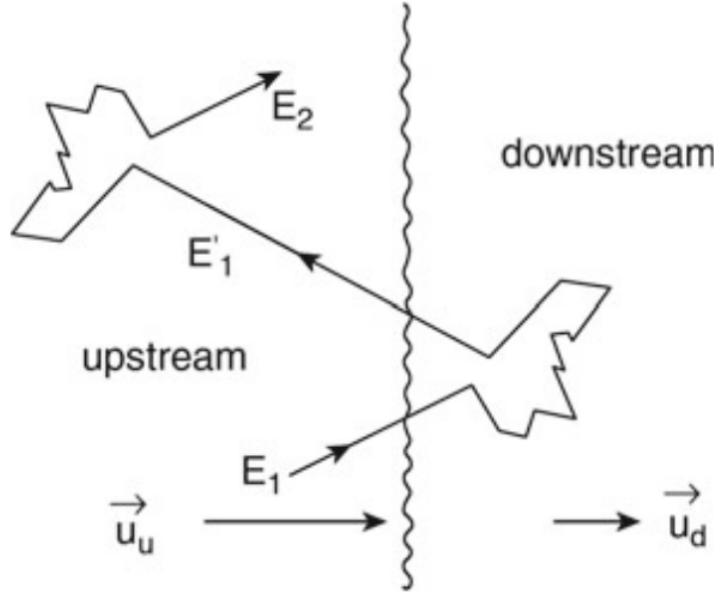


Figure 2.2: Fermi acceleration of a plane shock front[7].

175 essentially random. On average the energy of the particle after a collision with this boundary
 176 will increase by a factor (eq. 2.2):

$$\langle \frac{\Delta E}{E} \rangle \approx \frac{4}{3}\beta^2 \quad (2.2)$$

177 Where $\beta = V/c$, corresponding to the boundary velocity. This is *second order Fermi*
 178 *acceleration*, and is generally insufficient to explain the cosmic ray spectrum. If we instead
 179 assume a plane shock front, as in figure 2.4 , then the post-collision velocity of the particle
 180 is no longer randomly distributed, and we instead obtain the expression (eq. 2.3):

$$\langle \frac{\Delta E}{E} \rangle \approx \frac{4}{3}\beta \quad (2.3)$$

181 This describes *first order Fermi acceleration*. From here, we can calculate the expected
 182 flux measured at Earth to be (eq. 2.4):

$$\frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-\Gamma} \quad (2.4)$$

183 where $\Gamma = \alpha + 1$, $\alpha \approx \frac{3P_e}{4\beta}$, and P_e is the probability that a particle may escape the
 184 shock region (proportional to the velocity, V). Note that this is a power law with almost
 185 constant spectral index, as both β and P_e are proportional to V , similar to what is seen in
 186 the observed cosmic ray spectrum [7].

187 We can additionally derive constraints on the maximum energy to which a particle can be
 188 accelerated based off the magnetic field intensity, B , and the size of the accelerating region,
 189 R (eq. 2.5):

$$E_{max} = eBR \approx 10^{21} \left[\frac{R}{1 \text{ pc}} \right] \left[\frac{B}{1 \text{ Gauss}} \right] \text{ eV} \quad (2.5)$$

190 This is known as *Hillas' condition*, and can inform us about the properties of cosmic
 191 ray accelerators. We can plot this condition to show sources which satisfy this requirement
 192 for a particular energy, producing what is commonly referred to as Hillas' plot (Figure 2.3).
 193 Sources must exist above the diagonal thresholds in order to be valid accelerators of cosmic
 194 rays of certain energies. For example, sources capable of accelerating protons to 100 EeV
 195 energies must exist above the dashed line in figure 2.3. Sources below are either too small,
 196 or have magenetic fields that are too weak.

197 2.2 Photons

198 Perhaps the oldest and most well-studied astrophysical messenger, photons have the useful
 199 property of being electrically neutral. Unlike cosmic rays, their paths are not bent by
 200 magnetic fields, and the arrival directions of photons can be used to identify their origin,
 201 particularly at high energies due to their good angular resolution. Photons additionally span
 202 a large energy range, and can be used to explore a wide variety of astrophysical production
 203 processes. For the purposes of this work, we are most interested in the highest energy
 204 photons (gamma rays, with energies greater than 100 keV), which can be produced through
 205 either leptonic or hadronic processes.

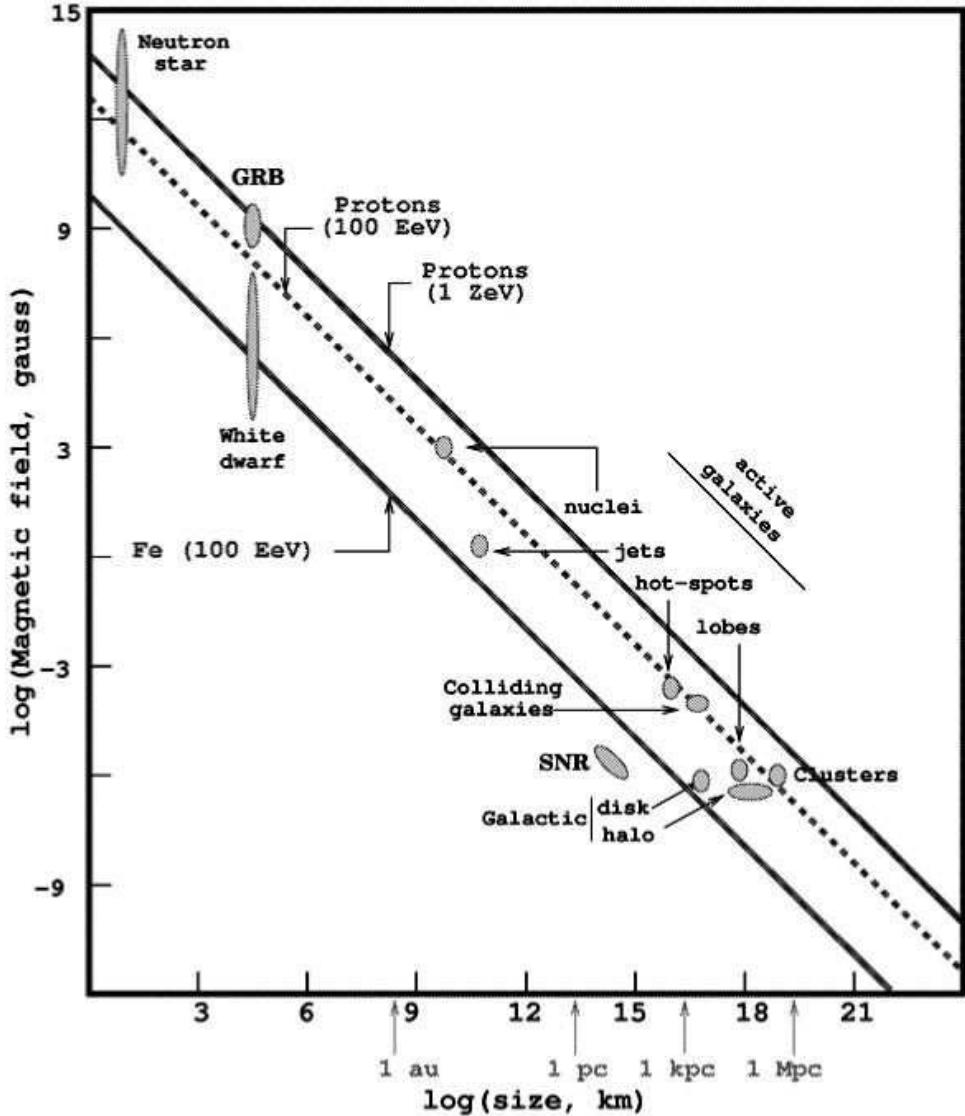


Figure 2.3: A Hillas plot, showing source populations capable of accelerating cosmic rays to a given energy, given their size and magnetic field strength [8].

206 2.2.1 Leptonic Radiative Processes

207 While photons cannot be directly accelerated by electric and magnetic fields, there are a
208 variety of mechanisms by which processes involving leptons can produce high energy photons.

- 209 • **Synchrotron radiation:** Radially accelerated relativistic particles will release syn-
210 chrotron radiation. This typically occurs as charged particles (such as electrons) spiral
211 through magnetic fields. The power loss, (dE/dT) , due to synchrotron radiation for a

212 particle of mass M and charge Z can be expressed as (eq. 2.6):

$$-\frac{dE}{dT} \approx 2.6 \left(\frac{Zm_e}{M} \right)^4 \left(\frac{E}{1 \text{ keV}} \right)^2 \left(\frac{B}{1 \text{ G}} \right)^2 \text{ keV s}^{-1} \quad (2.6)$$

213 Where m_e is the electron mass, E is the particle energy, and B is the magnetic field
 214 strength. From equation 2.6 it can be seen that synchrotron emission is significantly
 215 more relevant for electrons than protons, due to the $1/M^4$ term.

216 • **Compton Scattering and Inverse Compton Scattering:** Photons may scatter
 217 off an electron at rest, and in doing so experience a wavelength shift according to (2.7):

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\hbar\omega}{m_e c^2} (1 - \cos \alpha) \quad (2.7)$$

218 Where λ' and λ correspond to the final and initial photon wavelengths, and α is the
 219 deflection angle of the photon. If a low energy photon collides with a high energy
 220 electron, the energy of the photon may exit with more energy than it began with,
 221 providing an effective mechanism for increasing the photon energy.

222 A combination of these two processes can produce the two peak structure typically seen
 223 in the spectrum of observed gamma ray sources. The lower energy peak is due to synchrotron
 224 radiation, while the higher energy peak corresponds to low energy photons being scattered
 225 to higher energies via inverse compton scattering.

226 2.2.2 Hadronic Radiative Processes

227 While leptonic processes can produce high energy gamma rays, hadronic processes will pro-
 228 duce gamma rays in addition to neutrinos and cosmic rays. Protons accelerated to high
 229 energies in an astrophysical source may interact with other protons via (eq. 2.8):



230 Where X is the appropriate nucleus or hadron depending on the pions produced, and on
 231 average there is expected to be equal production of neutral and charged pions. Additionally,

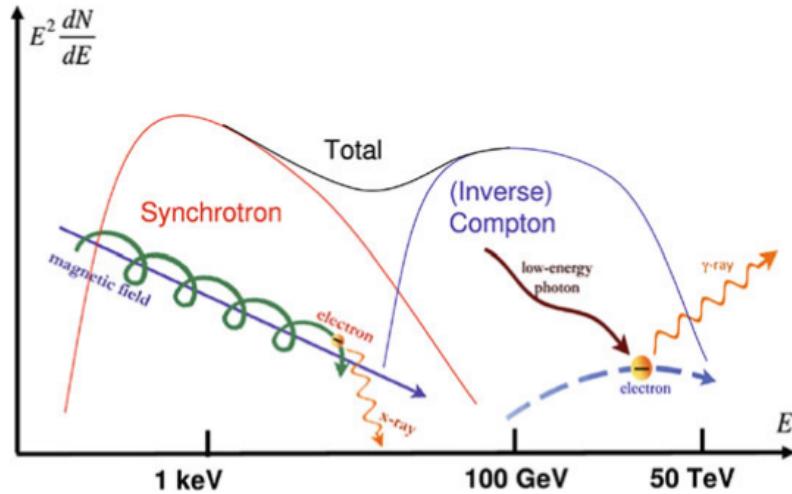


Figure 2.4: A cartoon plot of the gamma ray spectrum for a high energy gamma ray source. The two peak structure arises from the two production mechanisms of gamma rays. At lower energies, photons are produced by synchrotron radiation of electrons spiraling around magnetic field lines. The higher energy peak is then formed by the lower energy photons being scattered off energetic electrons via inverse compton scattering [9].

232 protons may interact with photons, providing an additional path to pion production (eq. 2.9
 233 and 2.10):



235 These resultant pions can subsequently decay, producing child particles depending on the
 236 particular pion type. Neutral pions will decay into gamma rays ($\pi^0 \rightarrow \gamma + \gamma$), and charged
 237 pions will decay to a lepton and their associated (anti)neutrino (most often muons and muon
 238 neutrinos: $\pi^+ \rightarrow \nu_\mu + \mu^+$, $\pi^- \rightarrow \bar{\nu}_\mu + \mu^-$). As such the sources of cosmic rays are expected
 239 to additionally produce a high energy gamma ray flux.

240 2.2.3 Gamma Rays as an Astrophysical Messenger

241 Gamma rays are a robust astrophysical messenger. As previously mentioned, photons are
 242 uncharged and their arrival directions can consequently be used to identify their origin.

243 Additionally, the gamma ray energy can be used to infer information about the primary
 244 particle energy for a known production mechanism. Gamma rays are also well studied, and
 245 modern detectors boast high event rates: Fermi-LAT, for example, observes enough events to
 246 be capable of studying the temporal variability of sources on the timescale of approximately
 247 a day.

248 Gamma rays are not without their limitations, however. Note that while at the high-
 249 est energies gamma rays may be produced by both leptonic and hadronic processes, only
 250 hadronic processes are expected to additionally produce neutrinos and cosmic rays. For this
 251 reason, high energy gamma rays alone cannot be used to identify the source of high energy
 252 cosmic rays.

253 Additionally, high energy gamma rays may also interact electromagnetically with the
 254 ambient radiation in the universe, undergoing pair production (eq. 2.11):

$$\gamma_{VHE} + \gamma_{CMB} \rightarrow e^+ + e^- \quad (2.11)$$

255 This interaction can occur between high energy gamma rays and photons from the CMB,
 256 leading to an event horizon for gamma rays of a particular energy. Figure 2.5 shows this
 257 horizon as a function of distance and photon energy. The photon event horizon prevents us
 258 from studying the highest energy photons from many AGN, and there is a significant region
 259 of parameter space below the highest observed cosmic ray energy that cannot be examined
 260 with photons alone.

261 2.3 Neutrinos

262 2.3.1 Production Mechanisms

263 Neutrinos in astrophysical sources are produced via charged pion decay from pions pro-
 264 duced from cosmic ray interactions with either photons or protons. The charge of the pion
 265 determines whether more neutrinos or anti-neutrinos are produced (eq. 2.12 and 2.13):

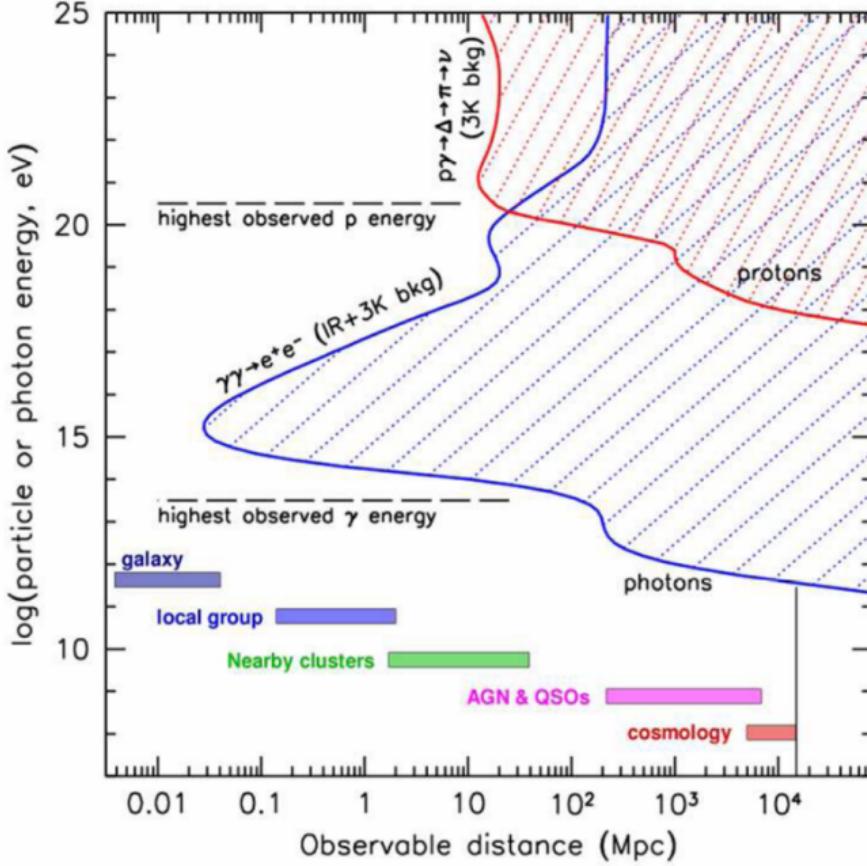


Figure 2.5: A plot of the photon and cosmic ray event horizons, showing the regions/energy scales visible using observations originating from photons (blue) and cosmic rays (red). Pair production of high energy photons with the ambient radiation in the universe prevents photon observations at long distances and high energies, including much of the energy scale below the highest observed cosmic ray energy. Also note that the photon event horizon can prevent us from studying the highest energy photons from AGN [10].

$$\pi^+ \rightarrow \nu_\mu + \mu^+ \rightarrow \nu_\mu + e^+ + \nu_e + \bar{\nu}_\mu \quad (2.12)$$

$$\pi^- \rightarrow \nu_\mu + \mu^- \rightarrow \bar{\nu}_\mu + e^- + \bar{\nu}_e + \nu_\mu \quad (2.13)$$

266 In either case, note that the expected flavor ratio of the neutrinos and anti-neutrinos
 267 produced at the source is $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. Combining this initial flavor ratio
 268 with neutrino oscillations over cosmic distances produces an expected observed astrophysical
 269 neutrino flavor ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$.

270 Neutrinos are also produced by cosmic rays interacting with the Earth's atmosphere. This
 271 is referred to as the atmospheric neutrino flux, and has contributions both from pion decay
 272 ($\pi^+ \rightarrow \nu_\mu + \mu^+$, $\pi^- \rightarrow \bar{\nu}_\mu + \mu^-$), as well as kaon decay ($K_L^0 \rightarrow \pi^\pm + e^\mp + \nu_e$, $K_L^0 \rightarrow \pi^\pm + \mu^\mp + \nu_\mu$,
 273 $K^+ \rightarrow \mu^+ + \nu_\mu$, $K^+ \rightarrow \pi^0 + e^+ + \nu_e$, $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$). Below 100 TeV, the resultant
 274 spectrum closely follows that of cosmic rays, with a spectral index of $\gamma_{atmos} = 2.7$. At higher
 275 energies the shower mesons are able to travel further in the atmosphere, giving them an
 276 increased opportunity to interact and steepening the spectral index to $\gamma_{atmos} = 3.7$. The
 277 atmospheric neutrino flux exists as an irreducible background for many studies of the origin
 278 of astrophysical neutrinos.

279 2.3.2 Observed Neutrino Spectrum

280 Both the astrophysical and atmospheric neutrino fluxes have been observed in data from
 281 the IceCube detector, with the astrophysical neutrino spectral index being measured to be
 282 $\gamma_{astro} \approx 2.28$ [11]. While astrophysical and atmospheric neutrinos are largely indistinguish-
 283 able on an individual event basis, the difference in spectra between the two populations
 284 ($\gamma_{astro} \approx 2.28$, $\gamma_{atmos} \approx 3.7$) allows us to distinguish between the two on a statistical ba-
 285 sis. Higher energy events are more likely to be astrophysical in origin, and an equal share
 286 of astrophysical and atmospheric neutrinos can be expected at ≈ 200 TeV. Currently, the
 287 source(s) of the astrophysical neutrino flux have yet to be identified. For this reason, the
 288 observed astrophysical neutrino flux is commonly referred to as the "diffuse" astrophysical
 289 neutrino flux (as opposed to a "point source" flux, where specific astrophysical objects that
 290 are producing neutrino are identified).

291 2.3.3 Neutrinos as Astrophysical Messengers

292 As noted in the previous section, hadronic processes in astrophysical sources can produce
 293 neutrinos via pion decay. Like gamma rays, neutrinos are electrically neutral, and thus
 294 their paths are not bent by magnetic fields. Unlike gamma rays, however, neutrinos do not

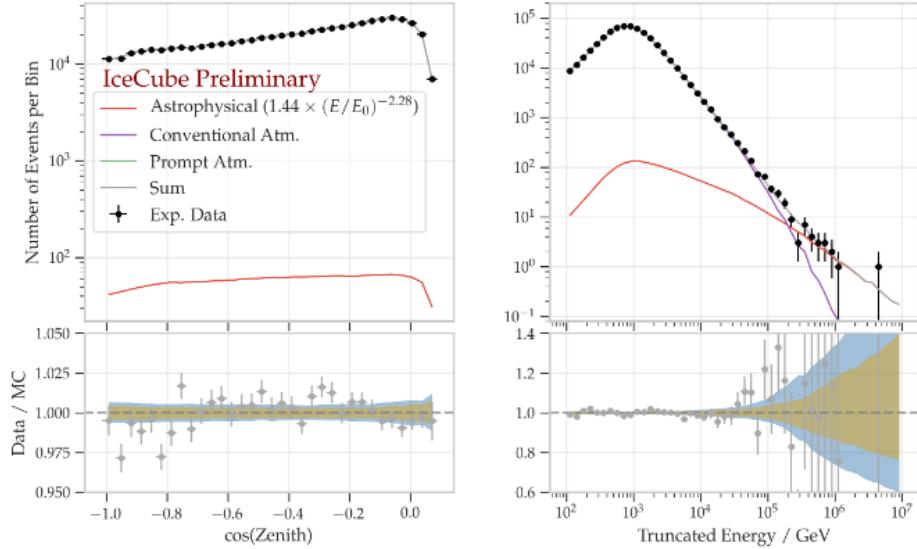


Figure 2.6: Observation of the astrophysical neutrino flux in through-going tracks detected by the IceCube Neutrino Observatory. The effect is most visible in the plot of the observed energy spectrum on the right, showing the atmospheric contribution (blue) and the astrophysical contribution (red). The excess of events at high energies is unable to be explained without including a non-zero astrophysical contribution [11].

295 interact electromagnetically, instead interacting only via the weak force. Because of this,
 296 the mean free path of neutrinos traveling through the universe is significantly greater than
 297 that of photons, allowing us to use neutrinos to probe regions normally opaque to gamma
 298 rays.

299 Astrophysical neutrinos are not without their drawbacks, however. The fact that neutrinos
 300 interact only weakly necessitates the construction of large detectors in order to observe
 301 an appreciable number of astrophysical events. Such detectors also observe a large, irre-
 302 ducible background of atmospheric neutrinos, particularly at lower (TeV) energies. For this
 303 reason, multi-messenger approaches that combine information from neutrinos and other as-
 304 trophysical messengers are commonly used to attempt to identify the sources of astrophysical
 305 neutrinos.

306 2.3.4 AGN as Neutrino Source Candidates

307 Since neutrinos can be used as an indicator of hadronic processes in astrophysical sources,
 308 identifying the source of the astrophysical neutrino flux is a promising route to also identi-
 309 fying the source of ultra-high energy cosmic rays. Though the source of the astrophysical
 310 neutrino flux has yet to be fully determined, in recent years there have been promising hints
 311 that extragalactic AGN are at least partially responsible.

312 An active galactic nucleus (AGN) is a compact region at the center of a galaxy displaying
 313 elevated electromagnetic emission, such that the additional luminosity cannot be attributed
 314 to stars. The luminosity is expected to originate from the accretion disk surrounding a
 315 supermassive black hole at the center of the galaxy, where conditions are thought to be ideal
 316 for high-energy particle acceleration and production.

317 The angle at which the AGN is viewed relative to its jet determines its exact classification,
 318 as can be seen in figure 2.7. If the AGN is viewed "down" the jet (i.e. the jet is directed at
 319 Earth), the AGN is seen to be particularly luminous and is referred to as a blazar. Blazars
 320 can be further divided into two subclasses: flat spectrum radio quasars (FSRQ), which
 321 have luminous broad emission lines and continuous thermal emission, and BL-Lacs, which
 322 have weak or absent broad emission lines. AGN viewed off-axis fall into a variety of other
 323 classifications, with further sub-classification being defined by the spectrum of the AGN in
 324 various wavelength bands.

325 2.3.4.1 TXS 0506+056

326 On September 22, 2017, the IceCube Neutrino Observatory detected a high energy neu-
 327 trino, IceCube-170922A, with an energy of approximately 290 TeV. The arrival direction of
 328 IceCube-170922A was consistent with the location of the blazar TXS 0506+056, and sparked
 329 multi-messenger followup from a variety of other telescopes and detectors. Notably, it was
 330 determined that TXS 0506+056 was flaring in gamma rays at the time IC-170922A was
 331 observed. The significance of the positional correlation of a high energy IceCube event and

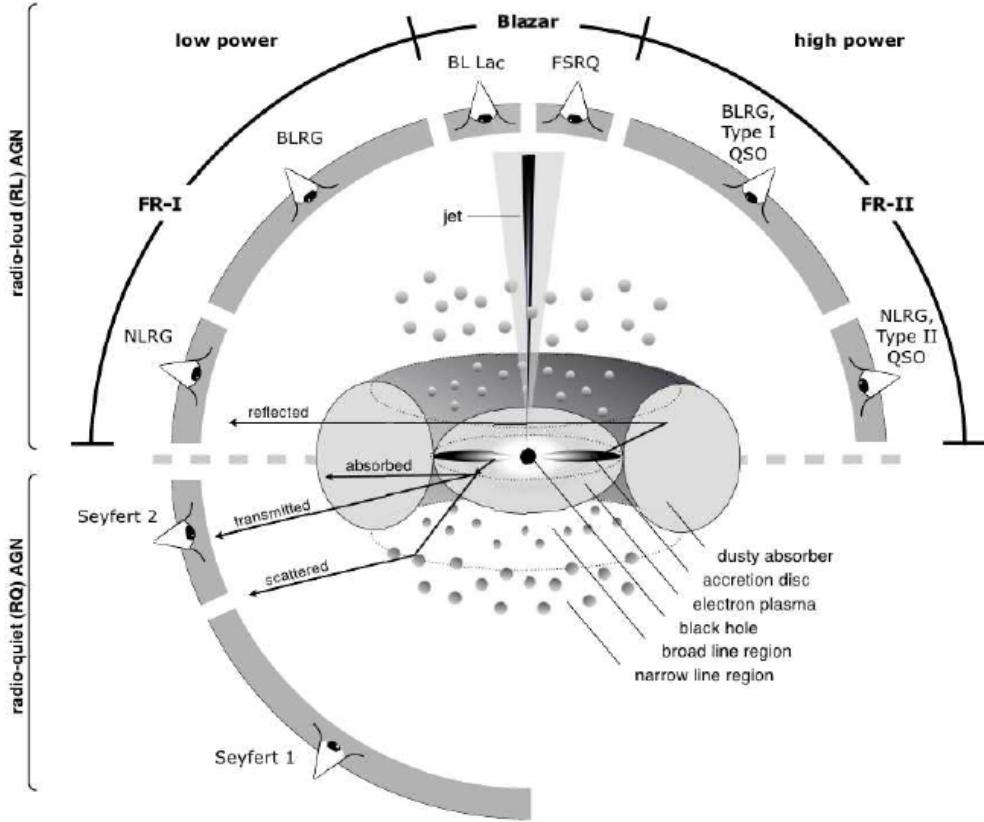


Figure 2.7: The unified AGN model, showing the various classifications of AGN for different observation angles. If the jet is directed at Earth, the object is viewed as a blazar, while other viewing angles can obscure the jet, leading to alternative classification (e.g. as a Seyfert galaxy) [12].

332 a flaring blazar was calculated to be 3.0σ , providing the first hint that blazars may be a
 333 source of high energy astrophysical neutrinos [2].

334 In addition to examining the multi-messenger correlation of a high energy IceCube event
 335 and a flaring blazar, an analysis of archival IceCube data was performed at the coordinates
 336 of IC170922A, examining the historical behavior of neutrino emission from this location of
 337 the sky over the previous nine years. In addition to the original high energy event observed
 338 in 2017, this archival analysis also revealed a 158 day period of elevated neutrino emission
 339 beginning in September 2014. The significance of this archival "neutrino flare" was calculated
 340 to be 3.5σ . Unlike the 2017 high energy event, the 2014 neutrino flare did not correspond
 341 to a flaring period for TXS 0506+056 in gamma rays. [3]

342 The combination of a 3.0σ multimessenger result with the 3.5σ "untriggered" neutrino

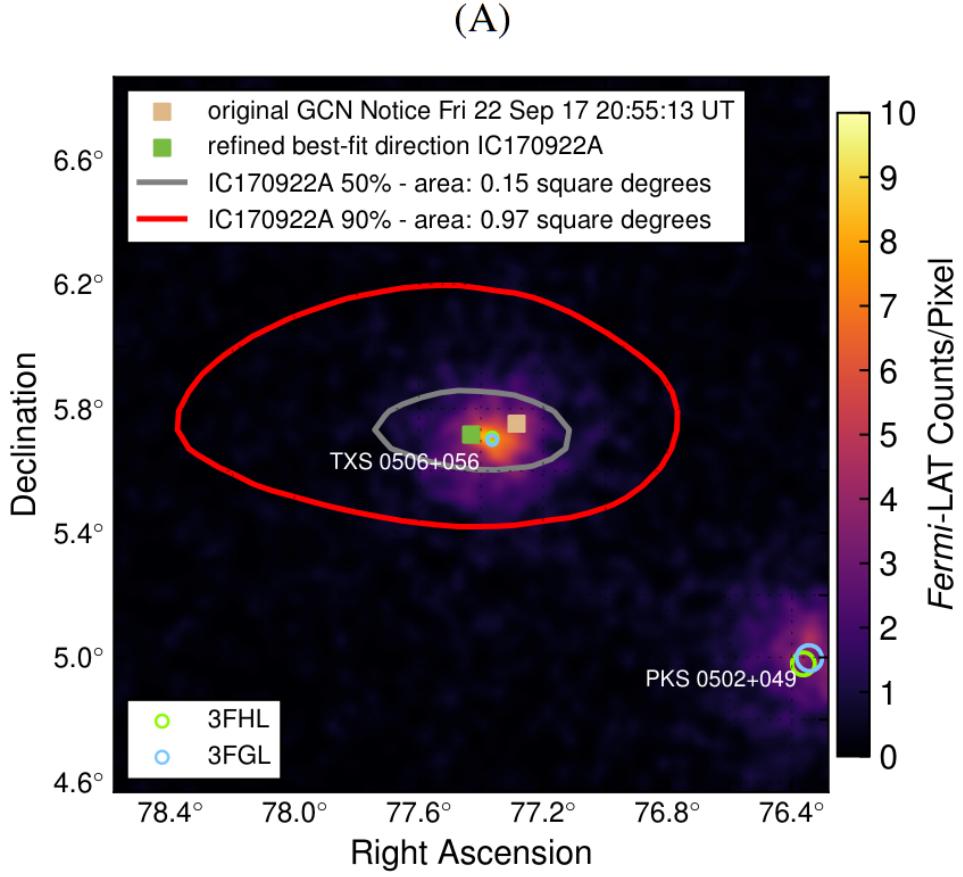


Figure 2.8: The Fermi-LAT skymap near the location of the observed high energy IceCube event IC170922A, with the 50% and 90% error contours associated with IC170922A plotted in grey and red, respectively. The significance associated with the spatial coincidence of the high energy event and TXS 0506+056 was found to be 3.0σ [2].

343 flare in 2014 suggest a significance of TXS 0506+056 as a neutrino source of at least 3σ .
 344 This makes for a strong argument for extragalactic blazars as interesting source candidates
 345 from the perspective of neutrino astronomy.

346 2.3.4.2 NGC 1068

347 In addition to the multimessenger and archival results associated with TXS 0506+056, the
 348 results of the 10-year time integrated IceCube analysis [4] also seem to suggest AGN as
 349 candidates for astrophysical neutrino emission. In this all-sky, untriggered, time integrated
 350 analysis, the most significant point in the northern sky appears to be spatially coincident

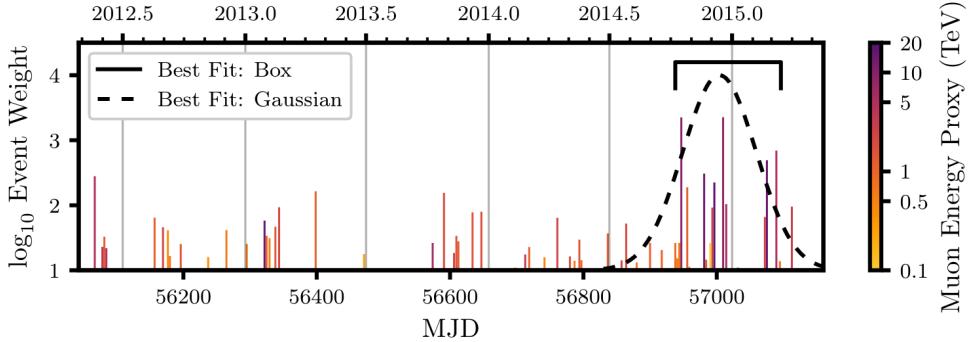


Figure 2.9: The result of the archival analysis of IceCube data near the location of TXS 0506+056, with the 3.5σ flare shown. This flare is separate from the high energy event observed in 2017, and there was no corresponding flare observed in gamma rays during this time period [3].

351 with the Seyfert II galaxy NGC 1068. Notably, NGC 1068 was also included in an associated
 352 time integrated catalog analysis. In this catalog analysis, the pre-trial significance of time-
 353 integrated neutrino emission from NGC 1068 was 1.8×10^{-5} , corresponding to a significance
 354 of 2.9σ . At a 14.4 Mpc distance, NGC 1068 is the most luminous Seyfert II galaxy detected
 355 by Fermi-LAT, and NGC 1068 had additionally been hypothesized as a candidate cosmic
 356 ray accelerator prior to this particular analysis [13][14][15].

357 It should additionally be noted that the catalog analysis mentioned above identified
 358 three other objects which, together with NGC 1068, collectively form a 3.3σ excess over
 359 the background expectation. These objects are the Seyfert II galaxy NGC 1068, the blazar
 360 TXS 0506+056, and the BL Lacs PKS 1424+240 and GB6 J1542+6129 [4], providing further
 361 indication of AGN as potential neutrino emitters.

362 2.3.5 Galactic Neutrino Sources

363 Though this work will primarily focus on searches for extra-galactic point sources of the
 364 astrophysical neutrino flux, potential neutrino emission from sources within our galaxy is
 365 an active area of study as well. Notably, one of the first confirmed sources of lower energy
 366 astrophysical neutrinos was Supernova 1987A [16], a supernova occurring within the Milky
 367 Way. Other galactic objects that are considered candidates for neutrino emission include

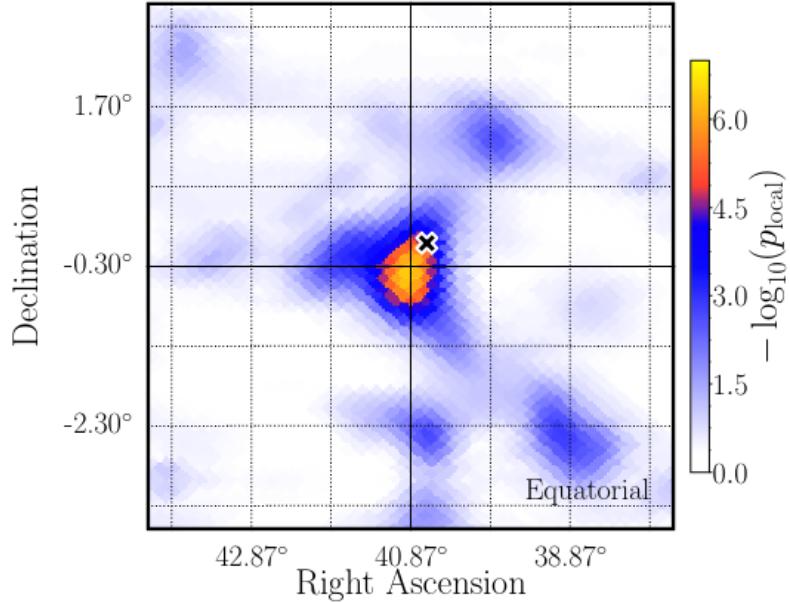


Figure 2.10: The results of the untriggered time integrated IceCube point source search analysis, showing the most significant spot in the northern sky, with the location of NGC 1068 plotted as the black "X" [4].

368 supernova remnants [17], as well as pulsar wind nebula [18], both of which are known to
 369 produce gamma rays.

370 The majority of the galactic plane lies in the southern sky. Since neutrino telescopes make
 371 use of the earth to block out atmospheric muon backgrounds (see subsequent chapter), this
 372 means that to best study the galactic plane, a neutrino observatory located in the northern
 373 hemisphere would be required. While there are several such telescopes either planned or
 374 under construction [19][20], they have yet to reach effective volumes comparable to the
 375 existing IceCube observatory (located in the southern hemisphere).

376 **2.3.6 Constraints on Neutrino Source Populations from Diffuse
 377 Measurements**

378 It is important to keep in mind that whatever the sources of astrophysical neutrinos may
 379 be, they must combine to reproduce the measured diffuse astrophysical neutrino flux. The

380 current best fit astrophysical neutrino spectrum, assuming a single power law in energy, is
 381 given as (eq. 2.14):

$$\frac{d\phi_{\nu+\bar{\nu}}}{dE} = 1.44^{+0.25}_{-0.24} \left(\frac{E}{100\text{TeV}}\right)^{-2.28^{+0.08}_{-0.09}} 10^{-18} \text{GeV}^{-1}\text{cm}^{-1}\text{s}^{-1}\text{sr}^{-1} \quad (2.14)$$

382 This places constraints on the density and luminosity of potential astrophysical neutrino
 383 source populations. If a candidate source population has sources that are too numerous
 384 and/or too bright, then that source population would produce a higher flux of astrophysical
 385 neutrinos than has been observed, and an explanation of the nondetection of the additional
 386 flux is required. Similarly, if sources are too sparse and/or too dim, then such a source class
 387 is incapable of explaining the entirety of the measured diffuse flux.

388 These constraints can be summarized in figure 2.11, showing the band of the measured
 389 diffuse flux in source density/luminosity space, as well as several commonly discussed source
 390 populations. It should be noted that in this plot, the luminosity values for various source
 391 populations are derived from the electromagnetic luminosity L_γ . In principle, the true ratio
 392 L_ν/L_γ remains unknown, and consequently the positions of sources along the luminosity axis
 393 may shift according to this value.

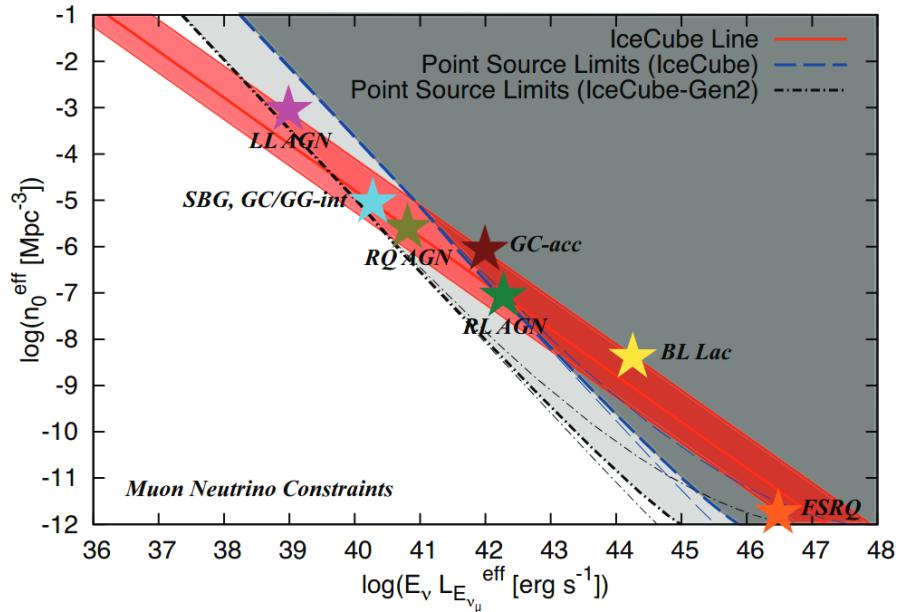


Figure 2.11: Constraints on potential populations of sources of the high energy astrophysical neutrino flux. The red band represents combinations of source density and luminosity that are consistent with the measured astrophysical neutrino flux. Source populations that lie above this band are too bright and/or too numerous, while source below this band are too dim/too sparse. Also shown are limits derived from the non-observation of neutrino sources in time-integrated analyses performed by the IceCube collaboration as of 2016, as well as a projected set of limits that would be associated with a non-observation from IceCube-Gen2 [21].

³⁹⁴ **Chapter 3**

³⁹⁵ **The IceCube Detector**

³⁹⁶ The IceCube Detector is a large (cubic kilometer scale) water cherenkov detector situated
³⁹⁷ underneath the ice at the south pole. Its large volume makes it ideal for studying high energy
³⁹⁸ (TeV and higher) neutrino events originating from either the atmosphere or astrophysical
³⁹⁹ sources.

⁴⁰⁰ **3.1 Detection Mechanism**

⁴⁰¹ The cross section for neutrino interaction in Earth can be seen in figure 3.1. These cross
⁴⁰² sections are small, but the Earth is also quite large. We can calculate the mean free path
⁴⁰³ for neutrinos traveling through Earth as (eq. 3.1):

$$\lambda = \frac{1}{\sigma_\nu \rho_{Earth}} \quad (3.1)$$

⁴⁰⁴ Using the neutrino cross section near 1 PeV ($\sigma_\nu \approx 10^{-33} \text{ cm}^2$), and the density of the
⁴⁰⁵ Earth ($\approx 5.5 \text{ g/cm}^3$, corresponding to a nucleon density of $\rho_{Earth} = 3.3 \times 10^{24} \text{ cm}^{-3}$), we
⁴⁰⁶ obtain an estimate of the neutrino mean free path of $\approx 3000 \text{ km}$. Since the diameter of the
⁴⁰⁷ Earth is approximately $1.3 \times 10^4 \text{ km}$, we can conclude that the Earth is opaque to high energy
⁴⁰⁸ neutrinos. We can detect the neutrinos that interacted in the Earth by way of identifying
⁴⁰⁹ their interaction products.

410 For the purposes of neutrino point source searches, we primarily focus on neutrinos
 411 interacting either through charged current (CC) or neutral current (NC) interactions. In
 412 both cases, the energy of the neutrinos observed by the IceCube detector is high enough
 413 that neutrinos interact through deep inelastic scattering with nucleons in the antarctic ice.
 414 In neutral current (NC) interactions, the process is mediated by a neutral Z boson, as shown
 415 on the left in figure 3.2. In this case, the product of the interaction is a neutrino, meaning
 416 the only visible signature of this interaction is the production of a hadronic particle cascade.

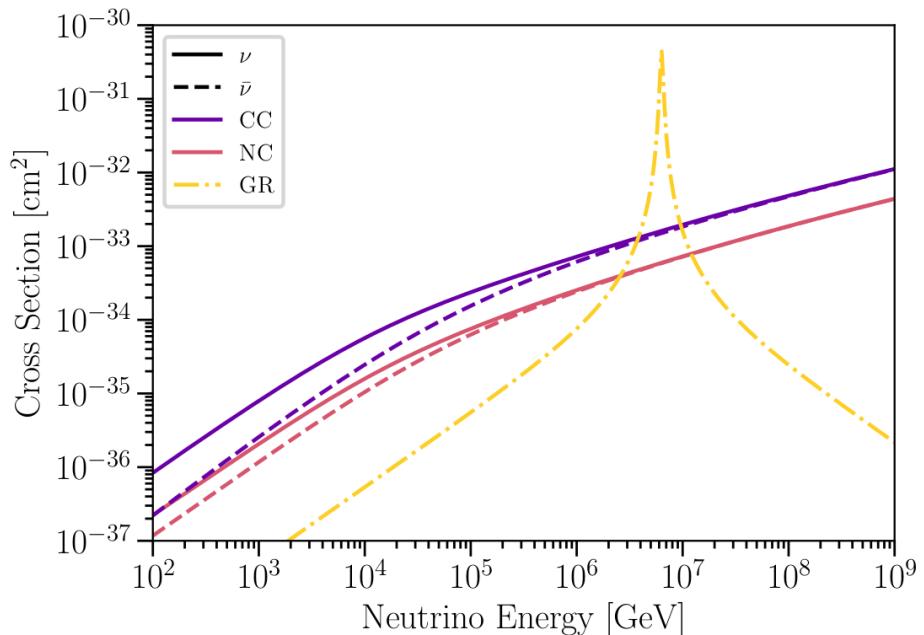


Figure 3.1: The cross sections of various neutrino interactions in the Earth. The yellow curve corresponds to Glashow Resonance [22], whereby an anti-electron neutrino combines with an atomic electron to produce a W+. These events are relatively rare in IceCube data, and are subsequently not used in the context of astrophysical neutrino source searches.

417 Charged current interactions are mediated by a charged W boson, and can be summarized
 418 as (eq. 3.2 and 3.3):

$$\nu_\ell + n \rightarrow \ell^- + p \quad (3.2)$$

$$\bar{\nu}_\ell + p \rightarrow \ell^+ + n \quad (3.3)$$

420 The Feynman diagrams for this process can be seen on the right in figure 3.2. Notably,
 421 charged current interactions produce an outgoing lepton ℓ in addition to a hadronic cascade.
 422 This is particularly useful in the case that the lepton produced is a muon, as muons can
 423 travel a sizeable distance (hundreds, or even thousands of meters) before decaying. If the
 424 lepton receives enough energy, it will subsequently produce cherenkov radiation, as it will
 425 be traveling faster than the local speed of light in ice. This radiation is emitted at an angle
 426 θ_c relative to the direction of travel, where θ_c is given by (eq. 3.4):

$$\cos(\theta_c) = \frac{1}{n\beta} \quad (3.4)$$

427 Where n is the index of refraction of the medium through which the particle is traveling
 428 (in this case ice), $\beta = \frac{v}{c}$, and v is the velocity of the traveling particle (the lepton, in this
 429 case). For ice, $n = 1.31$, and this emission angle is approximately 41 degrees. The number of
 430 photons expected per unit track length is given by the Frank-Tamm formula [23] (eq. 3.5):

$$\frac{dN}{dx d\lambda} = \frac{2\pi z\alpha}{\lambda^2} \sin^2(\theta_c) \quad (3.5)$$

431 Where z is the charge of the ionizing particle, λ is the wavelength of the emitted radiation,
 432 α is the fine structure constant ($\approx \frac{1}{137}$), and θ_c is the cherenkov angle given by eq. 3.4. Peak
 433 emission is found in the optical portion of the light spectrum, between 350 and 600 nm
 434 corresponding to a characteristic blue hue.

435 To summarize, neutrinos traveling through the antarctic ice will occasionally interact,
 436 producing child particles that will emit cherenkov radiation. We can then build an array
 437 of photon detectors to detect these photons, and subsequently infer information about the
 438 original incident particles.

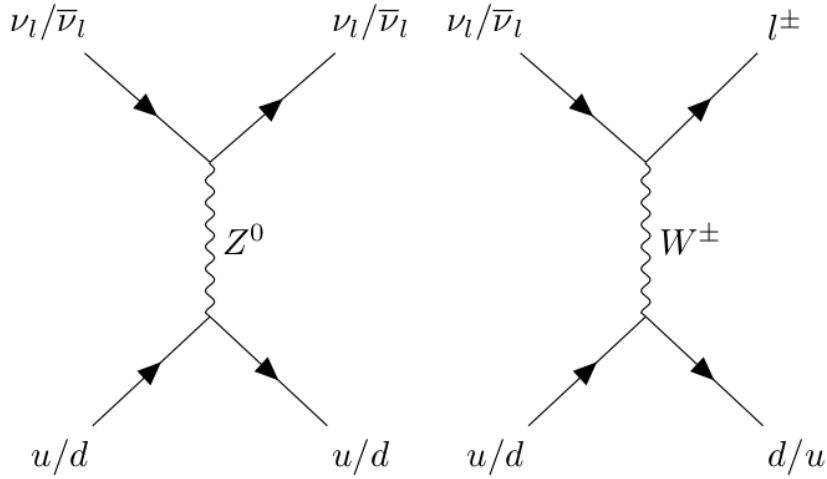


Figure 3.2: The Feynman diagrams corresponding to neutral current (left) and charged current (right) neutrino interactions. Charged current interactions can produce a detectable outgoing lepton in addition to a hadronic cascade, while neutral current interactions only produce a hadronic cascade.

439 3.2 The Physical Components of the IceCube

440 Detector

441 As outlined in the previous section, the strategy for detecting neutrinos with a water
 442 cherenkov detector is not to directly detect the neutrinos themselves, but rather to detect
 443 the cherenkov photons from the outgoing particles resulting from the neutrino interactions.
 444 The IceCube detector accomplishes this through the use of a large number of photomultiplier
 445 tubes (PMTs). At the very highest level, a PMT is a device that converts photons to an
 446 electrical signal that can then be read out by a set of associated electronics. The details of
 447 general PMT design and operation can be found in [24].

448 In IceCube, PMTs are housed in a single unit referred to as a digital optical module
 449 ("DOM"), seen in figure 3.3. Each module contains the PMT and its associated electronics,
 450 as well as a set of LED flashers that can be used to calibrate the detector once the DOMs have
 451 been lowered into the ice. DOMs are arranged onto 86 vertical strings, 78 of which contain
 452 60 DOMs spaced 17 meters apart along the length of the string. These strings are arranged

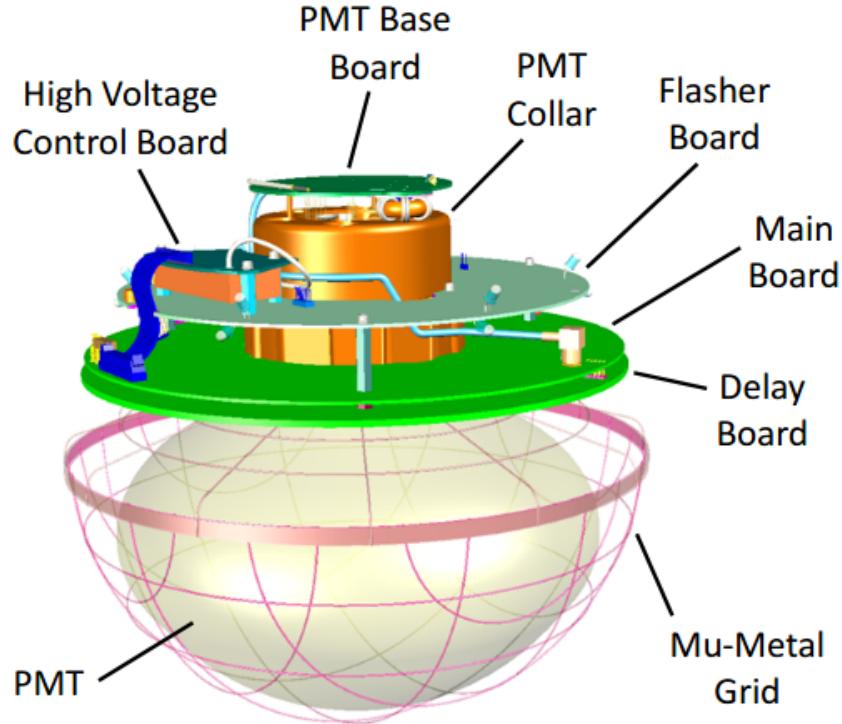


Figure 3.3: A diagram of a single IceCube digital optical module (DOM). The entire apparatus is encased in a pressurized glass sphere for protection (not shown). The PMT faces downward for optimal collection of photons from upgoing neutrino events. In addition to the PMT, each DOM also contains digitization electronics and an array of LED flashers that can be used for calibration [25].

453 into a hexagonal grid pattern under the south pole ice with a separation between strings
 454 of approximately 125 meters, resulting in a cubic kilometer of instrumented ice extending
 455 between 1400 and 2400m below the surface of the south pole ice sheet. The remaining 8
 456 strings form the DeepCore sub-array, a more densely instrumented region near the center of
 457 the detector, primarily used for studying lower (\approx 100s of GeV) scale events. PMTs that are
 458 part of the DeepCore portion of the detector have a higher quantum efficiency than those
 459 on the other 78 strings [26].

460 A diagram of the IceCube detector can be seen in figure 3.4. Due to the weakly interacting

461 nature of neutrinos combined with a power law spectrum, a large detector volume is critical
 462 to detecting the astrophysical neutrino flux. Prior to IceCube's construction, calculations
 463 indicated that a cubic-kilometer scale detector would be necessary to observe an appreciable
 464 event rate of astrophysical neutrinos [27][28]. The size and design of IceCube are well
 465 suited for detecting astrophysical neutrino events in the TeV+ energy range, as the large
 466 instrumented volume ensures a significant number of interactions in this energy range, and
 467 the detector's nanosecond scale timing resolution makes event reconstruction possible for
 468 events producing cherenkov radiation over the scale of ≈ 100 s of meters.

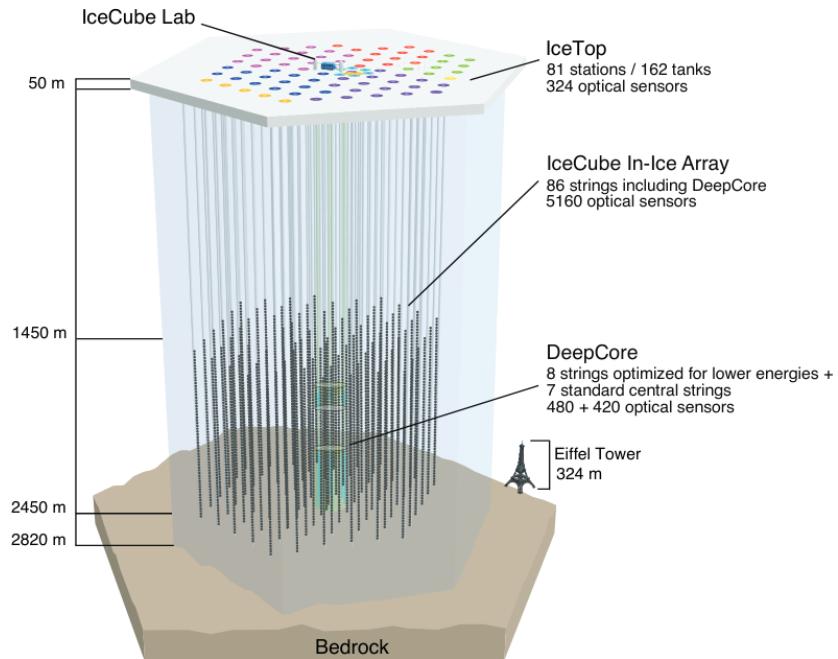


Figure 3.4: A diagram of the IceCube detector. The detector consists of 86 strings of DOMs lowered into holes drilled in the pattern of a hexagonal grid defined by a separation of 125 meters. Each string is formed by 60 DOMs connected by a cable, used for powering the DOMs and communication with the IceCube Lab (ICL) on the surface. The instrumented detector volume begins at a depth of 1450 meters, and extends down to 2450 meters, resulting in approximately a cubic kilometer of instrumented ice [25].

469 As the IceCube detector is situated in the south pole ice, a proper description of the
 470 optical properties of the ice is key for being able to accurately reconstruct events. While the
 471 ice is generally clear, it does contain several impurities and features that make it optically

472 nonuniform. The scattering and absorbtion of light are known to vary as a function of depth,
 473 as seen in figure 3.5. The large peak near a depth of 2000 meters is thought to correspond to
 474 a layer of dust deposited on the south pole ice sheet many years in the past, and is commonly
 475 referred to as "the dust layer" (physicists are not always the most creative with names).

476 In addition to varying as function of depth, the optical properties of the ice also vary as
 477 a function of azimuth. The largest effects along this axis are the ice tilt and ice anisotropy.
 478 The ice tilt refers to the layers of ice with similar optical properties being not perfectly
 479 horizontal, and results in a variation in scattering and absorption as light travels in different
 480 directions through the ice sheet. The ice anisotropy is similar, but has an observed additional
 481 axis of symmetry, leading to variations that are twice as frequent as a function of azimuth.
 482 Notably, while this effect was original observed in calibration data using LED flashers, it is
 483 visible in data from observed atmospheric muons, as seen in figure 3.6.

484 The final major optical nonuniformity of the detector ice is the hole ice. During the
 485 construction of the IceCube detector, holes were drilled in the south pole ice, and strings
 486 of DOMs were lowered down the appropriate depth. The water in these holes then refroze
 487 with different optical properties than the surrounding ice. The column of refrozen ice is
 488 referred to as the "hole ice", typically containing an inner region where air bubbles have
 489 been trapped referred to as the "bubble column". The hole ice has significantly shorter
 490 scattering distances, and is an area of active study within the IceCube collaboration.

491 3.3 Event Types

492 IceCube events can be generally be categorized into three different types: tracks, cascades,
 493 and double cascades. While neutral current events exclusively correspond to cascades,
 494 charged current events can produce any of the three, dependent on the variety of the out-
 495 going lepton. This gives IceCube the ability to identify the flavor of the incident neutrino,
 496 and this fact can be leveraged to do a large amount of interesting physics. However, for the
 497 purposes of this work, we are primarily interested in tracks, due to their excellent pointing

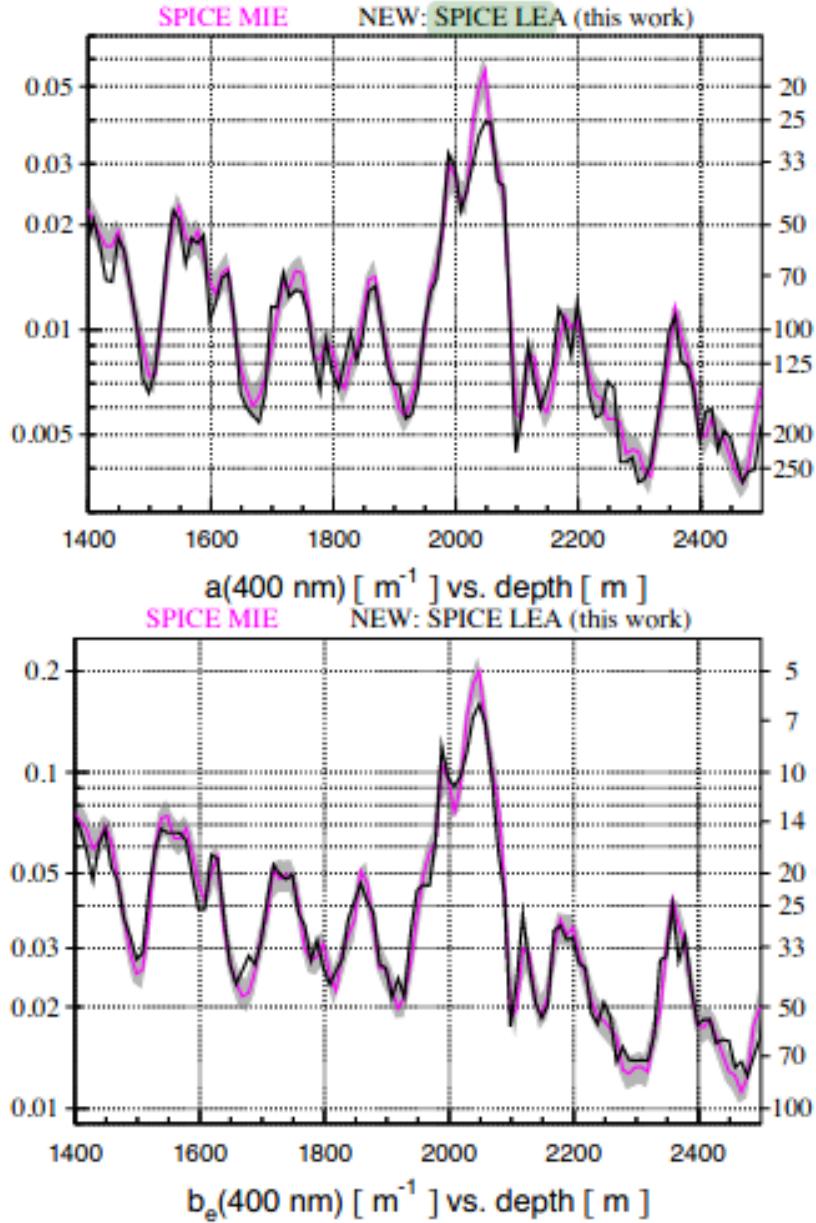


Figure 3.5: Plots of the scattering and absorption of light as a function of depth in the south pole ice. The peak near a depth of 2000 meters is referred to as "the dust layer", and is thought to correspond to a layer of dust deposited at the south pole at some point in the Earth's history. [29]

498 resolution as discussed below.

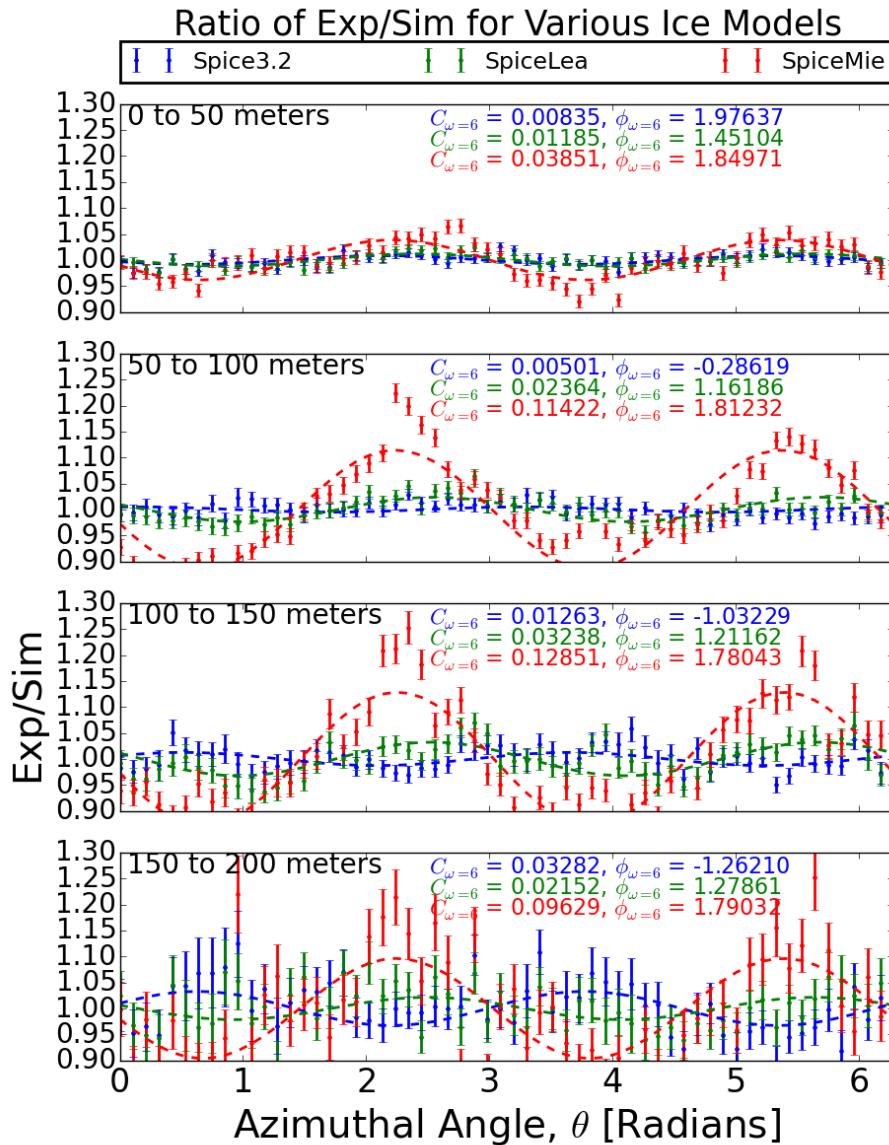


Figure 3.6: Plots of the ratio of light seen from atmospheric muons in experiment and simulation using various ice models (SpiceMie, SpiceLea, and Spice3.2), as a function of azimuthal angle between the muon track and the observing DOM. SpiceMie does not account for the ice anisotropy, resulting in a sinusoidal shape that grows with distance. SpiceLea and Spice3.2 both account for the anisotropy, and consequently the sinusoidal shape is reduced in amplitude when using these ice models.

499 3.3.1 Tracks

500 From equation 3.2 we can see that charged current interactions produce an outgoing lepton,
 501 which will in turn produce cherenkov radiation if it is energetic enough. In particular, if the
 502 outgoing particle is a muon, it will travel a significant distance before decaying or exiting the

503 detector. As the muon produces cherenkov radiation as it travels through the ice, the pattern
 504 of DOMs which detect this radiation will resemble a line, as shown in figure 3.7. These events
 505 are referred to as "tracks". If the neutrino interaction occurred inside the IceCube detector,
 506 then the track will appear to start within the detector, and the event can be referred to as
 507 a "starting track". If instead the neutrino interaction occurred outside the detector, and
 508 the resultant muon traveled through the detector, then the event is instead referred to as a
 509 "through-going track" (again, physicists are not the most creative). Through-going tracks
 510 can result from astrophysical neutrinos, as well as atmospheric neutrinos and atmospheric
 511 muons (muons resulting from cosmic ray interactions in the atmosphere). Starting tracks,
 512 however, can only be produced by muons originating from atmospheric or astrophysical
 513 neutrinos. For the purposes of this work, we will focus primarily on through-going tracks,
 514 however starting tracks are also scientifically interesting, and can be used for point-source
 515 and diffuse analyses of the neutrino flux as well [30].

IceCube Event Types

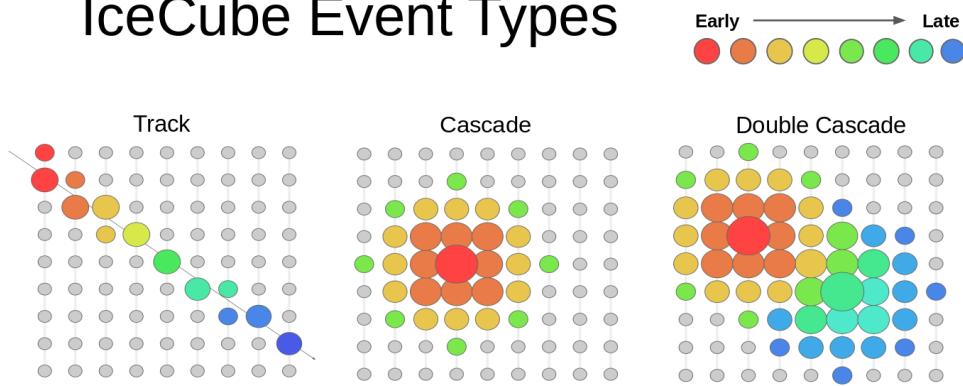


Figure 3.7: Cartoons in the x/z plane depicting the various event types seen by the IceCube detector. Grey circles are intended to represent individual DOMs, while colored circles correspond to DOMs that saw photoelectrons. The size of the circle corresponds to the amount of charge seen, while the color denotes the timing. Of most relevance to this work are tracks (left), as their long lever arm provides good pointing resolution.

516 Through-going tracks are notable for their good angular resolution (< 1 degree), par-
 517 ticularly at high energies (> 1 TeV), as seen in figure 4.1. This makes them an excellent
 518 candidate for attempting to do astronomy, as they can be expected to point back to their

519 point of origin with reasonable accuracy. Energy reconstruction of these events is somewhat
 520 more challenging, however, due to a combination of stochastic energy losses of the muon
 521 producing the cherenkov photons, and the fact that the event itself is often not entirely
 522 contained in the detector. The energy resolution of tracks in IceCube is approximately a
 523 factor of 2 at 10 TeV, though this increases for higher energy events [31].

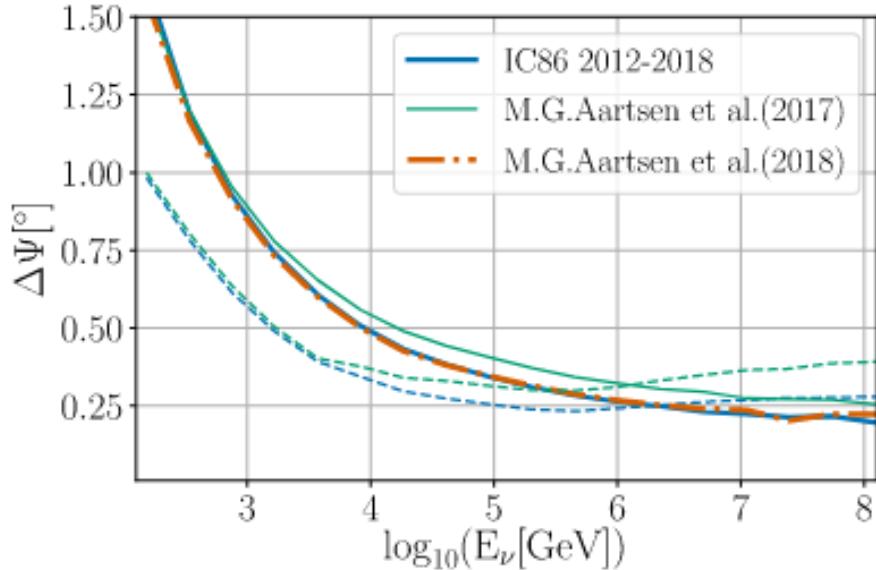


Figure 3.8: The median angle between simulated neutrino and reconstructed muon directions as a function of energy for various high-statistics IceCube samples composed of track events. The dark blue line corresponds to PSTracks v003p02, the light blue line corresponds to PSTracks v002p03, and the orange dashed line corresponds to NorthernTracks v002p06 (see section 1.7 for a description of these data samples). The solid lines describe events in the northern hemisphere, while the dashed lines are events in the southern hemisphere [4][32][33].

524 3.3.2 Cascades

525 In CC interactions, the initial interaction produces a hadronic cascade regardless of the vari-
 526 ety of outgoing lepton produced. If the outgoing lepton is an electron, the interaction length
 527 is relatively short, typically less than the 125 meter spacing of IceCube strings. Because
 528 of this, the resultant pattern of DOM hits in the detector is formed from the combined
 529 hadronic and electromagnetic showers. As the two showers occur at approximately the same

530 spatial position, this appears to the detector as a single cascade. While due to their spherical
 531 symmetry cascades tend to have poor (10 degree or more) pointing resolution, they have
 532 excellent energy resolution, as even for high energy events, the entirety of the cascade is often
 533 contained within the detector, making energy reconstruction as simple as counting the total
 534 number of cherenkov photons produced. The energy resolution of cascades is approximately
 535 15%, potentially even lower at higher energies [6].

536 3.3.3 Double Cascades

537 If the outgoing lepton from a CC interaction is a tau, then it may travel a relatively short
 538 distance before interacting again. If the initial tau is high enough energy, this can produce
 539 a signature of two spatially separated hadronic cascades in the detector. If the tau is lower
 540 energy, the two cascades may not be spatially resolvable, though may still be able to be
 541 identified by examining the waveforms observed by hit DOMs. This type of event was only
 542 recently observed in IceCube data [34]. Notably, tau events of these energies are not expected
 543 to be produced by atmospheric processes, and consequently this observation provides further
 544 evidence of an astrophysical neutrino flux.

545 3.4 Triggering

546 IceCube DOMs are subject to dark noise in the detector originating from PMTs emitting
 547 electrons from the cathode in the absence of an external photoelectron. This noise should
 548 not be correlated between nearby DOMs, and can be dramatically reduced by imposing the
 549 triggering requirement that neighboring DOMs also experience a signal with $1 \mu s$. These
 550 hits are classified as "Hard Local Coincidence" (HLC). An eight-channel simple majority
 551 trigger (SMT8) is used to trigger a readout window where DOM information is written to
 552 file. This trigger requires eight DOMs to have HLC hits within a $5 \mu s$ window.

553 Events are further filtered through several additional algorithms to pare down events
 554 based on temporal and spatial coincidence of DOM hits. The result is a "Level 2" event rate

555 of approximately 2.5 kHz. These events can be additionally filtered to select specifically for
 556 high quality track-like events (originating mostly from atmospheric muons, but also from
 557 atmospheric and astrophysical muon neutrinos as well), reducing the event rate to Hz. This
 558 level of data is often referred to within the collaboration as "Muon Level 3", and primarily
 559 consists of through-going track-like events.

560 3.5 Directional Reconstruction

561 In this section we discuss algorithms for reconstructing the direction of through-going track
 562 events observed by the IceCube detector. Different algorithms may be used for cascades,
 563 though understanding these is unimportant for the remaining content of this thesis. Those
 564 who are interested in the reconstruction of cascade and double cascade events can refer to
 565 [6].

566 3.5.1 LineFit

567 An initial guess for the reconstructed direction of the track can be obtained by ignoring the
 568 geometry of the cherenkov cone and the optical properties of the ice, and assuming a plane
 569 wave of light traveling along a straight line in the detector. The location of each DOM that
 570 observes photons (\mathbf{r}_i) can then be written as a line (eq. 3.6):

$$\mathbf{r}_i = \mathbf{r} + \mathbf{v} \cdot t_i \quad (3.6)$$

571 Where t_i is the time that the i th DOM observes photons, \mathbf{r} is the vertex of the track,
 572 and \mathbf{v} is the velocity of light in the ice. A χ^2 fit can then be preformed to determine the
 573 free parameters \mathbf{r} and \mathbf{v} (eq. 3.7):

$$\chi^2 \equiv \sum_{i=1}^{N_{tot}} (\mathbf{r}_i - \mathbf{r} - \mathbf{v} \cdot t_i)^2 \quad (3.7)$$

574 This can be solved analytically, resulting in fitted vectors for \mathbf{r} and \mathbf{v} (eq. 3.8 and 3.9):

$$\mathbf{r} = \langle \mathbf{r}_i \rangle - \mathbf{v} \cdot \langle t_i \rangle \quad (3.8)$$

$$\mathbf{v} = \frac{\langle \mathbf{r}_i \cdot t_i \rangle - \langle \mathbf{r}_i \rangle \cdot \langle t_i \rangle}{\langle t_i^2 \rangle - \langle t_i \rangle^2} \quad (3.9)$$

575 This corresponds to a vertex (\mathbf{r}) and a direction (\mathbf{v}) describing the path of the particle.

576 Further discussion on this topic can be found in [35].

577 3.5.2 SplineMPE

578 An improved reconstruction of the particle direction can be obtained by iterating on the
 579 initial LineFit reconstruction described above. The LineFit result is treated as a seed for
 580 a likelihood based reconstruction that takes the cherenkov angle and ice properties into
 581 account. A description of the specific likelihood components may be found in section 3 of
 582 [35]. In the simplest implementation of this likelihood method, only information from the
 583 first photoelectron observed by a particular DOM is used. This is referred to as a single
 584 photoelectron fit, or "SPEFit". For events that produce multiple hits on a single DOM,
 585 this information can be included in the likelihood as well, producing a multi-photoelectron
 586 (MPE) fit. Since photons arriving after the first are likely to have experienced at least some
 587 scattering, a proper description of the ice properties is necessary for an accurate MPE fit.
 588 This is incorporated into the likelihood described in [35] via the use of tabulated timing
 589 and light yield distributions for various DOM/photon configurations given an ice model
 590 developed from fits to LED flasher data [36]. The information in these tables is stored via
 591 a multi-dimensional spline, allowing these tables to be used directly as PDFs in the MPE
 592 likelihood, hence the name for this variety of reconstruction: "SplineMPE".

593 3.5.3 Paraboloid

594 There is some inherent uncertainty associated with the observation and reconstruction of a
 595 particular event. For an event originating from a particular true position, the reconstructed

position is expected to be drawn from distribution centered on the true position. Properly describing this distribution is key to characterizing the per-event uncertainty associated with the directional reconstruction, and is important for obtaining accurate results in a point source analysis.

A semi-analytic description of the angular error associated with a particular event can be obtained from the likelihood map associated with the direction reconstruction outlined in the previous section. An ellipse can be fit around the global minimum of the likelihood map, describing a 1σ containment region. The ellipse is parameterized by two variables, σ_1 and σ_2 describing the scales of the two axes of the ellipse, and an average circularized error can be computed as (eq. 3.10)[37]:

$$\sigma_{evt} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}} \quad (3.10)$$

This description can be further improved by comparing this error with the "true error" obtained from examining the difference between the simulated event direction and the corresponding reconstructed direction. By multiplying the above circularized error by the ratio of the reconstructed error to the "true" error, we remove potential bias due to effects not accounted for by the directional reconstruction algorithm. For example, one such effect would be the angle between the incident neutrino and the outgoing muon. Since the reconstruction algorithm only reconstructs the direction of the track associated with the muon, the direction of the neutrino is still technically unknown. While at high energies the muon and its parent neutrino can be assumed to be colinear, this angle becomes a significant source of uncertainty below 1 TeV.

3.6 Energy Reconstruction

While the direction of track event in IceCube can be reconstructed with good precision using the methods described previously, reconstructing the energy is more difficult. The primary reason for this is simply a lack of information: since many tracks originate from

620 events neutrino interactions outside of the instrumented volume, the entirety of the muon
 621 track is not contained within the detector. This prevents the detector from behaving as
 622 a calorimeter, and estimates must be made to extrapolate the initial event energy based
 623 on the portion of the track observed. Similar to the directional reconstruction, an energy
 624 reconstruction of a particular event can be obtained using a likelihood based approach based
 625 on the individual DOM observations.

626 At its core, this likelihood approach assumes that the number of detected photoelectrons
 627 (k) is expected to be poisson distributed with a mean directly proportional to the energy:
 628 $\lambda = E\Lambda$ (eq. 3.11), where Λ is the number of photons the event produces per unit energy:

$$\mathcal{L} = \frac{(E\Lambda)^k}{k!} e^{-E\Lambda} \quad (3.11)$$

629 Maximizing this likelihood over all j DOMs that see photoelectrons results in the relation
 630 (eq. 3.12):

$$E = \sum k_j / \sum \Lambda_j \quad (3.12)$$

631 Which is largely a statement that the event energy is proportional to the number of
 632 observed photons, scaled by some factor that is the sum of the light yield scaling functions
 633 Λ_j . These functions depend on a variety of factors including the geometry and ice optical
 634 properties. Much like in the case of the directional reconstruction, these functions are often
 635 obtained from tabulated monte carlo data, smoothed with a multi-dimensional spline.

636 This likelihood can then be applied to calculate the energy loss rate ($\langle dE/dx \rangle$) of a par-
 637 ticular muon. Above 1 TeV, this energy loss rate is roughly proportional to the muon energy.
 638 A robust estimate of this rate can be obtained by fitting segmented energy losses along the
 639 muon track using the methodology described in [38]. Once a list of segmented energy losses
 640 is calculated, there are several different approaches to converting to a reconstructed energy.

- 641 • **MuEX** takes the average of the segmented energy losses and uses that as an estimate
 642 of the energy

- 643 • **TruncatedEnergy** first removes the largest 40% of energy losses before calculating
 644 an average, ideally reducing the variance in the calculation of $\langle dE/dx \rangle$.

645 A comparison of these approaches can be found in [38], and both methods provide approx-
 646 imately 35% precision in $\log_{10} E$ for an initial muon energy of 10^4 GeV, improving slightly
 647 to approximately 30% in $\log_{10} E$ at higher energies.

648 3.7 IceCube Event Samples Overview

649 As the Icecube detector is capable of doing a wide variety of science, there exists a plethora
 650 of event samples used within the collaboration. Even within the context of searches for point
 651 sources of astrophysical neutrinos, there exist several event samples of IceCube events, each
 652 with their own selection criterion and set of reconstructed parameters. In many cases, the
 653 distinguishing features between event selections used in point source analyses are purity and
 654 event rate. An ideal point source event sample would have both a high purity of astrophysical
 655 events in addition to a high event rate. Unfortunately, this is not possible with the current
 656 iteration of the IceCube data, as we observe relatively few events above 100 TeV (where
 657 the highest purity of astrophysical events can be achieved), and below 100 TeV there is a
 658 significant irreducible background of atmospheric neutrinos.

659 For this reason, point source event samples in the IceCube collaboration generally fol-
 660 low two different philosophies. "Low statistics/high purity" samples contain relatively few
 661 events, but the events in these samples have a high probability of being astrophysical in
 662 origin [30] [39]. By contrast "high statistics/low purity" samples contain a higher number
 663 of events, including additional astrophysical events. However in doing so, these samples also
 664 include significantly more atmospheric (background) events [11] [4]. In the context of point
 665 source searches, these samples typically rely on advanced statistical methods in their analy-
 666 sis pipeline to distinguish between clustered astrophysical signal and isotropic atmospheric
 667 background.

668 Even within the category of high statistics/low purity samples used for neutrino source
 669 searches, there exist multiple samples that are regularly used within the IceCube collabora-
 670 tion. The following sections will briefly outline three such samples that are relevant to the
 671 analyses presented later in this work.

672 **3.7.1 PointSourceTracks v002p03**

673 Starting with Muon level 3 data, this sample applies an additional BDT to attempt to select
 674 for well-reconstructed muon neutrino interactions from mis-reconstructed atmospheric muon
 675 background. The variables used in this BDT were associated with a track-like event topology,
 676 and the BDT was trained using both background data, as well as simulation of both an $E^{-2.0}$
 677 and $E^{-2.7}$ signal. The result is a high-statistics sample of mostly track-like neutrino events
 678 in the northern sky, consisting of both atmospheric and astrophysical neutrinos [33]. Events
 679 in this sample use a SplineMPE reconstruction ("plain" settings, corresponding to a balance
 680 of computation time and precision) for the event direction, and a MuEX reconstruction for
 681 the event energy.

682 This sample covers seven years of IceCube data (2008-2015), and was used for the histor-
 683 ical untriggered flare analysis of TXS0506+056 [3], where a 3.5σ neutrino flare was identified
 684 in 2014.

685 **3.7.2 PointSourceTracks v003p02**

686 Though this sample is oftentimes treated as a "successor" to PointSourceTracks v002p03, it
 687 is in many ways a completely new event selection. This sample covers 10 years (2008-2018)
 688 of IceCube data. Like v002p03, this sample also starts with Muon level 3 data, however here
 689 the BDT is trained to additionally reject cascade-like events in addition to accepting track-
 690 like events. Additionally, an improved directional reconstruction is used: the SplineMPE
 691 reconstruction algorithm is applied twice, with the second application using the results of the
 692 first as a seed, and additionally including the energy estimation. This results in improved

693 angular resolution [40].

694 This sample additionally requires that events pass some precuts prior the the application
 695 of the BDT. Events are only selected if they satisfy the conditions below:

696 • The length of empty track (the portion of the reconstructed track that does not have
 697 any associated DOM hits) must be $\leq 400\text{m}$

698 • The reconstructed track length must be at least 200m

699 • The number of hit DOMs must be ≥ 12

700 • The number of DOMs that observe direct (non-scattered) photons must be ≥ 6

701 • $\cos(\theta_{geo}^2) < 0.2$, where θ_{geo} is the angle between independent reconstructions calculated
 702 for the first and second half of the track only. For a high quality track, both the first
 703 half reconstruction and the second half reconstruction should lie almost parallel.

704 This sample also has different requirements for events observed from the southern sky.

705 The minimum track length and maximum empty track length pre-cuts are applied as in
 706 the northern sky, in addition to an additional cut that the initial estimated uncertainty
 707 ($\sigma_{paraboloid}$) must be less than 5 degrees. A cut on the likelihood reconstructions is also
 708 enforced, requiring $R \log(\mathcal{L}) > 9$, where $R \log(\mathcal{L})$ is the "reduced log likelihood", where the
 709 maximum likelihood value, \mathcal{L} is divided by the number of degrees of freedom, n_{dof} ($n_{dof} = 5$
 710 in this particular case). Events in the southern sky are also required to have hits on more
 711 than 5 different IceCube strings if the number of direct DOM hits was fewer than 12 [40].

712 This sample is notable as it was the sample used for the 10-year all-sky IceCube time
 713 integrated analysis, where NGC 1068 was identified as having a significance of 3σ [4]. This
 714 sample has also been publicly released for use outside the IceCube collaboration [31].

715 **3.7.3 NorthernTracks v002p06**

716 The NorthernTracks sample is a high statistics, low purity sample of through-going tracks
 717 that was originally developed for the purpose of performing a diffuse fit of the atmospheric
 718 and astrophysical neutrino spectrum. As such, this sample has relatively good data/MC
 719 agreement (as this is essential to the diffuse fit analysis pipeline), but is restricted to only
 720 events in the northern sky. Additionally, this sample does not make use of data from
 721 DeepCore DOMs, a decision that was made in order to homogenize the detector. This
 722 sample covers 8 years of livetime (2009-2017) has been used for point source analyses as well
 723 as diffuse studies of the neutrino spectrum [32].

724 The precuts for this sample in the northern sky are identical to PointSourceTracks
 725 v003p02, however this selection then uses two separate BDTs to further filter events: one
 726 BDT to select for track-like events, and a separate BDT to reject cascade-like events. In
 727 practice the application of these two BDTs is similar, but not perfectly identical, to the
 728 application of the single BDT in PointSourceTracks v003p02. Also of note is that the BDTs
 729 used in the NorthernTracks selection are trained exclusively using simulation, for both signal
 730 and background.

731 This sample uses a SplineMPE directional reconstruction (with "max" settings, priori-
 732 tizing precision over computation speed), and a TruncatedEnergy estimator as the energy
 733 reconstruction.

734 A comparison of the three samples outlined here can be seen in table ??

	PSTracks v2	PSTracks v3	Northern Tracks
Pre-cuts	No	Yes	Yes
BDT 1	Selects tracks	Selects tracks and rejects cascades	Selects tracks
BDT 2	None	None	Rejects cascades
Signal training set	Simulation	Simulation	Simulation
Background training set	Data	Data	Simulation
Direction reconstruction	SplineMPE ("plain")	SplineMPE ("plain") $\times 2$	SplineMPE ("max")
Angular error estimator	Paraboloid	Paraboloid	Paraboloid
Energy estimator	MuEX	Yes	TruncatedEnergy
DeepCore included?	Yes	Yes	No
Livetime	7 years (2008-2015)	10 years (2008-2018)	8 years (2009-2017)

Table 3.1: A comparison of several similar high statistics/low purity event samples used by the IceCube collaboration

735 **Chapter 4**

736 **Statistical Methods in Neutrino**

737 **Astronomy**

738 Point source searches in neutrino astronomy are essentially weighted clustering analyses:
739 each event is determined by a set of coordinates (right ascension, declination, and time),
740 and a weight (energy), and the task is to determine if high weight points are clumped
741 beyond what is expected purely from statistical fluctuations. Most attempts to do this make
742 use of a similar approach, using data-driven background estimation and a likelihood-based
743 estimator of clustering. This section outlines the general framework of such an analysis, as
744 well as several variants of the likelihood-based clustering estimator that is commonly used.

745 **4.1 Clustering Analysis Outline for a Generalized TS**

746 There are many ways of constructing a test statistic in neutrino astronomy, in addition to
747 a variety of types of clustering to look for (for example, clustering near various source cata-
748 logs, clustering in space and time, clustering with weights according to source/event energy
749 weights, clustering about an extended spatial feature such as the galactic plane). However,
750 almost all clustering analyses in this area share several features in their construction:

- 751 1. A test statistic is formulated, which ideally tests the degree of clustering of astrophys-

752 ical events near a particular source candidate location. This test statistic is typically
753 a likelihood-based test statistic of a form similar to that which is described in the
754 following section, however this test statistic is often modified to reflect the specifics of
755 the clustering hypothesis being tested.

- 756 2. A background test statistic distribution is calculated using maps generated from data
757 that has the right ascension values of events randomized. This destroys any cluster-
758 ing that may have already been present, providing an excellent representation of the
759 null hypothesis (a purely diffuse astrophysical neutrino flux with no spatial or tem-
760 poral clustering). As this background estimation is data-driven, it is robust against
761 unknown backgrounds that may be present in the sample. Scrambling the data does
762 not affect the sample event content, and consequently the underlying distributions of
763 event declination, energy, and arrival time are unchanged.
- 764 3. Alternative hypothesis test statistic distributions may be obtained from simulations of
765 signal events. If the analysis is not sensitive to the total astrophysical neutrino flux,
766 simulated events can simply be added to the background maps generated above. If the
767 analysis is sensitive to the total astrophysical flux, then more sophisticated methods
768 (such as moving existing events instead of injecting new events) may be required. Once
769 maps containing signal have been generated, they can be used both to characterize the
770 performance of the analysis, as well as compute upper limits in the event of a null
771 result.
- 772 4. The performance of the test statistic being used is typically evaluated using the *sen-*
773 *sitivity* and/or *discovery potential*. The *sensitivity* refers to the amount of signal that
774 needs to be injected before 90% of the test statistic distribution is greater than the
775 median of the background test statistic distribution. Similarly the *discovery potential*
776 refers to the amount of signal that needs to be injected before 50% of the test statis-
777 tic distribution is greater than the $N\sigma$ significance threshold in the background test
778 statistic distribution (N is typically either 3 or 5, but can be other values. This is

779 typically clarified by stating that the value reported is the "N-sigma discovery potential".
 780 "Amount of signal" is intentionally ambiguous here, as different analyses are
 781 sensitive to different types of signal. The sensitivity and discovery potential can be
 782 cast in terms of any number of variables that may be relevant to a particular anal-
 783 ysis. For example, in the time-integrated case, these values are often reported as a
 784 flux, however time-dependent analyses may report a fluence, and analyses sensitive to
 785 source populations may even report their sensitivity as a curve in the space of source
 786 density and source luminosity.

787 5. The test statistic is calculated for the observed data sample, and this observed test
 788 statistic is compared to the distribution generated in step 2 to calculate a p-value.
 789 This p-value is then used to either accept/reject the null hypothesis of no clustering.

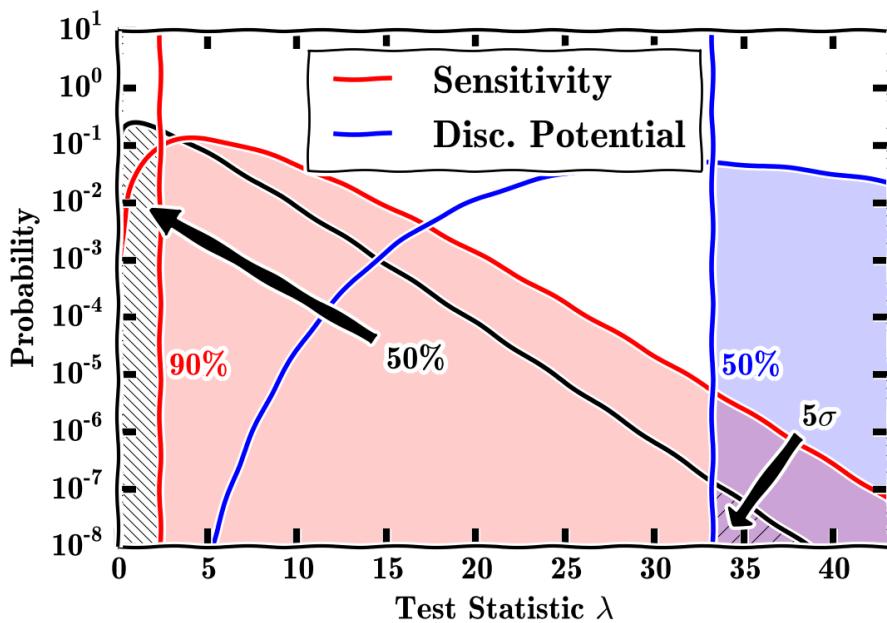


Figure 4.1: A toy plot showing the definitions of sensitivity and discovery potential. [41]

790 4.2 Unbinned Time Integrated Methods

791 Astrophysical point sources may be distinguished from the atmospheric neutrino background
 792 by way of clustering: if astrophysical sources are present in the data, events should be clus-
 793 tered near one another, while atmospheric events are expected to be isotropically distributed.
 794 Additionally, since high energy neutrino events are more likely to be astrophysical in origin,
 795 clustering of these events can be an even stronger indicator of the effect of neutrino point
 796 sources in the data. In short, we would like to construct a test statistic that reflects these
 797 observations. This can be done by way of a likelihood-based test statistic, using a likelihood
 798 composed of signal and background PDFs that describe the spatial and energy properties of
 799 events relative to a source candidate location. For a neutrino sample composed of N total
 800 events, the likelihood of the data for some number of signal (clustered astrophysical) events,
 801 n_s is (eq. 4.1)[42]:

$$802 \quad \mathcal{L}(n_s, \gamma) = \prod_{i=1}^N \left[\frac{n_s}{N} S_i + \left(1 - \frac{n_s}{N}\right) B_i \right] \quad (4.1)$$

803 Where here, S_i and B_i are PDFs that describe the spatial and energy distributions
 804 of signal and background events relative to the source candidate location. S_i and B_i are
 themselves composed of spatial and energy components (eq. 4.2 and 4.3).

$$805 \quad S_i = R(r_i) \times \mathcal{E}(E_i, \delta_i | \gamma) \quad (4.2)$$

$$806 \quad B_i = \frac{1}{\Omega} \times \mathcal{E}(E_i, \delta_i | Atm_\nu) \quad (4.3)$$

807 Note that the source spectral index, γ , enters as a free parameter in the energy portion
 of the signal PDF.

808 For the background PDF, the construction is relatively straightforward: background
 809 events originate from all directions equally, so the only effect that produces anisotropies is
 810 the detector acceptance. Since the IceCube detector acceptance does not vary as a function
 811 of right ascension, we can define B_i for a particular declination band to simply be $1/\Omega$,

where Ω is the solid angle of a declination band centered on the source candidate declination. Similarly, the background energy PDF can be obtained by measuring the energy distribution of observed events as a function of declination. 2D histograms of event counts as a function of declination and event energy are assembled, and these distributions are then splined to create PDFs that can be used to evaluate B_i for events at an arbitrary declination and energy.

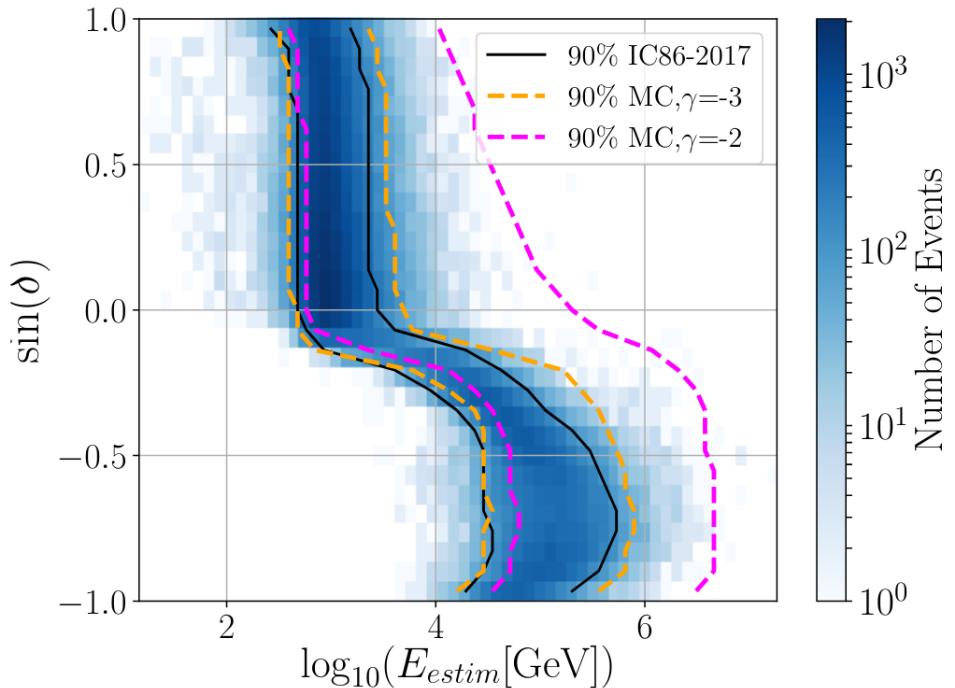


Figure 4.2: A histogram showing the distribution of events as a function of declination and energy for 1 year of the PointSourceTracks v003p02 data sample. The 90% energy range for the data is shown in black, and the 90% energy ranges of simulated astrophysical signal Monte-Carlo(MC) for both a E^2 and E^3 spectrum are shown in pink and orange, respectively[31]

Unlike background events, signal events are expected to be clustered near the source candidate location. For this reason, the spatial component of the signal PDF, $R(r_i)$, is assumed to be a 2-D gaussian centered on the source candidate location (eq. 4.4):

$$R(r_i) = \frac{1}{2\pi\sigma_i^2} e^{-\frac{r_i^2}{2\sigma_i^2}} \quad (4.4)$$

821 Where r_i is the angular distance between the i th event and the source candidate location
 822 and σ_i is the event angular error.

823 The energy component of S_i is obtained from simulation: Simulated events are weighted
 824 according to a particular astrophysical spectral index, creating a 2D PDF describing the sum
 825 of simulated event weights as a function of declination and energy. These maps are then
 826 divided by the background PDF described above to create a map describing the "signalness"
 827 of events at a particular energy and declination, given a spectral index. As these maps are
 828 created for a range of spectral index values (typically ranging between $\gamma = 1$ and $\gamma = 4$),
 829 a 3-D PDF of event energy, declination, and spectral index hypothesis results from this
 830 process. This is precisely the function $\mathcal{E}(E_i, \delta_i | \gamma)$ that enters the likelihood, and can be used
 831 to fit for γ , given events observed at declination δ with energy E_i .

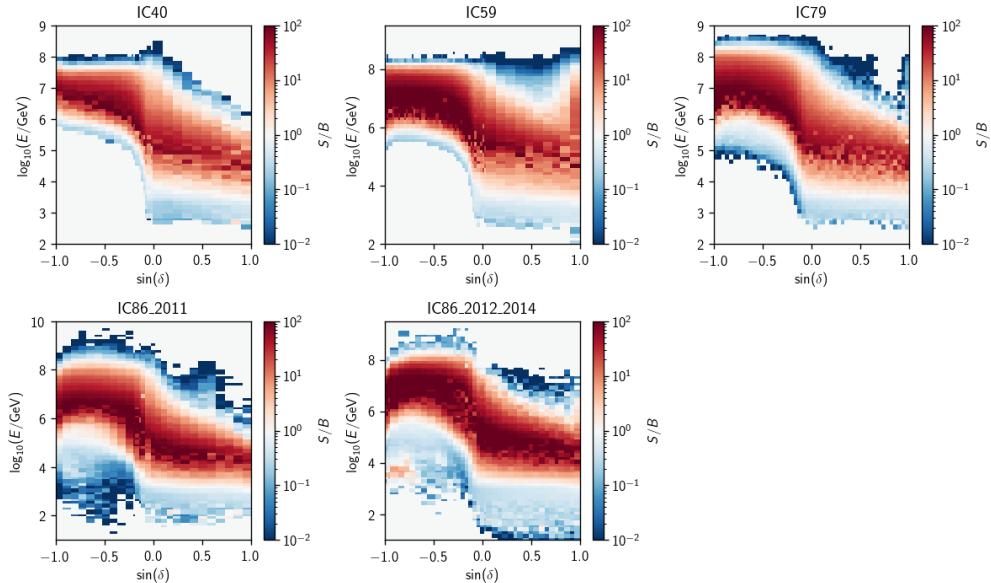


Figure 4.3: Signal PDFs associated with a $\gamma = 2.0$ spectral index hypothesis, assembled from PSTracks v002p03. Similar PDFs are constructed for spectral indices ranging from $\gamma = 1.0$ to $\gamma = 4.0$, thereby constructing a 3-D PDF that can be used in the point source likelihood.

832 The likelihood described in eq. 4.1 can be maximized as a function of s and γ , resulting
 833 in best fit parameters \hat{n}_s and $\hat{\gamma}$. We can then compute a test statistic for a given data
 834 sample from the likelihood ratio (4.5):

$$TS = -2 \log \frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s, \hat{\gamma})} \quad (4.5)$$

835 This test statistic can then be used in the generalized framework described in the sections
 836 above to test the hypothesis of spatial clustering of astrophysical neutrino events near a
 837 particular source location.

838 This framework can be applied to multiple locations at once, and the results combined by
 839 summing the individual test statistics calculated at each location (note that this is equivalent
 840 to calculating the product of the likelihoods). This process is often referred to as "source
 841 stacking", and can be used to improve sensitivity to dim sources, provided there are multiple
 842 emitters in the source candidate list.

843 Other methods of combining information across multiple source locations exist as well. In
 844 particular, the binomial test is a popular way of doing this. Give a list of source candidates
 845 and their associated p-values, we can search for the most significant combination of source
 846 candidates by employing the binomial test-statistic (eq. 4.6)

$$p(k) = \sum_{i=k}^{N_{eff}} \binom{N_{eff}}{i} p_k^i (1-p_k)^{N_{eff}-i} \quad (4.6)$$

847 Where $p(k)$ is the p-value associated with combining the results of the k most significant
 848 sources, N_{eff} is the effective number of trials (often simply equal to the total number of
 849 source candidates), and p_k is the p-value of the k th most significant source. The minimum
 850 of $p(k)$ can then be computed and treated as a test statistic, to obtain a p-value associated
 851 with the best-fit number of sources (k_{min}) in a particular catalog.

852 4.3 Unbinned Time-Dependent Methods

853 In addition to being clustered in space, astrophysical neutrino events may also be clustered in
 854 time (a neutrino "flare"). Accounting for this temporal clustering may allow us to identify
 855 sources that are insignificant under a corresponding time integrated analysis. Studying

856 the temporal variation of source candidates can also inform us of the specifics of the source
 857 dynamics of particle production, as the time scale of observed flares is related to the physical
 858 scale of the astrophysical objects in which particle production is occurring.

859 **4.3.1 Optimizing for a Single Flare**

860 We can modify our time-integrated likelihood to account for potential temporal clustering
 861 as well by simply appending a temporal PDF to the existing PDFs that describe the spatial
 862 and energy components of the analysis (eq. 4.7, 4.8):

$$S_i = R(r_i) \times \mathcal{E}(E_i, \delta_i | \gamma) \times \mathcal{T}(t_i) \quad (4.7)$$

$$B_i = \frac{1}{\Omega} \times \mathcal{E}(E_i, \delta_i | Atm_\nu) \times \frac{1}{\delta T} \quad (4.8)$$

863 Where δT is the full livetime of the sample, and the associated temporal background PDF
 864 is $1/\delta T$, as the background event rate should be constant. In practice, this is assembled in a
 865 data-driven manner by measuring the data sample event rate in each of the 8-hour segments
 866 ("runs") that are used to segment the data. In this way, seasonal variations of the detector
 867 event rate are accounted for by the likelihood, as the flare candidates are always being
 868 compared to the local event rate in time.

869 The temporal contribution to the signal PDF, $\mathcal{T}(t_i)$, can take various forms depending
 870 on the particular shape that one assumes for the time profile of flare candidates, but the
 871 most basic temporal profile can be used is a box (eq. 4.9):

$$\mathcal{T}(t_i | t_0, \Delta t) = \begin{cases} 0 & t_i < t_0 \\ \frac{1}{\Delta t} & t_0 \leq t_i \leq t_0 + \Delta t \\ 0 & t_i > t_0 + \Delta t \end{cases} \quad (4.9)$$

872 This box-shaped temporal PDF essentially acts as a mask, filtering for events that occur
 873 within the time window between t_0 and $t_0 + \Delta t$. The events within this time window then

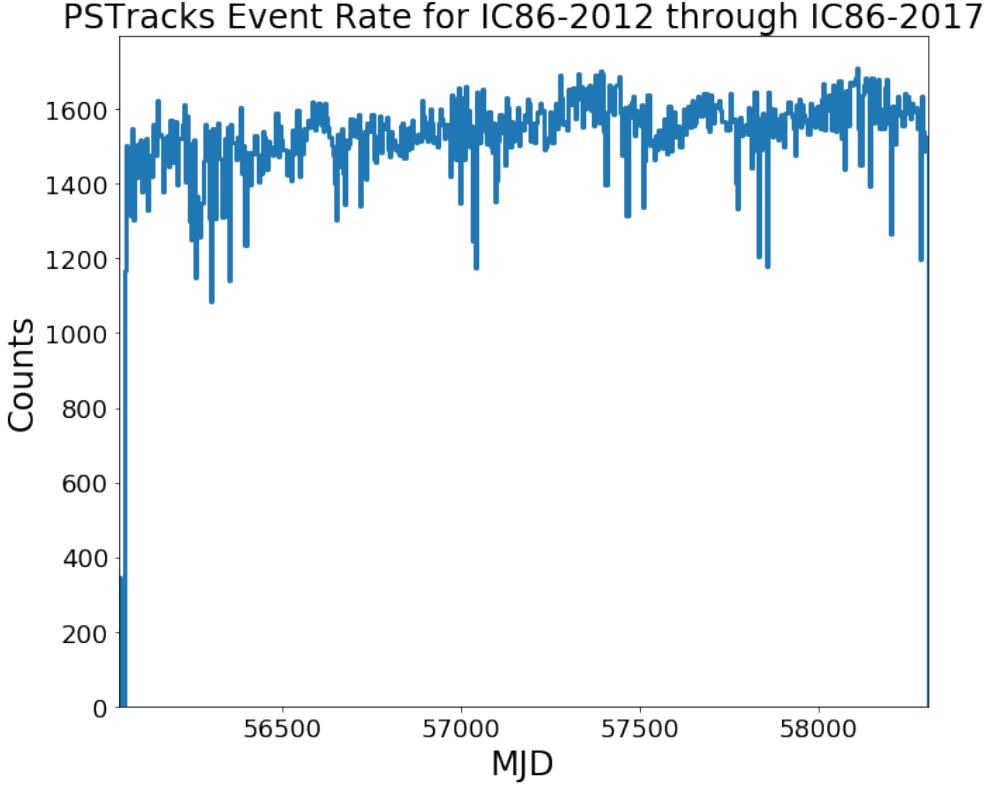


Figure 4.4: The event rate of the IC86 seasons of the PointSourceTracks v3 data sample. This event rate can be used to generate a background temporal PDF in the time-dependent clustering likelihood.

874 contribute to the likelihood in a similar manner to the time-integrated case. Note that by
 875 adding this temporal PDF to the likelihood, we have also introduced 2 new parameters that
 876 the likelihood can be maximized with respect to: t_0 and ΔT , which describe the flare start
 877 time and duration, respectively.

878 Similar to the time-integrated case, a likelihood ratio test statistic can be assembled from
 879 this likelihood, however this test statistic can be further improved in the time-dependent case
 880 by accounting for the duration of flare candidates that were scanned over (eq. 4.10):

$$TS = -2 \log \left[\frac{\Delta T}{\hat{\Delta}t} \frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s, \hat{\gamma}, \hat{t}_0, \hat{\Delta}t)} \right] \quad (4.10)$$

881 Where the factor $\frac{\Delta T}{\hat{\Delta}t}$ can be interpreted as a trial factor that accounts for the fact
 882 that there are significantly more short flare candidates that can be tested than long ones
 883 (e.g. there is only 1 potential flare candidate with $\Delta t = \Delta T$, but there are many flares that

884 can be made with $\Delta t = \Delta T/100$). This factor normalizes the test statistic scale between
 885 short and long flares [42].

886 In practice, this process is extremely computationally intensive due to the number of
 887 likelihood minimizations that need to be performed. This can be mitigated by seeding flares
 888 with events that already have a high probability of being signal in origin, based off their
 889 energy and arrival direction. An ensemble of seed events can be defined to the set of events
 890 with S_i/B_i greater than some threshold value, where S_i and B_i refer to the signal and
 891 background PDF components of the time integrated likelihood (describing only the spatial
 892 and energy properties of contributing events). Typically this threshold is set to be relatively
 893 low ($S_i/B_i > 1$), however higher values can also be used if computational requirements are
 894 particularly stringent (e.g. if running over the entire sky).

895 Many analyses compute the test statistic in eq. 4.10 for a variety of flare candidates,
 896 and then use the flare with the maximum test statistic as the "best-fit" flare. The test
 897 statistic associated with this "best-fit" flare is then compared to a distribution of "best-fit"
 898 flares obtained from the background case (generated from data scrambled in right ascension),
 899 resulting in a p-value that describes the strength of the clustering hypothesis relative the
 900 the null hypothesis of no clustering.

901 In simulated signal trials, the likelihood-based flare test statistic is able to reconstruct
 902 the injected number of signal events in each flare with reasonable accuracy, and the per-flare
 903 spectral index fits show similar agreement as well.

904 4.3.2 Fitting Ensembles of Multiple Flares

905 As the IceCube detector collects more data, it becomes increasingly useful to be able to test
 906 the hypothesis of multiple neutrino flares that may have occurred at a particular location.
 907 Fitting only the largest flare is advantageous if the source flare rate is expected to be low
 908 enough that sources only flare once over the data-taking period, however if sources flare
 909 multiple times the single-flare analysis potentially misses a significant amount of useful

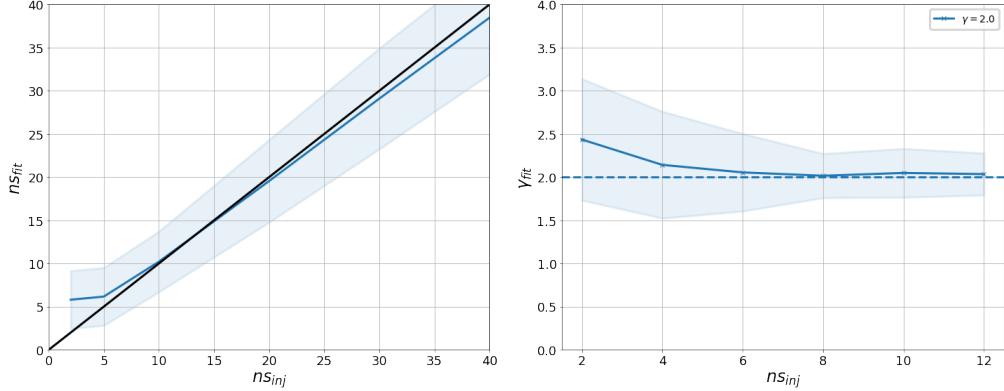


Figure 4.5: The best fit number of signal events in a flare, and the best fit spectral index versus the true number of signal events injected in simulated trials. On average, for flares with $>\approx 5$ events, the time-dependent likelihood is able to accurate estimate the true number of signal events.

910 information. By incorporating information from multiple flares at a particular location,
 911 an analysis can be sensitive to a smaller individual flare intensity.

912 The process of fitting multiple flares at a particular source candidate location is similar
 913 to the single-flare maximization procedure described above. Flare candidates are seeded by
 914 events with a high S_i/B_i ratio near the candidate location. For each flare candidate, the
 915 test statistic described in equation 4.10 is calculated. Unlike the single flare case, these
 916 flare candidates are then ordered by decreasing test statistic value, and any flares that
 917 overlap in time with a different flare that itself has a higher test statistic are removed.
 918 Additionally, any flares with a test statistic < 0 are removed. What remains is a temporally
 919 decorrelated ensemble of flare candidates, each with $TS > 0$. This ensemble functions as
 920 a neutrino "flare curve" (similar to light curves produced by photon-based telescopes), that
 921 describes the temporal variability of the signal originating from a source candidate location.
 922 A "multi-flare" test statistic describing the significance of the temporal variability of this
 923 source candidate can then be calculated as the sum of the individual flare candidate test
 924 statistics (eq. 4.11):

$$\widetilde{TS} = \sum_{j=0}^{N_{\text{flares}}} TS_j \quad (4.11)$$

925 Where here, j is an index that refers to the individual flares that compose the neutrino
 926 flare curve. This test statistic can then be compared to an ensemble of similar test statis-
 927 tics generated from right-ascension scrambled (background) data to obtain a final p-value
 928 describing the significance of the set of flares that were fit at a particular source candidate
 929 location.

930 An estimation of the total number of signal events can be obtained in a similar manner
 931 (eq. 4.12), where the total number of signal events associated with a particular source
 932 candidate is just the sum of the best-fit number of n_s in each contributing flare candidate:

$$\tilde{n}_s = \sum_{j=0}^{N_{\text{flares}}} \hat{n}_{sj} \quad (4.12)$$

933 Note that this similar to time-integrated source stacking mentioned above, however in-
 934 stead of stacking test statistics associated with spatially distinct locations, we are instead
 935 stacking spatially and temporally distinct *flares*.

936 The multi-flare algorithm is, in some sense, inclusive of the single flare algorithm: by
 937 fitting all the flares at a source candidate location, we have obviously also fit for the largest
 938 flare. It is thus trivial to obtain the single-flare significance once the multi-flare result is
 939 obtained. In order to do this, simply compare the highest flare candidate test statistic that
 940 was obtained with a distribution of single-flare test statistics obtained in a similar manner
 941 from background (right-ascension scrambled) data.

942 In addition to calculating the local significance of the largest flare, the local significance
 943 of the other flare candidates composing the flare curve fit by the multi-flare algorithm can
 944 also be obtained in a similar manner. We can define the "local significance" of a particular
 945 flare (not necessarily the largest) to be the fraction of flares in the background distribution
 946 with $TS_j > TS_{j,\text{observed}}$. Note however, that the calculation of multi-flare significance is done
 947 in the space of TS_j , not the space of the corresponding local significances p_j . This means
 948 that the multi-flare significance is *not* simply the product of the component flare p_j 's, as each
 949 flare is not entirely statistically independent from the others (e.g. once the largest flare is

950 fit, the remaining available livetime in which other flares can be fit is reduced by an amount
 951 equal to the duration of the largest flare.)

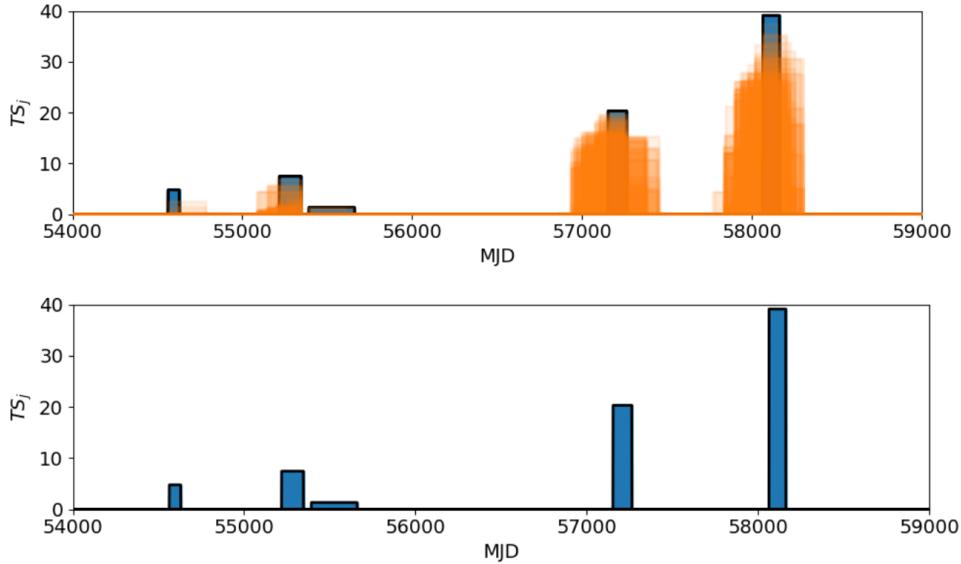


Figure 4.6: A graphical example of the progression of the multi-flare decorrelation algorithm. Colored boxes represent flare candidates that were tested, seeded by events with high S_i/B_i ratios. The height of each box corresponds to the individual flare candidate test statistic, TS_j . Orange flares overlap with another flare with a higher test statistic, and are consequently removed, leaving only the blue flares. The test statistics of the blue flares are then summed, and can be used for hypothesis testing of flaring neutrino emission at this source candidate location.

952 As mentioned above, the multi-flare algorithms is particularly useful in the case of several
 953 similarly sized flares. In this case, while the single flare algorithms will identify the correct
 954 number of events in the largest flare, the estimation of the total number of signal events
 955 associated with the source candidate will be incorrect (as there is a non-negligible portion
 956 of events that belong to flares that were not identified by the single flare algorithm). By
 957 contrast, the multiflare algorithm improves the estimation of n_s in the case of multiple
 958 similarly sized flares, as signal events in all flare candidates (not just the largest one) are
 959 able to contribute. This improvement is shown in figure 4.7.

960 As an example of a case where the multi-flare algorithm is advantageous, consider the
 961 case shown in 4.8, which has 3 flares injected at MJD=55246.3, 55807.4, and 57632.7 at
 962 a test location of $(\text{RA}, \text{Dec}) = (77.45^\circ, 5.61^\circ)$. The single flare method fits only the flare

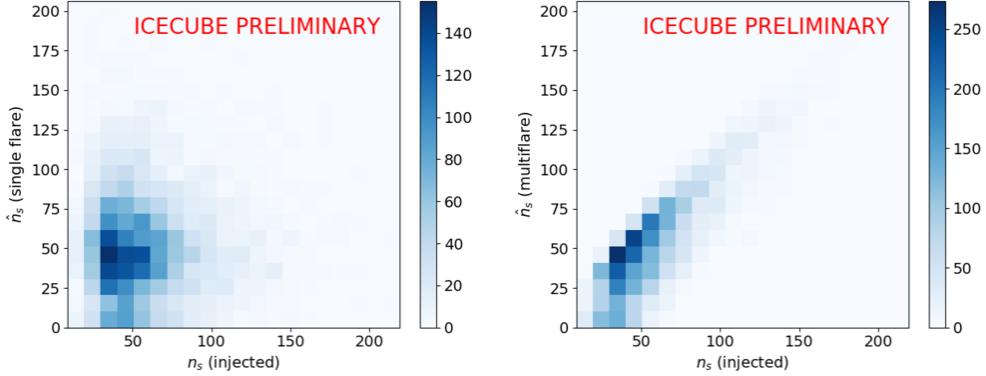


Figure 4.7: Left: the injected vs. fit number of signal events when attempting to use the single flare algorithm to describe an ensemble of neutrino flares. Right: The same plot, but using the multiflare estimation of n_s that accounts for signal events in all flare candidates.

963 at MJD = 55246.3 (with $\text{TS} = 22.66$), while the multiflare method fits all flares together,
 964 with a multi-flare test statistic of $\widetilde{\text{TS}} = 60.52$. While the largest individual flare only has
 965 a significance of 3.29σ , by combining information from all the flares that were fit at this
 966 location, we arrive at a multi-flare significance of 4.7σ .

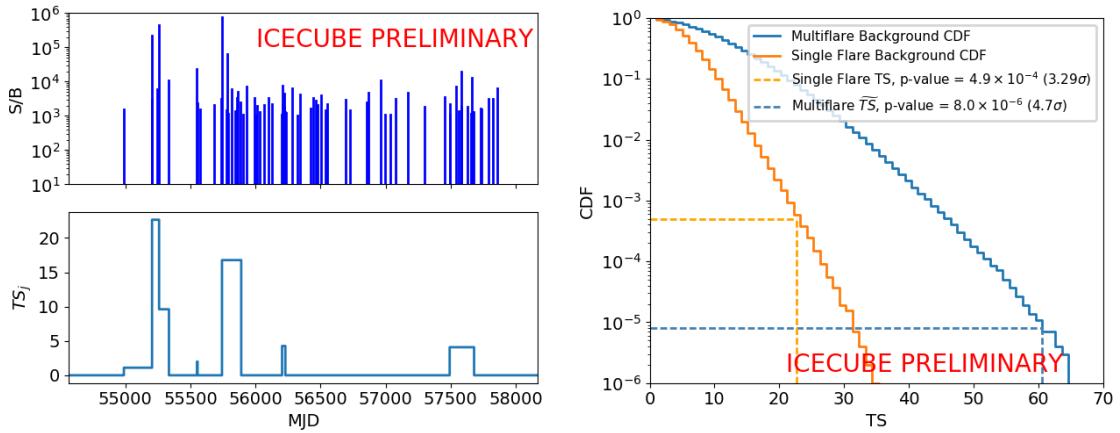


Figure 4.8: Left, top: Events that pass the S/B threshold cut for generating windows, for a source located at $(\text{ra}, \text{dec}) = (77.45^\circ, 5.61^\circ)$. There are 3 injected flares, centered at MJD=55246.3, 55807.4, and 57632.7. Left, bottom: The single flare method fits only the flare at MJD = 55246.3 (with $\text{TS} = 22.66$), while the multiflare method fits all flares together, with a global test statistic of $\widetilde{\text{TS}} = 60.52$. Right: The background test statistic distributions for a single source, located at declination = 5.61° . The background multiflare test statistic distribution is shown in blue, while the single flare method is shown in orange. The vertical lines represent the test statistics associated with the flare curve on the left. The single flare p-value for this source is 4.9×10^{-4} (3.29σ), while the multiflare p-value is 8.0×10^{-6} (4.7σ).

967 **Chapter 5**

968 **Applications of the Multi-flare
969 Algorithm to Source Catalogs**

970 The multi-flare algorithm introduced in the previous section may be applied to an ensemble
971 of sources that share common features. This is common practice in neutrino astronomy, as
972 examining emission from a catalog allows us to explore the possibility of neutrino emission
973 from a class of sources, rather than a specific individual source. Here, we explore two
974 catalogs designed to explore source features related to those associated with the analysis of
975 TXS 0506+056. Namely, the fact that the analysis was initially triggered by a high-energy
976 neutrino event, and also the fact that TXS 0506+056 is a blazar. For the former, we assemble
977 a catalog of high energy IceCube events to treat as "sources" (a "self-triggered" catalog
978 analysis), while for the latter we use the pre-existing catalog of Fermi 3LAC blazars [43].

979 In both cases, the multi-flare algorithm is applied at each source candidate location,
980 resulting in a multi-flare test statistic (and corresponding pre-trial p-value) associated with
981 each source candidate. To test for potential sub-populations of strong emitters within the
982 catalog, we additionally calculate a best-fit number of multi-flare sources via iteratively
983 summing the sources with the largest test statistics (5.1). For a given data set, the sources are
984 ordered by their multiflare test statistic, \widetilde{TS} . A p-value for $k = 1, 2, 3, \dots N_{srcs}$ is calculated,
985 and subsequently the k that produces the minimum p-value is selected (k_{best}).

$$\widetilde{TS}_{all} = \sum_{m=0}^{k_{best}} \widetilde{TS}_m \quad (5.1)$$

986 An additional trial factor is then calculated by applying this procedure to maps of data
 987 with randomized right ascension values to assemble a distribution of \widetilde{TS}_{all} representative of
 988 the null hypothesis. A final p-value can then be obtained by comparing an observed \widetilde{TS}_{all}
 989 with this null hypothesis distribution.

990 For all the analyses detailed in this section, the NorthernTracks v002p05 sample was used.
 991 This sample is described earlier in this document (section 2.5).

992 5.1 Self-Triggered Catalog

993 For this catalog, we select the locations of all the events in the NorthernTracks v002p05 data
 994 sample that have a reconstructed energy proxy greater than 200 TeV. This energy threshold
 995 is chosen as it is roughly the point where 50% or more of neutrino events are expected to
 996 be astrophysical in origin. We can consequently expect that this catalog contains a non-
 997 negligible number of astrophysical neutrino sources, and can test the hypothesis of additional
 998 time dependent neutrino emission from these locations. Events that have been selected as
 999 source candidates for the catalog are subsequently removed from the sample, and do not
 1000 otherwise contribute to the calculation of a multi-flare test statistic.

1001 This catalog contains 32 source candidates, all in the northern sky. Notably, this cat-
 1002 alog does not include IC170922A (the alert event that triggered the followup analysis on
 1003 TXS 0506+056), as the data sample does not extend past the 2015 season, and IC170922A
 1004 occurred in the 2017 season.

1005 Applying the multi-flare algorithm described above to this catalog of the locations of
 1006 32 high energy neutrino events results in a best-fit number of sources of $k_{best} = 4$, with
 1007 an associated p-value of $p = 0.017$ (2.13σ), shown in figure 5.5. Detailed results for all 32
 1008 source candidates, including fitted number of events and pre-trial p-values, can be viewed in
 1009 table 5.1.

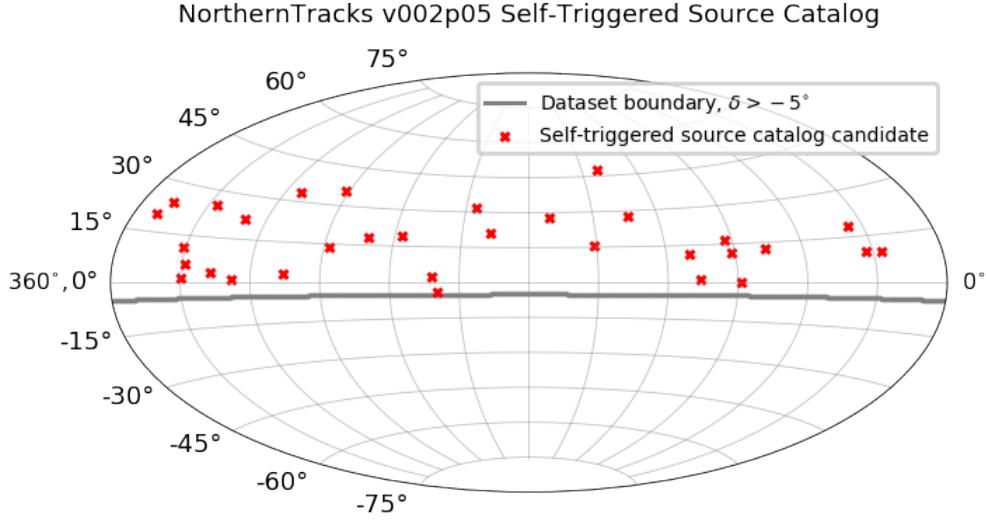


Figure 5.1: The self-triggered catalog assembled from the NorthernTracks v002p05 data sample, consisting of all events in the sample with reconstructed energy proxy greater than 200 TeV. The locations of these events are treated as source candidates for the purposes of applying the multi-flare algorithm, however the events themselves are removed from the sample prior to calculating a test statistic.

1010 As mentioned in previous sections, a significant advantage of the multi-flare algorithm is
 1011 the generation of neutrino "light curves" (or "flare curves") that show the historical activity
 1012 of a source candidate. The flare curves associated with the 4 most significant sources in the
 1013 self-triggered catalog can be seen in figure 5.5.

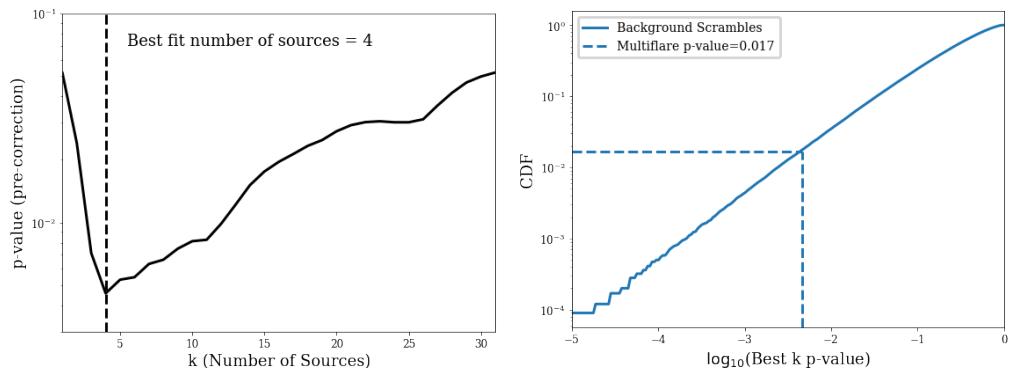


Figure 5.2: Left: The local significance of stacking the k highest test statistic sources, as a function of k . The best fit value of $k = 4$ can be seen as the minimum of this curve. Right: The trial corrected result for stacking the top 4 sources together, shown as a vertical line superimposed on the background distribution obtained by applying the algorithm to right-ascension scrambled data.

1014 Though not statistically significant, these results have several interesting features. Though

RA	Dec	\hat{n}_s	p (pre-trial)
36.69	18.32	47.21	0.00197
272.14	35.66	30.71	0.00729
170.19	27.85	45.75	0.00834
93.26	16.33	30.02	0.02667

Table 5.1: The top 4 most significant multi-flare source candidates in the self-triggered catalog composed of high energy IceCube neutrino events. Collectively, these sources form an excess with a significance of $p = 0.017$.

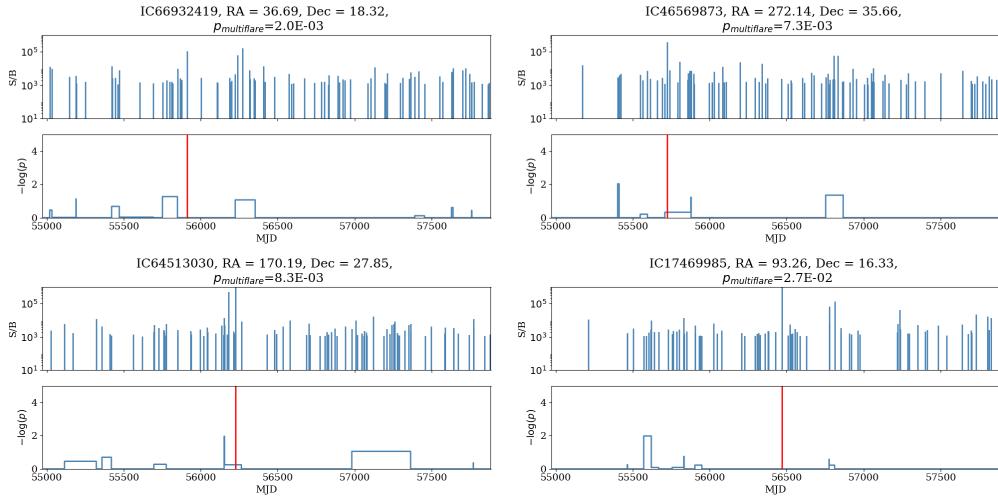


Figure 5.3: The neutrino "flare curves" associated with the 4 most significant multiflare source candidates in the self-triggered catalog. The top panel of each subplot shows the event weights, calculated by taking the ratio of the spatial and energy components of the signal and background PDFs described in equation 4.2 and 4.3, while the bottom panels show the fitted ensemble of decorrelated flares, with the local per-flare p-value plotted on the y-axis. The vertical red line denotes the arrival time of the high energy event used to define the source candidate location. This event is removed from the sample prior to applying the multiflare-algorithm, and consequently these events do not contribute to the flare curves shown in this figure.

1015 the high-energy "seed" events are removed prior to the calculation of a flare curve, 7 of the
 1016 flare curves generated include flares that would have included a high energy seed event.
 1017 Though not statistically significant, this is certainly above average, as only 11% of back-
 1018 ground trials have 7 or more flare curves with seed event/flare correlations.

1019 It is also potentially interesting to investigate the distributions of the parameters fit by
 1020 the multi-flare likelihood. By comparing the observed distributions with those expected from
 1021 background scrambles, we can check for inconsistencies of our data with the null hypothesis.

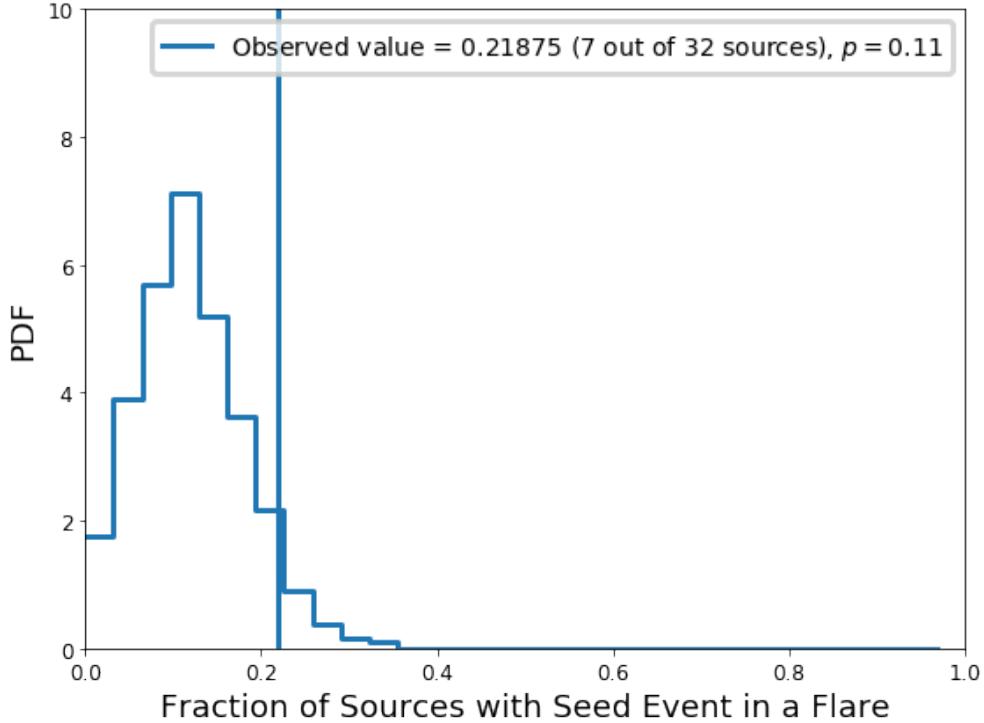


Figure 5.4: The distribution of number of flare curves that have a temporal correlation between the high energy seed event, and a fitted flare, obtained from sets of data where the right ascension of events has been randomized. The observed value (7 out of 32 sources, or 21.9%) is shown as a blue vertical line. 11% of trials have more than 7 correlations between seed events and fitted flares.

1022 5.2 Fermi 3LAC Blazars

1023 The 2014 TXS 0506+056 neutrino flare was notable not only for its association with a high
 1024 energy IceCube alert, but also for its spatial coincidence with the blazar TXs 0506+056. In
 1025 searching for additional neutrino sources, it is then not unreasonable to assemble a search
 1026 for neutrino flares associated with a large catalog of blazars. For this purpose, we use the
 1027 Fermi 3LAC catalog [43] to define a set of source candidates. Previous analyses searching
 1028 for spatial coincidence with earlier iterations of this catalog have been performed, but were
 1029 unable to identify any statistically significant excess [5]. Here, we instead specifically search
 1030 for transient emission using multi-flare algorithm described above.

1031 The Fermi 3LAC catalog is a catalog of AGNs detected by Fermi-LAT, consisting of
 1032 gamma ray sources in the third Fermi-LAT catalog (3FGL) [44] between 100 MeV and

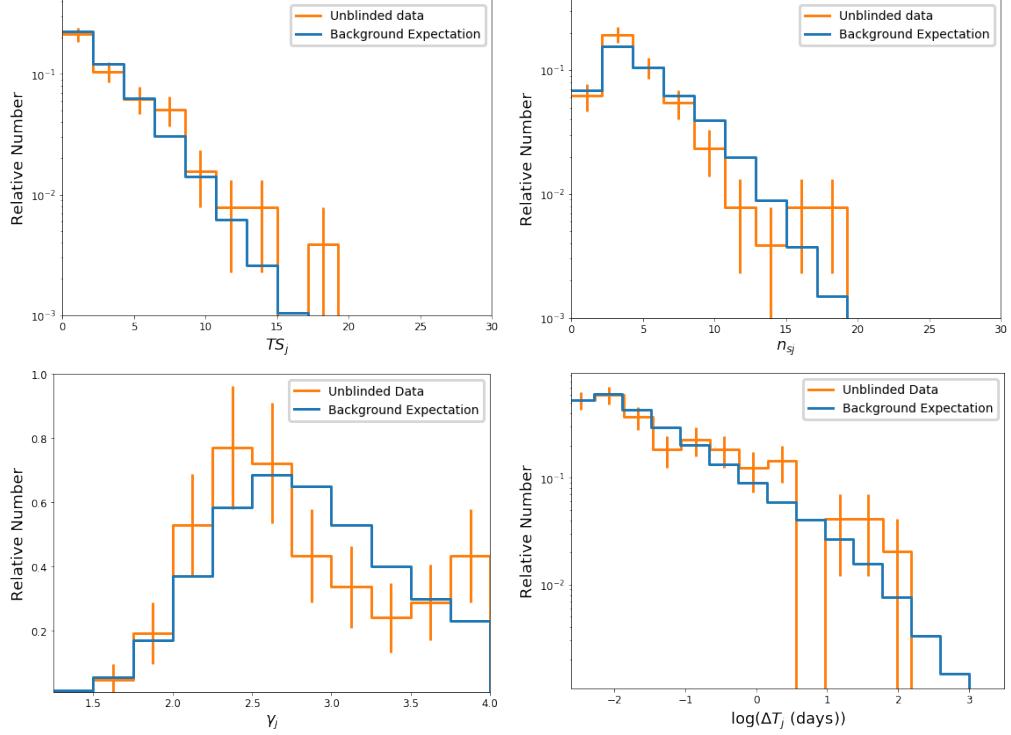


Figure 5.5: Distributions of fitted flare parameters for flares associated with the self-triggered catalog in both background scrambles (blue) and unblinded data (orange). While there is potentially some deformation in the distribution of fitted spectral indices (bottom left), a 2-sample K-S test comparing the observed and background distributions only returns a p-value of $p = 0.26$, indicating that the blue and the orange distributions are not significantly inconsistent with one another.

1033 300 GeV with a Fermi test statistic greater than 25 between August 4, 2008, and July 31,
 1034 2012 [43]. The catalog contains 1591 objects, the majority (98%) of which are blazars,
 1035 roughly evenly split between FSRQs and BL Lacs. Notably, TXS 0506+056 is a member of
 1036 this catalog.

1037 In constructing this analysis, we select for blazars at declinations greater than -5° , as
 1038 the IceCube data sample (NorthernTracks v002p05) does not extend below this point. We
 1039 impose no further cuts, and all sources are weighted equally when calculating a multi-flare
 1040 test statistic. The final catalog to be used for the multi-flare analysis consists of 1023 blazars,
 1041 the locations of which are shown in figure 5.6.

1042 The results of applying the multi-flare algorithm to this catalog of 3LAC blazars can be
 1043 summarized in figure 5.8. The optimization procedure for the most significant combination

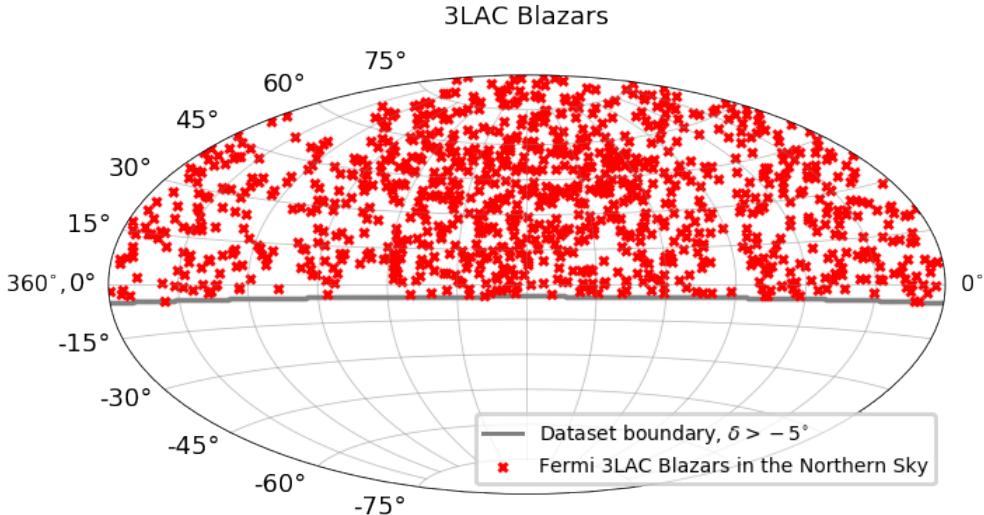


Figure 5.6: The locations of the 3LAC blazars that compose the catalog used for a multi-flare analysis. Blazars with $\delta < 5^\circ$ are not considered, as the IceCube NorthernTracks v002p05 data sample does not extend into this region. There are 1023 blazars that are considered as source candidates for the multi-flare analysis.

1044 of sources returned a best fit number of sources of $k = 125$, with an associated post-trial
 1045 p-value of $p = 0.06$. A list of these source candidates can be seen in table 5.2. As the
 1046 significance of this excess is consistent with the null hypothesis, we do not claim discovery
 1047 of neutrino flares associated with 3LAC blazars.

1048 Interestingly, despite the presence of the 2014 neutrino flare identified in [3], TXS 0506+056
 1049 is not the most significant source candidate, having a pre-trial p-value of only $p = 9.24 \times 10^{-3}$.
 1050 This is not unexpected, considering that there does not appear to much activity (in terms
 1051 of neutrino flares) at this location beyond the 2014 flare itself. As the multi-flare algorithm
 1052 performs will return a high significance when there are multiple, moderately significant flares
 1053 to stack together, it is unsurprising that there are other sources that have a higher multi-flare
 1054 significance. As an example, the source candidate with the highest multi-flare significance is
 1055 1RXS J154604.6+081912. Rather than having a single large flare, the multi-flare algorithm
 1056 fits multiple flares at this source candidate location, which combined have a significance of
 1057 $p = 8.1 \times 10^{-5}$, despite none of the individual flares having a local significance much greater
 1058 than $p = 0.01$.

1059 Similar to the self-triggered catalog, we can additionally examine the distributions of

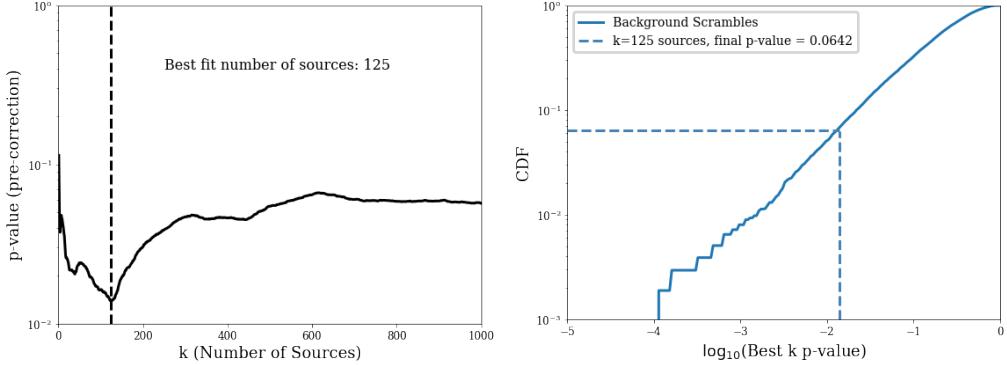


Figure 5.7: Left: The local significance of stacking the k highest test statistic sources in the 3LAC blazar catalog, as a function of k . The best fit value of $k = 125$ can be seen as the minimum of this curve. Right: The trial corrected result for stacking the top 125 sources together, shown compared the background distribution obtained by applying the algorithm to right-ascension scrambled data. As the final p-value associated with the top 125 blazar sources is only $p = 0.06$, there was no significant excess of neutrino flares observed to be associated with 3LAC blazars.

1060 the flare parameters that were fit for all the sources in the 3LAC catalog. No significant
 1061 deviations from the background expectation are observed in any of the flare parameter
 1062 distributions, as seen in figure 5.8.

1063 As with the self-triggered catalog, we also obtain neutrino flare curves describing the
 1064 historical variability of each source candidate. Figure 5.9 shows the flare curves of several
 1065 3LAC source candidates of note, including TXS 0506+056. While none of these sources are
 1066 significant enough to claim discovery, flare curves like the ones shown here are a potentially
 1067 valuable tool for multi-messenger analyses in the future that may seek to correlate neutrino
 1068 emission with other astrophysical messengers.

1069 Though the application of the multi-flare algorithm to this catalog of 3LAC blazars did
 1070 not result in a significant detection, it does allow us to constrain the behavior of neutrino
 1071 flares associated with 3LAC blazars. Given that this method stacks flares together, the lack
 1072 of a significant results suggest that there is an upper limit to how bright and numerous
 1073 blazar neutrino flares may be. We express these limits in terms of the flare rate (how many
 1074 flares occurred over the lifetime of the data sample), and the per-flare E^2 flux. This allows
 1075 us to compare to previously calculated limits obtained from a time-integrated analysis [5].

1076 These upper limits can be seen in figure ??

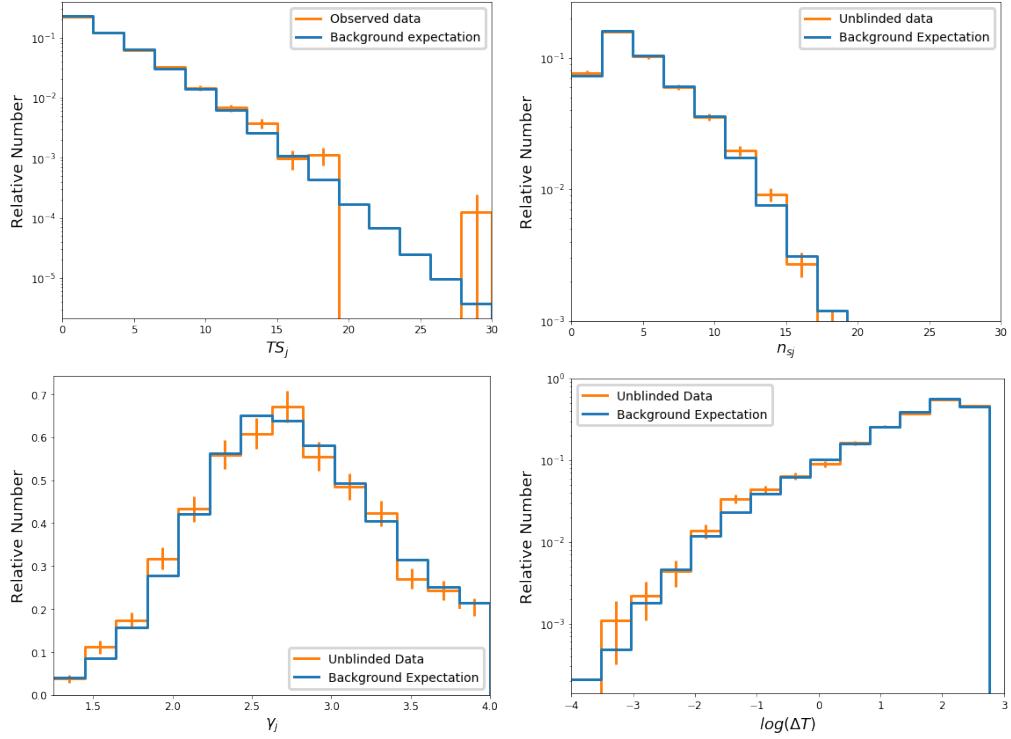


Figure 5.8: Distributions of fitted flare parameters for flares associated with the 3LAC blazar catalog in both background scrambles (blue) and unblinded data (orange). All distributions of fit parameters from observed data appear to be consistent with the background expectation. As this catalog includes TXS 0506+056, the 2014 neutrino flare originally discovered in the IC170922A follow-up analysis [3] is visible as the rightmost entry in the histogram of flare TS_j values in the plot in the upper left, having a value of $TS_j = 27.9$. There are no other flares that were fit in this catalog with $TS_j > 20$.

Table 5.2: The top 125 multiflare source candidates in the 3LAC blazar catalog.

Source Candidate Name	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1RXS J154604.6+081912	bll	236.52	8.32	53.49	8.11e-5
RBS 1467	bll	227.18	27.15	55.11	3.05e-4
GB6 J0723+2859	fsrq	110.98	28.99	40.48	4.58e-4
RBS 1558	bll	241.59	56.51	28.88	1.92e-3
PMN J2324+0801	bll	351.19	8.04	28.15	3.11e-3

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
B2 2214+24B	bll	334.25	24.36	36.90	3.79e-3
GB6 J0850+4855	bll	132.50	48.92	38.18	4.75e-3
4C +20.25	fsrq	171.49	20.10	27.83	5.02e-3
MG2 J094148+2728	fsrq	145.45	27.48	60.58	5.12e-3
TXS 2241+406	bll	341.05	40.95	22.61	5.23e-3
TXS 0213+619	bcu III	34.26	62.19	30.15	5.32e-3
GB6 J0100+0745	bll	15.09	7.76	37.33	6.45e-3
RX J0850.5+3455	bll	132.65	34.92	20.42	7.16e-3
B2 1436+37B	fsrq	219.72	37.18	49.47	7.83e-3
MG1 J165034+0824	fsrq	252.66	8.41	39.19	7.83e-3
PKS 0256+075	fsrq	44.86	7.79	42.53	9.04e-3
TXS 0506+056	bll	77.36	5.69	20.71	9.24e-3
1ES 1421+582	bll	215.66	58.03	28.49	0.0100
TXS 0518+211	bll	80.44	21.21	48.49	0.0118
W Comae	bll	185.38	28.23	30.32	0.0123
NVSS J141828+354250	bcu II	214.62	35.71	40.74	0.0128
PKS 1532+01	fsrq	233.72	1.52	30.78	0.0131
PKS 1424+240	bll	216.75	23.80	52.78	0.0148
B3 2319+444	fsrq	350.58	44.76	41.93	0.0160
PKS 0039+230	fsrq	10.52	23.33	48.89	0.0168
B3 2238+410	bll	340.28	41.34	21.24	0.0175
TXS 2315+189	bcu II	349.60	19.25	31.98	0.0197
B3 2322+396	bll	351.32	39.96	30.07	0.0199
NVSS J080637+774607	bcu II	121.66	77.77	52.32	0.0215
MG1 J010908+1816	bll	17.28	18.27	26.59	0.0220

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1H 0323+342	nlsy1	51.17	34.18	10.09	0.0221
NVSS J131921+775823	bcu II	199.84	77.97	16.99	0.0224
RX J1351.3+1115	bll	207.84	11.25	27.93	0.0228
RGB J1808+468	bll	272.00	46.83	29.19	0.0252
RX J1149.5+2439	bll	177.38	24.66	33.52	0.0266
TXS 2157+102	bll	330.03	10.50	47.04	0.0271
S5 1357+76	fsrq	209.48	76.72	35.44	0.0274
S5 1803+784	bll	270.19	78.47	27.73	0.0287
GB6 J1439+4958	bll	219.95	49.97	32.32	0.0291
B2 2234+28A	bll	339.09	28.48	25.36	0.0298
GB6 J0929+5013	bll	142.31	50.23	20.09	0.0306
RGB J2054+002	bll	313.74	0.26	26.79	0.0315
1ES 1028+511	bll	157.83	50.89	35.27	0.0333
GB6 J0331+6307	bcu II	52.97	63.14	26.86	0.0335
PKS 2320-035	fsrq	350.88	-3.28	15.57	0.0350
GB6 J0934+3926	bll	143.53	39.44	32.46	0.0352
B2 1811+31	bll	273.40	31.74	35.37	0.0356
GB6 J0937+5008	fsrq	144.30	50.15	27.81	0.0357
TXS 1015+057	fsrq	154.62	5.51	19.30	0.0361
TXS 1614+473	fsrq	243.92	47.19	16.66	0.0374
4C +04.42	fsrq	185.59	4.22	19.22	0.0376
MG2 J110606+2812	fsrq	166.53	28.21	41.99	0.0396
RX J1246.9+4423	bll	191.75	44.39	22.30	0.0396
B2 2308+34	fsrq	347.77	34.42	41.08	0.0408
TXS 2106-030	bll	317.19	-2.84	16.86	0.0410

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
NVSS J125820+612049	bll	194.59	61.35	36.36	0.0425
SBS 0812+578	bll	124.09	57.65	41.15	0.0426
WN B1609.6+8517	bcu II	240.13	85.16	37.14	0.0434
PMN J2227+0037	bll	336.99	0.62	28.56	0.0440
1RXS J234332.5+343957	bll	355.89	34.66	25.27	0.0440
GB6 J0529+0934	bcu II	82.26	9.58	32.69	0.0450
MG2 J131037+2447	bcu III	197.66	24.81	27.71	0.0458
3C 454.3	fsrq	343.49	16.15	37.56	0.0459
GB6 J0929+7304	bcu II	142.43	73.07	25.62	0.0459
ZS 0214+083	bll	34.32	8.62	36.08	0.0485
3C 264	rdg	176.27	19.61	36.47	0.0485
RXS J094620.5+010459	bll	146.58	1.08	19.50	0.0486
NRAO 512	fsrq	250.12	39.78	21.52	0.0490
1H 0323+022	bll	51.56	2.42	43.87	0.0494
GB6 J1027+7428	bcu II	156.85	74.47	31.92	0.0495
RX J0805.4+7534	bll	121.36	75.57	30.25	0.0505
4C +73.07	bcu II	142.25	72.95	22.60	0.0514
B3 1222+438	bll	186.21	43.59	28.13	0.0516
MG1 J120448+0408	fsrq	181.22	4.14	33.72	0.0525
GB6 J0148+5202	bcu III	27.08	52.03	19.63	0.0556
7C 1823+6856	bll	275.89	68.96	21.57	0.0561
MG2 J180948+2910	bll	272.44	29.17	34.89	0.0591
NVSS J022304+682154	bcu II	35.77	68.37	31.49	0.0591
OM 280	bll	177.58	24.30	42.49	0.0626
PKS 1717+177	bll	259.80	17.75	24.64	0.0627

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1RXS J133021.4+444117	bll	202.59	44.69	22.24	0.0650
S4 1726+45	fsrq	261.87	45.51	34.78	0.0664
RX J1702.6+3115	bll	255.66	31.26	25.34	0.0664
SBS 1410+530	bcu I	212.96	52.82	24.04	0.0666
MG1 J154628+1817	bll	236.60	18.29	40.11	0.0668
MG2 J190411+3627	bcu II	286.05	36.45	34.19	0.0713
TXS 2331+073	fsrq	353.55	7.61	27.29	0.0731
PKS 2047+098	bcu II	312.44	10.05	19.18	0.0734
GB6 J1542+6129	bll	235.74	61.50	34.32	0.0736
S5 0159+723	bll	30.89	72.55	30.54	0.0765
TXS 2032+117	fsrq	308.65	11.91	21.51	0.0777
S5 1027+74	bcu I	157.84	74.70	30.89	0.0777
RGB J1426+340	bll	216.53	34.07	45.49	0.0778
B3 0350+465	bcu III	58.63	46.72	22.88	0.0778
RGB J1742+597	bll	265.63	59.75	37.12	0.0781
1RXS J125117.4+103914	bll	192.82	10.65	18.18	0.0782
MG3 J184126+2910	bcu II	280.34	29.16	37.55	0.0787
TXS 1645+635	fsrq	251.49	63.50	36.91	0.0793
B3 1058+413	bcu III	165.35	41.06	44.91	0.0799
87GB 152947.5+574636	bll	232.74	57.61	21.01	0.0802
GB6 J0229+6706	bcu III	37.34	67.11	11.03	0.0806
MG1 J125348+0326	bll	193.45	3.44	27.69	0.0806
GB6 J0342+3858	fsrq	55.57	38.99	33.59	0.0806
PMN J0148+0129	bll	27.14	1.48	18.43	0.0818
87GB 164812.2+524023	bll	252.35	52.59	33.97	0.0827

Continued on next page

Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
GB6 J0024+0349	fsrq	6.19	3.82	31.06	0.0830
B3 1307+433	bll	197.36	43.08	22.38	0.0839
NVSS J092542+595812	bll	141.43	59.97	17.27	0.0913
TXS 1549+089	bll	238.01	8.85	30.66	0.0915
RX J0202.9-0223	bcu II	30.72	-2.39	11.81	0.0922
PKS 1203+04	ssrq	181.58	4.10	39.00	0.0926
3C 221	rdg	143.78	39.70	28.81	0.0931
RBS 0909	bll	162.86	39.72	30.20	0.0942
NVSS J121500+500216	bll	183.75	50.04	36.18	0.0956
RX J1101.3+4108	bll	165.35	41.15	39.73	0.0962
OX 131	fsrq	320.25	19.02	28.54	0.0970
PKS 0017+200	bll	4.91	20.36	25.43	0.0973
1ES 0647+250	bll	102.69	25.05	33.94	0.0990
TXS 0237+655	bcu II	40.34	65.72	22.76	0.102
NVSS J224753+441317	bll	341.97	44.22	29.52	0.102
OI 280	fsrq	117.72	12.52	22.89	0.102
B2 1348+30B	fsrq	207.72	30.58	27.45	0.102
TXS 1833+137	bcu III	278.90	13.81	23.62	0.105
RX J1027.4+6317	bll	156.85	63.30	31.32	0.105
PKS 2354-021	bll	359.35	-1.87	5.92	0.105

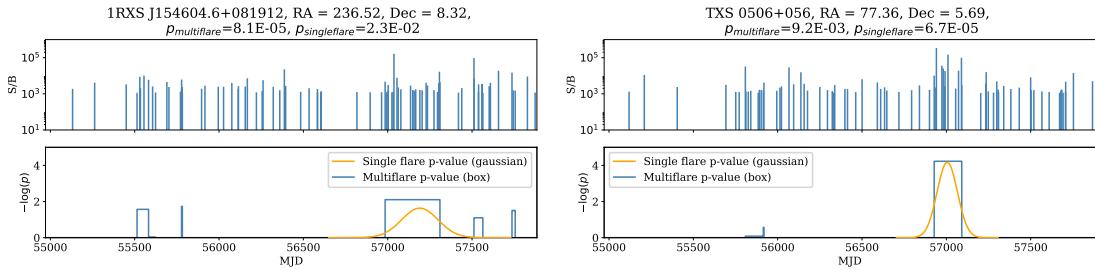


Figure 5.9: Neutrino flare curves for several sources of note in the multi-flare 3LAC blazar catalog analysis. The flare curves generated by the multi-flare algorithm are shown in blue, while the results of a corresponding single-flare analysis (that only fits the largest flare on each source) are shown in orange. Left: 1RXS J154604.6+081912, the most significant 3LAC blazar in the multi-flare analysis, having a pre-trial multi-flare p-value of $p = 8.11 \times 10^{-5}$. Note that for this particular source, the multi-flare significance driven by a set of five moderately significant flares, none of which are particularly significant on their own. The significance of this source in the single-flare analysis was only $p = 0.02$. Right: The flare curve for TXS 0506+056, showing the sizeable 2014 neutrino flare originally observed in [3]. The multi-flare p-value for TXS 0506+056 is only $p = 9.24 \times 10^{-3}$, as other than the 2014 neutrino flare, there are no other particularly significant neutrino flare candidates at this location.

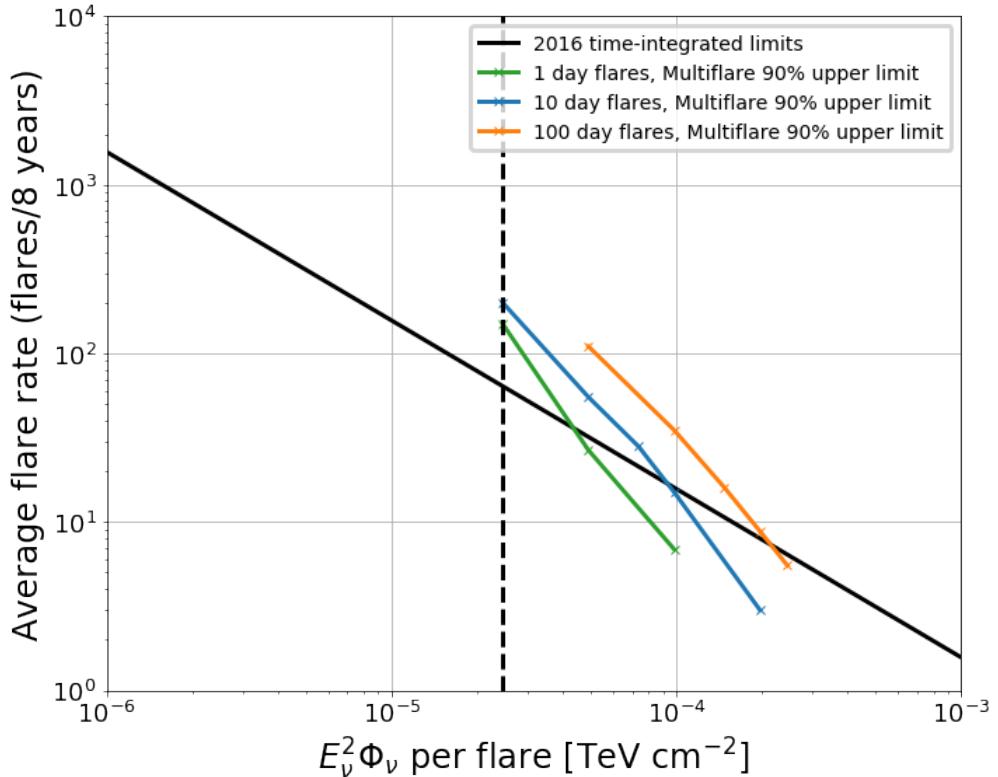


Figure 5.10: The 90% upper limits associated with the non-detection of a multi-flare signal using the 3LAC blazar catalog. The solid black line represents the time integrated limits obtained from [5]. Colored curves represent limits associated with various flare durations, as the multi-flare method imposes more stringent limits in the case that flares are shorter (for a fixed E^2 flux per flare). Note that the parameter space that is realistically accessible to a time-dependent analysis is somewhat compressed: points below the x-axis do not produce more than 1 flare in the data sample, on average, and points to the left of the vertical dashed line have an average flare size of < 1 event.

1078 **Chapter 6**

1079 **An All-Sky Multi-flare Analysis**

1080 **6.1 Motivation**

1081 While the previous section outlined the application of the multi-flare algorithm to a pair
1082 of source catalogs, an additional obvious application of the method is to simply apply the
1083 algorithm across the entire neutrino sky. While catalog searches have the advantage of a
1084 greatly reduced trial factor, they also rely on having a mostly-correct guess of the underlying
1085 source population. By contrast, an all-sky search could potentially reveal sources without
1086 requiring any prior knowledge of what the specific sources could be, and could additionally
1087 even identify astrophysical neutrino sources with no multi-messenger counterpart. Given the
1088 results of a temporally untriggered neutrino flare associated with TXS 0506+056 [3], it is
1089 only natural to ask whether other such flares exist in IceCube’s astrophysical neutrino data.
1090 An all-sky multi-flare analysis would be an appropriate approach of detecting a population
1091 of TXS-like flares, should they exist.

1092 Similar to the FAVA analysis produced by the Fermi collaboration [45], an IceCube
1093 multi-flare skymap provides a description of the temporal variability of every point in the
1094 sky. Even in the absence of a statistically significant population of neutrino flares, these
1095 neutrino “flare curves” may be of use for multi-messenger analyses in the future, similar to
1096 what was done for TXS 0506+056 [3][2].

1097 6.2 Analysis Construction

1098 Conceptually, the construction of an all-sky multi-flare analysis is fairly straightforward: a
 1099 grid of pixels with is defined over the entire sky using the HEALPY software package [46].
 1100 Using a HEALPY grid with a `Nside=256` results in a grid of 786,432 pixels, each with a
 1101 radius of 0.12° . Pixels with declinations $\delta > 85^\circ$ or $\delta < -85^\circ$ are excluded, as the data-
 1102 driven method of estimating background using data scrambled in right ascension does not
 1103 perform well in this region due to the small statistics of declination bands near the poles.
 1104 The multi-flare algorithm is then applied to the central location of each pixel, resulting in a
 1105 map of multi-flare test statistics, each associated with an individual pixel.

1106 The significance of each pixel can then be calculated by comparing the observed multi-
 1107 flare test statistic to a distribution of similar test statistics obtained by applying the above
 1108 procedure to maps of IceCube events that have had their right ascension locations random-
 1109 ized, providing a description of the null hypothesis. Pixels are divided into 40 declination
 1110 bands, and for each declination band a chi-squared distribution is fit to the distribution of
 1111 test statistics in that declination range. A pre-trial local significance for each pixel is calcu-
 1112 lated by comparing a particular pixel test statistic to the chi-squared distribution that was
 1113 fit in the corresponding declination band. Once this has been done for every pixel, a map of
 1114 multi-flare p-values has been obtained, describing the local multi-flare significance of every
 1115 pixel in the sky.

1116 Once a p-value map has been generated, several basic tests can be conducted. The most
 1117 obvious test is to simply check if the most significant pixel is more significant than is expected
 1118 from the background case (a "hotspot" test). A distribution of most significant multi-flare
 1119 p-values is created by applying the procedure above to a set of background maps. The most
 1120 significant pixel in the observed multi-flare map can then be compared to this distribution to
 1121 obtain a final hotspot significance that has been trial-corrected for the all sky trial factor. As
 1122 the event selection is different in the northern and southern skies, this process is conducted
 1123 separately for declinations $\delta > -5^\circ$ and $\delta < -5^\circ$

1124 Populations of multi-flare sources may also be tested for using a binomial test. A set of
 1125 ”spatially independent” local hotspots may be obtained by defining a list of all pixels that
 1126 are at least 1° away from a more significant pixel. The binomial test statistic p-value of the
 1127 population test is then defined as (eq. 6.1):

$$p(k) = \sum_{i=k}^{N_{eff}} \binom{N_{eff}}{i} p_k^i (1 - p_k)^{N_{eff}-i} \quad (6.1)$$

1128 Here, $p(k)$ is correlated with the significance of observing k hot spots with a p-value of p_k
 1129 or less, and N_{eff} is the effective number of trials associated with the list of hot spots, chosen
 1130 to produce proper containment of the final binomial p-values (e.g. a final binomial p-value
 1131 of $p = 0.1$ or less should only occur in 10% of background trials). In this case, $N_{eff} = N_{pixels}$
 1132 produces proper containment. Hot spots are ordered by decreasing significance, and k is
 1133 varied to identify the most significant combination. The $p(k)$ associated with the best fit k
 1134 is then compared to a distribution of $p(k)$ ’s obtained in a similar manner from data scrambled
 1135 in right ascension, resulting in a final post-trial binomial p-value. Like with the study of
 1136 the most significant pixel, this process is conducted separately in the northern and southern
 1137 skies.

1138 Since the multi-flare algorithm fits for every flare candidate at a particular pixel, it is
 1139 trivial to extract the single-flare skymap results from this process as well. The procedure
 1140 is almost identical to that which was outlined above, except instead of using the multi-flare
 1141 test statistic at each pixel, the test statistic of the flare with the highest test statistic at each
 1142 pixel is used. The hotspot and population tests then proceed as normal.

1143 This particular analysis uses the PSTracks003p02 data set.

1144 **6.3 Results**

1145 **6.3.1 Hotspots and Populations Analysis**

1146 The multi-flare pre-trial p-value map can be seen in figure 6.1, with the locations of the
 1147 brightest multi-flare pixels marked. The most significant locations identified by the multi-
 1148 flare algorithm have pre-trial p-values of $p = 9.2 \times 10^{-6}$, located at (RA, Dec)=(145.02°, 36.42°)
 1149 and $p = 3.5 \times 10^{-7}$, located at (RA, Dec)=(126.21°, -24.81°). These pre-trial p-values can be
 1150 corrected to account for the all-sky trial factor using the process described above, resulting
 1151 in post-trial p-values of $p = 0.69$ for the northern sky hot spot and $p = 0.06$ for the southern
 1152 sky hot spot.

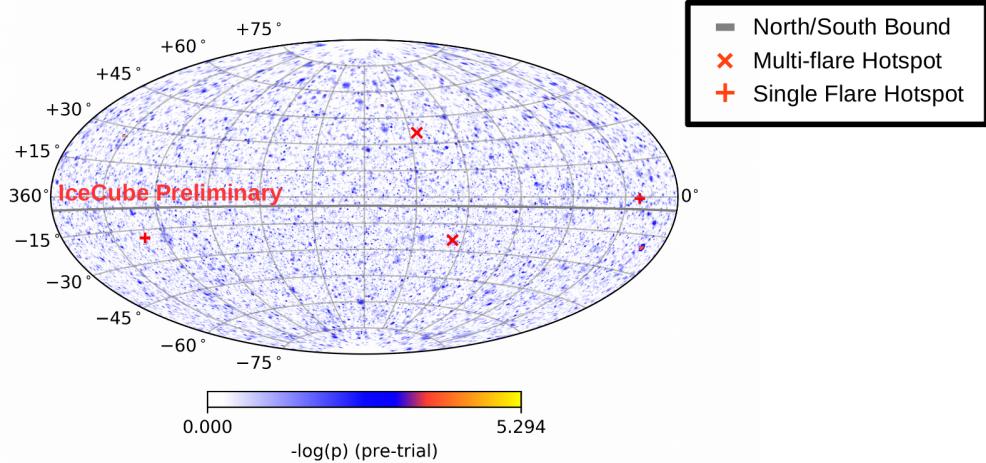


Figure 6.1: The p-value map produced by applying the multi-flare algorithm to the entire neutrino sky between $-85^\circ < \delta < 85^\circ$. The locations of the most significant multi-flare pixels in each hemisphere are shown as red "x's", while the locations of the most significant individual flares in each hemisphere are shown as red "+"s". The gray line denotes the boundary between what is considered the "northern" and "southern" sky by the IceCube data sample that was used.

1153 The flare curves for the multi-flare hotspots can be seen in figure 6.3. Note that the
 1154 multi-flare test statistic is a measure of the activity of a source integrated over the entire
 1155 livetime of the data sample. While no individual flares that were fit at either the northern
 1156 or southern hotspot are particularly significant on their own, the combination of many
 1157 moderately significant flares is what makes these pixels the brightest multi-flare pixels in

1158 their respective hemispheres.

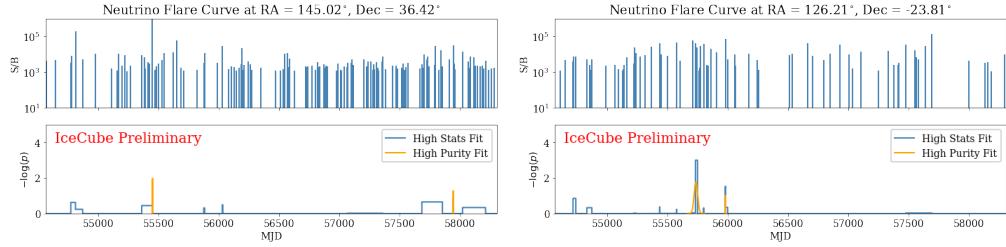


Figure 6.2: The flare curves returned by the multi-flare algorithm for the most significant pixel in the northern sky (left), and southern sky (right). For comparison, a complementary high-purity fit that uses a gaussian hypothesis and more stringent cuts on flare decorrelation is shown in orange. In both the northern and southern sky, the multi-flare significance is fueled not by individual flares with high local significance, but rather by a large number of moderately significant flares.

1159 Since the application of the high-statistics multi-flare analysis involves fitting every pos-
 1160 sible flare in the data, it is trivial to additionally calculate the significance of the largest in-
 1161 dividual flare candidate that was fit in both the northern and southern sky. We find that the
 1162 most significant flare candidate in the northern sky is located at $(\text{RA}, \text{Dec}) = (21.97^\circ, -0.60^\circ)$
 1163 (recall that the "northern sky" refers to declinations between -5° and 85°), and has a pre-
 1164 trial significance of $p = 5.08 \times 10^{-6}$ ($p = 0.82$ post-trial). The most significant flare candidate
 1165 in the southern sky is located at $(\text{RA}, \text{Dec}) = (311.66^\circ, -18.84^\circ)$, and has a pre-trial signifi-
 1166 cance of $p = 6.8 \times 10^{-6}$ ($p = 0.53$ post-trial).

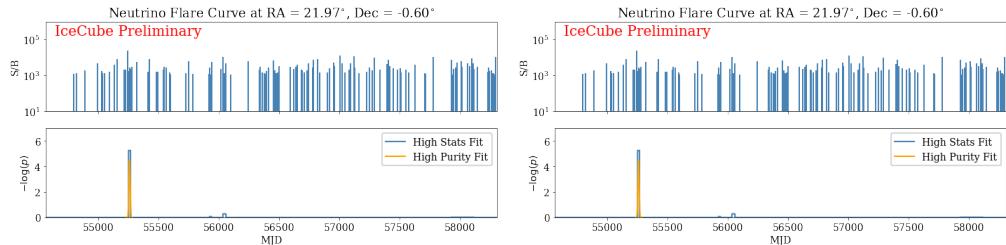


Figure 6.3: The flare curves returned by the multi-flare algorithm for the most significant flares identified in the northern sky (left), and southern sky (right). For comparison, a complementary high-purity fit that uses a gaussian hypothesis and more stringent cuts on flare decorrelation is shown in orange. Since neither of these locations appear to display significant activity beyond the large individual flares, these do not constitute particularly significant locations from a multi-flare perspective.

1167 In addition to examining the most significant pixels that were seen, it is also informative
 1168 to conduct a population analysis on the set of spatially independent ($> 1^\circ$ separation)

1169 hotspots using the binomial test framework described above. In both the northern and
 1170 southern sky, the binomial test was conducted for the set of spatially independent hotspots,
 1171 defined by both the local multi-flare and single-flare significance. In all cases, the best-fit
 1172 combination of hotspots was $k = 1$, tagging only the most significant pixel, and none of the
 1173 associated binomial p-values were significant (results may be viewed in table 6.1).

Analysis	Search	Hemisphere	Pre-trial p-value	Post-trial p-value
Multi-flare	Hottest spot	North	9.2×10^{-6}	0.69
		South	3.5×10^{-7}	0.06
	Population test	North	0.98	0.98
		South	0.12	0.12
Single Flare	Hottest spot	North	5.08×10^{-6}	0.82
		South	6.70×10^{-6}	0.53
	Population test	North	0.88	0.88
		South	0.91	0.91

Table 6.1: Summary of the tests that were performed on the unblinded multi-flare skymap. Both hotspot and population analyses were conducted using both the local multi-flare significance (summing contributions from all flare candidates at a particular location) and the local single-flare significance (taking only the largest flare fit at each location). There were no significant transient neutrino sources identified by any of the tests conducted.

1174 The results of the population analysis are unsurprising, given the distribution of per-
 1175 pixel local p-values observed from the data. In the presence of a significant population of
 1176 flaring neutrino sources, some deformation of the local p-value distribution should be visible,
 1177 however as can be seen in figure 6.5, the observed distribution is entirely consistent with
 1178 what is expected from background maps.

1179 Similar to the p-value distributions, the distributions of fitted flare parameters are also
 1180 consistent with the background expectation, as can be seen in figure 6.6. There does not
 1181 appear to be a significant sub-population of transient sources with a particular flare duration,
 1182 spectral index, flare rate, or flare flux.

1183 The observed diffuse astrophysical neutrino flux [11] places restrictions on how bright
 1184 and numerous neutrino flares may be. The non-detection of a population of transient as-
 1185 trophysical neutrino sources can be used to further constrain this space. These limits can
 1186 be seen in figure ??, which shows 90% upper limits on source flare rate and burst energy in

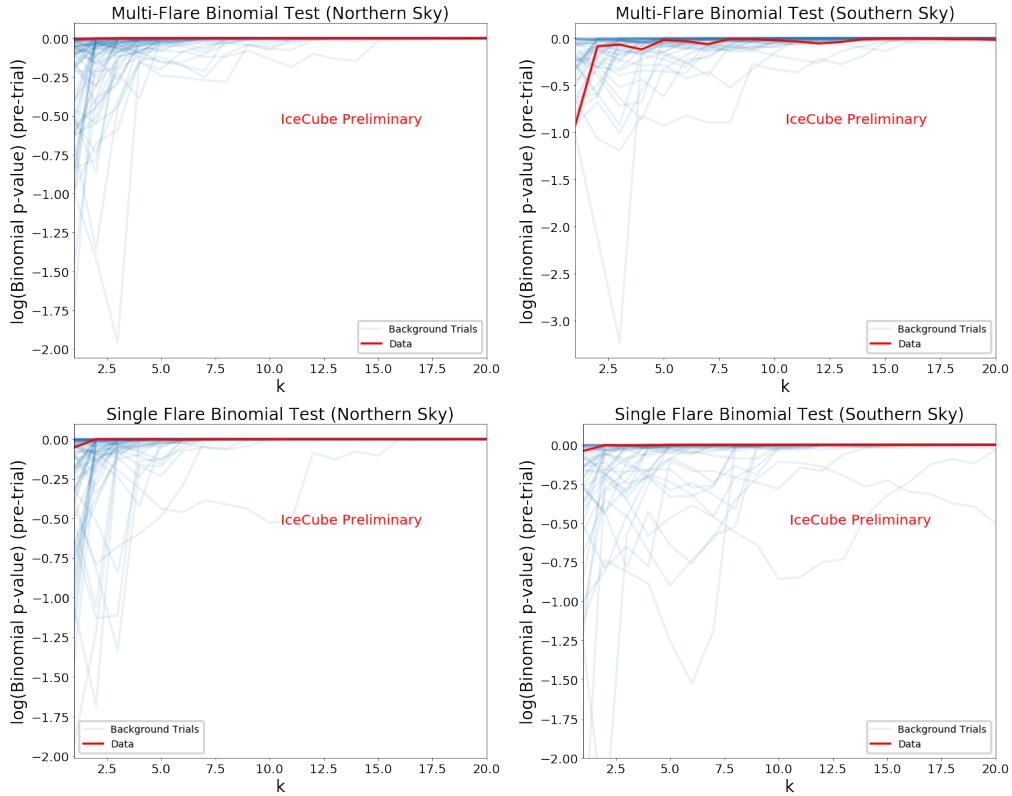


Figure 6.4: The local significance of the binomial test for various values of k , the fitted number of hotspots, for both single and multi-flare binomial tests conducted in both the northern and southern hemispheres. Unblinded data is shown in red, while results obtained from background maps are shown in blue. In all cases using unblinded data, the observed binomial test curve has a global minimum at $k = 1$, with an associated p-value that is consistent with the background expectation.

1187 neutrinos, as well as limits that can be placed on the per-source flare rate. Note that these
 1188 limits are a statistical statement about the data itself: If the "true" parameters describing
 1189 a population of transient neutrino sources lies above these lines, the data would have had a
 1190 larger population of significant flares than what was observed.

1191 6.3.2 Locations of External Interest

1192 Though this analysis was designed to be spatially untriggered (scanning over the entire sky,
 1193 rather than the locations of a select few sources), the neutrino "flare curves" produced are a
 1194 potentially powerful tool for exploring the historical behavior of source candidates identified
 1195 though other methods. The skymap produced here is similar to the TXS 0506+056 followup

¹<https://github.com/ChrisCFTung/FIRESONG>

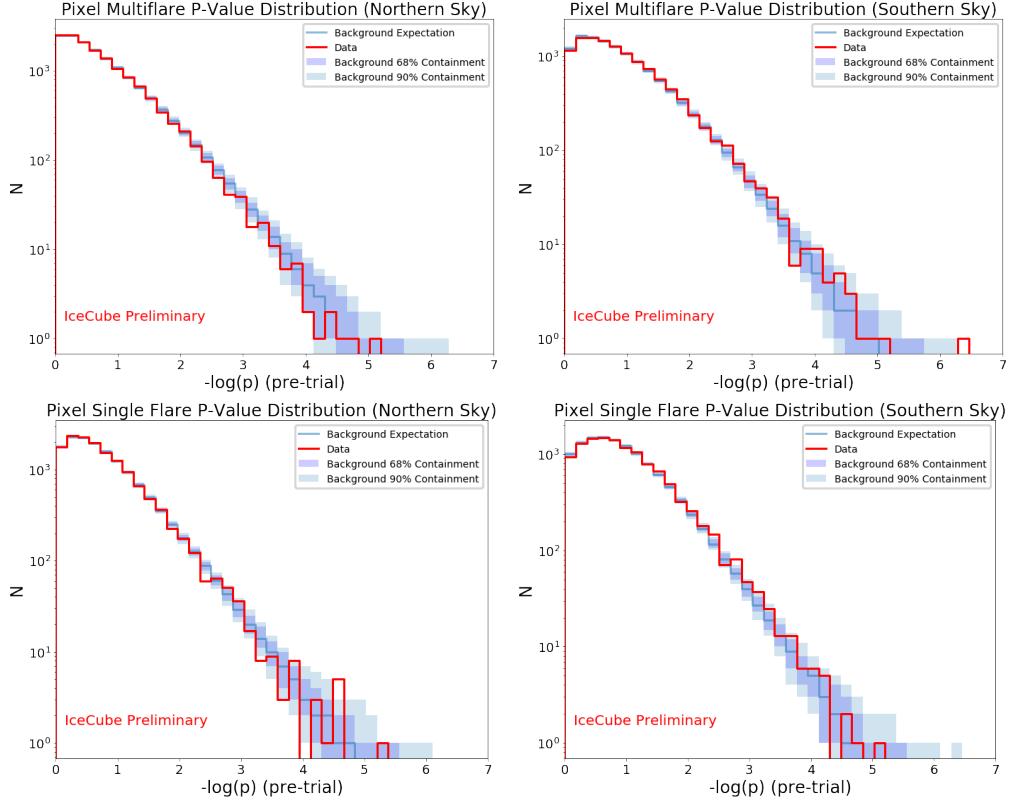


Figure 6.5: The local p-value distributions for the local multi-flare pixel significance (top) and the local single-flare pixel significance (bottom), for both data (red) and background maps (blue bands). In all cases, the observed distribution is consistent with the background expectation.

1196 analysis [3], except that this analysis has now been performed at almost every location in
 1197 the sky. This allows for easy followup of external triggers, as identifying potential transient
 1198 neutrino emission associated with a source candidate is as easy as examining the flare curve
 1199 at that location in the sky.

1200 To demonstrate the use of the all-sky flare curve map produced here, we examine the
 1201 flare curves of several time-integrated source candidates identified by [4] in order to explore
 1202 the potential of temporally clustered neutrino emission from these locations.

1203 6.3.2.1 NGC 1068

1204 The most significant location identified by the all-sky time-integrated analysis is spatially
 1205 coincident with the Seyfert II galaxy NGC 1068. This source was also tested as part of
 1206 a catalog of 110 candidate neutrino emitters, and was the brightest object in the catalog,

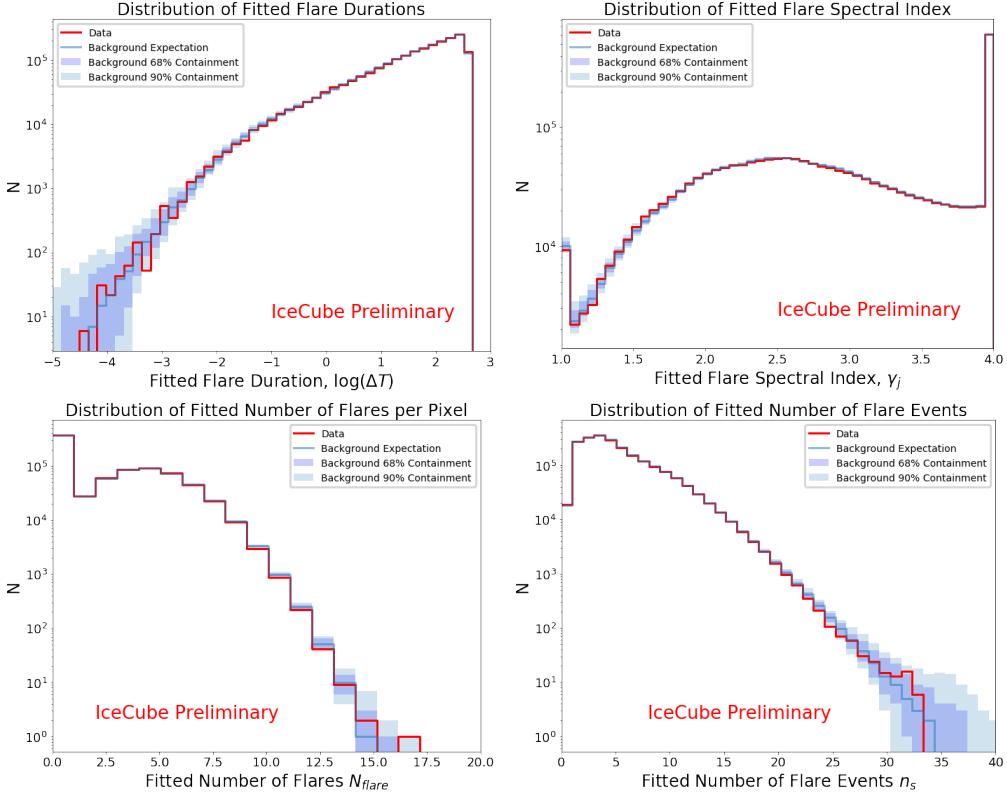


Figure 6.6: Distributions of fitted flare parameters: flare duration (top, left), spectral index (top, right), number of flares fit at a particular candidate location

1207 with an associated post-trial significance of 2.9σ [4]. These significances all arise from time-
 1208 integrated tests that do not take into account the arrival time of the contributing neutrino
 1209 events. It is then potentially interesting to explore the temporal structure of this excess with
 1210 the flare curves that were produced as part of the multi-flare skymap.

1211 The flare curve generated at the location of NGC 1068 can be seen in figure 6.8. Though
 1212 this location is significant under the time-integrated analysis, the multi-flare p-value is only
 1213 $p = 0.016$ (pre-trial), with no individual flare candidate having a local significance greater
 1214 than $p = 0.072$. Note however, that these results are not in tension with the time-integrated
 1215 significance, as the time-integrated analysis tests only for an excess of events over the entire
 1216 livetime of the sample. By contrast, a source with high multi-flare significance requires that
 1217 events not only be clustered spatially, but temporally as well. As such, sources with high
 1218 time-integrated significance do not necessarily have a correspondingly high multi-flare signif-
 1219 icance, particularly if there is not significant temporal clustering of events that contribute

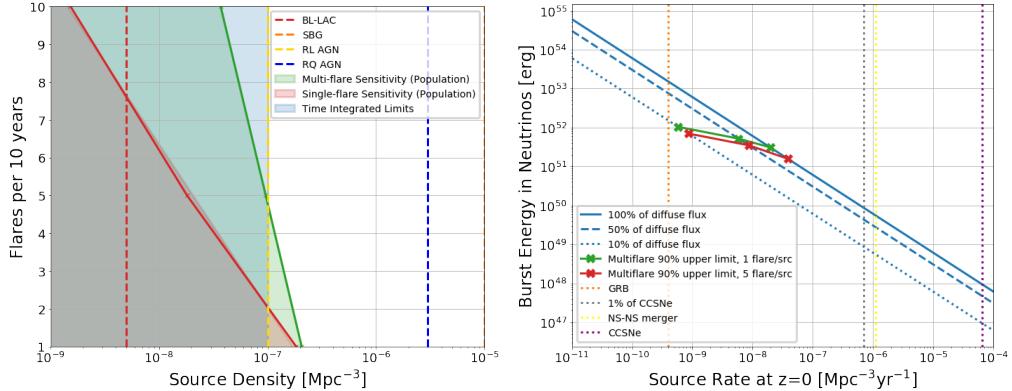


Figure 6.7: Left: 90% upper limits on neutrino source density and per-source flare rate obtained from the all-sky multi-flare analysis. In this plot, transient sources are assumed to compose 100% of the astrophysical neutrino flux, with the FIRESONG simulation package¹ being used to distribute the flux among sources according to the Hopkins and Beacom (2006) source evolution model [47]. Flares are simulated to have a duration of 20 days, and a spectral index of $\gamma = 2.28$. All flares on the same source are assumed to have the same intrinsic intensity, equal to the time integrated source flux divided by the number of flares. Right: The 90% upper limits on the source rate (source density \times flare rate) as a function of neutrino burst energy. Blue lines correspond to combinations of source rate and neutrino burst energy that reproduce specific fractions of the diffuse astrophysical neutrino flux. In both plots, the parameters associated with several candidate source populations are shown for comparison..

1220 to the time-integrated result.

1221 Nonetheless, we have here provided a description of the historical behavior of the neutrino
 1222 emission associated with NGC 1068. Despite the time-integrated excess observed in previous
 1223 analyses, there does not appear to be significant temporal clustering of events at this location.

1224 6.3.2.2 Other Time-Integrated Candidates: PKS 1424+240 and GB6

1225 J1542+6129

1226 In addition to identifying NGC 1068 as a potential source candidate, the population analysis
 1227 component of the time integrated analysis [4] also identified an excess of 3.3σ associated
 1228 with the combination of the sources NGC 1068, TXS 0506+056, PKS 1424+240, and GB6
 1229 J1542+6129. Figure 6.9 shows the flare curves that were obtained at the locations of PKS
 1230 1424+240 and GB6 J1542+6129.

1231 Similar to NGC 1068, PKS 1424+240 does not display significant temporal clustering
 1232 of neutrino events, and has a local multi-flare p-value of $p = 0.085$. By contrast, GB6
 1233 J1542+612 seems to have a somewhat significant flare candidate beginning at MJD=57564.878

1234 and ending at MJD=57944.512. The pre-trial significance of this flare candidate alone is
 1235 $p = 0.00173$ (2.9σ). Combining all the flares candidates at the location of GB6 J1542+612
 1236 using the multi-flare test statistics results in a local multi-flare significance of $p = 0.0098$
 1237 (2.3σ). While this is not particularly significant (especially when the all-sky trial factor is
 1238 considered), it does inform us that the period between MJD=57564.878 and MJD=57944.512
 1239 was likely a large contributor to the time-integrated significance for this particular source. If
 1240 there are further multi-messenger signals from this location in the future, the neutrino flare
 1241 candidates at this location may become of interest.

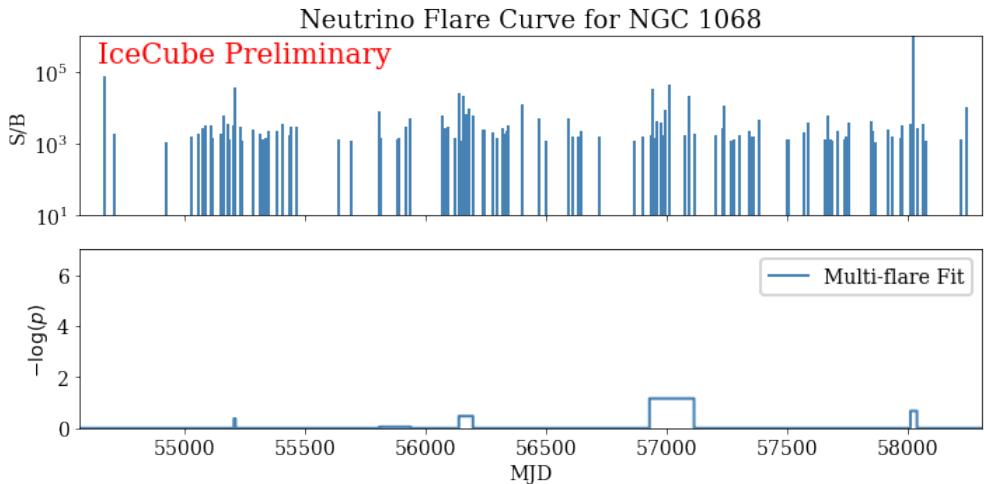


Figure 6.8: The flare curve at the location of NGC 1068, a source candidate that was identified as potentially interesting in the time-integrated analysis [4]. While this is the brightest spot in the time-integrated analysis, the multi-flare analysis does not reveal any significant temporal clustering of events at this location, resulting in a local multi-flare significance of $p = 0.016$

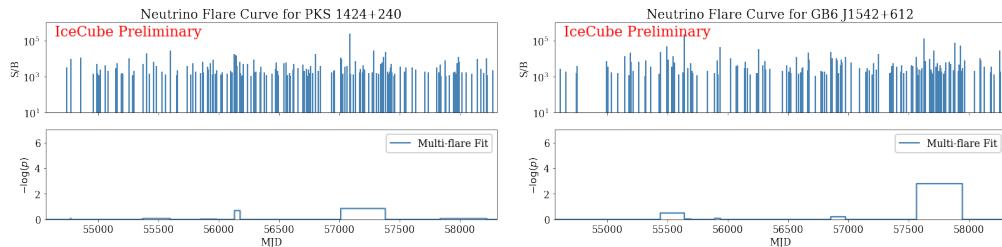


Figure 6.9: The flare curves at the locations of PKS 1424+240 and GB6 J1542+612. While PKS 1424+240 does not display any significant temporal clustering (having a local multi-flare p-value of $p = 0.085$), GB6 J1542+612 has a potentially interesting flare candidate beginning at MJD=57564.878 and ending at MJD=57944.512. This flare candidate has a local flare significance of $p = 0.00173$ (2.9σ), contributing to the overall pre-trial multi-flare significance for GB6 J1542+612 of $p = 0.0098$ (2.3σ).

1242 **6.3.2.3 TXS 0506+056**

1243 In addition to being part of the excess that was identified in the all-sky time-integrated
 1244 analysis, TXS 0506+056 is notable for both the multi-messenger coincidence sparked by the
 1245 high energy IceCube alert IC-170922A [2], as well as the archival analysis that revealed a
 1246 3.5σ neutrino flare that occurred in 2014, prior to the 2017 high energy alert event [3].

1247 The all-sky multi-flare analysis presented in this work includes the location of TXS
 1248 0506+056, and we can consequently generate a flare curve for this source candidate. The
 1249 flare curve observed in the all-sky multi-flare analysis can be seen in figure 6.12. TXS
 1250 0506+056 has a local multi-flare significance of $p = 3.37 \times 10^{-4}$ (3.4σ), with the main
 1251 contributor being the 2014 neutrino flare candidate, beginning on MJD=56927.86 and ending
 1252 on MJD=57072.99.

1253 Readers who have been paying close attention to the content of this thesis (a group which
 1254 at this point probably includes my thesis committee and like, no one else) may notice that
 1255 the flare curve shown in figure 6.12 seems to be in disagreement with the published result
 1256 with regards to the 2014 TXS flare . While the result shown in [3] reports a significance for
 1257 the 2014 flare candidate of 3.5σ , the all-sky multi-flare analysis presented here observes a
 1258 local significance for this flare of only $p = 0.0054$ (2.5σ).

1259 The difference in significance between the two results can be explained by the different
 1260 event selections that were used in each analysis. Descriptions of these event samples may
 1261 be found in section 3 of this thesis. While [2] used the PointSourceTracks v002p03 event
 1262 sample, the all-sky multi-flare analysis presented here uses the PointSourceTracks v003p02
 1263 event sample. The most relevant difference between the two event samples in this case are
 1264 the pre-cuts: PointSourceTracks v003p02 introduces a cut prior to the event selection BDT
 1265 that requires events to have a reconstructed track length greater than 200 meters. This
 1266 pre-cut does not exist in PointSourceTracks v002p03.

1267 As a result of the track length pre-cut that was introduced in PointSourceTracks v003p02,
 1268 two events (referred to by their event IDs: 40914587 and 56262988) that were major contrib-

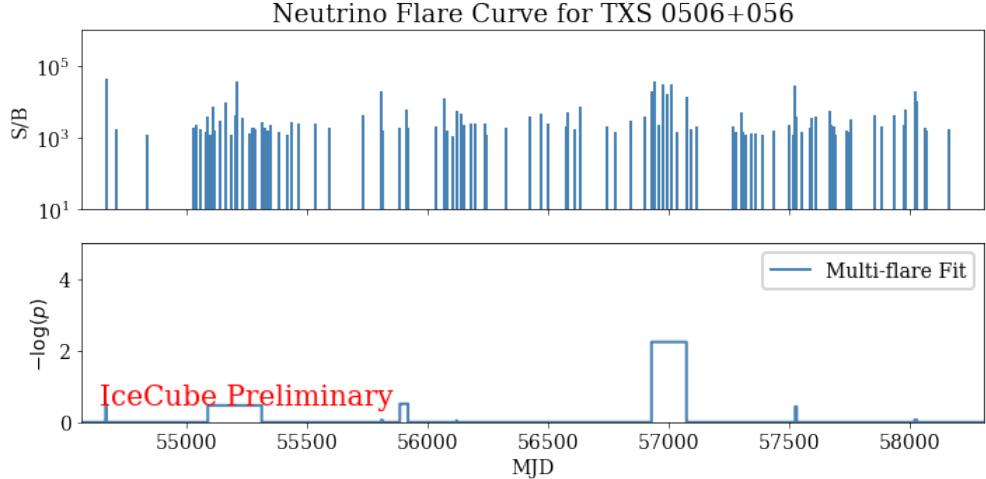


Figure 6.10: The flare curve obtained from the all-sky multi-flare analysis for TXS 0506+056. The pre-trial multi-flare significance of this location is $p = 3.37 \times 10^{-4}$ (3.4σ). Interestingly, the 2014 flare candidate is not seen at the original significance of 3.5σ that was reported in [2]. While the 2014 flare candidate is present, this analysis obtains an associated local significance of only $p = 0.0054$ (2.5σ).

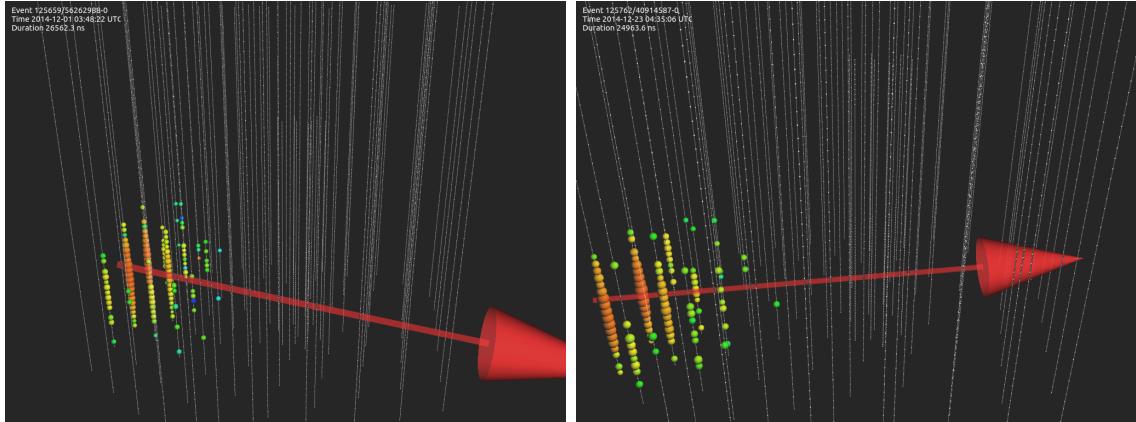


Figure 6.11: The two cascade-like events (event IDs: 40914587 and 56262988) that contributed to the 2014 TXS 0506+056 neutrino flare candidate in PointSourceTracks v002p03, but are not present in PointSourceTracks v003p02. The red arrows show the directional reconstruction associated with the track reconstruction that was used, however given the cascade topology of these events, this reconstruction is likely to be inaccurate.

1269 utors to the 2014 TXS 0506+056 neutrino flare candidate were removed. These events have
 1270 a cascade-like topology, though in PointSourceTracks v002p03, they are reconstructed as if
 1271 they were tracks, leading to potentially inaccurate descriptions of the event direction and
 1272 energy. Since these events are cascade-like, they have a short reconstructed track length,
 1273 and as such these events do not exist in PointSourceTracks v003p02.

1274 Manually removing the two cascade events from the PointSourceTracks v002p03 data

sample and recomputing the significance of the 2014 TXS 0506+056 neutrino flare results in a drop in significance comparable to what was seen when using PointSourceTracks v003p02, as can be seen in table 6.2. The drop in significance cannot be otherwise adequately explained by changes to the angular reconstruction or other differences between the two versions of the data sample. The time-integrated sensitivity of the two samples is comparable, and favors PointSourceTracks v3 at the declination of TXS 0506+056. As such, the drop in significance of the 2014 TXS 0506+056 flare candidate is unlikely to have been caused by PointSourceTracks v003p02 being an overall less sensitive sample.

Untriggered Flare Cross-check Results

Sample	p (pre-trial)	T_{start}	T_{stop}	n_s	γ
PSTricks v2 [3]	7.0e-5	56937.81	57096.22	14.39	2.20
PSTricks v2 w/o cascades	1.17e-3	56937.81	57112.65	12.22	2.26
PSTricks v3 (multi-flare skymap)	5.4e-3	56927.86	57072.99	11.87	2.22

Table 6.2: The results of repeating the untriggered flare analysis preformed in [3], but using PSTricks v3 in place of PSTricks v2, the dataset that was originally used. The apparent drop in significance when using PSTricks v3 can be explained by cascade-like events present in v2 that have been removed from v3.

It should be noted that the result published in [3] is not incorrect, despite using an older data sample that contains unforeseen background events. Since the methods used for the original untriggered flare analysis use a data-driven background estimation, the cascade background in PointSourceTracks v002p03 is accounted for in the original 3.5σ significance that was reported for the 2014 neutrino flare candidate.

Though the cascade background is handled differently in the PointSourceTracks v002p03 analysis versus the PointSourceTracks v003p02 analysis, the two results are not inconsistent with one another. Figure 6.13 shows the results of comparing the results of simulated flares comparable to the 2014 TXS flare injected into both PointSourceTracks v002p03 and PointsourceTracks v003p02. As the background test statistic distributions for both samples are similar, comparing the test statistic values obtained using each sample is a good proxy for comparing the flare local significance. For this test, trials of simulated signal at the location of TXS 0506+056 were injected into background maps generated using PointSourceTracks

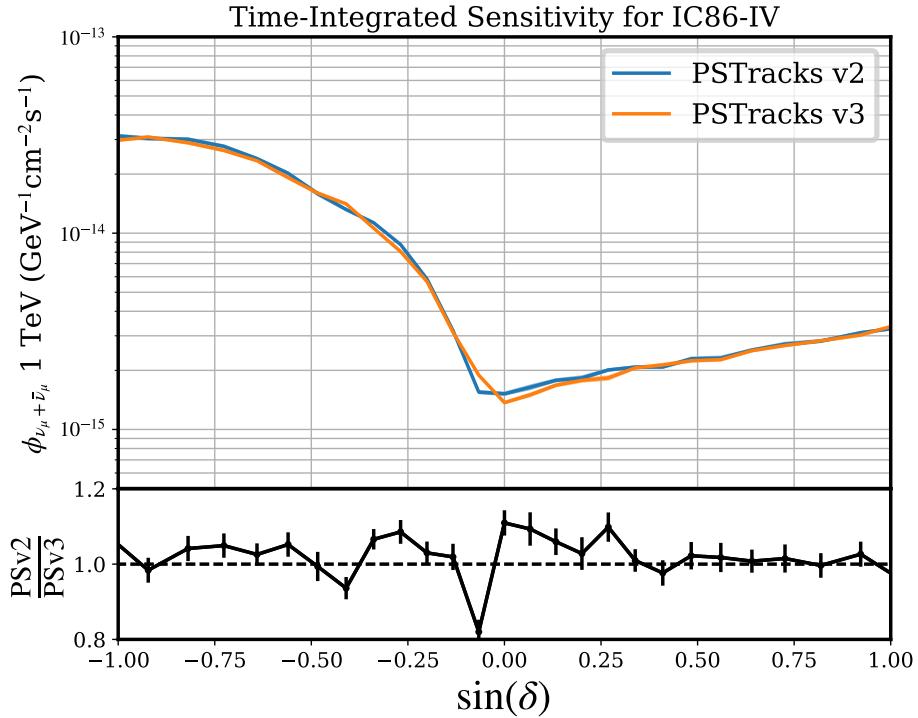


Figure 6.12: A comparison of the time integrated sensitivities of PointSourceTracks v002p03 and PointSourceTracks v003p02. At the declination of TXS 0506+056 ($\delta = 5.7^\circ$), PointSourceTracks v003p02 has a slightly better sensitivity.

1296 v002p03. The injected signal was chosen to replicate the best fit parameters in the original
 1297 PointSourceTracks v002p03 untriggered flare analysis [3]: 13 events with a spectral index
 1298 of $\gamma = 2.2$, injected over a time window of length $\Delta t = 158$ days. A test statistic for each
 1299 of these PointSourceTracks v002p03 signal trials was then calculated. The same simulated
 1300 signal events were then injected into background trials generated with PointsourceTracks
 1301 v003p02, but any events that do not pass the PointSourceTracks v003p02 event selection
 1302 were removed. A new PointSourceTracks v003p02 test statistic is then calculated for each
 1303 of these trials with injected signal.

1304 Figure 6.13 shows the results of this crosscheck, showing the distribution of test statistics
 1305 differences between the v2 and v3 test statistics. While on average the test statistic associ-
 1306 ated with a signal of this type is higher in PointSourceTracks v003p02, the distribution is
 1307 rather wide, and it is not uncommon for the PointSourceTracks v003p02 result to be lower

1308 significance than the PointSourceTracks v002p03 result. A drop in significance comparable
 1309 or greater than what was seen in the observed v2 and v3 results ($TS_{v3} - TS_{v2} = 11.84$)
 1310 occurs in 8% of the simulated trials.

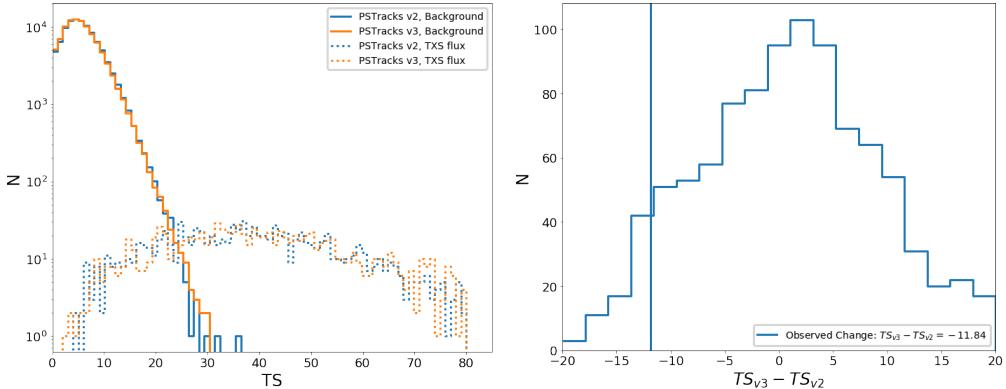


Figure 6.13: Simulated trials comparing the significance of a flare comparable to the 2014 TXS 0506+056 flare in both PointSourceTracks v002p03 and PointSourceTracks v003p02. Left: the test statistic distributions for both datasets in the background (solid) and injected signal (dotted) cases. The PointSourceTracks v003p02 signal is identical to the PointSourceTracks v002p03 signal, except with events that do not pass the v3 selection removed. Right: A comparison of the v2 and v3 test statistics for each simulated signal trial, with the observed test statistic difference plotted as a vertical line. A drop in significance at least as large as what was seen in observed data occurs in 8% of simulated trials.

1311 While there appears to be a drop in significance for the 2014 TXS 0506+056 untrig-
 1312 gered neutrino flare when using to the newer PointSourceTracks v003p02 data sample, it
 1313 is important to realize that the updated v3 result is not inconsistent with the previous v2
 1314 result. Additionally, the 2014 flare composes only part of the ensemble of results that make
 1315 TXS 0506+056 an interesting source candidate, as it is the combination of the 2014 untrig-
 1316 gered neutrino flare with the 2017 high energy alert event and multi-messenger observations
 1317 that suggest TXS 0506+056 as a source of astrophysical neutrinos. The significance of the
 1318 2014 TXS 0506+056 flare will likely continue to be monitored as the IceCube collaboration
 1319 improves its data samples and reconstruction techniques, hopefully converging on a more
 1320 precise measurement to the flare significance.

1321 Chapter 7

1322 Outlook

1323 In this work, we have introduced a novel method for fitting ensembles of neutrino flare
1324 candidates at a particular source location. This method was applied to two catalogs of
1325 source candidates motivated by TXS 0506+056: a catalog of northern-sky 3LAC blazars, and
1326 a "self-triggered" catalog of high energy IceCube events. Though neither catalog displayed
1327 a significant excess of neutrino flares, this allowed us to further constrain potential transient
1328 neutrino emission from blazars.

1329 The multi-flare algorithm introduced here was also applied on an all-sky scale. No signif-
1330 icant population of neutrino flares was observed, though flare curves were produced at every
1331 location in the sky, providing a potentially useful tool for future multi-messenger searches
1332 wishing to incorporate information on the historical neutrino emission from source candidate
1333 locations, similar to what was done in [3].

1334 The non-detection of a significant population of transient neutrino sources is somewhat
1335 interesting from the perspective of constraining astrophysical neutrino source populations.
1336 Whatever the sources of astrophysical neutrinos may be, they cannot produce neutrino flares
1337 that are so bright or numerous that the data becomes inconsistent with the background
1338 expectation on an all-sky scale. This places limits on the source density, flare rate, and
1339 neutrino burst energy that are not associated with any particular source class, serving as a
1340 "universal limit" of sorts on the behavior of potential astrophysical neutrino transients.

1341 It should be noted, however, that a non-detection of neutrino sources on an all-sky scale
1342 is not entirely unexpected. The all-sky trial factor is quite large, and the most significant
1343 neutrino source candidates to date (TXS 0506+056 and NGC 1068) were both originally
1344 identified in spatially triggered analyses, where the candidate location was already identified
1345 as potentially interesting by other astrophysical messengers. Future, spatially triggered
1346 searches may be sensitive to individual sources that have been obscured by the all-sky trial
1347 factor. Ideally, the neutrino flare curves introduced in this work would aid with this, as the
1348 historical neutrino information provides an additional piece to the multi-messenger puzzle.

1349 Though the first applications of the multi-flare algorithm did not yield any significant dis-
1350 covery of astrophysical neutrino sources, the future of neutrino astronomy is still quite bright.
1351 With two interesting source candidates already identified, the answer to the question of the
1352 source of astrophysical neutrinos (and consequently, the source of high energy cosmic rays) is
1353 potentially beginning to become more clear. The upcoming years bring improvements to the
1354 IceCube detector calibration and event reconstruction, providing a more accurate descrip-
1355 tion of existing data, as well as the construction of IceCube Gen2 which will boast increased
1356 effective area at high energies, thereby reducing the atmospheric neutrino background. Ad-
1357 ditionally, upcoming radio-based neutrino experiments such as RNO-G, BEACON, IceCube
1358 Gen2-Radio, PUEO, and POEMMA will provide a view into the ultra-high energy neutrino
1359 regime, beyond what the current iteration of IceCube is realistically capable of measuring.
1360 As the identification of candidate neutrino emission from TXS 0506+056 was triggered by a
1361 single high energy neutrino event, the expansion of neutrino detectors into the high energy
1362 regime could lead the way to identifying future neutrino sources.

¹³⁶³ **Chapter 8**

¹³⁶⁴ **Appendix: Statistics Background**

¹³⁶⁵ Much of the material presented in this thesis requires an understanding of a few key areas
¹³⁶⁶ of statistics. This material is typically covered in almost any introductory level statistics
¹³⁶⁷ course, but is generally not part of the standard undergraduate physics curriculum. For
¹³⁶⁸ this reason, I've collected some of the key concepts in this section, to potentially serve as a
¹³⁶⁹ starting point for people who may not already be familiar with the statistical underpinnings
¹³⁷⁰ of recent source searches in neutrino astronomy. An interactive ipython notebook showing
¹³⁷¹ examples of these concepts can also be found as part of the materials for the 2019-2021
¹³⁷² IceCube bootcamps¹.

¹³⁷³ **8.1 Estimators**

¹³⁷⁴ An estimator is a procedure applied to the data sample which gives a numerical value for a
¹³⁷⁵ property of the parent population, or, as appropriate, a property or parameter of the parent
¹³⁷⁶ distribution function.

¹³⁷⁷ This is a general definition: The sample of interest may have been drawn from a large
¹³⁷⁸ parent population, for which some property is to be inferred. Alternatively, it may have
¹³⁷⁹ been generated from a distribution function, arising from a basic law being investigated.

¹<https://events.icecube.wisc.edu/event/105/contributions/1054/>

1380 This distribution function also has parameters and properties, which one may also wish to
 1381 calculate (e.g. σ for a 1 dimensional gaussian distribution).

1382 Estimators are generally not classified as "right" or "wrong", rather we tend to classify
 1383 estimators as "good" or "bad" based on their properties. Some properties that are often
 1384 relevant when describing estimators:

- 1385 • An estimator is *consistent* if it tends to the true value as the number of data values
 1386 tends to infinity.
- 1387 • An estimator is *unbiased* if its expectation value is equal to the true value.
- 1388 • An estimator is *efficient* if its variance is small.

1389 Estimators can be any number of the above, but good estimators should fulfill all three
 1390 descriptions [48].

1391 8.2 Likelihoods

1392 One class of estimators that is commonly used in neutrino astronomy are *likelihood-based*
 1393 *estimators*. For a set of data, $x_1, x_2, x_3 \dots x_N$ produced from some underlying distribution
 1394 with parameter a , the probability of obtaining that particular data set is the product of
 1395 the individual probabilities of obtaining each of the x_i . This product is referred to as the
 1396 *likelihood* (eq. 8.1 and 8.2):

$$L(x_1, x_2, x_3 \dots x_N, a) = P(x_1; a)P(x_2; a) \dots P(x_N; a) \quad (8.1)$$

$$L(x_1, x_2, x_3 \dots x_N, a) = \prod_{i=1}^N P(x_i; a) \quad (8.2)$$

1397 The likelihood is typically used to fit the model parameter a , given some observed data
 1398 $x_1, x_2, x_3 \dots x_N$ by finding the value of a which maximizes the likelihood function. Note that
 1399 a can be a vector of values in the case that we have a model with multiple free parameters.

1400 For computational reasons, the process of maximizing the likelihood is oftentimes done
 1401 by minimizing the negative log-likelihood instead. Taking the logarithm of the likelihood
 1402 function turns the product into a sum, making the derivative more intuitive and easier to
 1403 work with numerically, and the switch from maximization of the likelihood to minimizing
 1404 the negative likelihood is purely convention.

1405 8.3 Confidence Intervals

1406 Suppose we want to know the value of a parameter X and have estimated it from the data,
 1407 giving result x . We know the resolution of our underlying distribution, and thus the variance,
 1408 $V(x)$ and the standard deviation σ . Our goal is then to create a "confidence interval": An
 1409 interval within which there is a C % chance that a single observation will lie within the
 1410 interval, where C is our "confidence level". Formally, the confidence interval (x_-, x_+) for a
 1411 given confidence level C obeys the requirement that (eq. 8.3):

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x)dx = C \quad (8.3)$$

1412 There are a few possible ways to construct this interval:

1413 1. The **symmetric interval**: x_- and x_+ are equidistant from the mean, μ , i.e $x_+ - \mu =$
 1414 $\mu - x_-$

1415 2. The **central interval**: The probability above and below the interval are equal: $\int_{-\infty}^{x_-} P(x)dx =$
 1416 $\int_{x_+}^{\infty} P(x)dx = (1 - C)/2$

1417 Common values for C are 68% (1σ), 90% (1.64σ), and 99% (2.58σ). There is a trade
 1418 off between a narrow interval and low confidence. You can say with great confidence that a
 1419 value will lie within a very wide interval, but if one wants to tie it down more precisely the
 1420 confidence lessens.

1421 8.4 Hypothesis Testing

1422 Many problems in science can be reduced to attempting to distinguish between 2 distinct
 1423 possibilities, where the observed data has different properties under each. In many cases,
 1424 we are attempting to distinguish between a "null hypothesis", which is used to refer to a
 1425 "default" model (e.g. the absence of neutrino point sources), and an "alternative hypothesis",
 1426 which represents the addition of some new physics, model parameter, or disagreement with
 1427 the "default" case (e.g. the presence of neutrino point sources). Statistical hypothesis testing
 1428 is a framework for comparing these two hypotheses, given some set of observed data.

1429 We can roughly break down the process of hypothesis testing into the following steps:

- 1430 1. State relevant null and alternative hypotheses
- 1431 2. State the test statistic
- 1432 3. Derive the distribution of the test statistic under the null hypothesis
- 1433 4. Select significance level (α) at which the null hypothesis will be rejected
- 1434 5. Compute an observed test statistic from the data
- 1435 6. Calculate a p-value and decide whether to reject the appropriate hypothesis

1436 8.4.1 State relevant null and alternative hypotheses

1437 When constructing an analysis, we should be careful to state the null and alternative hy-
 1438 potheses. Typically, the null hypothesis (H_o) refers to a statement or default position that
 1439 there is no relationship between two phenomena, that there is no signal, or that our cur-
 1440 rent models describe the data well. The alternative (H_a) hypothesis usually describes the
 1441 opposite: that there is a relationship between two phenomena, that there is a signal, or that
 1442 our models to NOT describe the data well. Here are some example pairs of hypotheses one
 1443 might encounter when working with IceCube data:

1444 • H_o : Neutrino event arrival directions are not significantly correlated with the positions
 1445 of blazars identified in Fermi's 3FGL catalog

1446 • H_a : Neutrino event arrival directions are correlated with the positions of blazars iden-
 1447 tified in Fermi's 3FGL catalog

1448 • H_o : A 3-flavor oscillation model describes the IceCube data well

1449 • H_a : IceCube data is inconsistent with a 3-flavor model

1450 It is important to be careful when formulating null and alternative hypotheses so as to
 1451 not overreach with the claims being made should the alternative hypothesis be accepted.

1452 In general, it's best to keep in mind the specifics of how a particular analysis is being
 1453 constructed, and what the property of the data being measured actually is. For example, in
 1454 the first pair of hypotheses above, it would be improper for our alternative hypothesis to be
 1455 "blazars are a source of astrophysical neutrinos" if we were only examining the positions of
 1456 the 3FGL catalog, which covers only a subset of all observed blazars.

1457 8.4.2 State a test statistic

1458 A *test statistic* is a quantity derived from the sample, intended to be a numerical summary
 1459 of a dataset that reduces the data to one value that can be used for hypothesis testing. In
 1460 general, the test statistic is selected in such a way as to quantify, within observed data,
 1461 behaviors that would distinguish the null and alternative hypotheses. These are very similar
 1462 to estimators, and we can oftentimes use estimators as test statistics.

1463 An important property of the test statistic is that its sampling distribution under the
 1464 null hypothesis must be calculable either exactly or approximately. There are many types of
 1465 test statistics that you can use, and what the exact form of the test statistic is will usually be
 1466 determined by your analysis. One example of a test statistic is the chi-squared test statistic
 1467 intended to compare counts in an oscillation analysis (eq. 8.4):

$$\chi^2 = \sum_{i \in bins} \frac{(n_i^{\nu+\mu_{atm}} - n_i^{data})^2}{(\sigma_i^{data})^2 + (\sigma_{\nu+\mu_{atm},i})^2} \quad (8.4)$$

1468 Where χ^2 is the test statistic, i refers to bins in zenith and azimuth, $n_i^{\nu+\mu_{atm}}$ and n_i^{data} are
 1469 the number of events in a particular bin in both simulation and data, and $\sigma_{\nu+\mu_{atm},i}$ and σ_i^{data}
 1470 are similar the per-bin standard deviations in both simulation and data. This test statistic
 1471 is used to measure the agreement of the data with a "nominal" model that can be simulated
 1472 to obtain the values $n_i^{\nu+\mu_{atm}}$ and $\sigma_{\nu+\mu_{atm},i}$.

1473 Another example of a commonly used test statistic in neutrino astronomy is the point
 1474 source likelihood (eq. 8.5):

$$TS = -2 \log\left(\frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s)}\right) \quad (8.5)$$

1475 This is a likelihood ratio test statistic, where \mathcal{L} is a likelihood function that describes
 1476 how clustered events are near a particular location. In its simplest form, this likelihood has
 1477 1 parameter, n_s , that describes the number of excess events seen near the location being
 1478 examined.

1479 The likelihood ratio is a very powerful test statistic. Given two hypotheses H_o and $H_a(a)$
 1480 where $H_o = H_a(\tilde{a})$ (they are *nested* models), then the Neyman-Pearson lemma states that
 1481 the likelihood ratio test has the strongest statistical power (i.e. correctly reject H_o when H_1
 1482 is true). This is the case for many point source searches, where the null hypothesis ("There
 1483 are zero signal events, $n_s = 0$) is nested in the alternative hypothesis ("There are a nonzero
 1484 number of signal events, $n_s > 0$ ").

1485 Note that test statistics are essentially estimators of the alternative hypothesis: a "more
 1486 extreme" test statistic should correspond to the data being better described by the alter-
 1487 native hypothesis. In fact, most test statistics are in fact based on estimators of some
 1488 statistical property of the data. The point source test statistic, for example, is based around
 1489 a likelihood estimator of the number of clustered neutrino events.

1490 **8.4.3 Select a significance level at which the null hypothesis will**
 1491 **be rejected**

1492 Since in testing our hypothesis, we have two options for our result (accept/reject the null
 1493 hypothesis), and there are two options for the "actual" state of our null hypothesis (either
 1494 it is true or it isn't), there are four possible outcomes describing whether we were correct or
 1495 not:

	H_o rejected	Accept (Fail to reject) H_o
H_o False	Correct	Type II error (false negative)
H_o True	Type II error (false positive)	Correct

1497 If you reject a true null hypothesis, then this is a type II error. If you accept a false
 1498 null hypothesis, then this is a type I error. "Type I" and "type II" are needlessly confusing
 1499 names, so let's instead refer to these with the more intuitive names "false positive" and
 1500 "false negative".

1501 Both types of errors are bound to happen sometimes, but we're typically more concerned
 1502 with false positives: We don't want to make any claims of discovery when in fact it was
 1503 simply a background fluctuation. We can control the rate at which this happens with the
 1504 "significance" of the test (α). The *significance* refers to the probability of obtaining a false
 1505 positive.

1506 Given a hypothesis test with some test statistic (where we can calculate the distribution
 1507 of this test statistic under the null hypothesis, see above!), we can divide the range of our
 1508 test statistic into the acceptance region, containing test statistics less extreme than some
 1509 pre-defined significance threshold, and the rejection region, containing test statistics more
 1510 extreme than this threshold. If our observed test statistic falls in the acceptance region, we
 1511 will accept the null hypothesis to be true. If our observed test statistic falls into the rejection
 1512 region, we will reject the null hypothesis.

1513 The probability of a false positive is then simply the integral of our null hypothesis test
 1514 statistic distribution over the rejection region. This is the "significance" of the test. In
 1515 principle, we can set the significance of our test to be whatever we want, simply by changing
 1516 the acceptance and rejection regions. In practice, we typically want to set a high bar for
 1517 announcing a result (this corresponds to a low significance). In particle physics, the threshold
 1518 for rejecting the null hypothesis is typically set such that the significance is 2.87×10^{-7} (the
 1519 one-sided five sigma threshold). In other disciplines, the convention is different, with social
 1520 scientists and biologists typically rejecting the null hypothesis at the .01 or .05 significance
 1521 level.

1522 You might sometimes hear significance referred to in units of "sigma" (e.g. " 5σ " or " 3σ ").
 1523 This can be obtained by the process shown in figure 8.1, by mapping the significance to a
 1524 normal distribution. This simply provides a standardized way of describing how "extreme"
 1525 the observed significance is: more "sigmas" corresponds to a result that is less likely to be
 1526 seen under the null hypothesis.

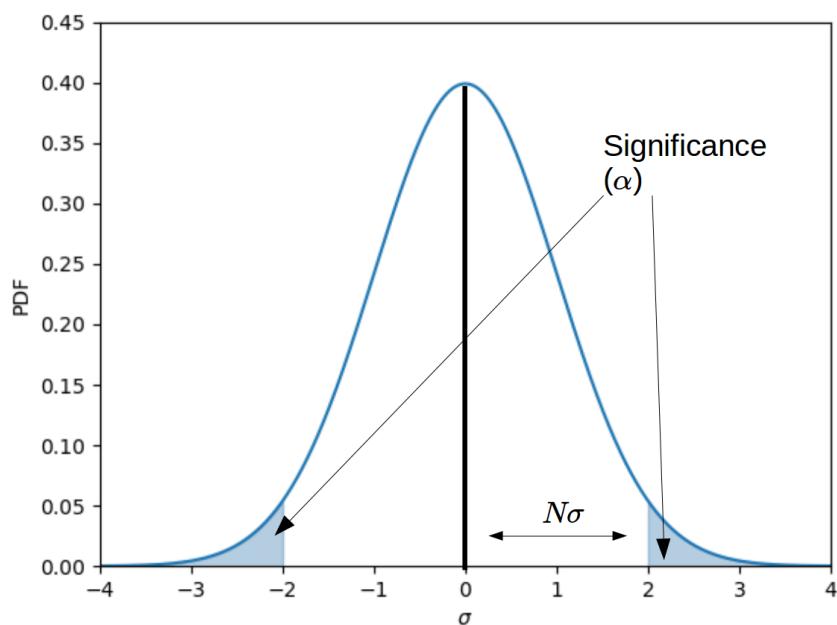


Figure 8.1: The mapping of significance into units of sigma using a normal distribution, for a two tailed test.

1527 **8.4.4 Compute the TS from observed data**

1528 This step is fairly straightforward, as the test statistic has already been defined. Simply
 1529 run your test statistic algorithm using the observed data generated from your experimental
 1530 apparatus.

1531 **8.4.5 Decide whether to accept or reject the null hypothesis**

1532 There are two different, but equivalent constructions for deciding whether to reject the null
 1533 hypothesis or not. The first construction involves pre-defining an *acceptance region* prior
 1534 to calculating the test statistic associated with the observed data. The acceptance region
 1535 corresponds to the range of test statistic values for which the null hypothesis cannot be
 1536 rejected. This is typically chosen to correspond to a particular significance (e.g. 5%, 1%,
 1537 0.01% etc.). If the observed test statistic lies outside of the acceptance region corresponding
 1538 to $X\%$ significance, then we can say that the null hypothesis is rejected at the $X\%$ signif-
 1539 icance level. Note that since we had to pick our significance threshold (and consequently
 1540 define an acceptance region) prior to computing an observed test statistic, we can make no
 1541 further claims on how extreme the result is: if our acceptance region is $TS > 12$, it does not
 1542 matter if we observe a $TS = 12.1$ or if we observe a $TS = 99.0$. In both instances we will
 1543 reject the null hypothesis as the significance level we initially chose.

1544 An alternative, but mathematically equivalent, construction involves the calculation of a
 1545 *p-value*. A p-value is the probability, under the null hypothesis, of obtaining a test statistic
 1546 at least as extreme as the one we have observed from data. If our p-value is smaller than a
 1547 pre-determined significance threshold, then we can reject the null hypothesis. The p-value
 1548 can be calculated by simply integrating our null hypothesis test statistic distribution over
 1549 the range of values more extreme than what was observed. For the one-tailed case where a
 1550 larger test statistic is correlated with stronger evidence of the alternative hypothesis, this is
 1551 (eq. 8.6):

$$p = \int_{TS_{obs}}^{\infty} PDF_{H_0} \quad (8.6)$$

1552 Where TS_{obs} is our observed test statistic and PDF_{H_0} is the distribution of test statistics
 1553 under the null hypothesis. It is important to note that the above equation is only valid for
 1554 the positive, one-tailed case (the alternative hypothesis is only associated with test statistic
 1555 values on one side of the distribution). A more general formula could be expressed as (eq.
 1556 8.7):

$$p = \int_{R|TS_{obs}} PDF_{H_0} \quad (8.7)$$

1557 Where here $R|TS_{obs}$ is the rejection region of the test statistic distribution associated
 1558 with a particular observed test statistic (i.e. it is the region of the background test statistic
 1559 distribution containing values more extreme than what was observed). This distinction is
 1560 particularly important when conducting two-tailed tests, where test statistics that are either
 1561 above an upper bound, or below a lower bound may be cause for rejecting the null hypothesis.

1562 8.5 Trial Factors

1563 Each time you obtain the answer of an analysis from real data, you have done one experi-
 1564 mental trial. It is common to perform multiple trials when working with a specific analysis
 1565 technique.

1566 • In the neutrino sources group, we might test the probability that one location is con-
 1567 sistent with background expectation. If we were to apply this framework to a list of
 1568 30 source candidate locations, then this would be 30 trials.

1569 • Repeating an analysis with new cuts. We might count the number of neutrinos with
 1570 $E > 1$ TeV in 1 year, then count the number of neutrinos with $E > 10$ TeV in 1 year.
 1571 This would be 2 trials.

- 1572 • Repeating with different signal hypotheses. You might look for emission from blazars
 1573 using both the hypothesis of steady emission and a flare hypothesis (2 trials)

1574 Trials can be both independent, or correlated:

- 1575 • **Independent:** Search for a new source/signal in 1 year of data, repeat the search in
 1576 the next year of data (separately). This would be 2 independent trials, as the results
 1577 of the first year will not affect the results of the second year
- 1578 • **Correlated:** Search for a new source/signal in 1 year of data, repeat the search with
 1579 2 years of data, including the first year. This would be 2 trials, but they would be
 1580 correlated, since the results of the first 2 years of data will be affected by the results
 1581 of the first year alone.

1582 Accounting for trials is important, as given enough trials even the null hypothesis can
 1583 produce a significant result. A simple and conservative way to account for trials is to assume
 1584 that trials are uncorrelated and apply the Sidak correction (eq. 8.8):

$$P_{post} = 1 - (1 - P_{pre})^N \quad (8.8)$$

1585 Where P_{post} is the trial-corrected p-value, P_{pre} is the smallest p-value observed across
 1586 all trials performed, and N is the number of trials. This formula is intended to describe
 1587 uncorrelated trials, however it can also be used to obtain a conservative trial correction
 1588 for correlated trials as well. More sophisticated methods of accounting for correlated trials
 1589 typically involve simulation of the null hypothesis, and are oftentimes specific to a particular
 1590 experiment.

1591 When reporting a trial corrected p-value, it is additionally important to specifically state
 1592 which trials are being accounted for. Accounting for trials across all sources in a catalog
 1593 will result in a different p-value than accounting for all analyses performed by the IceCube
 1594 collaboration, yet both are "post-trial" p-values.

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