

1 Every Flare, Everywhere: Untriggered Searches for Astrophysical Neutrino Transients Using Data

2 From the IceCube Detector

3 *by*

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20 Abstract

21 Recent results from the IceCube collaboration regarding the blazar TXS 0506+056 suggest
22 the presence of neutrino flares that are not temporally coincident with a significant corre-
23 sponding gamma ray flare. Such flares are particularly difficult to identify, as their presence
24 must be inferred from the temporal distribution of neutrino data alone. While previously
25 existing statistical methods fit for only the largest neutrino flare candidate at a particular
26 location, this work introduces an extension to fit for the contributions from multiple flare
27 candidates at once, thereby increasing the sensitivity to sub-threshold neutrino flares. The
28 need for a multiple flare fit is further motivated by the increasing duration of the IceCube
29 data available for analysis. In this work, this "multi-flare" method is applied to data from
30 the IceCube Neutrino Observatory using two different source catalogs to explore emission
31 from candidates similar to TXS 0506+056. This method is additionally used to create a
32 "multi-flare" skymap, describing the temporal variability of IceCube data at every point in
33 the neutrino sky. These results can be used to place further constraints on potential pop-
34 ulations of transient neutrino sources, serving as a complement to existing time-integrated
35 and time-dependent methods.

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₁₂₃ **Chapter 1**

₁₂₄ **Introduction**

₁₂₅ Though cosmic rays were first discovered in 1912, their origins have to date remained a
₁₂₆ mystery. Since cosmic rays are charged particles and their paths through the universe are
₁₂₇ bent by magnetic fields, identifying the sources of cosmic rays is not as simple as tracing
₁₂₈ back their arrival directions at Earth. The discovery of the astrophysical neutrino flux in
₁₂₉ 2013 [1] provides a new avenue for investigating this problem, as the sources of astrophysical
₁₃₀ neutrinos are suspected to be the same as high energy cosmic rays, but neutrinos are not
₁₃₁ charged, and thus their arrival directions point directly back to their source. Identifying
₁₃₂ the sources of astrophysical neutrinos (and consequently cosmic rays) would provide an
₁₃₃ entirely new view of the universe, potentially allowing us to examine regions opaque to more
₁₃₄ traditional astrophysical messengers such as photons.

₁₃₅ With the IceCube Neutrino Observatory having collected over 10 years of data, the
₁₃₆ first hints of astrophysical neutrino sources are beginning to be visible: a high energy alert
₁₃₇ event from the direction of the blazar TXS 0506+056 sparked multi-messenger followup that
₁₃₈ suggests neutrino emission from that location [2][3], and recent studies of time integrated
₁₃₉ neutrino emission from candidate blazars reveal a 3.5σ excess of astrophysical neutrino events
₁₄₀ from the direction of the Seyfert II galaxy NGC 1068 [4].

₁₄₁ These first hints of astrophysical neutrino sources raise several further questions: Could
₁₄₂ there be more sources like TXS 0506+056 or NGC 1068? How many? Is neutrino emission

143 time-dependent, or steady? Perhaps most importantly, if there are more sources, how can
144 we identify them? Though neutrinos are an excellent astrophysical messenger due to the
145 correlation of their arrival directions with their source of origin, neutrino astronomy is made
146 significantly more difficult by the existence of the atmospheric neutrino flux, which produces
147 a large, irreducible background for searches of astrophysical neutrino sources. Because of
148 this, high energy neutrino astronomy relies either on triggering off a multi-messenger signal
149 (such as a blazar that was also detected in gamma rays), or leveraging knowledge about the
150 pattern of neutrino emission (for example, the arrival directions of astrophysical neutrinos
151 should be clustered near their sources of origin, while the atmospheric background should
152 be isotropically distributed).

153 This work will focus specifically on exploring the possibility of time-dependent neutrino
154 emission, motivated in part by the archival followup that was performed by IceCube at the
155 location of the blazar TXS 0506+056 [3]. A novel method for fitting decorrelated ensembles of
156 neutrino flare candidates is introduced and subsequently applied to source catalogs assembled
157 according properties of the TXS 0506+056 result that could potentially define a class of
158 astrophysical neutrino sources. This new method of fitting ensembles of neutrino flares is
159 intended to fill a methodological gap that exists in the field, as it provides a framework for
160 combining ("stacking") neutrino flare candidates.

161 This method is additionally applied to the entire neutrino sky, allowing for a more gen-
162 eral examination of potential clustering of neutrino data in both space and time. If neutrino
163 data is clustered beyond what is expected from the background hypothesis (isotropic arrival
164 directions distributed uniformly in time), then that would be evidence of a population of
165 astrophysical neutrino sources. Even in the case of a non-discovery, the method introduced
166 here produces neutrino "light-curves" (here referred to as "flare curves") which are of po-
167 tential use for future multi-messenger searches that may wish to incorporate information on
168 historical neutrino emission from a particular source candidate.

	Time Independent	Untriggered Time-Dependent
Single-Source Searches	Time-integrated skymap analysis [4]	Single flare analysis [3]
Source Stacking	Blazar catalog analysis [5]	This work!

Figure 1.1: Examples of some of the types of astrophysical neutrino source searches that have been performed using IceCube data. Time-integrated analyses search for an excess of events over the entire data sample livetime, ignoring any information about potential temporal clustering. By contrast, untriggered, time-dependent searches attempt to fit for temporal neutrino clusters ("flares") without using a multi-messenger lightcurve as a template. Single source searches report the most significant result from a small number of source candidate locations, corrected by a trial factor, while source stacking analyses combine information from many source locations under the assumption that each source candidate is a weak emitter, thereby increasing the search sensitivity to low individual source flux.

¹⁶⁹ Chapter 2

¹⁷⁰ Astrophysical Messengers

¹⁷¹ This work will focus primarily on cosmic rays, photons, and neutrinos, however other mes-
¹⁷² sengers, such as gravitational waves, can provide unique information as well.

¹⁷³ 2.1 Cosmic Rays

¹⁷⁴ 2.1.1 Energy Spectrum

¹⁷⁵ Cosmic rays are high energy protons and atomic nuclei, originating from outside the Earth's
¹⁷⁶ atmosphere. It is difficult to do traditional astronomy using cosmic rays (in the sense of using
¹⁷⁷ their arrival directions to identify sources) due to the fact that cosmic rays are charged, and
¹⁷⁸ consequently their paths are bent by magnetic fields in the universe. However, the cosmic
¹⁷⁹ ray spectrum is well studied, with many measurements having been made over the past
¹⁸⁰ century since their initial discovery. In general, the spectrum can be modeled as a power
¹⁸¹ law (eq. 2.1):

$$N(E)dE \propto E^{-x}dE \quad (2.1)$$

¹⁸² where N describes the number of cosmic ray particles, E is their energy, and x is an energy-
¹⁸³ dependent spectral index. The spectrum has several notable features, shown in figure 2.1.

At lower energies (below 10^{16} eV), the spectrum is well described by an $E^{-2.7}$ flux. At 10^{16} eV, the spectrum softens to $E^{-3.1}$, commonly referred to as the "knee". At even higher energies (around $10^{18.5}$ eV), the spectrum hardens again. This region is referred to as the "ankle", and is thought to correspond to the point where extragalactic cosmic rays begin to dominate the flux.

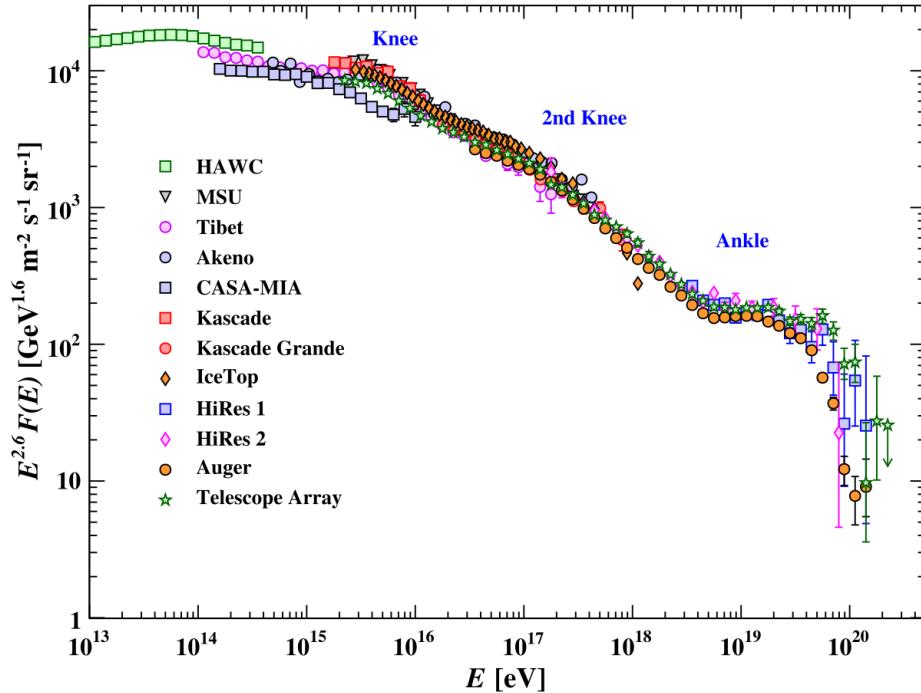


Figure 2.1: A plot of the cosmic ray spectrum assembled from observations from a variety of experiments. Major features visible are the knee ($\approx 10^{16}$ eV), and the ankle ($\approx 10^{19}$ eV).
[5]

2.1.2 Acceleration Mechanism

Though the exact origin of cosmic rays is still unknown, we can infer how these particles are being accelerated based on our knowledge of how charged particles behave.

Suppose a particle encounters a moving cloud of plasma. If it is large enough, the cloud will act as a massive scatterer. The overall collision is the result of a large number of individual scatterings inside the cloud, and the outgoing angle of the original particle is

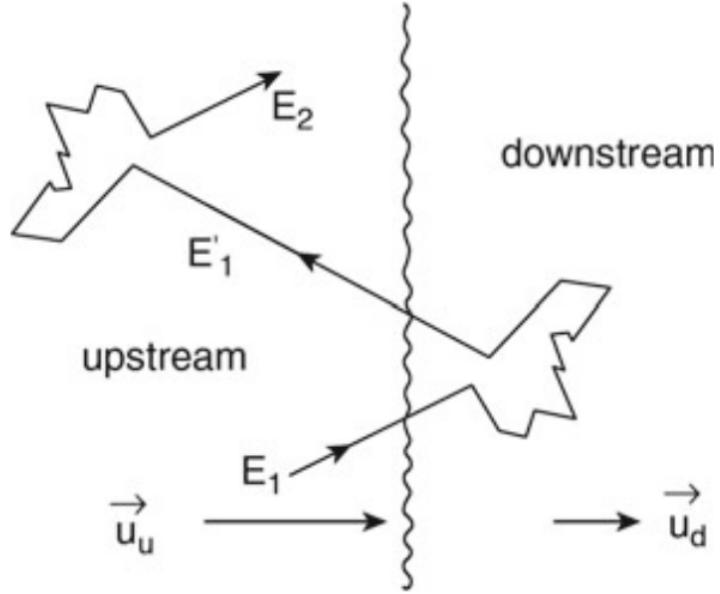


Figure 2.2: Fermi acceleration of a plane shock front[6].

195 essentially random. On average the energy of the particle after a collision with this boundary
 196 will increase by a factor (eq. 2.2):

$$\langle \frac{\Delta E}{E} \rangle \approx \frac{4}{3}\beta^2 \quad (2.2)$$

197 Where $\beta = V/c$, corresponding to the boundary velocity. This is *second order Fermi*
 198 *acceleration*, and is generally insufficient to explain the cosmic ray spectrum. If we instead
 199 assume a plane shock front, as in figure 2.4 , then the post-collision velocity of the particle
 200 is no longer randomly distributed, and we instead obtain the expression (eq. 2.3):

$$\langle \frac{\Delta E}{E} \rangle \approx \frac{4}{3}\beta \quad (2.3)$$

201 This describes *first order Fermi acceleration*. From here, we can calculate the expected
 202 flux measured at Earth to be (eq. 2.4):

$$\frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-\Gamma} \quad (2.4)$$

203 where $\Gamma = \alpha + 1$, $\alpha \approx \frac{3P_e}{4\beta}$, and P_e is the probability that a particle may escape the
 204 shock region (proportional to the velocity, V). Note that this is a power law with almost
 205 constant spectral index, as both β and P_e are proportional to V , similar to what is seen in
 206 the observed cosmic ray spectrum [6].

207 We can additionally derive constraints on the maximum energy to which a particle can be
 208 accelerated based off the magnetic field intensity, B , and the size of the accelerating region,
 209 R (eq. 2.5):

$$E_{max} = eBR \approx 10^{21} \left[\frac{R}{1 \text{ pc}} \right] \left[\frac{B}{1 \text{ Gauss}} \right] \text{ eV} \quad (2.5)$$

210 This is known as *Hillas' condition*, and can inform us about the properties of cosmic
 211 ray accelerators. We can plot this condition to show sources which satisfy this requirement
 212 for a particular energy, producing what is commonly referred to as Hillas' plot (Figure 2.3).
 213 Sources must exist above the diagonal thresholds in order to be valid accelerators of cosmic
 214 rays of certain energies. For example, sources capable of accelerating protons to 100 EeV
 215 energies must exist above the dashed line in figure 2.3. Sources below are either too small,
 216 or have magenetic fields that are too weak.

217 2.2 Photons

218 Perhaps the oldest and most well-studied astrophysical messenger, photons have the useful
 219 property of being electrically neutral. Unlike cosmic rays, their paths are not bent by
 220 magnetic fields, and the arrival directions of photons can be used to identify their origin,
 221 particularly at high energies due to their good angular resolution. Photons additionally span
 222 a large energy range, and can be used to explore a wide variety of astrophysical production
 223 processes. For the purposes of this work, we are most interested in the highest energy
 224 photons (gamma rays, with energies greater than 100 keV), which can be produced through
 225 either leptonic or hadronic processes.

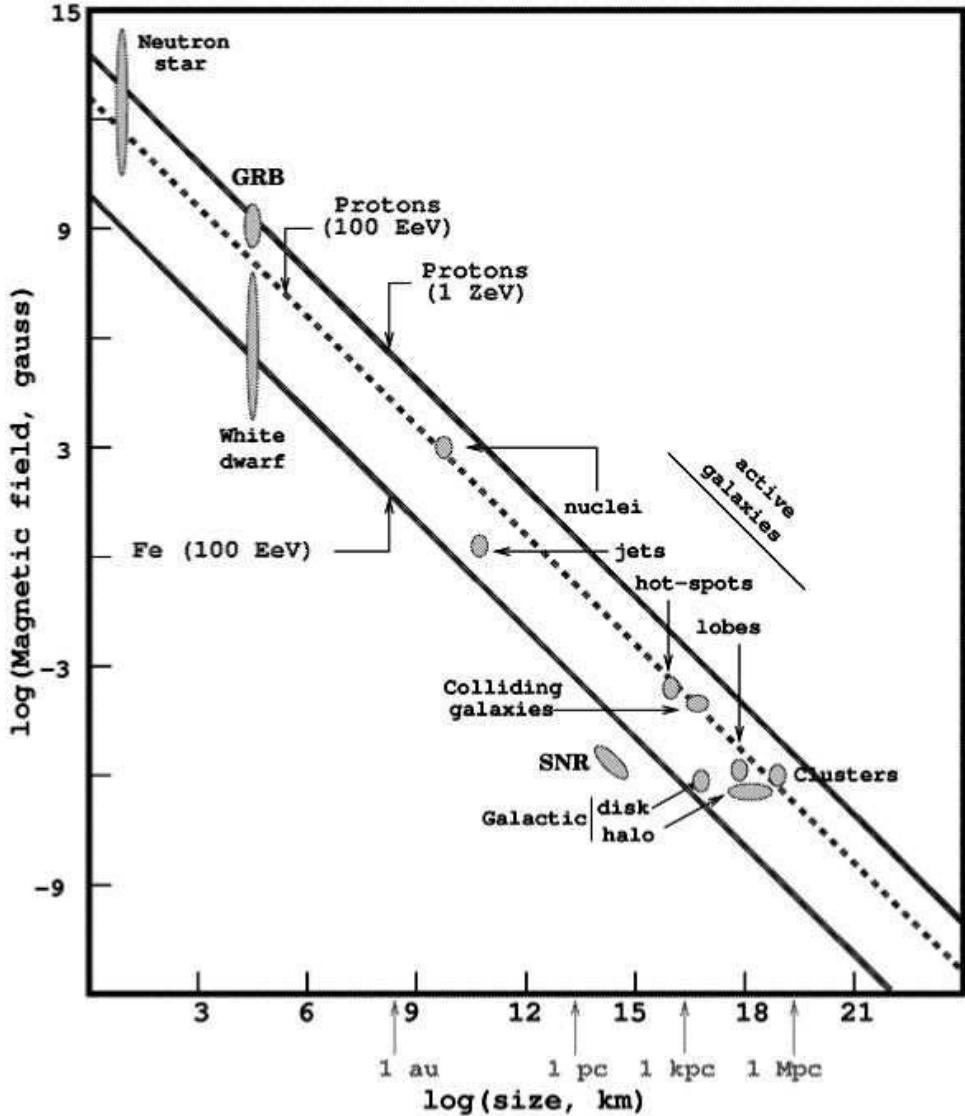


Figure 2.3: A Hillas plot, showing source populations capable of accelerating cosmic rays to a given energy, given their size and magnetic field strength [7].

226 2.2.1 Leptonic Radiative Processes

227 While photons cannot be directly accelerated by electric and magnetic fields, there are a
228 variety of mechanisms by which processes involving leptons can produce high energy photons.

- 229 • **Synchrotron radiation:** Radially accelerated relativistic particles will release syn-
230 chrotron radiation. This typically occurs as charged particles (such as electrons) spiral
231 through magnetic fields. The power loss, (dE/dT) , due to synchrotron radiation for a

232 particle of mass M and charge Z can be expressed as (eq. 2.6):

$$-\frac{dE}{dT} \approx 2.6 \left(\frac{Zm_e}{M} \right)^4 \left(\frac{E}{1 \text{ keV}} \right)^2 \left(\frac{B}{1 \text{ G}} \right)^2 \text{ keV s}^{-1} \quad (2.6)$$

233 Where m_e is the electron mass, E is the particle energy, and B is the magnetic field
 234 strength. From equation 2.6 it can be seen that synchrotron emission is significantly
 235 more relevant for electrons than protons, due to the $1/M^4$ term.

236 • **Compton Scattering and Inverse Compton Scattering:** Photons may scatter
 237 off an electron at rest, and in doing so experience a wavelength shift according to (2.7):

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\hbar\omega}{m_e c^2} (1 - \cos \alpha) \quad (2.7)$$

239 Where λ' and λ correspond to the final and initial photon wavelengths, and α is the
 240 deflection angle of the photon. If a low energy photon collides with a high energy
 241 electron, the energy of the photon may exit with more energy than it began with,
 242 providing an effective mechanism for increasing the photon energy.

243 A combination of these two processes can produce the two peak structure typically seen
 244 in the spectrum of observed gamma ray sources. The lower energy peak is due to synchrotron
 245 radiation, while the higher energy peak corresponds to low energy photons being scattered
 246 to higher energies via inverse compton scattering.

247 2.2.2 Hadronic Radiative Processes

248 While leptonic processes can produce high energy gamma rays, hadronic processes will pro-
 249 duce gamma rays in addition to neutrinos and cosmic rays. Protons accelerated to high
 250 energies in an astrophysical source may interact with other protons via (eq. 2.8):



251 Where X is the appropriate nucleus or hadron depending on the pions produced, and on
 252 average there is expected to be equal production of neutral and charged pions. Additionally,

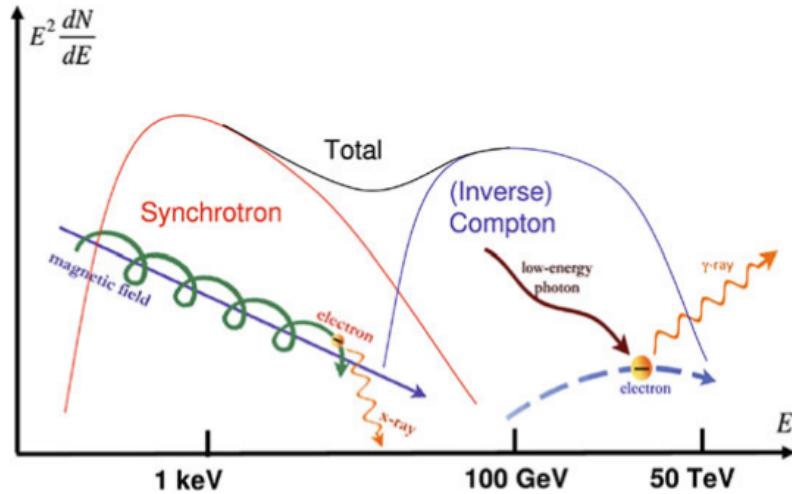


Figure 2.4: A cartoon plot of the gamma ray spectrum for a high energy gamma ray source. The two peak structure arises from the two production mechanisms of gamma rays. At lower energies, photons are produced by synchrotron radiation of electrons spiraling around magnetic field lines. The higher energy peak is then formed by the lower energy photons being scattered off energetic electrons via inverse compton scattering [8].

253 protons may interact with photons, providing an additional path to pion production (eq. 2.9
 254 and 2.10):



255



256 These resultant pions can subsequently decay, producing child particles depending on the
 257 particular pion type. Neutral pions will decay into gamma rays ($\pi^0 \rightarrow \gamma + \gamma$), and charged
 258 pions will decay to a lepton and their associated (anti)neutrino (most often muons and muon
 259 neutrinos: $\pi^+ \rightarrow \nu_\mu + \mu^+$, $\pi^- \rightarrow \bar{\nu}_\mu + \mu^-$). As such the sources of cosmic rays are expected
 260 to additionally produce a high energy gamma ray flux.

261 **2.2.3 Gamma Rays as an Astrophysical Messenger**

262 Gamma rays are a robust astrophysical messenger. As previously mentioned, photons are
 263 uncharged and their arrival directions can consequently be used to identify their origin.
 264 Additionally, the gamma ray energy can be used to infer information about the primary
 265 particle energy for a known production mechanism. Gamma rays are also well studied, and
 266 modern detectors boast high event rates: Fermi-LAT, for example, observes enough events to
 267 be capable of studying the temporal variability of sources on the timescale of approximately
 268 a day.

269 Gamma rays are not without their limitations, however. Note that while at the high-
 270 est energies gamma rays may be produced by both leptonic and hadronic processes, only
 271 hadronic processes are expected to additionally produce neutrinos and cosmic rays. For this
 272 reason, high energy gamma rays alone cannot be used to identify the source of high energy
 273 cosmic rays.

274 Additionally, high energy gamma rays may also interact electromagnetically with the
 275 ambient radiation in the universe, undergoing pair production (eq. 2.11):

$$\gamma_{VHE} + \gamma_{CMB} \rightarrow e^+ + e^- \quad (2.11)$$

276 This interaction can occur between high energy gamma rays and photons from the CMB,
 277 leading to an event horizon for gamma rays of a particular energy. Figure 2.5 shows this
 278 horizon as a function of distance and photon energy. The photon event horizon prevents us
 279 from studying the highest energy photons from many AGN, and there is a significant region
 280 of parameter space below the highest observed cosmic ray energy that cannot be examined
 281 with photons alone.

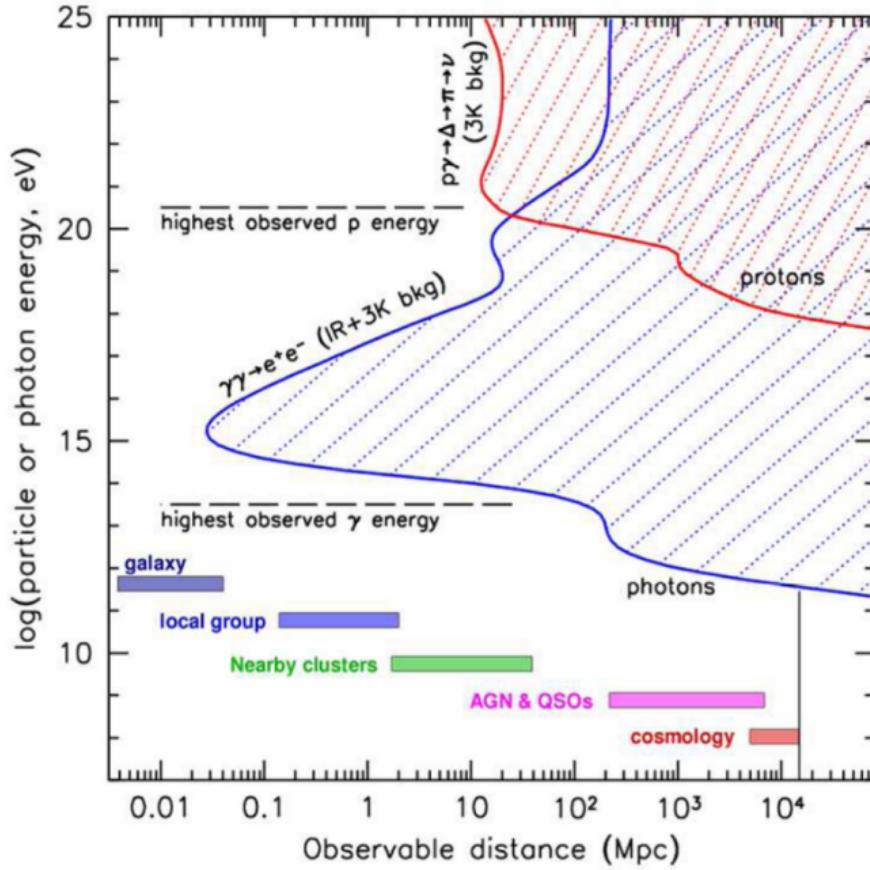


Figure 2.5: A plot of the photon and cosmic ray event horizons, showing the regions/energy scales visible using observations originating from photons (blue) and cosmic rays (red). Pair production of high energy photons with the ambient radiation in the universe prevents photon observations at long distances and high energies, including much of the energy scale below the highest observed cosmic ray energy. Also note that the photon event horizon can prevent us from studying the highest energy photons from AGN [9].

282 2.3 Neutrinos

283 2.3.1 Production Mechanisms

284 Neutrinos in astrophysical sources are produced via charged pion decay from pions pro-
 285 duced from cosmic ray interactions with either photons or protons. The charge of the pion
 286 determines whether more neutrinos or anti-neutrinos are produced (eq. 2.12 and 2.13):

$$\pi^+ \rightarrow \nu_\mu + \mu^+ \rightarrow \nu_\mu + e^+ + \nu_e + \bar{\nu}_\mu \quad (2.12)$$

$$\pi^- \rightarrow \nu_\mu + \mu^- \rightarrow \bar{\nu}_\mu + e^- + \bar{\nu}_e + \nu_\mu \quad (2.13)$$

287 In either case, note that the expected flavor ratio of the neutrinos and anti-neutrinos
 288 produced at the source is $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. Combining this initial flavor ratio
 289 with neutrino oscillations over cosmic distances produces an expected observed astrophysical
 290 neutrino flavor ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$.

291 Neutrinos are also produced by cosmic rays interacting with the Earth's atmosphere. This
 292 is referred to as the atmospheric neutrino flux, and has contributions both from pion decay
 293 ($\pi^+ \rightarrow \nu_\mu + \mu^+$, $\pi^- \rightarrow \bar{\nu}_\mu + \mu^-$), as well as kaon decay ($K_L^0 \rightarrow \pi^\pm + e^\mp + \nu_e$, $K_L^0 \rightarrow \pi^\pm + \mu^\mp + \nu_\mu$,
 294 $K^+ \rightarrow \mu^+ + \nu_\mu$, $K^+ \rightarrow \pi^0 + e^+ + \nu_e$, $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$). Below 100 TeV, the resultant
 295 spectrum closely follows that of cosmic rays, with a spectral index of $\gamma_{atmos} = 2.7$. At higher
 296 energies the shower mesons are able to travel further in the atmosphere, giving them an
 297 increased opportunity to interact and steepening the spectral index to $\gamma_{atmos} = 3.7$. The
 298 atmospheric neutrino flux exists as an irreducible background for many studies of the origin
 299 of astrophysical neutrinos.

300 2.3.2 Observed Neutrino Spectrum

301 Both the astrophysical and atmospheric neutrino fluxes have been observed in data from
 302 the IceCube detector, with the astrophysical neutrino spectral index being measured to be
 303 $\gamma_{astro} \approx 2.28$ [10]. While astrophysical and atmospheric neutrinos are largely indistinguish-
 304 able on an individual event basis, the difference in spectra between the two populations
 305 ($\gamma_{astro} \approx 2.28$, $\gamma_{atmos} \approx 3.7$) allows us to distinguish between the two on a statistical ba-
 306 sis. Higher energy events are more likely to be astrophysical in origin, and an equal share
 307 of astrophysical and atmospheric neutrinos can be expected at ≈ 200 TeV. Currently, the
 308 source(s) of the astrophysical neutrino flux have yet to be identified. For this reason, the
 309 observed astrophysical neutrino flux is commonly referred to as the "diffuse" astrophysical
 310 neutrino flux (as opposed to a "point source" flux, where specific astrophysical objects that

311 are producing neutrino are identified).

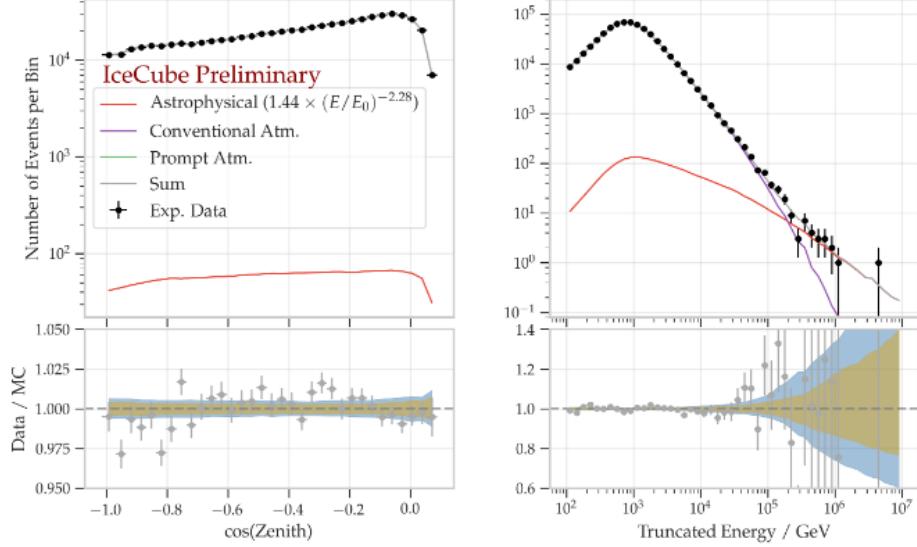


Figure 2.6: Observation of the astrophysical neutrino flux in through-going tracks detected by the IceCube Neutrino Observatory. The effect is most visible in the plot of the observed energy spectrum on the right, showing the atmospheric contribution (blue) and the astrophysical contribution (red). The excess of events at high energies is unable to be explained without including a non-zero astrophysical contribution [10].

312 2.3.3 Neutrinos as Astrophysical Messengers

313 As noted in the previous section, hadronic processes in astrophysical sources can produce
 314 neutrinos via pion decay. Like gamma rays, neutrinos are electrically neutral, and thus
 315 their paths are not bent by magnetic fields. Unlike gamma rays, however, neutrinos do not
 316 interact electromagnetically, instead interacting only via the weak force. Because of this,
 317 the mean free path of neutrinos traveling through the universe is significantly greater than
 318 that of photons, allowing us to use neutrinos to probe regions normally opaque to gamma
 319 rays.

320 Astrophysical neutrinos are not without their drawbacks, however. The fact that neutrinos
 321 interact only weakly necessitates the construction of large detectors in order to observe
 322 an appreciable number of astrophysical events. Such detectors also observe a large, irre-

ducible background of atmospheric neutrinos, particularly at lower (TeV) energies. For this reason, multi-messenger approaches that combine information from neutrinos and other astrophysical messengers are commonly used to attempt to identify the sources of astrophysical neutrinos.

2.3.4 AGN as Neutrino Source Candidates

Since neutrinos can be used as an indicator of hadronic processes in astrophysical sources, identifying the source of the astrophysical neutrino flux is a promising route to also identifying the source of ultra-high energy cosmic rays. Though the source of the astrophysical neutrino flux has yet to be fully determined, in recent years there have been promising hints that extragalactic AGN are at least partially responsible.

An active galactic nucleus (AGN) is a compact region at the center of a galaxy displaying elevated electromagnetic emission, such that the additional luminosity cannot be attributed to stars. The luminosity is expected to originate from the accretion disk surrounding a supermassive black hole at the center of the galaxy, where conditions are thought to be ideal for high-energy particle acceleration and production.

The angle at which the AGN is viewed relative to its jet determines its exact classification, as can be seen in figure 2.7. If the AGN is viewed "down" the jet (i.e. the jet is directed at Earth), the AGN is seen to be particularly luminous and is referred to as a blazar. Blazars can be further divided into two subclasses: flat spectrum radio quasars (FSRQ), which have luminous broad emission lines and continuous thermal emission, and BL-Lacs, which have weak or absent broad emission lines. AGN viewed off-axis fall into a variety of other classifications, with further sub-classification being defined by the spectrum of the AGN in various wavelength bands.

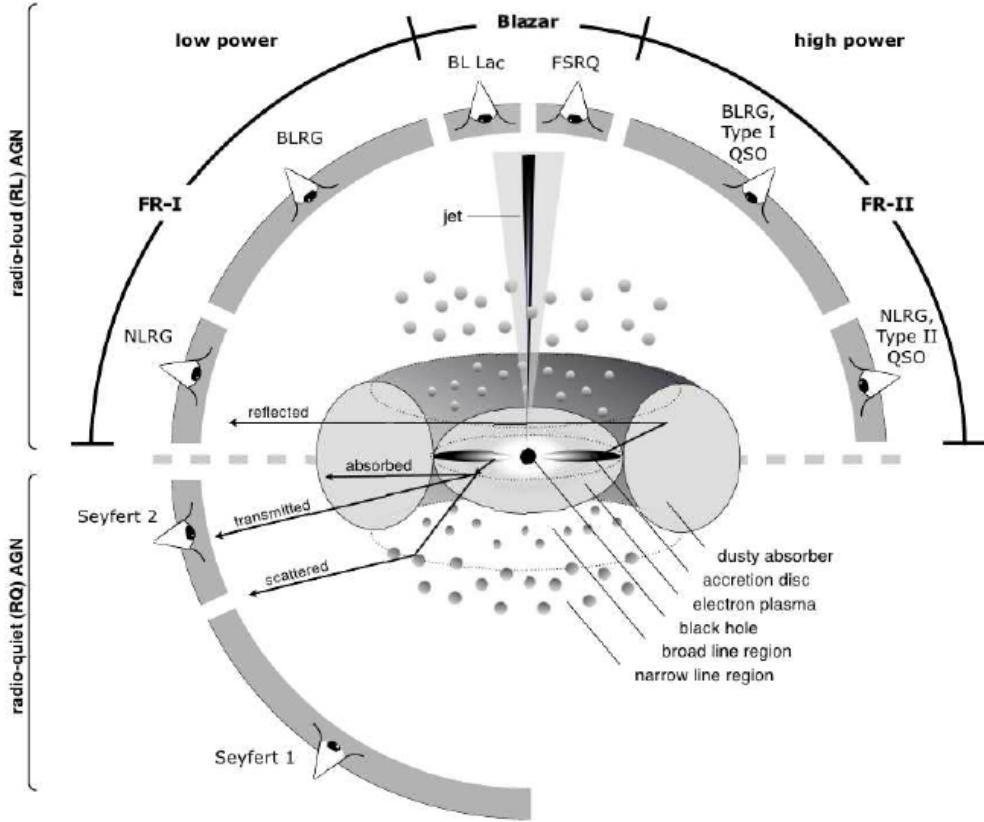


Figure 2.7: The unified AGN model, showing the various classifications of AGN for different observation angles. If the jet is directed at Earth, the object is viewed as a blazar, while other viewing angles can obscure the jet, leading to alternative classification (e.g. as a Seyfert galaxy) [11].

346 2.3.4.1 TXS 0506+056

347 On September 22, 2017, the IceCube Neutrino Observatory detected a high energy neu-
 348 trino, IceCube-170922A, with an energy of approximately 290 TeV. The arrival direction of
 349 IceCube-170922A was consistent with the location of the blazar TXS 0506+056, and sparked
 350 multi-messenger followup from a variety of other telescopes and detectors. Notably, it was
 351 determined that TXS 0506+056 was flaring in gamma rays at the time IC-170922A was
 352 observed. The significance of the positional correlation of a high energy IceCube event and
 353 a flaring blazar was calculated to be 3.0σ , providing the first hint that blazars may be a
 354 source of high energy astrophysical neutrinos [2].

355 In addition to examining the multi-messenger correlation of a high energy IceCube event
 356 and a flaring blazar, an analysis of archival IceCube data was performed at the coordinates

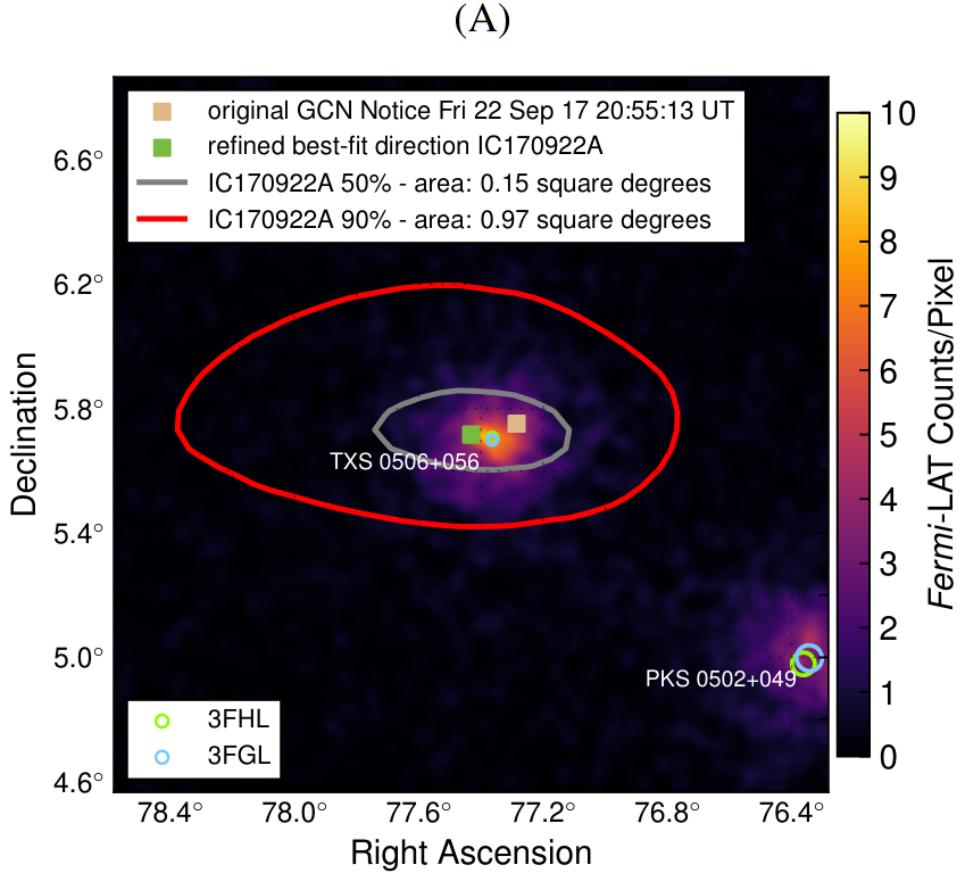


Figure 2.8: The Fermi-LAT skymap near the location of the observed high energy IceCube event IC170922A, with the 50% and 90% error contours associated with IC170922A plotted in grey and red, respectively. The significance associated with the spatial coincidence of the high energy event and TXS 0506+056 was found to be 3.0σ [2].

of IC170922A, examining the historical behavior of neutrino emission from this location of the sky over the previous nine years. In addition to the original high energy event observed in 2017, this archival analysis also revealed a 158 day period of elevated neutrino emission beginning in September 2014. The significance of this archival "neutrino flare" was calculated to be 3.5σ . Unlike the 2017 high energy event, the 2014 neutrino flare did not correspond to a flaring period for TXS 0506+056 in gamma rays. [3]

The combination of a 3.0σ multimessenger result with the 3.5σ "untriggered" neutrino flare in 2014 suggest a significance of TXS 0506+056 as a neutrino source of at least 3σ . This makes for a strong argument for extragalactic blazars as interesting source candidates

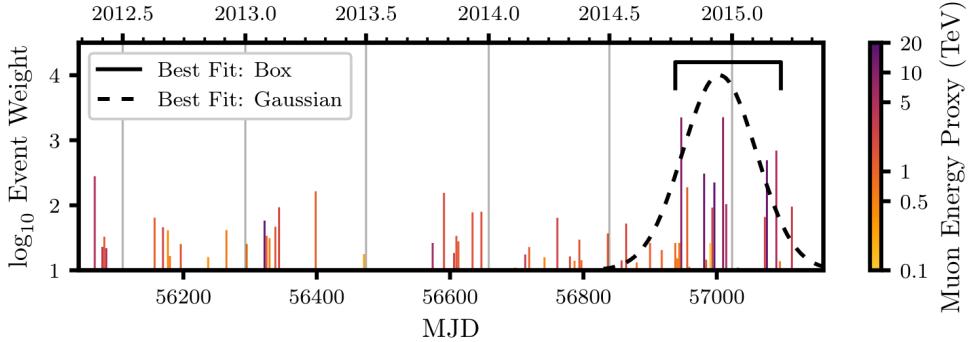


Figure 2.9: The result of the archival analysis of IceCube data near the location of TXS 0506+056, with the 3.5σ flare shown. This flare is separate from the high energy event observed in 2017, and there was no corresponding flare observed in gamma rays during this time period [3].

366 from the perspective of neutrino astronomy.

367 2.3.4.2 NGC 1068

368 In addition to the multimessenger and archival results associated with TXS 0506+056, the
 369 results of the 10-year time integrated IceCube analysis [4] also seem to suggest AGN as
 370 candidates for astrophysical neutrino emission. In this all-sky, untriggered, time integrated
 371 analysis, the most significant point in the northern sky appears to be spatially coincident
 372 with the Seyfert II galaxy NGC 1068. Notably, NGC 1068 was also included in an associated
 373 time integrated catalog analysis. In this catalog analysis, the pre-trial significance of time-
 374 integrated neutrino emission from NGC 1068 was 1.8×10^{-5} , corresponding to a significance
 375 of 2.9σ . At a 14.4 Mpc distance, NGC 1068 is the most luminous Seyfert II galaxy detected
 376 by Fermi-LAT, and NGC 1068 had additionally been hypothesized as a candidate cosmic
 377 ray accelerator prior to this particular analysis [12][13][14].

378 It should additionally be noted that the catalog analysis mentioned above identified
 379 three other objects which, together with NGC 1068, collectively form a 3.3σ excess over
 380 the background expectation. These objects are the Seyfert II galaxy NGC 1068, the blazar
 381 TXS 0506+056, and the BL Lacs PKS 1424+240 and GB6 J1542+6129 [4], providing further
 382 indication of AGN as potential neutrino emitters.

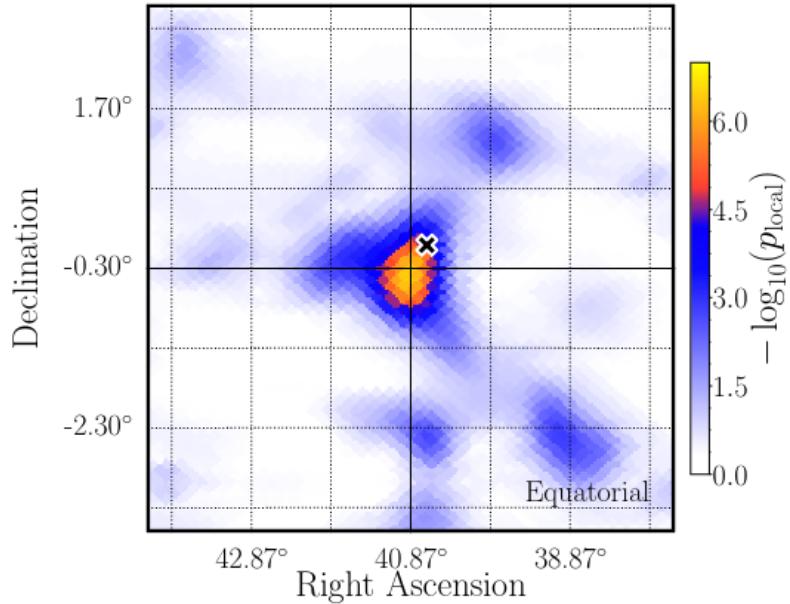


Figure 2.10: The results of the untriggered time integrated IceCube point source search analysis, showing the most significant spot in the northern sky, with the location of NGC 1068 plotted as the black "X" [4].

383 2.3.5 Galactic Neutrino Sources

384 Though this work will primarily focus on searches for extra-galactic point sources of the
 385 astrophysical neutrino flux, potential neutrino emission from sources within our galaxy is
 386 an active area of study as well. Notably, one of the first confirmed sources of lower energy
 387 astrophysical neutrinos was Supernova 1987A [15], a supernova occurring within the Milky
 388 Way. Other galactic objects that are considered candidates for neutrino emission include
 389 supernova remnants [16], as well as pulsar wind nebula [17], both of which are known to
 390 produce gamma rays.

391 The majority of the galactic plane lies in the southern sky. Since neutrino telescopes make
 392 use of the earth to block out atmospheric muon backgrounds (see subsequent chapter), this
 393 means that to best study the galactic plane, a neutrino observatory located in the northern
 394 hemisphere would be required. While there are several such telescopes either planned or
 395 under construction [18][19], they have yet to reach effective volumes comparable to the

396 existing IceCube observatory (located in the southern hemisphere).

397 **2.3.6 Constraints on Neutrino Source Populations from Diffuse
398 Measurements**

399 It is important to keep in mind that whatever the sources of astrophysical neutrinos may
400 be, they must combine to reproduce the measured diffuse astrophysical neutrino flux. The
401 current best fit astrophysical neutrino spectrum, assuming a single power law in energy, is
402 given as (eq. 2.14):

$$\frac{d\phi_{\nu+\bar{\nu}}}{dE} = 1.44^{+0.25}_{-0.24} \left(\frac{E}{100\text{TeV}}\right)^{-2.28^{+0.08}_{-0.09}} 10^{-18} \text{GeV}^{-1}\text{cm}^{-1}\text{s}^{-1}\text{sr}^{-1} \quad (2.14)$$

403 This places constraints on the density and luminosity of potential astrophysical neutrino
404 source populations. If a candidate source population has sources that are too numerous
405 and/or too bright, then that source population would produce a higher flux of astrophysical
406 neutrinos than has been observed, and an explanation of the nondetection of the additional
407 flux is required. Similarly, if sources are too sparse and/or too dim, then such a source class
408 is incapable of explaining the entirety of the measured diffuse flux.

409 These constraints can be summarized in figure 2.11, showing the band of the measured
410 diffuse flux in source density/luminosity space, as well as several commonly discussed source
411 populations. It should be noted that in this plot, the luminosity values for various source
412 populations are derived from the electromagnetic luminosity L_γ . In principle, the true ratio
413 L_ν/L_γ remains unknown, and consequently the positions of sources along the luminosity axis
414 may shift according to this value.

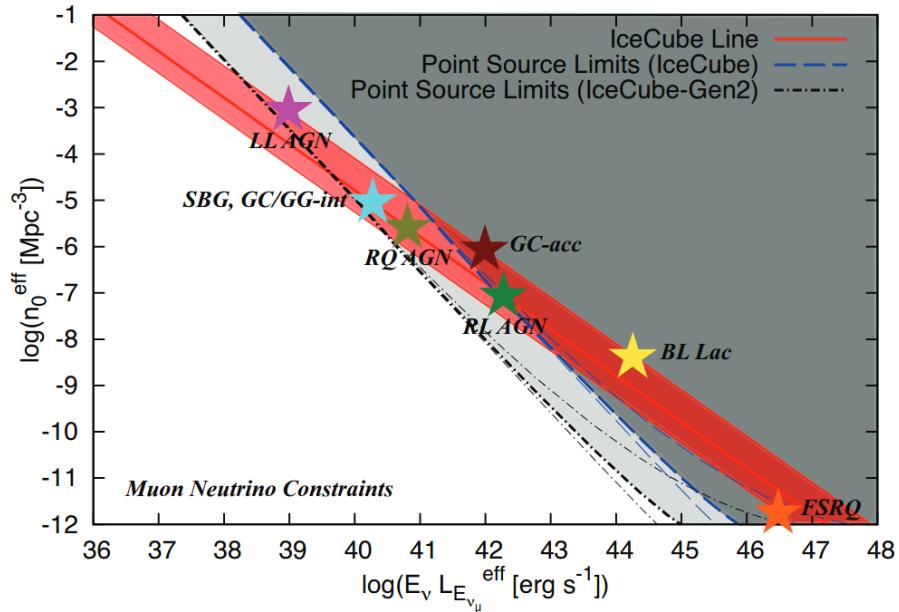


Figure 2.11: Constraints on potential populations of sources of the high energy astrophysical neutrino flux. The red band represents combinations of source density and luminosity that are consistent with the measured astrophysical neutrino flux. Source populations that lie above this band are too bright and/or too numerous, while source below this band are too dim/too sparse. Also shown are limits derived from the non-observation of neutrino sources in time-integrated analyses performed by the IceCube collaboration as of 2016, as well as a projected set of limits that would be associated with a non-observation from IceCube-Gen2 [20].

415 Chapter 3

416 The IceCube Detector

417 The IceCube Detector is a large (cubic kilometer scale) water cherenkov detector situated
418 underneath the ice at the south pole. Its large volume makes it ideal for studying high energy
419 (TeV and higher) neutrino events originating from either the atmosphere or astrophysical
420 sources.

421 3.1 Detection Mechanism

422 The cross section for neutrino interaction in Earth can be seen in figure 3.1. These cross
423 sections are small, but the Earth is also quite large. We can calculate the mean free path
424 for neutrinos traveling through Earth as (eq. 3.1):

$$\lambda = \frac{1}{\sigma_\nu \rho_{Earth}} \quad (3.1)$$

425 Using the neutrino cross section near 1 PeV ($\sigma_\nu \approx 1.15 \times 10^{-33} \text{ cm}^2$) [21], and the density
426 of the Earth ($\approx 5.5 \text{ g/cm}^3$, corresponding to a nucleon density of $\rho_{Earth} = 3.3 \times 10^{24} \text{ cm}^{-3}$),
427 we obtain an estimate of the neutrino mean free path of $\approx 2600 \text{ km}$. Since the diameter
428 of the Earth is approximately $1.3 \times 10^4 \text{ km}$, we can conclude that the Earth is opaque to
429 high energy neutrinos. We can detect the neutrinos that interacted in the Earth by way of
430 identifying their interaction products.

431 For the purposes of neutrino point source searches, we primarily focus on neutrinos
 432 interacting either through charged current (CC) or neutral current (NC) interactions. In
 433 both cases, the energy of the neutrinos observed by the IceCube detector is high enough
 434 that neutrinos interact through deep inelastic scattering with nucleons in the antarctic ice.
 435 In neutral current (NC) interactions, the process is mediated by a neutral Z boson, as shown
 436 on the left in figure 3.2. In this case, the product of the interaction is a neutrino, meaning
 437 the only visible signature of this interaction is the production of a hadronic particle cascade.

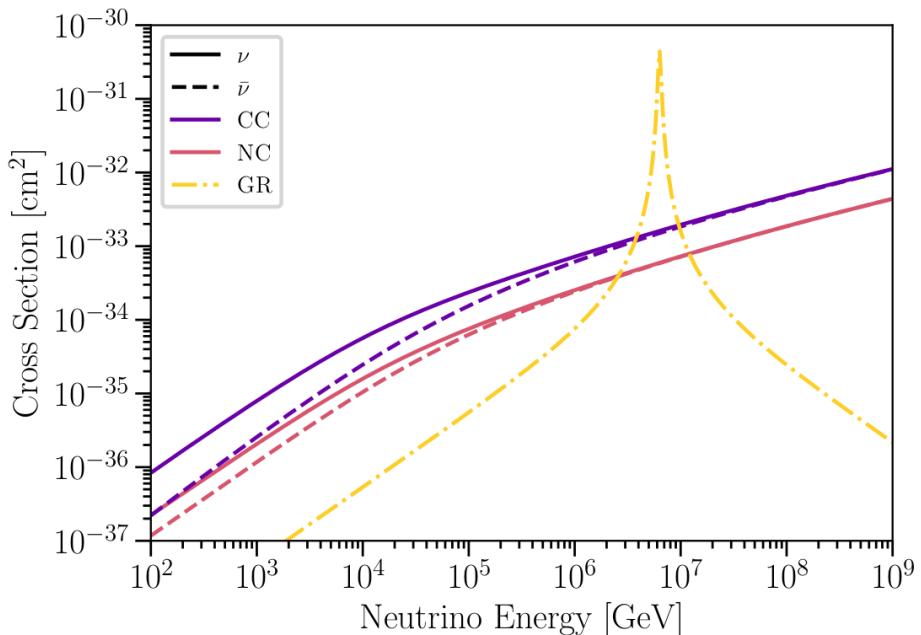


Figure 3.1: The cross sections of various neutrino interactions in the Earth. The yellow curve corresponds to Glashow Resonance [22], whereby an anti-electron neutrino combines with an atomic electron to produce a W+. These events are relatively rare in IceCube data, and are subsequently not used in the context of astrophysical neutrino source searches.

438 Charged current interactions are mediated by a charged W boson, and can be summarized
 439 as (eq. 3.2 and 3.3):

$$\nu_\ell + n \rightarrow \ell^- + p \quad (3.2)$$

440

$$\bar{\nu}_\ell + p \rightarrow \ell^+ + n \quad (3.3)$$

441 The Feynman diagrams for this process can be seen on the right in figure 3.2. Notably,
 442 charged current interactions produce an outgoing lepton ℓ in addition to a hadronic cascade.
 443 This is particularly useful in the case that the lepton produced is a muon, as muons can
 444 travel a sizeable distance (hundreds, or even thousands of meters) before decaying. If the
 445 lepton receives enough energy, it will subsequently produce cherenkov radiation, as it will
 446 be traveling faster than the local speed of light in ice. This radiation is emitted at an angle
 447 θ_c relative to the direction of travel, where θ_c is given by (eq. 3.4):

$$\cos(\theta_c) = \frac{1}{n\beta} \quad (3.4)$$

448 Where n is the index of refraction of the medium through which the particle is traveling
 449 (in this case ice), $\beta = \frac{v}{c}$, and v is the velocity of the traveling particle (the lepton, in this
 450 case). For ice, $n = 1.31$, and this emission angle is approximately 41 degrees. The number of
 451 photons expected per unit track length is given by the Frank-Tamm formula [23] (eq. 3.5):

$$\frac{dN}{dx d\lambda} = \frac{2\pi z\alpha}{\lambda^2} \sin^2(\theta_c) \quad (3.5)$$

452 Where z is the charge of the ionizing particle, λ is the wavelength of the emitted radiation,
 453 α is the fine structure constant ($\approx \frac{1}{137}$), and θ_c is the cherenkov angle given by eq. 3.4. Peak
 454 emission is found in the optical portion of the light spectrum, between 350 and 600 nm
 455 corresponding to a characteristic blue hue.

456 To summarize, neutrinos traveling through the antarctic ice will occasionally interact,
 457 producing child particles that will emit cherenkov radiation. We can then build an array
 458 of photon detectors to detect these photons, and subsequently infer information about the
 459 original incident particles.

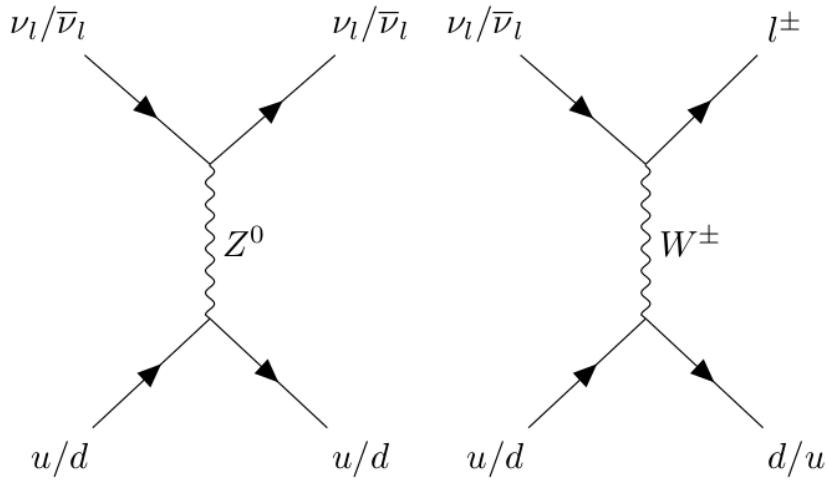


Figure 3.2: The Feynman diagrams corresponding to neutral current (left) and charged current (right) neutrino interactions. Charged current interactions can produce a detectable outgoing lepton in addition to a hadronic cascade, while neutral current interactions only produce a hadronic cascade.

460 3.2 The Physical Components of the IceCube

461 Detector

462 As outlined in the previous section, the strategy for detecting neutrinos with a water
 463 cherenkov detector is not to directly detect the neutrinos themselves, but rather to detect
 464 the cherenkov photons from the outgoing particles resulting from the neutrino interactions.
 465 The IceCube detector accomplishes this through the use of a large number of photomultiplier
 466 tubes (PMTs). At the very highest level, a PMT is a device that converts photons to an
 467 electrical signal that can then be read out by a set of associated electronics. The details of
 468 general PMT design and operation can be found in [24].

469 In IceCube, PMTs are housed in a single unit referred to as a digital optical module
 470 ("DOM"), seen in figure 3.3. Each module contains the PMT and its associated electronics,
 471 as well as a set of LED flashers that can be used to calibrate the detector once the DOMs have
 472 been lowered into the ice. DOMs are arranged onto 86 vertical strings, 78 of which contain
 473 60 DOMs spaced 17 meters apart along the length of the string. These strings are arranged

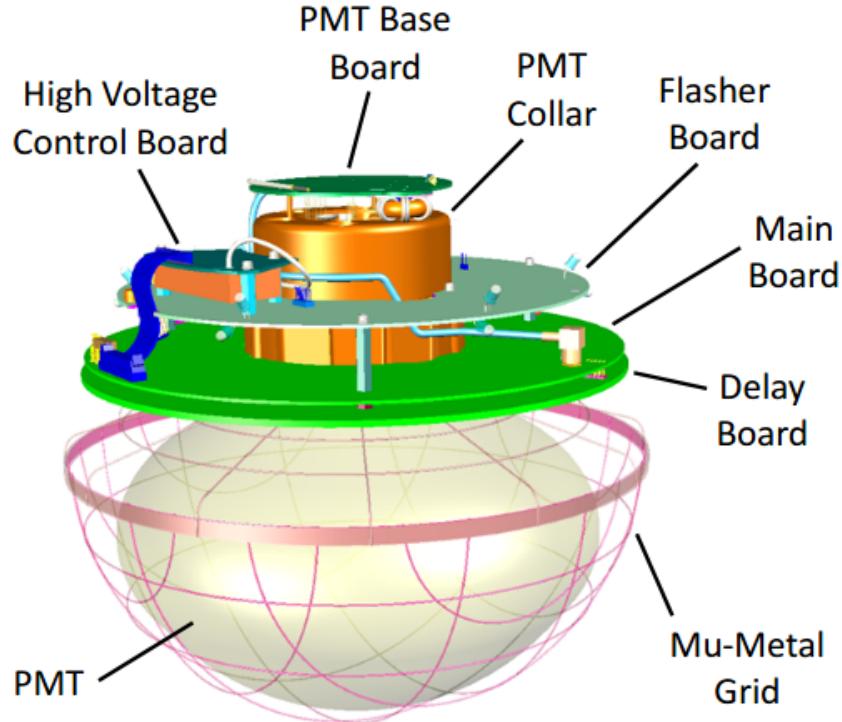


Figure 3.3: A diagram of a single IceCube digital optical module (DOM). The entire apparatus is encased in a pressurized glass sphere for protection (not shown). The PMT faces downward for optimal collection of photons from upgoing neutrino events. In addition to the PMT, each DOM also contains digitization electronics and an array of LED flashers that can be used for calibration [25].

474 into a hexagonal grid pattern under the south pole ice with a separation between strings
 475 of approximately 125 meters, resulting in a cubic kilometer of instrumented ice extending
 476 between 1450 and 2450m below the surface of the south pole ice sheet. The remaining 8
 477 strings form the DeepCore sub-array, a more densely instrumented region near the center of
 478 the detector, primarily used for studying lower (\approx 100s of GeV) scale events. PMTs that are
 479 part of the DeepCore portion of the detector have a higher quantum efficiency than those
 480 on the other 78 strings [26].

481 A diagram of the IceCube detector can be seen in figure 3.4. Due to the weakly interacting

482 nature of neutrinos combined with a power law spectrum, a large detector volume is critical
 483 to detecting the astrophysical neutrino flux. Prior to IceCube's construction, calculations
 484 based on the likely linkage of neutrinos to the observed high energy cosmic ray flux indicated
 485 that a cubic-kilometer scale detector would be necessary to observe an appreciable event
 486 rate of astrophysical neutrinos [27][28]. The size and design of IceCube are well suited for
 487 detecting astrophysical neutrino events in the TeV+ energy range, as the large instrumented
 488 volume ensures a significant number of interactions in this energy range, and the detector's
 489 nanosecond scale timing resolution makes event reconstruction possible for events producing
 490 cherenkov radiation over the scale of ≈ 100 s of meters.

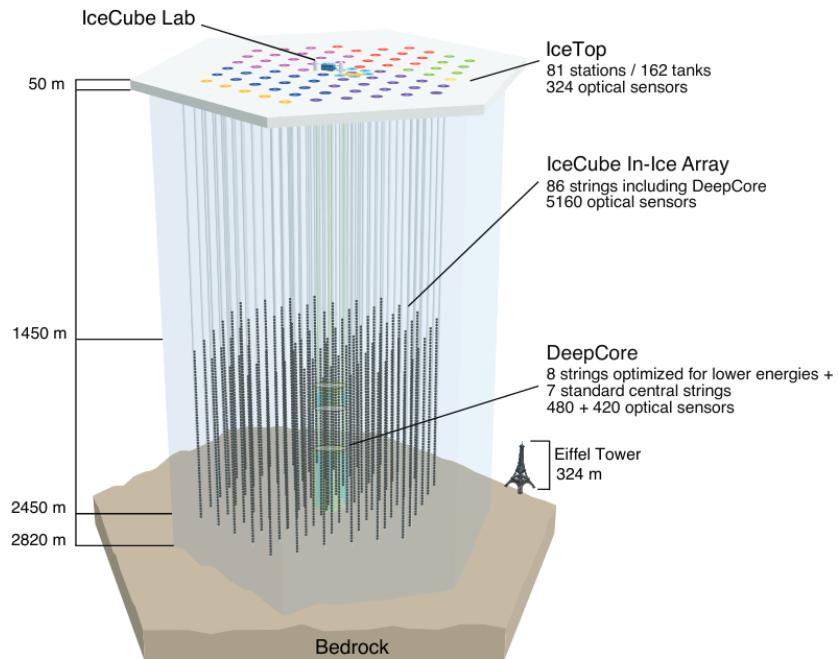


Figure 3.4: A diagram of the IceCube detector. The detector consists of 86 strings of DOMs lowered into holes drilled in the pattern of a hexagonal grid defined by a separation of 125 meters. Each string is formed by 60 DOMs connected by a cable, used for powering the DOMs and communication with the IceCube Lab (ICL) on the surface. The instrumented detector volume begins at a depth of 1450 meters, and extends down to 2450 meters, resulting in approximately a cubic kilometer of instrumented ice [25].

491 As the IceCube detector is situated in the south pole ice, a proper description of the
 492 optical properties of the ice is key for being able to accurately reconstruct events. While the

ice is generally clear, it does contain several impurities and features that make it optically nonuniform. The scattering and absorbtion of light are known to vary as a function of depth, as seen in figure 3.5. The large peak near a depth of 2000 meters is thought to correspond to a layer of dust deposited on the south pole ice sheet many years in the past, and is commonly referred to as "the dust layer" (physicists are not always the most creative with names).

In addition to varying as function of depth, the optical properties of the ice also vary as a function of azimuth. The largest effects along this axis are the ice tilt and ice anisotropy. The ice tilt refers to the layers of ice with similar optical properties being not perfectly horizontal, and results in a variation in scattering and absorption as light travels in different directions through the ice sheet. The ice anisotropy is similar, but has an observed additional axis of symmetry, leading to variations that are twice as frequent as a function of azimuth. Notably, while this effect was original observed in calibration data using LED flashers, it is visible in data from observed atmospheric muons, as seen in figure 3.6.

The final major optical nonuniformity of the detector ice is the hole ice. During the construction of the IceCube detector, holes were drilled in the south pole ice, and strings of DOMs were lowered down the appropriate depth. The water in these holes then refroze with different optical properties than the surrounding ice. The column of refrozen ice is referred to as the "hole ice", typically containing an inner region where air bubbles have been trapped referred to as the "bubble column". The hole ice has significantly shorter scattering distances, and is an area of active study within the IceCube collaboration.

3.3 Event Types

IceCube events can be generally be categorized into three different types: tracks, cascades, and double cascades. While neutral current events exclusively correspond to cascades, charged current events can produce any of the three, dependent on the variety of the outgoing lepton. This gives IceCube the ability to identify the flavor of the incident neutrino, and this fact can be leveraged to do a large amount of interesting physics. However, for the

519 purposes of this work, we are primarily interested in tracks, due to their excellent pointing
 520 resolution as discussed below.

521 **3.3.1 Tracks**

522 From equation 3.2 we can see that charged current interactions produce an outgoing lepton,
 523 which will in turn produce cherenkov radiation if it is energetic enough. In particular, if the
 524 outgoing particle is a muon, it will travel a significant distance before decaying or exiting the
 525 detector. As the muon produces cherenkov radiation as it travels through the ice, the pattern
 526 of DOMs which detect this radiation will resemble a line, as shown in figure 3.7. These events
 527 are referred to as "tracks". If the neutrino interaction occurred inside the IceCube detector,
 528 then the track will appear to start within the detector, and the event can be referred to as
 529 a "starting track". If instead the neutrino interaction occurred outside the detector, and
 530 the resultant muon traveled through the detector, then the event is instead referred to as a
 531 "through-going track" (again, physicists are not the most creative). Through-going tracks
 532 can result from astrophysical neutrinos, as well as atmospheric neutrinos and atmospheric
 533 muons (muons resulting from cosmic ray interactions in the atmosphere). Starting tracks,
 534 however, can only be produced by muons originating from atmospheric or astrophysical
 535 neutrinos. For the purposes of this work, we will focus primarily on through-going tracks,
 536 however starting tracks are also scientifically interesting, and can be used for point-source
 537 and diffuse analyses of the neutrino flux as well [30].

538 Through-going tracks are notable for their good angular resolution (up to 0.25 degrees
 539 at high energies), particularly at high energies (> 1 TeV), as seen in figure 4.1. This makes
 540 them an excellent candidate for attempting to do astronomy, as they can be expected to
 541 point back to their point of origin with reasonable accuracy. Energy reconstruction of these
 542 events is somewhat more challenging, however, due to a combination of stochastic energy
 543 losses of the muon producing the cherenkov photons, and the fact that the event itself is
 544 often not entirely contained in the detector. The energy resolution of tracks in IceCube is

545 approximately a factor of 2 at 10 TeV, though this increases for higher energy events [31].

546 3.3.2 Cascades

547 Neutral current neutrino interactions will always produce a hadronic cascade and an outgoing
 548 neutrino which is not detectable. Charged current interactions will produce a hadronic
 549 cascade in addition to an outgoing lepton. If the outgoing lepton is a muon (or potentially an
 550 extremely energetic tau), the track morphology described above is produced. If the incident
 551 neutrino is a ν_e , resulting in an outgoing electron, which in turn results in an electromagnetic
 552 shower that has an extension of ≈ 10 meters. The cascade morphology produced in this case
 553 is indistinguishable from the hadronic cascade produced by charged current interactions.
 554 While cascades tend to have relatively poor ($\approx 5^\circ$) pointing resolution [34] due to their
 555 spherical symmetry, they have excellent energy resolution, as even for high energy events, the
 556 entirety of the cascade is often contained within the detector, making energy reconstruction
 557 as simple as counting the total number of cherenkov photons produced. The energy resolution
 558 of cascades is approximately 15%, potentially even lower at higher energies [5].

559 3.3.3 Double Cascades

560 If the outgoing lepton from a CC interaction is a tau, then the tau will decay with an
 561 average decay length of 50 meters/PeV, potentially observable above energies of 200 TeV. If
 562 the initial tau is high enough energy, this can produce a signature of two spatially separated
 563 hadronic cascades in the detector. If the tau is lower energy, the two cascades may not be
 564 spatially resolvable, though may still be able to be identified by examining the waveforms
 565 observed by hit DOMs. This type of event was only recently observed in IceCube data
 566 [35]. Notably, tau events of these energies are not expected to be produced by atmospheric
 567 processes, and consequently this observation provides further evidence of an astrophysical
 568 neutrino flux.

569 3.4 Triggering

570 IceCube DOMs are subject to dark noise in the detector originating from PMTs emitting
 571 electrons from the cathode in the absence of an external photoelectron. Dark noise below 0°
 572 celsius is primarily caused by radioactivity in the PMT and DOM glass. This noise should
 573 not be correlated between nearby DOMs, and can be dramatically reduced by imposing the
 574 triggering requirement that neighboring DOMs also experience a signal with $1 \mu s$. These
 575 hits are classified as "Hard Local Coincidence" (HLC). An eight-channel simple majority
 576 trigger (SMT8) is used to trigger a readout window where DOM information is written to
 577 file. This trigger requires eight DOMs to have HLC hits within a $5 \mu s$ window.

578 Events are further filtered through several additional algorithms to pare down events
 579 based on temporal and spatial coincidence of DOM hits. The result is a "Level 2" event rate
 580 of approximately 2.5 kHz. These events can be additionally filtered to select specifically for
 581 high quality track-like events (originating mostly from atmospheric muons, but also from
 582 atmospheric and astrophysical muon neutrinos as well), reducing the event rate to a few
 583 Hz. The most relevant cuts that are used to achieve this are cuts on zenith and total
 584 charge deposited, which together form the IceCube "MuonFilter" [36]. This level of data
 585 is often referred to within the collaboration as "Muon Level 3", and primarily consists of
 586 through-going track-like events.

587 3.5 Directional Reconstruction

588 In this section we discuss algorithms for reconstructing the direction of through-going track
 589 events observed by the IceCube detector. Different algorithms may be used for cascades,
 590 though understanding these is unimportant for the remaining content of this thesis. Those
 591 who are interested in the reconstruction of cascade and double cascade events can refer to
 592 [5].

593 **3.5.1 LineFit**

594 An initial guess for the reconstructed direction of the track can be obtained by ignoring the
 595 geometry of the cherenkov cone and the optical properties of the ice, and assuming a plane
 596 wave of light traveling along a straight line in the detector. The location of each DOM that
 597 observes photons (\mathbf{r}_i) can then be written as a line (eq. 3.6):

$$\mathbf{r}_i = \mathbf{r} + \mathbf{v} \cdot t_i \quad (3.6)$$

598 Where t_i is the time that the i th DOM observes photons, \mathbf{r} is the vertex of the track,
 599 and \mathbf{v} is the velocity of light in the ice. A χ^2 fit can then be preformed to determine the
 600 free parameters \mathbf{r} and \mathbf{v} (eq. 3.7):

$$\chi^2 \equiv \sum_{i=1}^{N_{tot}} (\mathbf{r}_i - \mathbf{r} - \mathbf{v} \cdot t_i)^2 \quad (3.7)$$

601 This can be solved analytically, resulting in fitted vectors for \mathbf{r} and \mathbf{v} (eq. 3.8 and 3.9):

$$\mathbf{r} = \langle \mathbf{r}_i \rangle - \mathbf{v} \cdot \langle t_i \rangle \quad (3.8)$$

$$\mathbf{v} = \frac{\langle \mathbf{r}_i \cdot t_i \rangle - \langle \mathbf{r}_i \rangle \cdot \langle t_i \rangle}{\langle t_i^2 \rangle - \langle t_i \rangle^2} \quad (3.9)$$

602 This corresponds to a vertex (\mathbf{r}) and a direction (\mathbf{v}) describing the path of the particle.
 603 Further discussion on this topic can be found in [37].

604 **3.5.2 SplineMPE**

605 An improved reconstruction of the particle direction can be obtained by iterating on the
 606 initial LineFit reconstruction described above. The LineFit result is treated as a seed for
 607 a likelihood based reconstruction that takes the cherenkov angle and ice properties into
 608 account. A description of the specific likelihood components may be found in section 3 of
 609 [37]. In the simplest implementation of this likelihood method, only information from the

610 first photoelectron observed by a particular DOM is used. This is referred to as a single
 611 photoelectron fit, or "SPEFit". For events that produce multiple hits on a single DOM,
 612 this information can be included in the likelihood as well, producing a multi-photoelectron
 613 (MPE) fit. Since photons arriving after the first are likely to have experienced at least some
 614 scattering, a proper description of the ice properties is necessary for an accurate MPE fit.
 615 This is incorporated into the likelihood described in [37] via the use of tabulated timing
 616 and light yield distributions for various DOM/photon configurations given an ice model
 617 developed from fits to LED flasher data [38]. The information in these tables is stored via
 618 a multi-dimensional spline, allowing these tables to be used directly as PDFs in the MPE
 619 likelihood, hence the name for this variety of reconstruction: "SplineMPE".

620 3.5.3 Paraboloid

621 There is some inherent uncertainty associated with the observation and reconstruction of a
 622 particular event. For an event originating from a particular true position, the reconstructed
 623 position is expected to be drawn from distribution centered on the true position. Properly
 624 describing this distribution is key to characterizing the per-event uncertainty associated
 625 with the directional reconstruction, and is important for obtaining accurate results in a
 626 point source analysis.

627 A semi-analytic description of the angular error associated with a particular event can
 628 be obtained from the likelihood map associated with the direction reconstruction outlined
 629 in the previous section. An ellipse can be fit around the global minimum of the likelihood
 630 map, describing a 1σ containment region. The ellipse is parameterized by two variables, σ_1
 631 and σ_2 describing the scales of the two axes of the ellipse, and an average circularized error
 632 can be computed as (eq. 3.10)[39]:

$$\sigma_{evt} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}} \quad (3.10)$$

633 This description can be further improved by comparing this error with the "true error"

634 obtained from examining the difference between the simulated event direction and the cor-
 635 responding reconstructed direction. By multiplying the above circularized error by the ratio
 636 of the reconstructed error to the "true" error, we remove potential bias due to effects not ac-
 637 counted for by the directional reconstruction algorithm. For example, one such effect would
 638 be the angle between the incident neutrino and the outgoing muon. Since the reconstruc-
 639 tion algorithm only reconstructs the direction of the track associated with the muon, the
 640 direction of the neutrino is still technically unknown. While at high energies the muon and
 641 it's parent neutrino can be assumed to be colinear, this angle becomes a significant source
 642 of uncertainty below 1 TeV.

643 3.6 Energy Reconstruction

644 While the direction of track event in IceCube can be reconstructed with good precision
 645 using the methods described previously, reconstructing the energy is more difficult. The
 646 primary reason for this is simply a lack of information: since many tracks originate from
 647 events neutrino interactions outside of the instrumented volume, the entirety of the muon
 648 track is not contained within the detector. This prevents the detector from behaving as
 649 a calorimeter, and estimates must be made to extrapolate the initial event energy based
 650 on the portion of the track observed. Similar to the directional reconstruction, an energy
 651 reconstruction of a particular event can be obtained using a likelihood based approach based
 652 on the individual DOM observations.

653 At its core, this likelihood approach assumes that the number of detected photoelectrons
 654 (k) is expected to be poisson distributed with a mean directly proportional to the energy:
 655 $\lambda = E\Lambda$ (eq. 3.11), where Λ is the number of photons the event produces per unit energy:

$$\mathcal{L} = \frac{(E\Lambda)^k}{k!} e^{-E\Lambda} \quad (3.11)$$

656 Maximizing this likelihood over all j DOMs that see photoelectrons results in the relation
 657 (eq. 3.12):

$$E = \sum k_j / \sum \Lambda_j \quad (3.12)$$

658 Which is largely a statement that the event energy is proportional to the number of
 659 observed photons, scaled by some factor that is the sum of the light yield scaling functions
 660 Λ_j . These functions depend on a variety of factors including the geometry and ice optical
 661 properties. Much like in the case of the directional reconstruction, these functions are often
 662 obtained from tabulated monte carlo data, smoothed with a multi-dimensional spline.

663 This likelihood can then be applied to calculate the energy loss rate ($\langle dE/dx \rangle$) of a par-
 664 ticular muon. Above 1 TeV, this energy loss rate is roughly proportional to the muon energy.
 665 A robust estimate of this rate can be obtained by fitting segmented energy losses along the
 666 muon track using the methodology described in [40]. Once a list of segmented energy losses
 667 is calculated, there are several different approaches to converting to a reconstructed energy.

668 • **MuEX** takes the average of the segmented energy losses and uses that as an estimate
 669 of the energy

670 • **TruncatedEnergy** first removes the largest 40% of energy losses before calculating
 671 an average, ideally reducing the variance in the calculation of $\langle dE/dx \rangle$.

672 A comparison of these approaches can be found in [40], and both methods provide approx-
 673 imately 35% precision in $\log_{10} E$ for an initial muon energy of 10^4 GeV, improving slightly
 674 to approximately 30% in $\log_{10} E$ at higher energies.

675 3.7 IceCube Event Samples Overview

676 As the Icecube detector is capable of doing a wide variety of science, there exists a plethora
 677 of event samples used within the collaboration. Even within the context of searches for point
 678 sources of astrophysical neutrinos, there exist several event samples of IceCube events, each
 679 with their own selection criterion and set of reconstructed parameters. In many cases, the

680 distinguishing features between event selections used in point source analyses are purity and
 681 event rate. An ideal point source event sample would have both a high purity of astrophysical
 682 events in addition to a high event rate. Unfortunately, this is not possible with the current
 683 iteration of the IceCube data, as we observe relatively few events above 100 TeV (where
 684 the highest purity of astrophysical events can be achieved), and below 100 TeV there is a
 685 significant irreducible background of atmospheric neutrinos.

686 For this reason, point source event samples in the IceCube collaboration generally fol-
 687 low two different philosophies. "Low statistics/high purity" samples contain relatively few
 688 events, but the events in these samples have a high probability of being astrophysical in
 689 origin [30] [41]. By contrast "high statistics/low purity" samples contain a higher number
 690 of events, including additional astrophysical events. However in doing so, these samples also
 691 include significantly more atmospheric (background) events [10] [4]. In the context of point
 692 source searches, these samples typically rely on advanced statistical methods in their analy-
 693 sis pipeline to distinguish between clustered astrophysical signal and isotropic atmospheric
 694 background.

695 Even within the category of high statistics/low purity samples used for neutrino source
 696 searches, there exist multiple samples that are regularly used within the IceCube collabora-
 697 tion. The following sections will briefly outline three such samples that are relevant to the
 698 analyses presented later in this work. A comparison of the three samples outlined here can
 699 be seen in table ?? and figure 3.9

700 3.7.1 PointSourceTracks v002p03

701 Starting with Muon level 3 data, this sample applies an additional BDT to attempt to select
 702 for well-reconstructed muon neutrino interactions from mis-reconstructed atmospheric muon
 703 background. The variables used in this BDT were associated with a track-like event topology,
 704 and the BDT was trained using both background data, as well as simulation of both an $E^{-2.0}$
 705 and $E^{-2.7}$ signal. The result is a high-statistics sample of mostly track-like neutrino events

706 in the northern sky, consisting of both atmospheric and astrophysical neutrinos [33]. Events
 707 in this sample use a SplineMPE reconstruction (“plain” settings, corresponding to a balance
 708 of computation time and precision) for the event direction, and a MuEX reconstruction for
 709 the event energy.

710 This sample covers seven years of IceCube data (2008-2015), and was used for the histor-
 711 ical untriggered flare analysis of TXS0506+056 [3], where a 3.5σ neutrino flare was identified
 712 in 2014.

713 3.7.2 PointSourceTracks v003p02

714 Though this sample is oftentimes treated as a “successor” to PointSourceTracks v002p03, it
 715 is in many ways a completely new event selection. This sample covers 10 years (2008-2018)
 716 of IceCube data. Like v002p03, this sample also starts with Muon level 3 data, however here
 717 the BDT is trained to additionally reject cascade-like events in addition to accepting track-
 718 like events. Additionally, an improved directional reconstruction is used: the SplineMPE
 719 reconstruction algorithm is applied twice, with the second application using the results of the
 720 first as a seed, and additionally including the energy estimation. This results in improved
 721 angular resolution [42].

722 This sample additionally requires that events pass some precuts prior the the application
 723 of the BDT. Events are only selected if they satisfy the conditions below:

- 724 • The length of empty track (the portion of the reconstructed track that does not have
 725 any associated DOM hits) must be $\leq 400\text{m}$
- 726 • The reconstructed track length must be at least 200m
- 727 • The number of hit DOMs must be ≥ 12
- 728 • The number of DOMs that observe direct (non-scattered) photons must be ≥ 6

- 729 • $\cos(\theta_{geo}^2) < 0.2$, where θ_{geo} is the angle between independent reconstructions calculated
 730 for the first and second half of the track only. For a high quality track, both the first
 731 half reconstruction and the second half reconstruction should lie almost parallel.

732 This sample also has different requirements for events observed from the southern sky.
 733 The minimum track length and maximum empty track length pre-cuts are applied as in
 734 the northern sky, in addition to an additional cut that the initial estimated uncertainty
 735 ($\sigma_{paraboloid}$) must be less than 5 degrees. A cut on the likelihood reconstructions is also
 736 enforced, requiring $R \log(\mathcal{L}) > 9$, where $R \log(\mathcal{L})$ is the "reduced log likelihood", where the
 737 maximum likelihood value, \mathcal{L} is divided by the number of degrees of freedom, n_{dof} ($n_{dof} = 5$
 738 in this particular case). Events in the southern sky are also required to have hits on more
 739 than 5 different IceCube strings if the number of direct DOM hits was fewer than 12 [42].

740 This sample is notable as it was the sample used for the 10-year all-sky IceCube time
 741 integrated analysis, where NGC 1068 was identified as having a significance of 3σ [4]. This
 742 sample has also been publicly released for use outside the IceCube collaboration [31].

743 3.7.3 NorthernTracks v002p06

744 The NorthernTracks sample is a high statistics, low purity sample of through-going tracks
 745 that was originally developed for the purpose of performing a diffuse fit of the atmospheric
 746 and astrophysical neutrino spectrum. As such, this sample has relatively good data/MC
 747 agreement (as this is essential to the diffuse fit analysis pipeline), but is restricted to only
 748 events in the northern sky. Additionally, this sample does not make use of data from
 749 DeepCore DOMs, a decision that was made in order to homogenize the detector. This
 750 sample covers 8 years of livetime (2009-2017) has been used for point source analyses as well
 751 as diffuse studies of the neutrino spectrum [32].

752 The precuts for this sample in the northern sky are identical to PointSourceTracks
 753 v003p02, however this selection then uses two separate BDTs to further filter events: one
 754 BDT to select for track-like events, and a separate BDT to reject cascade-like events. In

755 practice the application of these two BDTs is similar, but not perfectly identical, to the
756 application of the single BDT in PointSourceTracks v003p02. Also of note is that the BDTs
757 used in the NorthernTracks selection are trained exclusively using simulation, for both signal
758 and background.

759 This sample uses a SplineMPE directional reconstruction (with "max" settings, priori-
760 tizing precision over computation speed), and a TruncatedEnergy estimator as the energy
761 reconstruction.

	PSTracks v2	PSTracks v3	NorthernTracks
Pre-cuts	No	Yes	Yes
BDT 1	Selects tracks	Selects tracks and rejects cascades	Selects tracks
BDT 2	None	None	Rejects cascades
Signal training set	Simulation	Simulation	Simulation
Background training set	Data	Data	Simulation
Direction reconstruction	SplineMPE ("plain")	SplineMPE ("plain") $\times 2$	SplineMPE ("max")
Angular error estimator	Paraboloid	Paraboloid	Paraboloid
Energy estimator	MuEX	MuEX	TruncatedEnergy
DeepCore included?	Yes	Yes	No
Livetime	7 years (2008-2015)	10 years (2008-2018)	8 years (2009-2017)
Declination Range	$-90^\circ < \delta < 90^\circ$	$-90^\circ < \delta < 90^\circ$	$-5^\circ < \delta < 90^\circ$
Events	711,878	1,134,451	493,252

Table 3.1: A comparison of several similar high statistics/low purity event samples used by the IceCube collaboration

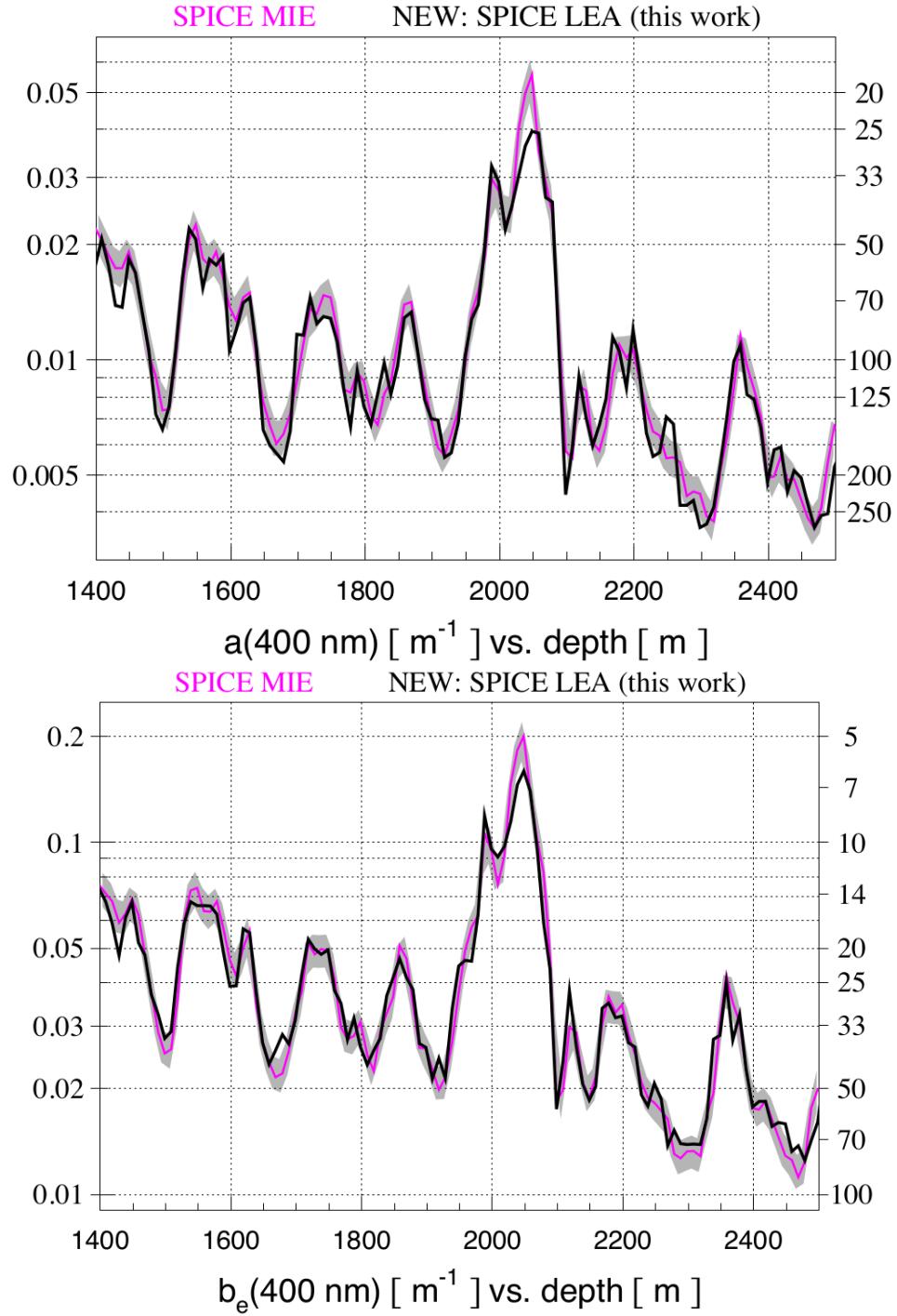


Figure 3.5: Plots of the absorption (top) and scattering (bottom) of 400 nm light as a function of depth in the south pole ice. The peak near a depth of 2000 meters is referred to as "the dust layer", and is thought to correspond to a layer of dust deposited at the south pole at some point in the Earth's history. [29]

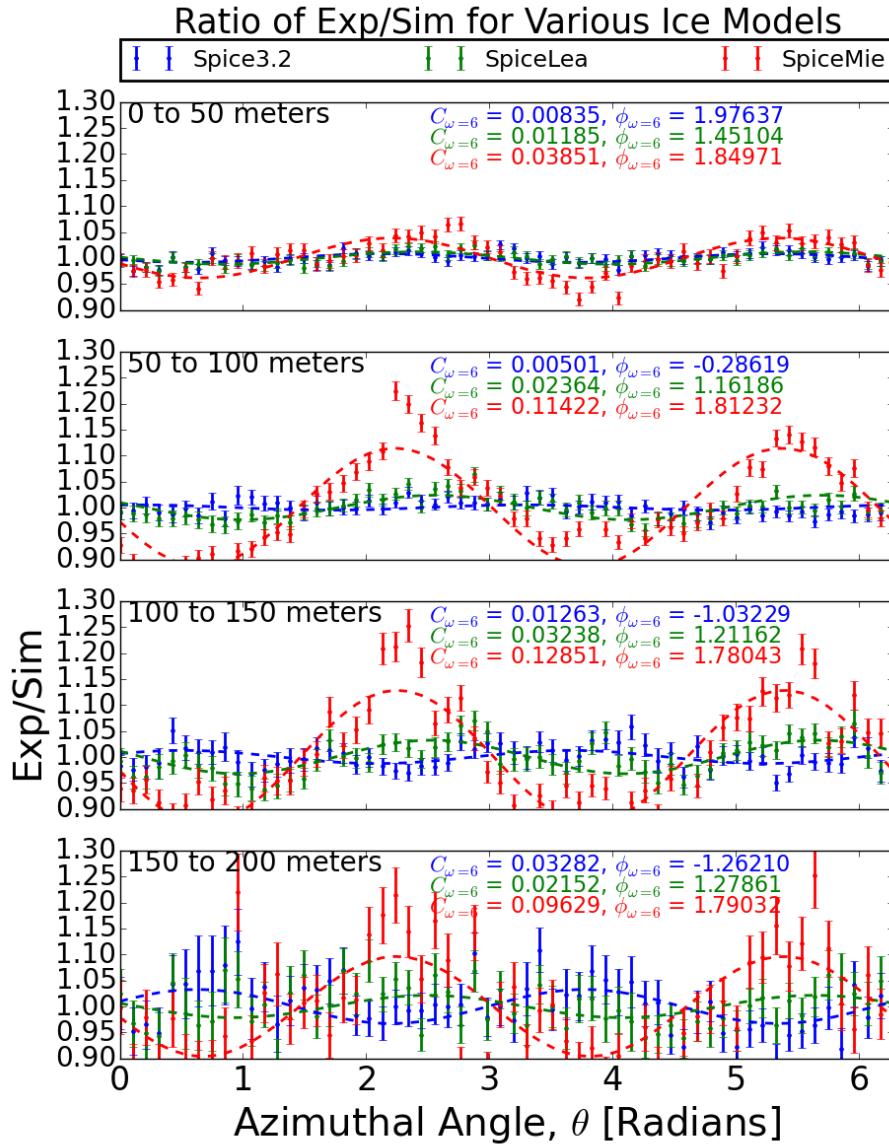


Figure 3.6: Plots of the ratio of light seen from atmospheric muons in experiment and simulation using various ice models (SpiceMie, SpiceLea, and Spice3.2), as a function of azimuthal angle between the muon track and the observing DOM. SpiceMie does not account for the ice anisotropy, resulting in a sinusoidal shape that grows with distance. SpiceLea and Spice3.2 both account for the anisotropy, and consequently the sinusoidal shape is reduced in amplitude when using these ice models.

IceCube Event Types

Early → Late

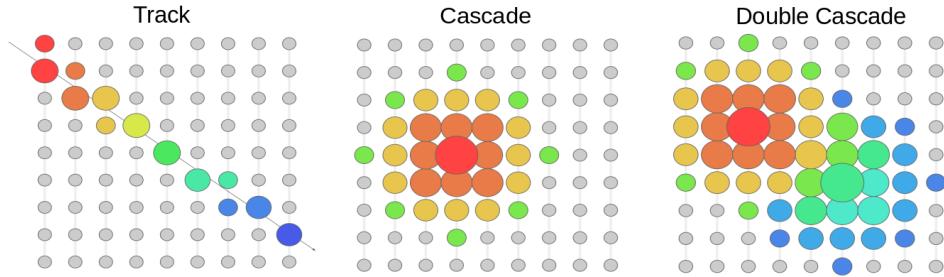



Figure 3.7: Cartoons in the x/z plane depicting the various event types seen by the IceCube detector. Grey circles are intended to represent individual DOMs, while colored circles correspond to DOMs that saw photoelectrons. The size of the circle corresponds to the amount of charge seen, while the color denotes the timing. Of most relevance to this work are tracks (left), as their long lever arm provides good pointing resolution.

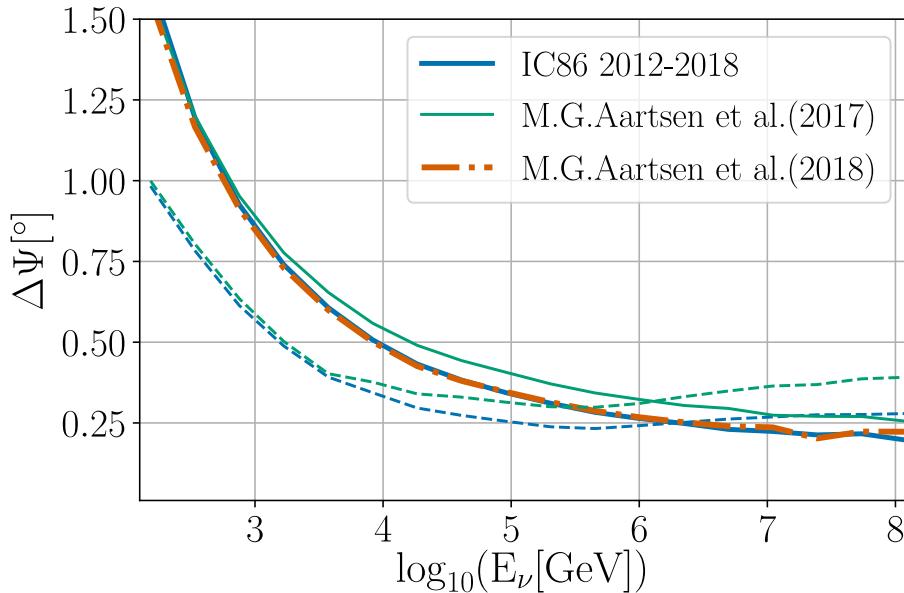


Figure 3.8: The median angle between simulated neutrino and reconstructed muon directions as a function of energy for various high-statistics IceCube samples composed of track events. The dark blue line corresponds to PSTracks v003p02, the light blue line corresponds to PSTracks v002p03, and the orange dashed line corresponds to NorthernTracks v002p06 (see section 1.7 for a description of these data samples). The solid lines describe events in the northern hemisphere, while the dashed lines are events in the southern hemisphere [4][32][33].

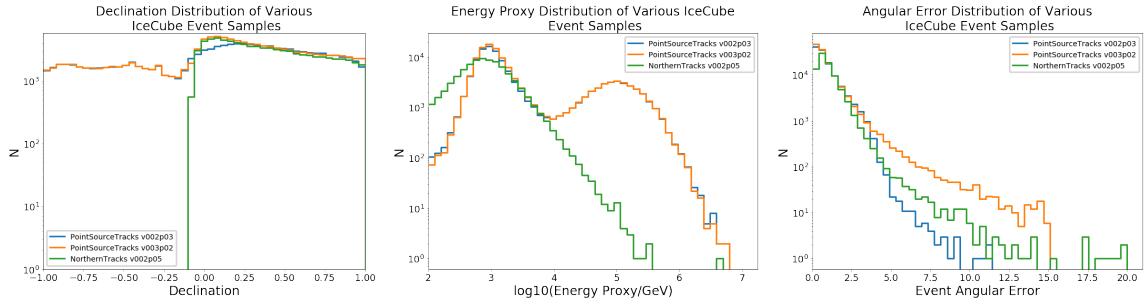


Figure 3.9: Distributions of the event variables in the three samples discussed in this section. The NorthernTracks sample does not contain events in the southern sky, as can be seen in the declination distribution on the left. This additionally has the effect of altering the overall energy proxy distribution for the sample, as the higher-energy peak in the energy proxy distribution for the PointSourceTracks samples is composed primarily of high energy southern sky events.

762 **Chapter 4**

763 **Review and Improvements to**
764 **Statistical Methods in Neutrino**
765 **Astronomy**

766 Point source searches in neutrino astronomy are essentially weighted clustering analyses:
767 each event is determined by a set of coordinates (right ascension, declination, angular error,
768 and time), and a weight (energy), and the task is to determine if high weight points are
769 clumped beyond what is expected purely from statistical fluctuations. Most attempts to
770 do this make use of a similar approach, using data-driven background estimation and a
771 likelihood-based estimator of clustering. This section outlines the general framework of such
772 an analysis, as well as several variants of the likelihood-based clustering estimator that is
773 commonly used. In the simplest case, only spatial clustering is examined, and temporal
774 information is excluded entirely ("time-integrated" analyses), however it can be additionally
775 interesting to explore the possibility of temporal clustering of events as well, increasing the
776 dimensionality of the clustering problem by one. Later in this section we introduce an
777 improved method for fitting ensembles of spatial and temporal clusters of events ("flares")
778 to characterize the data, which is an improvement over existing methods which only make
779 use of information from the most significant flare. This fills in the methodological gap that

780 can be seen in table 1.1.

781 4.1 Clustering Analysis Outline for an Arbitrary Test 782 Statistic

783 There are many ways of constructing a test statistic in neutrino astronomy, in addition to
784 a variety of types of clustering to look for (for example, clustering near various source cata-
785 logs, clustering in space and time, clustering with weights according to source/event energy
786 weights, clustering about an extended spatial feature such as the galactic plane). However,
787 almost all clustering analyses in this area share several features in their construction:

- 788 1. A test statistic is formulated, which ideally tests the degree of clustering of astrophys-
789 ical events near a particular source candidate location. This test statistic is typically
790 a likelihood-based test statistic of a form similar to that which is described in the
791 following section, however this test statistic is often modified to reflect the specifics of
792 the clustering hypothesis being tested.
- 793 2. A background test statistic distribution is calculated using maps generated from data
794 that has the right ascension values of events randomized. This destroys any cluster-
795 ing that may have already been present, providing an excellent representation of the
796 null hypothesis (a purely diffuse astrophysical neutrino flux with no spatial or tem-
797 poral clustering). As this background estimation is data-driven, it is robust against
798 unknown backgrounds that may be present in the sample. Scrambling the data does
799 not affect the sample event content, and consequently the underlying distributions of
800 event declination, energy, and arrival time are unchanged.
- 801 3. Alternative hypothesis test statistic distributions may be obtained from simulations of
802 signal events. If the analysis is not sensitive to the total astrophysical neutrino flux,
803 simulated events can simply be added to the background maps generated above. If the

analysis is sensitive to the total astrophysical flux, then more sophisticated methods (such as moving existing events instead of injecting new events) may be required. Once maps containing signal have been generated, they can be used both to characterize the performance of the analysis, as well as compute upper limits in the event of a null result.

4. The performance of the test statistic being used is typically evaluated using the *sensitivity* and/or *discovery potential*. The *sensitivity* refers to the amount of signal that needs to be injected before 90% of the test statistic distribution is greater than the median of the background test statistic distribution. Similarly the *discovery potential* refers to the amount of signal that needs to be injected before 50% of the test statistic distribution is greater than the $N\sigma$ significance threshold in the background test statistic distribution (N is typically either 3 or 5, but can be other values. This is typically clarified by stating that the value reported is the "N-sigma discovery potential"). "Amount of signal" is intentionally ambiguous here, as different analyses are sensitive to different types of signal. The sensitivity and discovery potential can be cast in terms of any number of variables that may be relevant to a particular analysis. For example, in the time-integrated case, these values are often reported as a flux, however time-dependent analyses may report a fluence, and analyses sensitive to source populations may even report their sensitivity as a curve in the space of source density and source luminosity.

5. The test statistic is calculated for the observed data sample, and this observed test statistic is compared to the distribution generated in step 2 to calculate a p-value. This p-value is then used to either accept/reject the null hypothesis of no clustering.

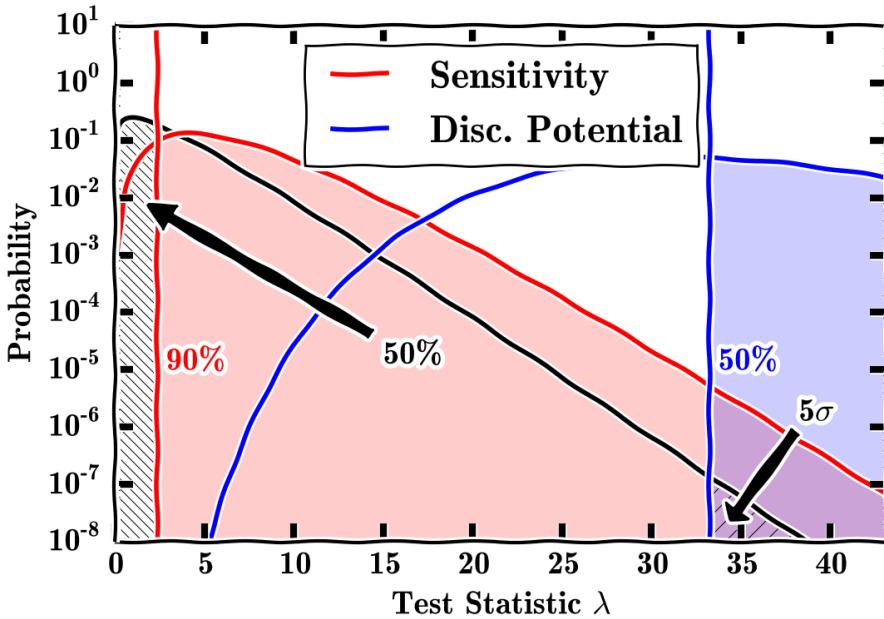


Figure 4.1: A toy plot showing the definitions of sensitivity and discovery potential. [43]

827 4.2 Unbinned Time Integrated Methods

828 Astrophysical point sources may be distinguished from the atmospheric neutrino background
 829 by way of clustering: if astrophysical sources are present in the data, events should be clus-
 830 tered near one another, while atmospheric events are expected to be isotropically distributed.
 831 Additionally, since high energy neutrino events are more likely to be astrophysical in origin,
 832 clustering of these events can be an even stronger indicator of the effect of neutrino point
 833 sources in the data. In short, we would like to construct a test statistic that reflects these
 834 observations. This can be done by way of a likelihood-based test statistic, using a likelihood
 835 composed of signal and background PDFs that describe the spatial and energy properties of
 836 events relative to a source candidate location. For a neutrino sample composed of N total
 837 events, the likelihood of the data for some number of signal (clustered astrophysical) events,
 838 n_s is (eq. 4.1)[44]:

$$\mathcal{L}(n_s, \gamma) = \prod_{i=1}^N \left[\frac{n_s}{N} S_i + \left(1 - \frac{n_s}{N}\right) B_i \right] \quad (4.1)$$

839 Where here, S_i and B_i are PDFs that describe the spatial and energy distributions

840 of signal and background events relative to the source candidate location. S_i and B_i are
 841 themselves composed of spatial and energy components (eq. 4.2 and 4.3).

$$S_i = R(r_i) \times \mathcal{E}(E_i, \delta_i | \gamma) \quad (4.2)$$

842

$$B_i = \frac{1}{\Omega} \times \mathcal{E}(E_i, \delta_i | Atm_\nu) \quad (4.3)$$

843 Note that the source spectral index, γ , enters as a free parameter in the energy portion
 844 of the signal PDF.

845 For the background PDF, the construction is relatively straightforward: background
 846 events originate from all directions equally, so the only effect that produces anisotropies is
 847 the detector acceptance. Since the IceCube detector acceptance does not vary as a function
 848 of right ascension, we can define B_i for a particular declination band to simply be $1/\Omega$,
 849 where Ω is the solid angle of a declination band centered on the source candidate declination.
 850 Similarly, the background energy PDF can be obtained by measuring the energy distribution
 851 of observed events as a function of declination. 2D histograms of event counts as a function
 852 of declination and event energy are assembled, and these distributions are then splined to
 853 create PDFs that can be used to evaluate B_i for events at an arbitrary declination and
 854 energy.

855 Unlike background events, signal events are expected to be clustered near the source
 856 candidate location. For this reason, the spatial component of the signal PDF, $R(r_i)$, is
 857 assumed to be a 2-D gaussian centered on the source candidate location (eq. 4.4):

$$R(r_i) = \frac{1}{2\pi\sigma_i^2} e^{-\frac{r_i^2}{2\sigma_i^2}} \quad (4.4)$$

858 Where r_i is the angular distance between the i th event and the source candidate location
 859 and σ_i is the event angular error.

860 The energy component of S_i is obtained from simulation: Simulated events are weighted
 861 according to a particular astrophysical spectral index, creating a 2D PDF describing the sum

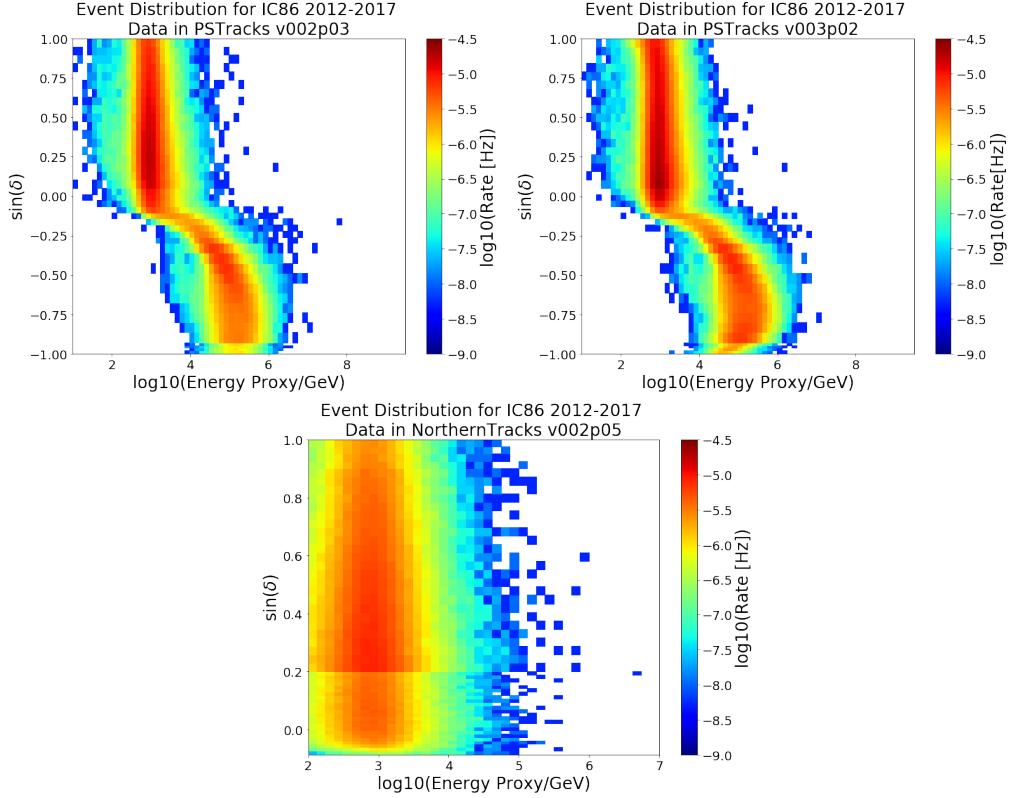


Figure 4.2: 2D histograms of declination and energy proxy for the 3 IceCube track event samples discussed in this document: PointSourceTracks v002p03 (left), PointSourceTracks v003p02 (right) and NorthernTracks v005p02 (bottom). Splines of these distributions are used as background PDFs in the clustering likelihood described in equation 4.1[31]. The NorthernTracks dataset is restricted to the northern sky, and additionally the splines associated with this dataset use finer binning near the horizon, hence the seeming discontinuity near $\sin(\delta) = 0.2$

862 of simulated event weights as a function of declination and energy. These maps are then
 863 divided by the background PDF described above to create a map describing the "signalness"
 864 of events at a particular energy and declination, given a spectral index. As these maps are
 865 created for a range of spectral index values (typically ranging between $\gamma = 1$ and $\gamma = 4$),
 866 a 3-D PDF of event energy, declination, and spectral index hypothesis results from this
 867 process. This is precisely the function $\mathcal{E}(E_i, \delta_i | \gamma)$ that enters the likelihood, and can be used
 868 to fit for γ , given events observed at declination δ with energy E_i .

869 The likelihood described in eq. 4.1 can be maximized as a function of s and γ , resulting
 870 in best fit parameters \hat{n}_s and $\hat{\gamma}$. We can then compute a test statistic for a given data sample
 871 from the likelihood ratio (4.5):

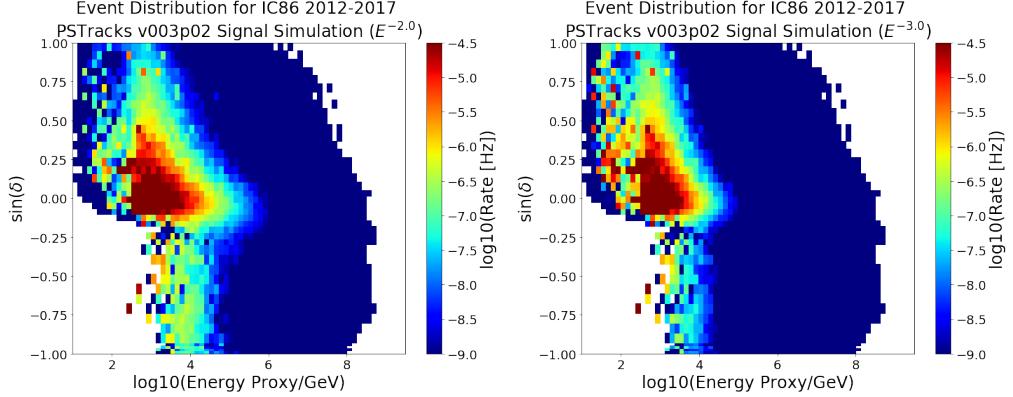


Figure 4.3: Distributions of simulated signal events in PointSourceTracks v003p02 for $E^{-2.0}$ (left) and $E^{-3.0}$ (right) spectra. The ratio of these distributions to the background expectation is then used to calculate the combined spatial and energy 2D PDF ratio ($S(r_i, E_i)/B(r_i, E_i)$) for a particular spectral index hypothesis. Similar distributions can be made for spectral indices ranging from $\gamma = 1.0$ to $\gamma = 4.0$, allowing for the signal spectral index to be fit by the likelihood.

$$TS = -2 \log \frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s, \hat{\gamma})} \quad (4.5)$$

872 This test statistic can then be used in the generalized framework described in the sections
 873 above to test the hypothesis of spatial clustering of astrophysical neutrino events near a
 874 particular source location.

875 This framework can be applied to multiple locations at once, and the results combined by
 876 summing the individual test statistics calculated at each location (note that this is equivalent
 877 to calculating the product of the likelihoods). This process is often referred to as "source
 878 stacking", and can be used to improve sensitivity to dim sources, provided there are multiple
 879 emitters in the source candidate list.

880 Other methods of combining information across multiple source locations exist as well. In
 881 particular, the binomial test is a popular way of doing this. Given a list of source candidates
 882 and their associated p-values, we can search for the most significant combination of source
 883 candidates by employing the binomial test-statistic (eq. 4.6)

$$p(k) = \sum_{i=k}^{N_{eff}} \binom{N_{eff}}{i} p_k^i (1-p_k)^{N_{eff}-i} \quad (4.6)$$

884 Where $p(k)$ is the p-value associated with combining the results of the k most significant
 885 sources, N_{eff} is the effective number of trials (often simply equal to the total number of
 886 source candidates), and p_k is the p-value of the k th most significant source. The minimum
 887 of $p(k)$ can then be computed and treated as a test statistic, to obtain a p-value associated
 888 with the best-fit number of sources (k_{min}) in a particular catalog.

889 4.3 Unbinned Time-Dependent Methods

890 In addition to being clustered in space, astrophysical neutrino events may also be clustered in
 891 time (a neutrino "flare"). Accounting for this temporal clustering may allow us to identify
 892 sources that are insignificant under a corresponding time integrated analysis. Studying
 893 the temporal variation of source candidates can also inform us of the specifics of the source
 894 dynamics of particle production, as the time scale of observed flares is related to the physical
 895 scale of the astrophysical objects in which particle production is occurring. This method
 896 was used in [3] to identify the 2014 neutrino flare candidate with a significance of 3.5σ .

897 4.3.1 Optimizing for a Single Flare

898 We can modify our time-integrated likelihood to account for potential temporal clustering
 899 as well by simply appending a temporal PDF to the existing PDFs that describe the spatial
 900 and energy components of the analysis (eq. 4.7, 4.8):

$$S_i = R(r_i) \times \mathcal{E}(E_i, \delta_i | \gamma) \times \mathcal{T}(t_i) \quad (4.7)$$

$$B_i = \frac{1}{\Omega} \times \mathcal{E}(E_i, \delta_i | Atm_\nu) \times \frac{1}{\Delta T} \quad (4.8)$$

901 Where ΔT is the full livetime of the sample, and the associated temporal background
 902 PDF is $1/\Delta T$, as the background event rate should be constant. In practice, this is assembled
 903 in a data-driven manner by measuring the data sample event rate in each of the 8-hour

904 segments ("runs") that are used to segment the data. In this way, seasonal variations of the
 905 detector event rate are accounted for by the likelihood, as the flare candidates are always
 906 being compared to the local event rate in time.

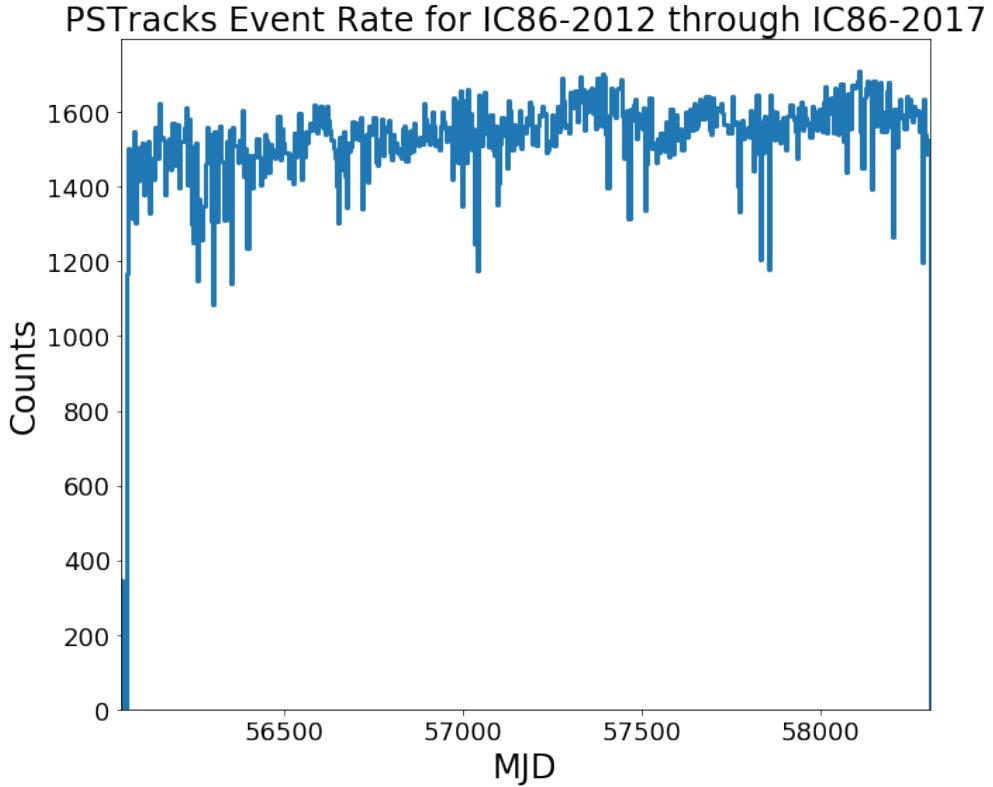


Figure 4.4: The event rate of the IC86 seasons of the PointSourceTracks v3 data sample. This event rate can be used to generate a background temporal PDF in the time-dependent clustering likelihood.

907 The temporal contribution to the signal PDF, $\mathcal{T}(t_i)$, can take various forms depending
 908 on the particular shape that one assumes for the time profile of flare candidates, but the
 909 most basic temporal profile can be used is a box (eq. 4.9):

$$\mathcal{T}(t_i|t_0, \Delta t) = \begin{cases} 0 & t_i < t_0 \\ \frac{1}{\Delta t} & t_0 \leq t_i \leq t_0 + \Delta t \\ 0 & t_i > t_0 + \Delta t \end{cases} \quad (4.9)$$

910 This box-shaped temporal PDF essentially acts as a mask, filtering for events that occur
 911 within the time window between t_0 and $t_0 + \Delta t$. The events within this time window then

contribute to the likelihood in a similar manner to the time-integrated case. Note that by adding this temporal PDF to the likelihood, we have also introduced 2 new parameters that the likelihood can be maximized with respect to: t_0 and ΔT , which describe the flare start time and duration, respectively.

Similar to the time-integrated case, a likelihood ratio test statistic can be assembled from this likelihood, however this test statistic can be further improved in the time-dependent case by accounting for the duration of flare candidates that were scanned over (eq. 4.10):

$$TS = -2 \log \left[\frac{\Delta T}{\hat{\Delta}t} \frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s, \hat{\gamma}, \hat{t}_0, \hat{\Delta}t)} \right] \quad (4.10)$$

Where the factor $\frac{\Delta T}{\hat{\Delta}t}$ can be interpreted as a trial factor that accounts for the fact that there are significantly more short flare candidates that can be tested than long ones (e.g. there is only 1 potential flare candidate with $\Delta t = \Delta T$, but there are many flares that can be made with $\Delta t = \Delta T/100$). This factor normalizes the test statistic scale between short and long flares [44].

In practice, this process is extremely computationally intensive due to the number of likelihood minimizations that need to be performed. This can be mitigated by seeding flares with events that already have a high probability of being signal in origin, based off their energy and arrival direction. An ensemble of seed events can be defined to the set of events with S_i/B_i greater than some threshold value, where S_i and B_i refer to the signal and background PDF components of the time integrated likelihood (describing only the spatial and energy properties of contributing events). Typically this threshold is set to be relatively low ($S_i/B_i > 1$), however higher values can also be used if computational requirements are particularly stringent (e.g. if running over the entire sky).

Many analyses compute the test statistic in eq. 4.10 for a variety of flare candidates, and then use the flare with the maximum test statistic as the "best-fit" flare. The test statistic associated with this "best-fit" flare is then compared to a distribution of "best-fit" flares obtained from the background case (generated from data scrambled in right ascension),

resulting in a p-value that describes the strength of the clustering hypothesis relative the null hypothesis of no clustering.

In simulated signal trials, the likelihood-based flare test statistic is able to reconstruct the injected number of signal events in each flare with reasonable accuracy, and the per-flare spectral index fits show similar agreement as well.

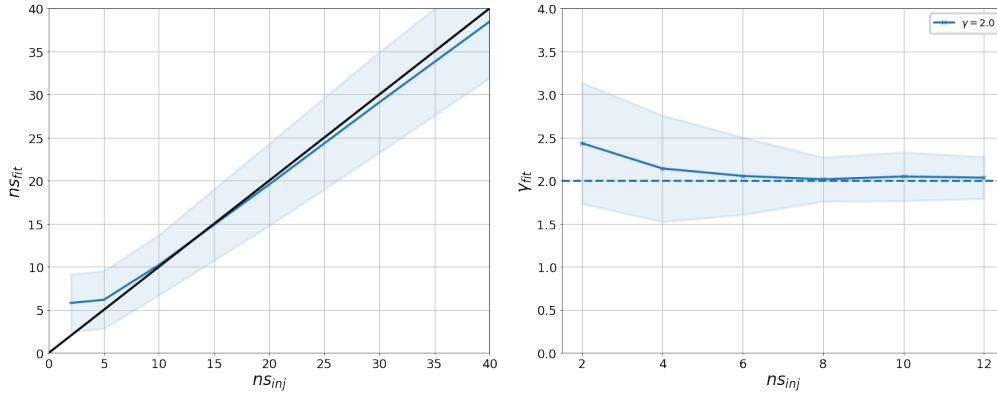


Figure 4.5: The best fit number of signal events in a flare, and the best fit spectral index versus the true number of signal events injected in simulated trials. On average, for flares with $>\approx 5$ events, the time-dependent likelihood is able to accurately estimate the true number of signal events.

4.3.2 Improving the Search for Temporal Clustering: Fitting Ensembles of Multiple Flares

As the IceCube detector collects more data, it becomes increasingly useful to be able to test the hypothesis of multiple neutrino flares that may have occurred at a particular location. Fitting only the largest flare is advantageous if the source flare rate is expected to be low enough that sources only flare once over the data-taking period, however if sources flare multiple times the single-flare analysis potentially misses a significant amount of useful information. By incorporating information from multiple flares at a particular location, an analysis can be sensitive to a smaller individual flare intensity. This is analogous to spatial stacking analyses, as can be seen in table 1.1, but we have additionally improved our algorithm to stack *flares* instead of just spatial source candidates. In this section we discuss

953 the construction of this method, and the following sections describe the results of the first
 954 applications of this method to various source catalogs.

955 The process of fitting multiple flares at a particular source candidate location is similar
 956 to the single-flare maximization procedure described above. Flare candidates are seeded by
 957 events with a high S_i/B_i ratio near the candidate location. For each flare candidate, the
 958 test statistic described in equation 4.10 is calculated. Unlike the single flare case, these
 959 flare candidates are then ordered by decreasing test statistic value, and any flares that
 960 overlap in time with a different flare that itself has a higher test statistic are removed.
 961 Additionally, any flares with a test statistic < 0 are removed. What remains is a temporally
 962 decorrelated ensemble of flare candidates, each with $TS > 0$. This ensemble functions as
 963 a neutrino "flare curve" (similar to light curves produced by photon-based telescopes), that
 964 describes the temporal variability of the signal originating from a source candidate location.
 965 A "multi-flare" test statistic describing the significance of the temporal variability of this
 966 source candidate can then be calculated as the sum of the individual flare candidate test
 967 statistics (eq. 4.11):

$$\widetilde{TS} = \sum_{j=0}^{N_{flares}} TS_j \quad (4.11)$$

968 Where here, j is an index that refers to the individual flares that compose the neutrino
 969 flare curve. This test statistic can then be compared to an ensemble of similar test statis-
 970 tics generated from right-ascension scrambled (background) data to obtain a final p-value
 971 describing the significance of the set of flares that were fit at a particular source candidate
 972 location.

973 An estimation of the total number of signal events can be obtained in a similar manner
 974 (eq. 4.12), where the total number of signal events associated with a particular source
 975 candidate is just the sum of the best-fit number of n_s in each contributing flare candidate:

$$\widetilde{n}_s = \sum_{j=0}^{N_{flares}} \hat{n}_{sj} \quad (4.12)$$

976 Note that this similar to time-integrated source stacking mentioned above, however in-
 977 stead of stacking test statistics associated with spatially distinct locations, we are instead
 978 stacking spatially and temporally distinct *flares*.

979 The multi-flare algorithm is, in some sense, inclusive of the single flare algorithm: by
 980 fitting all the flares at a source candidate location, we have obviously also fit for the largest
 981 flare. It is thus trivial to obtain the single-flare significance once the multi-flare result is
 982 obtained. In order to do this, simply compare the highest flare candidate test statistic that
 983 was obtained with a distribution of single-flare test statistics obtained in a similar manner
 984 from background (right-ascension scrambled) data.

985 In addition to calculating the local significance of the largest flare, the local significance
 986 of the other flare candidates composing the flare curve fit by the multi-flare algorithm can
 987 also be obtained in a similar manner. We can define the "local significance" of a particular
 988 flare (not necessarily the largest) to be the fraction of flares in the background distribution
 989 with $TS_j > TS_{j,observed}$. Note however, that the calculation of multi-flare significance is done
 990 in the space of TS_j , not the space of the corresponding local significances p_j . This means
 991 that the multi-flare significance is *not* simply the product of the component flare p_j 's, as each
 992 flare is not entirely statistically independent from the others (e.g. once the largest flare is
 993 fit, the remaining available livetime in which other flares can be fit is reduced by an amount
 994 equal to the duration of the largest flare.)

995 As mentioned above, the multi-flare algorithms is particularly useful in the case of several
 996 similarly sized flares. In this case, while the single flare algorithms will identify the correct
 997 number of events in the largest flare, the estimation of the total number of signal events
 998 associated with the source candidate will be incorrect (as there is a non-negligible portion
 999 of events that belong to flares that were not identified by the single flare algorithm). By
 1000 contrast, the multiflare algorithm improves the estimation of n_s in the case of multiple
 1001 similarly sized flares, as signal events in all flare candidates (not just the largest one) are
 1002 able to contribute. This improvement is shown in figure 4.7.

1003 As an example of a case where the multi-flare algorithm is advantageous, consider the

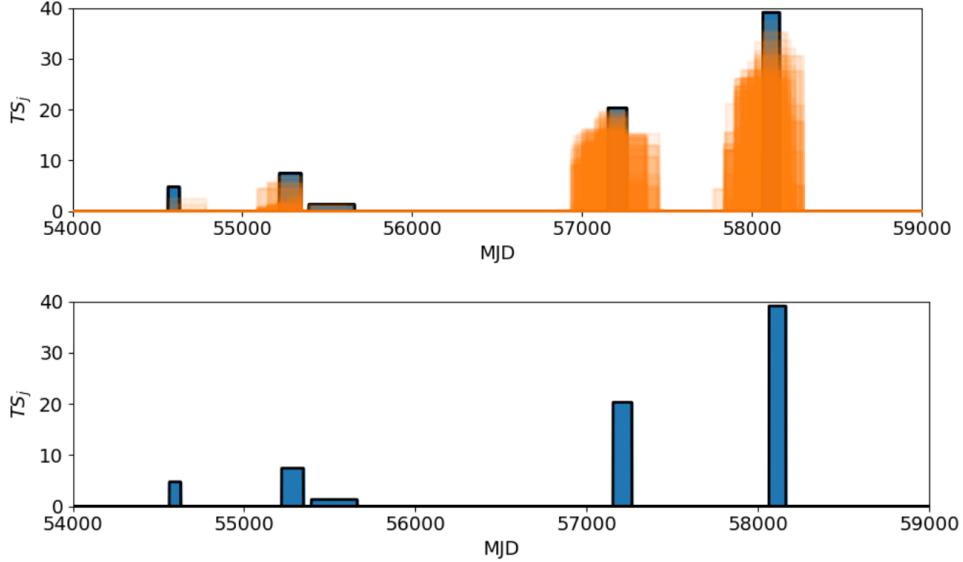


Figure 4.6: A graphical example of the progression of the multi-flare decorrelation algorithm. Colored boxes represent flare candidates that were tested, seeded by events with high S_i/B_i ratios. The height of each box corresponds to the individual flare candidate test statistic, TS_j . Orange flares overlap with another flare with a higher test statistic, and are consequently removed, leaving only the blue flares. The test statistics of the blue flares are then summed, and can be used for hypothesis testing of flaring neutrino emission at this source candidate location.

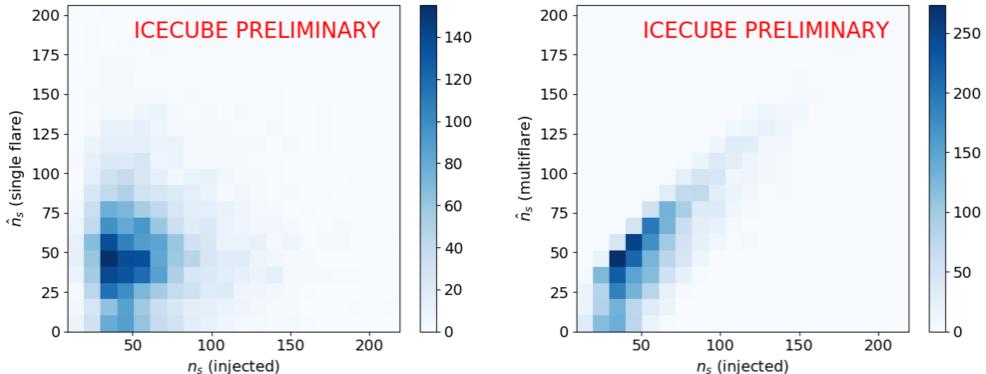


Figure 4.7: Left: the injected vs. fit number of signal events when attempting to use the single flare algorithm to describe an ensemble of neutrino flares. Right: The same plot, but using the multiflare estimation of n_s that accounts for signal events in all flare candidates.

1004 case shown in 4.8, which has 3 flares injected at MJD=55246.3, 55807.4, and 57632.7 at
 1005 a test location of (RA, Dec) = (77.45°, 5.61°). The single flare method fits only the flare
 1006 at MJD = 55246.3 (with TS = 22.66), while the multiflare method fits all flares together,
 1007 with a multi-flare test statistic of $\widetilde{TS} = 60.52$. While the largest individual flare only has

1008 a significance of 3.29σ , by combining information from all the flares that were fit at this
 1009 location, we arrive at a multi-flare significance of 4.7σ .

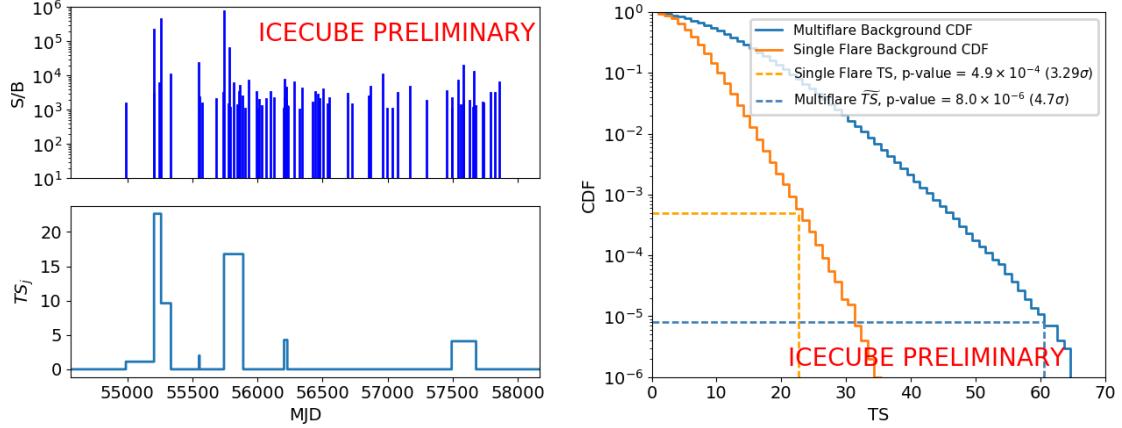


Figure 4.8: Left, top: Events that pass the S/B threshold cut for generating windows, for a source located at $(\text{ra}, \text{dec}) = (77.45^\circ, 5.61^\circ)$. There are 3 injected flares, centered at $\text{MJD}=55246.3$, 55807.4 , and 57632.7 . Left, bottom: The single flare method fits only the flare at $\text{MJD} = 55246.3$ (with $\text{TS} = 22.66$), while the multiflare method fits all flares together, with a global test statistic of $\bar{\text{TS}} = 60.52$. Right: The background test statistic distributions for a single source, located at declination = 5.61° . The background multiflare test statistic distribution is shown in blue, while the single flare method is shown in orange. The vertical lines represent the test statistics associated with the flare curve on the left. The single flare p-value for this source is 4.9×10^{-4} (3.29σ), while the multiflare p-value is 8.0×10^{-6} (4.7σ).

1010 **Chapter 5**

1011 **Applications of the Multi-flare
1012 Algorithm to Source Catalogs**

1013 The multi-flare algorithm introduced in the previous section may be applied to an ensemble
1014 of sources that share common features. This is common practice in neutrino astronomy, as
1015 examining emission from a catalog allows us to explore the possibility of neutrino emission
1016 from a class of sources, rather than a specific individual source. Here, we explore two
1017 catalogs designed to explore source features related to those associated with the analysis of
1018 TXS 0506+056. Namely, the fact that the analysis was initially triggered by a high-energy
1019 neutrino event, and also the fact that TXS 0506+056 is a blazar. For the former, we assemble
1020 a catalog of high energy IceCube events to treat as "sources" (a "self-triggered" catalog
1021 analysis), while for the latter we use the pre-existing catalog of Fermi 3LAC blazars [45].

1022 In both cases, the multi-flare algorithm is applied at each source candidate location,
1023 resulting in a multi-flare test statistic (and corresponding pre-trial p-value) associated with
1024 each source candidate. To test for potential sub-populations of strong emitters within the
1025 catalog, we additionally calculate a best-fit number of multi-flare sources via iteratively
1026 summing the sources with the largest test statistics (5.1). For a given data set, the sources are
1027 ordered by their multiflare test statistic, \widetilde{TS} . A p-value for $k = 1, 2, 3, \dots N_{srcs}$ is calculated,
1028 and subsequently the k that produces the minimum p-value is selected (k_{best}).

$$\widetilde{TS}_{all} = \sum_{m=0}^{k_{best}} \widetilde{TS}_m \quad (5.1)$$

1029 An additional trial factor is then calculated by applying this procedure to maps of data
 1030 with randomized right ascension values to assemble a distribution of \widetilde{TS}_{all} representative of
 1031 the null hypothesis. A final p-value can then be obtained by comparing an observed \widetilde{TS}_{all}
 1032 with this null hypothesis distribution.

1033 For all the analyses detailed in this section, the NorthernTracks v002p05 sample was used.
 1034 This sample is described earlier in this document (section 2.5).

1035 5.1 Self-Triggered Catalog

1036 For this catalog, we select the locations of all the events in the NorthernTracks v002p05 data
 1037 sample that have a reconstructed energy proxy greater than 200 TeV. This energy threshold
 1038 is chosen as it is roughly the point where 50% or more of neutrino events are expected to
 1039 be astrophysical in origin. We can consequently expect that this catalog contains a non-
 1040 negligible number of astrophysical neutrino sources, and can test the hypothesis of additional
 1041 time dependent neutrino emission from these locations. Events that have been selected as
 1042 source candidates for the catalog are subsequently removed from the sample, and do not
 1043 otherwise contribute to the calculation of a multi-flare test statistic.

1044 This catalog contains 32 source candidates, all in the northern sky. Notably, this cat-
 1045 alog does not include IC170922A (the alert event that triggered the followup analysis on
 1046 TXS 0506+056), as the data sample does not extend past the 2015 season, and IC170922A
 1047 occurred in the 2017 season.

1048 Applying the multi-flare algorithm described above to this catalog of the locations of
 1049 32 high energy neutrino events results in a best-fit number of sources of $k_{best} = 4$, with
 1050 an associated p-value of $p = 0.017$ (2.13σ), shown in figure 5.5. Detailed results for all 32
 1051 source candidates, including fitted number of events and pre-trial p-values, can be viewed in
 1052 table 5.1.

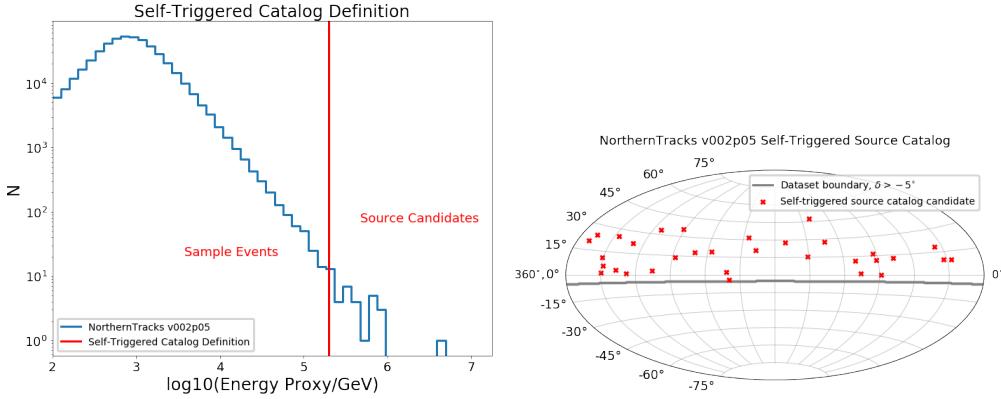


Figure 5.1: Left: The self-triggered catalog definition: Events with energy proxies to the right of the vertical red line (200 TeV) are treated as source candidates. Events to the left of the red line are then investigated for clustering around the locations of the source candidate events. Right: The self-triggered catalog assembled from the NorthernTracks v002p05 data sample, consisting of all events in the sample with reconstructed energy proxy greater than 200 TeV. The locations of these events are treated as source candidates for the purposes of applying the multi-flare algorithm, however the events themselves are removed from the sample prior to calculating a test statistic.

1053 As mentioned in previous sections, a significant advantage of the multi-flare algorithm is

1054 the generation of neutrino "light curves" (or "flare curves") that show the historical activity

1055 of a source candidate. The flare curves associated with the 4 most significant sources in the

1056 self-triggered catalog can be seen in figure 5.3.

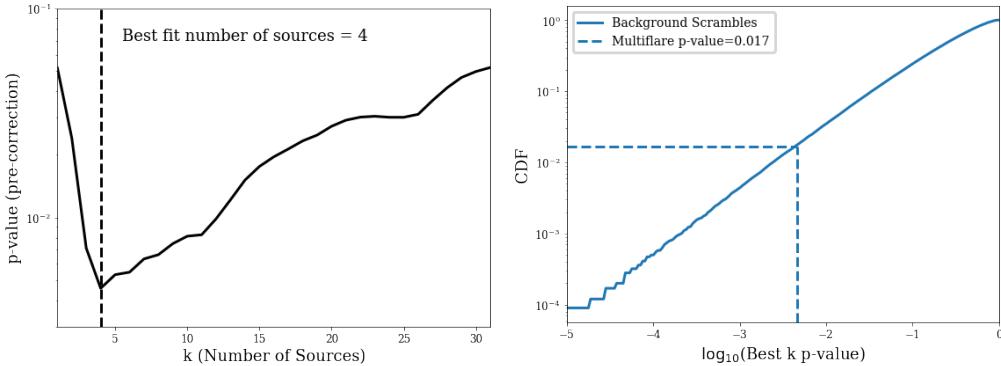


Figure 5.2: Left: The local significance of stacking the k highest test statistic sources, as a function of k . The best fit value of $k = 4$ can be seen as the minimum of this curve. Right: The trial corrected result for stacking the top 4 sources together, shown as a vertical line superimposed on the background distribution obtained by applying the algorithm to right-ascension scrambled data.

1057 Though not statistically significant, these results have several interesting features. Though

RA	Dec	\hat{n}_s	p (pre-trial)
36.69	18.32	47.21	0.00197
272.14	35.66	30.71	0.00729
170.19	27.85	45.75	0.00834
93.26	16.33	30.02	0.02667

Table 5.1: The top 4 most significant multi-flare source candidates in the self-triggered catalog composed of high energy IceCube neutrino events. Collectively, these sources form an excess with a significance of $p = 0.017$.

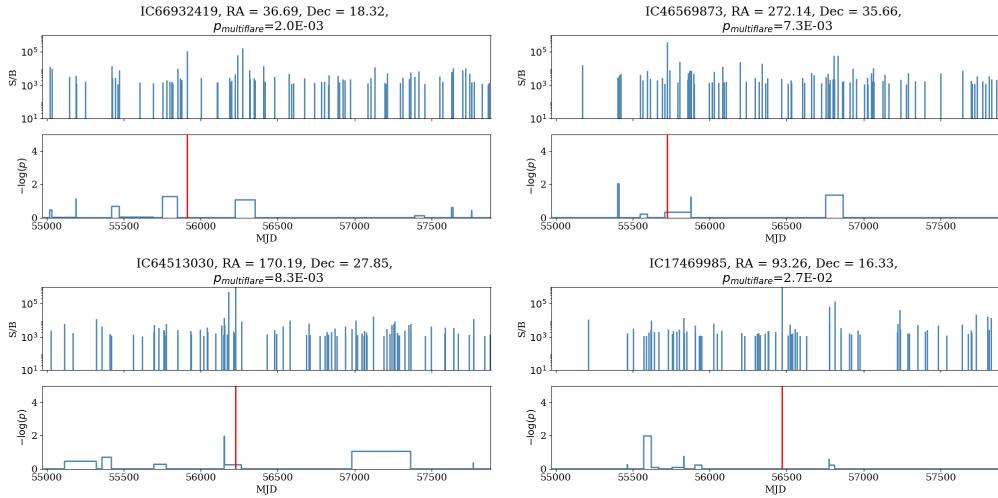


Figure 5.3: The neutrino "flare curves" associated with the 4 most significant multiflare source candidates in the self-triggered catalog. The top panel of each subplot shows the event weights, calculated by taking the ratio of the spatial and energy components of the signal and background PDFs described in equation 4.2 and 4.3, while the bottom panels show the fitted ensemble of decorrelated flares, with the local per-flare p-value plotted on the y-axis. The vertical red line denotes the arrival time of the high energy event used to define the source candidate location. This event is removed from the sample prior to applying the multiflare-algorithm, and consequently these events do not contribute to the flare curves shown in this figure.

1058 the high-energy "seed" events are removed prior to the calculation of a flare curve, 7 of the
 1059 flare curves generated include flares that would have included a high energy seed event.
 1060 Though not statistically significant, this is certainly above average, as only 11% of back-
 1061 ground trials have 7 or more flare curves with seed event/flare correlations.

1062 It is also potentially interesting to investigate the distributions of the parameters fit by
 1063 the multi-flare likelihood. By comparing the observed distributions with those expected from
 1064 background scrambles, we can check for inconsistencies of our data with the null hypothesis.

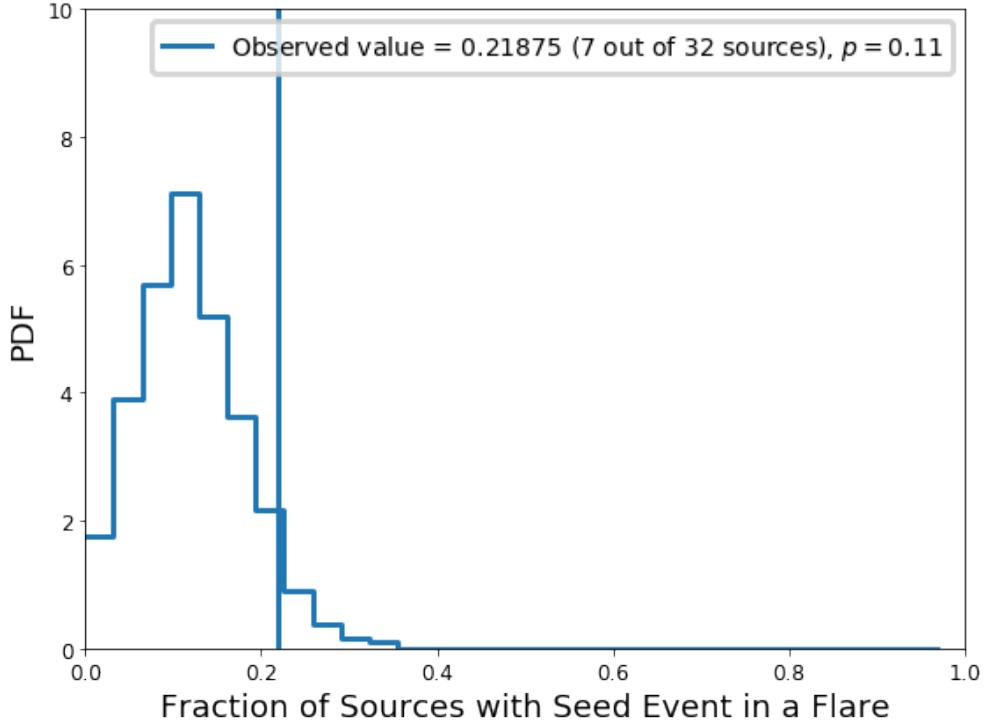


Figure 5.4: The distribution of number of flare curves that have a temporal correlation between the high energy seed event, and a fitted flare, obtained from sets of data where the right ascension of events has been randomized. The observed value (7 out of 32 sources, or 21.9%) is shown as a blue vertical line. 11% of trials have more than 7 correlations between seed events and fitted flares.

1065 5.2 Fermi 3LAC Blazars

1066 The 2014 TXS 0506+056 neutrino flare was notable not only for its association with a high
 1067 energy IceCube alert, but also for its spatial coincidence with the blazar TXS 0506+056. In
 1068 searching for additional neutrino sources, it is then not unreasonable to assemble a search
 1069 for neutrino flares associated with a large catalog of blazars. For this purpose, we use the
 1070 Fermi 3LAC catalog [45] to define a set of source candidates. Previous analyses searching
 1071 for spatial coincidence with earlier iterations of this catalog have been performed, but were
 1072 unable to identify any statistically significant excess [46]. Here, we instead specifically search
 1073 for transient emission using multi-flare algorithm described above.

1074 The Fermi 3LAC catalog is a catalog of AGNs detected by Fermi-LAT, consisting of
 1075 gamma ray sources in the third Fermi-LAT catalog (3FGL) [47] between 100 MeV and

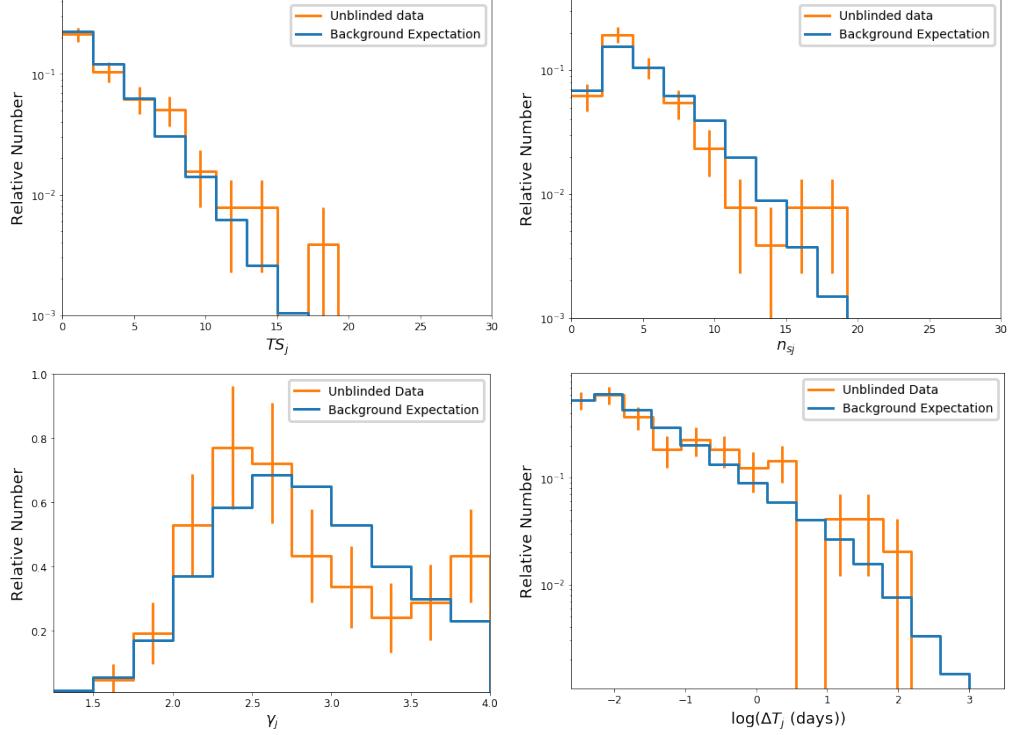


Figure 5.5: Distributions of fitted flare parameters for flares associated with the self-triggered catalog in both background scrambles (blue) and unblinded data (orange). While there is potentially some deformation in the distribution of fitted spectral indices (bottom left), a 2-sample K-S test comparing the observed and background distributions only returns a p-value of $p = 0.26$, indicating that the blue and the orange distributions are not significantly inconsistent with one another.

1076 300 GeV with a Fermi test statistic greater than 25 between August 4, 2008, and July 31,
 1077 2012 [45]. The catalog contains 1591 objects, the majority (98%) of which are blazars,
 1078 roughly evenly split between FSRQs and BL Lacs. Notably, TXS 0506+056 is a member of
 1079 this catalog.

1080 In constructing this analysis, we select for blazars at declinations greater than -5° , as
 1081 the IceCube data sample (NorthernTracks v002p05) does not extend below this point. We
 1082 impose no further cuts, and all sources are weighted equally when calculating a multi-flare
 1083 test statistic. The final catalog to be used for the multi-flare analysis consists of 1023 blazars,
 1084 the locations of which are shown in figure 5.6.

1085 The results of applying the multi-flare algorithm to this catalog of 3LAC blazars can be
 1086 summarized in figure 5.8. The optimization procedure for the most significant combination

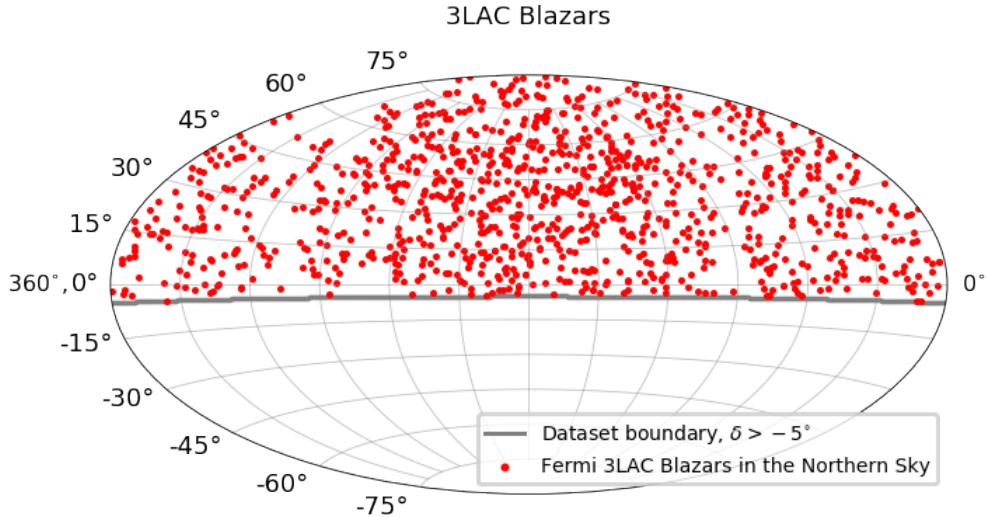


Figure 5.6: The locations of the 3LAC blazars that compose the catalog used for a multi-flare analysis. Blazars with $\delta < 5^\circ$ are not considered, as the IceCube NorthernTracks v002p05 data sample does not extend into this region. There are 1023 blazars that are considered as source candidates for the multi-flare analysis.

1087 of sources returned a best fit number of sources of $k = 125$, with an associated post-trial
 1088 p-value of $p = 0.06$. A list of these source candidates can be seen in table 5.2. As the
 1089 significance of this excess is consistent with the null hypothesis, we do not claim discovery
 1090 of neutrino flares associated with 3LAC blazars.

1091 Interestingly, despite the presence of the 2014 neutrino flare identified in [3], TXS 0506+056
 1092 is not the most significant source candidate, having a pre-trial p-value of only $p = 9.24 \times 10^{-3}$.
 1093 This is not unexpected, considering that there does not appear to much activity (in terms
 1094 of neutrino flares) at this location beyond the 2014 flare itself. As the multi-flare algorithm
 1095 performs will return a high significance when there are multiple, moderately significant flares
 1096 to stack together, it is unsurprising that there are other sources that have a higher multi-flare
 1097 significance. As an example, the source candidate with the highest multi-flare significance is
 1098 1RXS J154604.6+081912. Rather than having a single large flare, the multi-flare algorithm
 1099 fits multiple flares at this source candidate location, which combined have a significance of
 1100 $p = 8.1 \times 10^{-5}$, despite none of the individual flares having a local significance much greater
 1101 than $p = 0.01$.

1102 Similar to the self-triggered catalog, we can additionally examine the distributions of

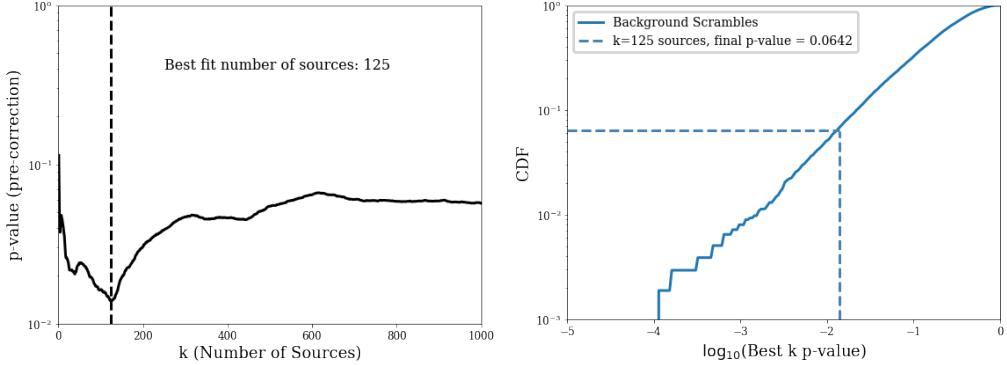


Figure 5.7: Left: The local significance of stacking the k highest test statistic sources in the 3LAC blazar catalog, as a function of k . The best fit value of $k = 125$ can be seen as the minimum of this curve. Right: The trial corrected result for stacking the top 125 sources together, shown compared the background distribution obtained by applying the algorithm to right-ascension scrambled data. As the final p-value associated with the top 125 blazar sources is only $p = 0.06$, there was no significant excess of neutrino flares observed to be associated with 3LAC blazars.

1103 the flare parameters that were fit for all the sources in the 3LAC catalog. No significant
 1104 deviations from the background expectation are observed in any of the flare parameter
 1105 distributions, as seen in figure 5.8.

1106 As with the self-triggered catalog, we also obtain neutrino flare curves describing the
 1107 historical variability of each source candidate. Figure 5.9 shows the flare curves of several
 1108 3LAC source candidates of note, including TXS 0506+056. While none of these sources are
 1109 significant enough to claim discovery, flare curves like the ones shown here are a potentially
 1110 valuable tool for multi-messenger analyses in the future that may seek to correlate neutrino
 1111 emission with other astrophysical messengers.

1112 Though the application of the multi-flare algorithm to this catalog of 3LAC blazars did
 1113 not result in a significant detection, it does allow us to constrain the behavior of neutrino
 1114 flares associated with 3LAC blazars. Given that this method stacks flares together, the lack
 1115 of a significant results suggest that there is an upper limit to how bright and numerous
 1116 blazar neutrino flares may be. We express these limits in terms of the flare rate (how many
 1117 flares occurred over the lifetime of the data sample), and the per-flare E^2 flux. This allows
 1118 us to compare to previously calculated limits obtained from a time-integrated analysis [46].

1119 These upper limits can be seen in figure ??

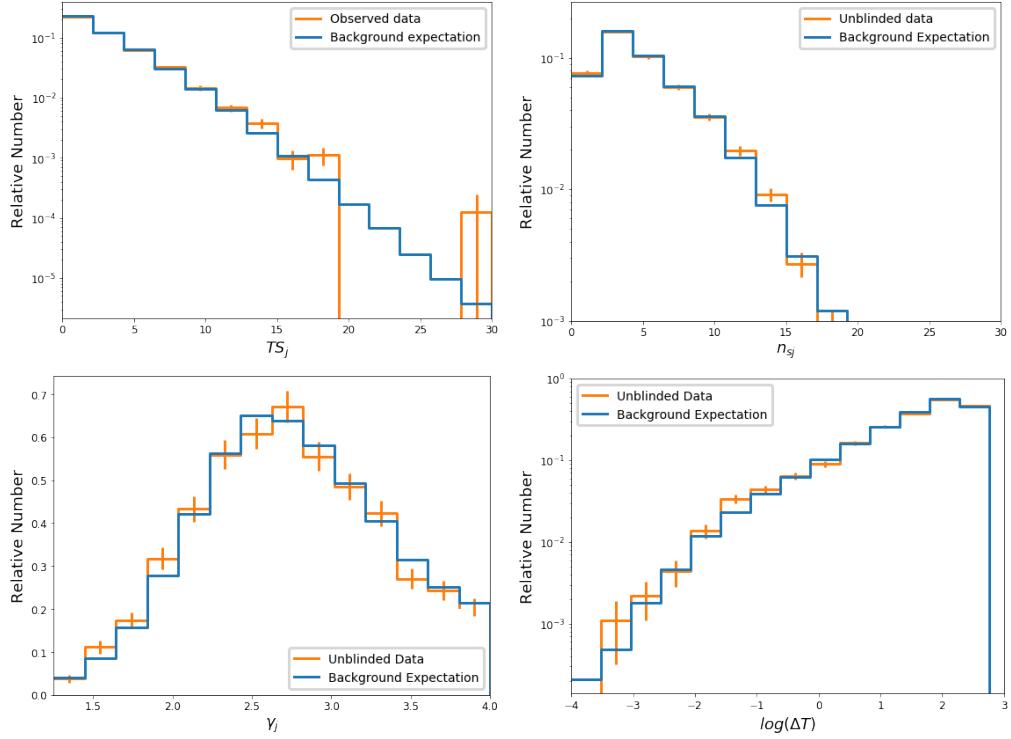


Figure 5.8: Distributions of fitted flare parameters for flares associated with the 3LAC blazar catalog in both background scrambles (blue) and unblinded data (orange). All distributions of fit parameters from observed data appear to be consistent with the background expectation. As this catalog includes TXS 0506+056, the 2014 neutrino flare originally discovered in the IC170922A follow-up analysis [3] is visible as the rightmost entry in the histogram of flare TS_j values in the plot in the upper left, having a value of $TS_j = 27.9$. There are no other flares that were fit in this catalog with $TS_j > 20$.

Table 5.2: The top 125 multiflare source candidates in the 3LAC blazar catalog.

Source Candidate Name	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1RXS J154604.6+081912	bll	236.52	8.32	53.49	8.11e-5
RBS 1467	bll	227.18	27.15	55.11	3.05e-4
GB6 J0723+2859	fsrq	110.98	28.99	40.48	4.58e-4
RBS 1558	bll	241.59	56.51	28.88	1.92e-3
PMN J2324+0801	bll	351.19	8.04	28.15	3.11e-3

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
B2 2214+24B	bll	334.25	24.36	36.90	3.79e-3
GB6 J0850+4855	bll	132.50	48.92	38.18	4.75e-3
4C +20.25	fsrq	171.49	20.10	27.83	5.02e-3
MG2 J094148+2728	fsrq	145.45	27.48	60.58	5.12e-3
TXS 2241+406	bll	341.05	40.95	22.61	5.23e-3
TXS 0213+619	bcu III	34.26	62.19	30.15	5.32e-3
GB6 J0100+0745	bll	15.09	7.76	37.33	6.45e-3
RX J0850.5+3455	bll	132.65	34.92	20.42	7.16e-3
B2 1436+37B	fsrq	219.72	37.18	49.47	7.83e-3
MG1 J165034+0824	fsrq	252.66	8.41	39.19	7.83e-3
PKS 0256+075	fsrq	44.86	7.79	42.53	9.04e-3
TXS 0506+056	bll	77.36	5.69	20.71	9.24e-3
1ES 1421+582	bll	215.66	58.03	28.49	0.0100
TXS 0518+211	bll	80.44	21.21	48.49	0.0118
W Comae	bll	185.38	28.23	30.32	0.0123
NVSS J141828+354250	bcu II	214.62	35.71	40.74	0.0128
PKS 1532+01	fsrq	233.72	1.52	30.78	0.0131
PKS 1424+240	bll	216.75	23.80	52.78	0.0148
B3 2319+444	fsrq	350.58	44.76	41.93	0.0160
PKS 0039+230	fsrq	10.52	23.33	48.89	0.0168
B3 2238+410	bll	340.28	41.34	21.24	0.0175
TXS 2315+189	bcu II	349.60	19.25	31.98	0.0197
B3 2322+396	bll	351.32	39.96	30.07	0.0199
NVSS J080637+774607	bcu II	121.66	77.77	52.32	0.0215
MG1 J010908+1816	bll	17.28	18.27	26.59	0.0220

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1H 0323+342	nlsy1	51.17	34.18	10.09	0.0221
NVSS J131921+775823	bcu II	199.84	77.97	16.99	0.0224
RX J1351.3+1115	bll	207.84	11.25	27.93	0.0228
RGB J1808+468	bll	272.00	46.83	29.19	0.0252
RX J1149.5+2439	bll	177.38	24.66	33.52	0.0266
TXS 2157+102	bll	330.03	10.50	47.04	0.0271
S5 1357+76	fsrq	209.48	76.72	35.44	0.0274
S5 1803+784	bll	270.19	78.47	27.73	0.0287
GB6 J1439+4958	bll	219.95	49.97	32.32	0.0291
B2 2234+28A	bll	339.09	28.48	25.36	0.0298
GB6 J0929+5013	bll	142.31	50.23	20.09	0.0306
RGB J2054+002	bll	313.74	0.26	26.79	0.0315
1ES 1028+511	bll	157.83	50.89	35.27	0.0333
GB6 J0331+6307	bcu II	52.97	63.14	26.86	0.0335
PKS 2320-035	fsrq	350.88	-3.28	15.57	0.0350
GB6 J0934+3926	bll	143.53	39.44	32.46	0.0352
B2 1811+31	bll	273.40	31.74	35.37	0.0356
GB6 J0937+5008	fsrq	144.30	50.15	27.81	0.0357
TXS 1015+057	fsrq	154.62	5.51	19.30	0.0361
TXS 1614+473	fsrq	243.92	47.19	16.66	0.0374
4C +04.42	fsrq	185.59	4.22	19.22	0.0376
MG2 J110606+2812	fsrq	166.53	28.21	41.99	0.0396
RX J1246.9+4423	bll	191.75	44.39	22.30	0.0396
B2 2308+34	fsrq	347.77	34.42	41.08	0.0408
TXS 2106-030	bll	317.19	-2.84	16.86	0.0410

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
NVSS J125820+612049	bll	194.59	61.35	36.36	0.0425
SBS 0812+578	bll	124.09	57.65	41.15	0.0426
WN B1609.6+8517	bcu II	240.13	85.16	37.14	0.0434
PMN J2227+0037	bll	336.99	0.62	28.56	0.0440
1RXS J234332.5+343957	bll	355.89	34.66	25.27	0.0440
GB6 J0529+0934	bcu II	82.26	9.58	32.69	0.0450
MG2 J131037+2447	bcu III	197.66	24.81	27.71	0.0458
3C 454.3	fsrq	343.49	16.15	37.56	0.0459
GB6 J0929+7304	bcu II	142.43	73.07	25.62	0.0459
ZS 0214+083	bll	34.32	8.62	36.08	0.0485
3C 264	rdg	176.27	19.61	36.47	0.0485
RXS J094620.5+010459	bll	146.58	1.08	19.50	0.0486
NRAO 512	fsrq	250.12	39.78	21.52	0.0490
1H 0323+022	bll	51.56	2.42	43.87	0.0494
GB6 J1027+7428	bcu II	156.85	74.47	31.92	0.0495
RX J0805.4+7534	bll	121.36	75.57	30.25	0.0505
4C +73.07	bcu II	142.25	72.95	22.60	0.0514
B3 1222+438	bll	186.21	43.59	28.13	0.0516
MG1 J120448+0408	fsrq	181.22	4.14	33.72	0.0525
GB6 J0148+5202	bcu III	27.08	52.03	19.63	0.0556
7C 1823+6856	bll	275.89	68.96	21.57	0.0561
MG2 J180948+2910	bll	272.44	29.17	34.89	0.0591
NVSS J022304+682154	bcu II	35.77	68.37	31.49	0.0591
OM 280	bll	177.58	24.30	42.49	0.0626
PKS 1717+177	bll	259.80	17.75	24.64	0.0627

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1RXS J133021.4+444117	bll	202.59	44.69	22.24	0.0650
S4 1726+45	fsrq	261.87	45.51	34.78	0.0664
RX J1702.6+3115	bll	255.66	31.26	25.34	0.0664
SBS 1410+530	bcu I	212.96	52.82	24.04	0.0666
MG1 J154628+1817	bll	236.60	18.29	40.11	0.0668
MG2 J190411+3627	bcu II	286.05	36.45	34.19	0.0713
TXS 2331+073	fsrq	353.55	7.61	27.29	0.0731
PKS 2047+098	bcu II	312.44	10.05	19.18	0.0734
GB6 J1542+6129	bll	235.74	61.50	34.32	0.0736
S5 0159+723	bll	30.89	72.55	30.54	0.0765
TXS 2032+117	fsrq	308.65	11.91	21.51	0.0777
S5 1027+74	bcu I	157.84	74.70	30.89	0.0777
RGB J1426+340	bll	216.53	34.07	45.49	0.0778
B3 0350+465	bcu III	58.63	46.72	22.88	0.0778
RGB J1742+597	bll	265.63	59.75	37.12	0.0781
1RXS J125117.4+103914	bll	192.82	10.65	18.18	0.0782
MG3 J184126+2910	bcu II	280.34	29.16	37.55	0.0787
TXS 1645+635	fsrq	251.49	63.50	36.91	0.0793
B3 1058+413	bcu III	165.35	41.06	44.91	0.0799
87GB 152947.5+574636	bll	232.74	57.61	21.01	0.0802
GB6 J0229+6706	bcu III	37.34	67.11	11.03	0.0806
MG1 J125348+0326	bll	193.45	3.44	27.69	0.0806
GB6 J0342+3858	fsrq	55.57	38.99	33.59	0.0806
PMN J0148+0129	bll	27.14	1.48	18.43	0.0818
87GB 164812.2+524023	bll	252.35	52.59	33.97	0.0827

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
GB6 J0024+0349	fsrq	6.19	3.82	31.06	0.0830
B3 1307+433	bll	197.36	43.08	22.38	0.0839
NVSS J092542+595812	bll	141.43	59.97	17.27	0.0913
TXS 1549+089	bll	238.01	8.85	30.66	0.0915
RX J0202.9-0223	bcu II	30.72	-2.39	11.81	0.0922
PKS 1203+04	ssrq	181.58	4.10	39.00	0.0926
3C 221	rdg	143.78	39.70	28.81	0.0931
RBS 0909	bll	162.86	39.72	30.20	0.0942
NVSS J121500+500216	bll	183.75	50.04	36.18	0.0956
RX J1101.3+4108	bll	165.35	41.15	39.73	0.0962
OX 131	fsrq	320.25	19.02	28.54	0.0970
PKS 0017+200	bll	4.91	20.36	25.43	0.0973
1ES 0647+250	bll	102.69	25.05	33.94	0.0990
TXS 0237+655	bcu II	40.34	65.72	22.76	0.102
NVSS J224753+441317	bll	341.97	44.22	29.52	0.102
OI 280	fsrq	117.72	12.52	22.89	0.102
B2 1348+30B	fsrq	207.72	30.58	27.45	0.102
TXS 1833+137	bcu III	278.90	13.81	23.62	0.105
RX J1027.4+6317	bll	156.85	63.30	31.32	0.105
PKS 2354-021	bll	359.35	-1.87	5.92	0.105

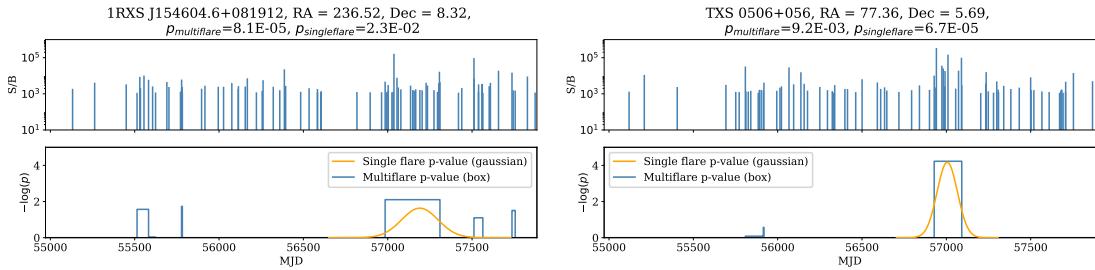


Figure 5.9: Neutrino flare curves for several sources of note in the multi-flare 3LAC blazar catalog analysis. The flare curves generated by the multi-flare algorithm are shown in blue, while the results of a corresponding single-flare analysis (that only fits the largest flare on each source) are shown in orange. Left: 1RXS J154604.6+081912, the most significant 3LAC blazar in the multi-flare analysis, having a pre-trial multi-flare p-value of $p = 8.11 \times 10^{-5}$. Note that for this particular source, the multi-flare significance driven by a set of five moderately significant flares, none of which are particularly significant on their own. The significance of this source in the single-flare analysis was only $p = 0.02$. Right: The flare curve for TXS 0506+056, showing the sizeable 2014 neutrino flare originally observed in [3]. The multi-flare p-value for TXS 0506+056 is only $p = 9.24 \times 10^{-3}$, as other than the 2014 neutrino flare, there are no other particularly significant neutrino flare candidates at this location.

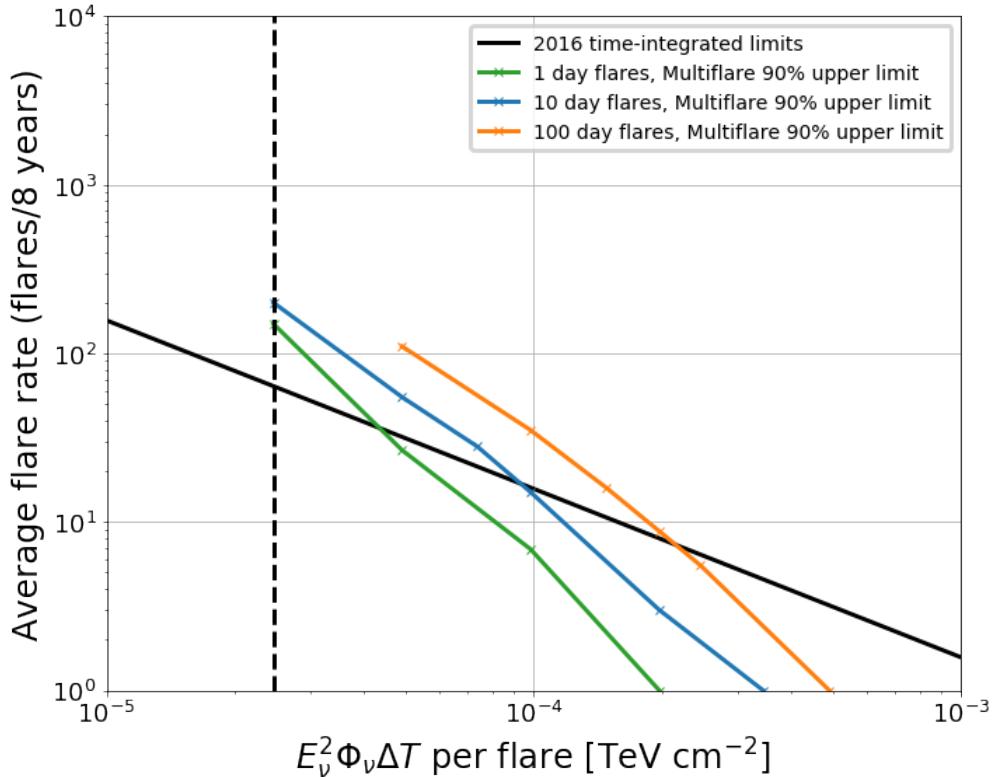


Figure 5.10: The 90% upper limits associated with the non-detection of a multi-flare signal using the 3LAC blazar catalog. The solid black line represents the time integrated limits obtained from [46]. Colored curves represent limits associated with various flare durations, as the multi-flare method imposes more stringent limits in the case that flares are shorter (for a fixed E^2 flux per flare). Note that the parameter space that is realistically accessible to a time-dependent analysis is somewhat compressed: points below the x-axis do not produce more than 1 flare in the data sample, on average, and points to the left of the vertical dashed line have an average flare size of < 1 event.

1121 Chapter 6

1122 An All-Sky Multi-flare Analysis

1123 6.1 Motivation

1124 While the previous section outlined the application of the multi-flare algorithm to a pair
1125 of source catalogs, an additional obvious application of the method is to simply apply the
1126 algorithm across the entire neutrino sky. While catalog searches have the advantage of a
1127 greatly reduced trial factor, they also rely on having a mostly-correct guess of the underlying
1128 source population. By contrast, an all-sky search could potentially reveal sources without
1129 requiring any prior knowledge of what the specific sources could be, and could additionally
1130 even identify astrophysical neutrino sources with no multi-messenger counterpart. Given the
1131 results of a temporally untriggered neutrino flare associated with TXS 0506+056 [3], it is
1132 only natural to ask whether other such flares exist in IceCube’s astrophysical neutrino data.
1133 An all-sky multi-flare analysis would be an appropriate approach of detecting a population
1134 of TXS-like flares, should they exist.

1135 Similar to the FAVA analysis produced by the Fermi collaboration [48], an IceCube
1136 multi-flare skymap provides a description of the temporal variability of every point in the
1137 sky. Even in the absence of a statistically significant population of neutrino flares, these
1138 neutrino “flare curves” may be of use for multi-messenger analyses in the future, similar to
1139 what was done for TXS 0506+056 [3][2].

1140 6.2 Analysis Construction

1141 Conceptually, the construction of an all-sky multi-flare analysis is fairly straightforward: a
 1142 grid of pixels with is defined over the entire sky using the HEALPY software package [49].
 1143 Using a HEALPY grid with a `Nside=256` results in a grid of 786,432 pixels, each with a
 1144 radius of 0.12° . Pixels with declinations $\delta > 85^\circ$ or $\delta < -85^\circ$ are excluded, as the data-
 1145 driven method of estimating background using data scrambled in right ascension does not
 1146 perform well in this region due to the small statistics of declination bands near the poles.
 1147 The multi-flare algorithm is then applied to the central location of each pixel, resulting in a
 1148 map of multi-flare test statistics, each associated with an individual pixel.

1149 The significance of each pixel can then be calculated by comparing the observed multi-
 1150 flare test statistic to a distribution of similar test statistics obtained by applying the above
 1151 procedure to maps of IceCube events that have had their right ascension locations random-
 1152 ized, providing a description of the null hypothesis. Pixels are divided into 40 declination
 1153 bands, and for each declination band a chi-squared distribution is fit to the distribution of
 1154 test statistics in that declination range. A pre-trial local significance for each pixel is calcu-
 1155 lated by comparing a particular pixel test statistic to the chi-squared distribution that was
 1156 fit in the corresponding declination band. Once this has been done for every pixel, a map of
 1157 multi-flare p-values has been obtained, describing the local multi-flare significance of every
 1158 pixel in the sky.

1159 Once a p-value map has been generated, several basic tests can be conducted. The most
 1160 obvious test is to simply check if the most significant pixel is more significant than is expected
 1161 from the background case (a "hotspot" test). A distribution of most significant multi-flare
 1162 p-values is created by applying the procedure above to a set of background maps. The most
 1163 significant pixel in the observed multi-flare map can then be compared to this distribution to
 1164 obtain a final hotspot significance that has been trial-corrected for the all sky trial factor. As
 1165 the event selection is different in the northern and southern skies, this process is conducted
 1166 separately for declinations $\delta > -5^\circ$ and $\delta < -5^\circ$

1167 Populations of multi-flare sources may also be tested for using a binomial test. A set of
 1168 "spatially independent" local hotspots may be obtained by defining a list of all pixels that
 1169 are at least 1° away from a more significant pixel. The binomial test statistic p-value of the
 1170 population test is then defined as (eq. 6.1):

$$p(k) = \sum_{i=k}^{N_{eff}} \binom{N_{eff}}{i} p_k^i (1 - p_k)^{N_{eff}-i} \quad (6.1)$$

1171 Here, $p(k)$ is correlated with the significance of observing k hot spots with a p-value of p_k
 1172 or less, and N_{eff} is the effective number of trials associated with the list of hot spots, chosen
 1173 to produce proper containment of the final binomial p-values (e.g. a final binomial p-value
 1174 of $p = 0.1$ or less should only occur in 10% of background trials). In this case, $N_{eff} = N_{pixels}$
 1175 produces proper containment. Hot spots are ordered by decreasing significance, and k is
 1176 varied to identify the most significant combination. The $p(k)$ associated with the best fit k
 1177 is then compared to a distribution of $p(k)$'s obtained in a similar manner from data scrambled
 1178 in right ascension, resulting in a final post-trial binomial p-value. Like with the study of
 1179 the most significant pixel, this process is conducted separately in the northern and southern
 1180 skies.

1181 Since the multi-flare algorithm fits for every flare candidate at a particular pixel, it is
 1182 trivial to extract the single-flare skymap results from this process as well. The procedure
 1183 is almost identical to that which was outlined above, except instead of using the multi-flare
 1184 test statistic at each pixel, the test statistic of the flare with the highest test statistic at each
 1185 pixel is used. The hotspot and population tests then proceed as normal.

1186 This particular analysis uses the PSTracks003p02 data set.

1187 **6.3 Results**

1188 **6.3.1 Hotspots and Populations Analysis**

1189 The multi-flare pre-trial p-value map can be seen in figure 6.1, with the locations of the
 1190 brightest multi-flare pixels marked. The most significant locations identified by the multi-
 1191 flare algorithm have pre-trial p-values of $p = 9.2 \times 10^{-6}$, located at (RA, Dec)=($145.02^\circ, 36.42^\circ$)
 1192 and $p = 3.5 \times 10^{-7}$, located at (RA, Dec)=($126.21^\circ, -24.81^\circ$). These pre-trial p-values can be
 1193 corrected to account for the all-sky trial factor using the process described above, resulting
 1194 in post-trial p-values of $p = 0.69$ for the northern sky hot spot and $p = 0.06$ for the southern
 1195 sky hot spot.

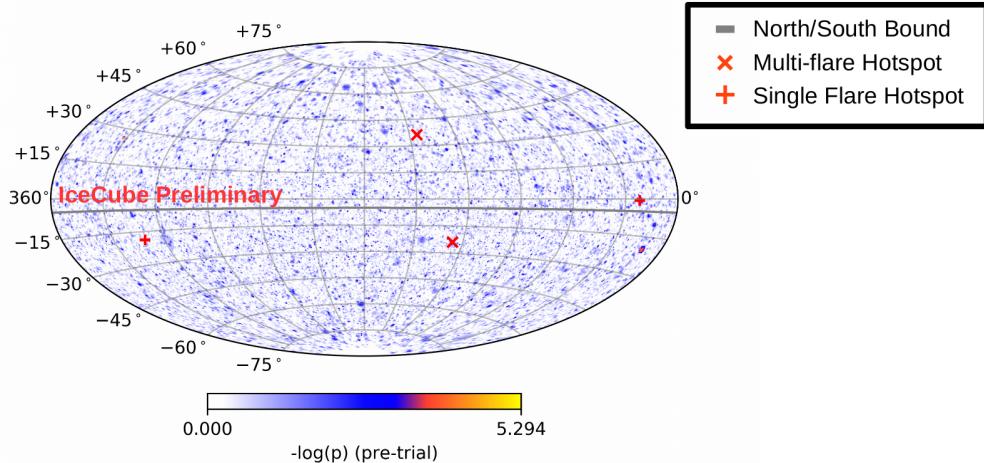


Figure 6.1: The p-value map produced by applying the multi-flare algorithm to the entire neutrino sky between $-85^\circ < \delta < 85^\circ$. The locations of the most significant multi-flare pixels in each hemisphere are shown as red "x's", while the locations of the most significant individual flares in each hemisphere are shown as red "+"s". The gray line denotes the boundary between what is considered the "northern" and "southern" sky by the IceCube data sample that was used.

1196 The flare curves for the multi-flare hotspots can be seen in figure 6.3. Note that the
 1197 multi-flare test statistic is a measure of the activity of a source integrated over the entire
 1198 livetime of the data sample. While no individual flares that were fit at either the northern
 1199 or southern hotspot are particularly significant on their own, the combination of many

1200 moderately significant flares is what makes these pixels the brightest multi-flare pixels in
 1201 their respective hemispheres.

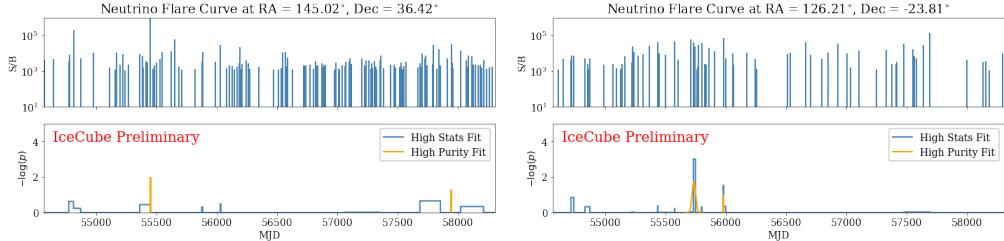


Figure 6.2: The flare curves returned by the multi-flare algorithm for the most significant pixel in the northern sky (left), and southern sky (right). For comparison, a complementary high-purity fit that uses a gaussian hypothesis and more stringent cuts on flare decorrelation is shown in orange. In both the northern and southern sky, the multi-flare significance is fueled not by individual flares with high local significance, but rather by a large number of moderately significant flares.

1202 Since the application of the high-statistics multi-flare analysis involves fitting every pos-
 1203 sible flare in the data, it is trivial to additionally calculate the significance of the largest in-
 1204 dividual flare candidate that was fit in both the northern and southern sky. We find that the
 1205 most significant flare candidate in the northern sky is located at $(\text{RA}, \text{Dec}) = (21.97^\circ, -0.60^\circ)$
 1206 (recall that the "northern sky" refers to declinations between -5° and 85°), and has a pre-
 1207 trial significance of $p = 5.08 \times 10^{-6}$ ($p = 0.82$ post-trial). The most significant flare candidate
 1208 in the southern sky is located at $(\text{RA}, \text{Dec}) = (311.66^\circ, -18.84^\circ)$, and has a pre-trial signifi-
 1209 cance of $p = 6.8 \times 10^{-6}$ ($p = 0.53$ post-trial).

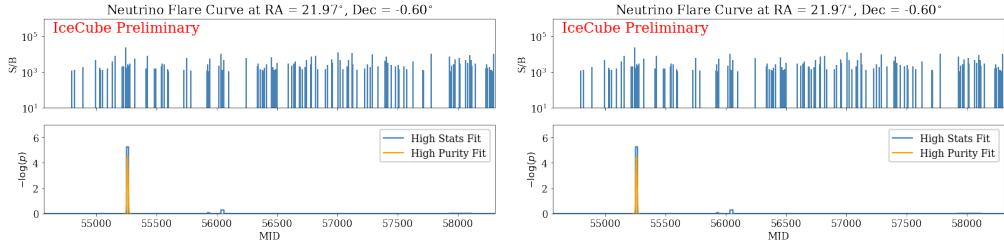


Figure 6.3: The flare curves returned by the multi-flare algorithm for the most significant flares identified in the northern sky (left), and southern sky (right). For comparison, a complementary high-purity fit that uses a gaussian hypothesis and more stringent cuts on flare decorrelation is shown in orange. Since neither of these locations appear to display significant activity beyond the large individual flares, these do not constitute particularly significant locations from a multi-flare perspective.

1210 In addition to examining the most significant pixels that were seen, it is also informative
 1211 to conduct a population analysis on the set of spatially independent ($> 1^\circ$ separation)
 1212 hotspots using the binomial test framework described above. In both the northern and
 1213 southern sky, the binomial test was conducted for the set of spatially independent hotspots,
 1214 defined by both the local multi-flare and single-flare significance. In all cases, the best-fit
 1215 combination of hotspots was $k = 1$, tagging only the most significant pixel, and none of the
 1216 associated binomial p-values were significant (results may be viewed in table 6.1).

Analysis	Search	Hemisphere	Pre-trial p-value	Post-trial p-value
Multi-flare	Hottest spot	North	9.2×10^{-6}	0.69
		South	3.5×10^{-7}	0.06
	Population test	North	0.98	0.98
		South	0.12	0.12
Single Flare	Hottest spot	North	5.08×10^{-6}	0.82
		South	6.70×10^{-6}	0.53
	Population test	North	0.88	0.88
		South	0.91	0.91

Table 6.1: Summary of the tests that were performed on the unblinded multi-flare skymap. Both hotspot and population analyses were conducted using both the local multi-flare significance (summing contributions from all flare candidates at a particular location) and the local single-flare significance (taking only the largest flare fit at each location). There were no significant transient neutrino sources identified by any of the tests conducted.

1217 The results of the population analysis are unsurprising, given the distribution of per-
 1218 pixel local p-values observed from the data. In the presence of a significant population of
 1219 flaring neutrino sources, some deformation of the local p-value distribution should be visible,
 1220 however as can be seen in figure 6.5, the observed distribution is entirely consistent with
 1221 what is expected from background maps.

1222 Similar to the p-value distributions, the distributions of fitted flare parameters are also
 1223 consistent with the background expectation, as can be seen in figure 6.6. There does not
 1224 appear to be a significant sub-population of transient sources with a particular flare duration,
 1225 spectral index, flare rate, or flare flux.

1226 The observed diffuse astrophysical neutrino flux [10] places restrictions on how bright

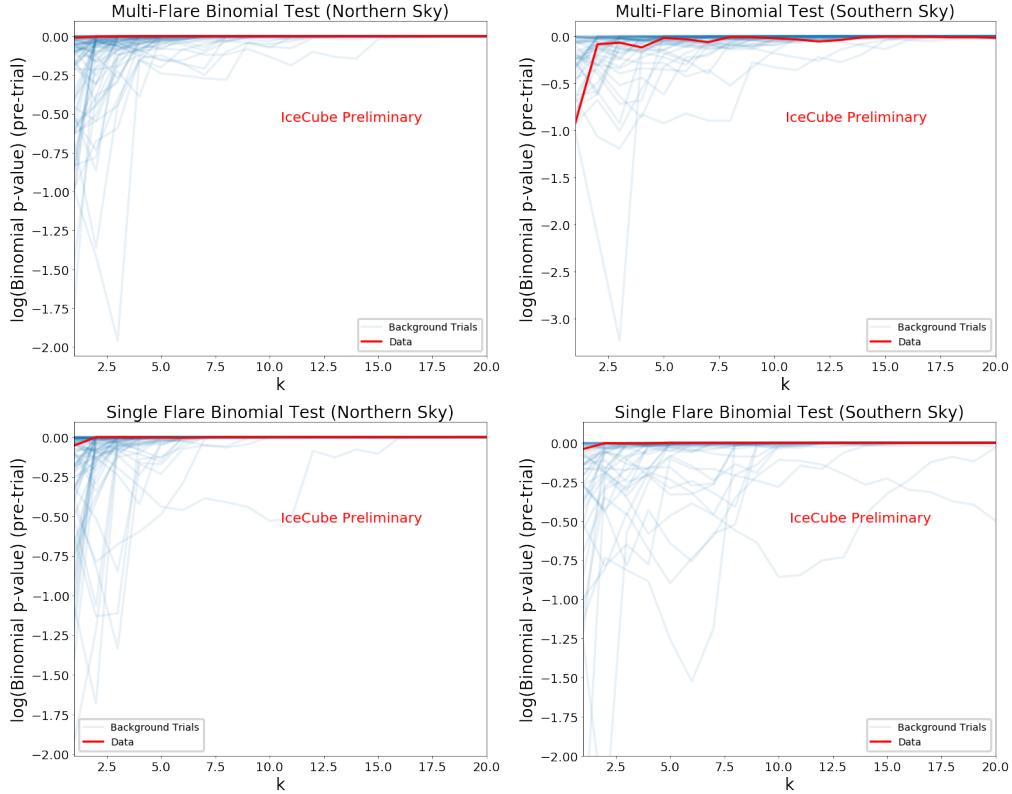


Figure 6.4: The local significance of the binomial test for various values of k , the fitted number of hotspots, for both single and multi-flare binomial tests conducted in both the northern and southern hemispheres. Unblinded data is shown in red, while results obtained from background maps are shown in blue. In all cases using unblinded data, the observed binomial test curve has a global minimum at $k = 1$, with an associated p-value that is consistent with the background expectation.

and numerous neutrino flares may be. The non-detection of a population of transient astrophysical neutrino sources can be used to further constrain this space. These limits can be seen in figure 6.7, which shows 90% upper limits on source flare rate and burst energy in neutrinos, as well as limits that can be placed on the per-source flare rate. Note that these limits are a statistical statement about the data itself: If the "true" parameters describing a population of transient neutrino sources lies above these lines, the data would have had a larger population of significant flares than what was observed.

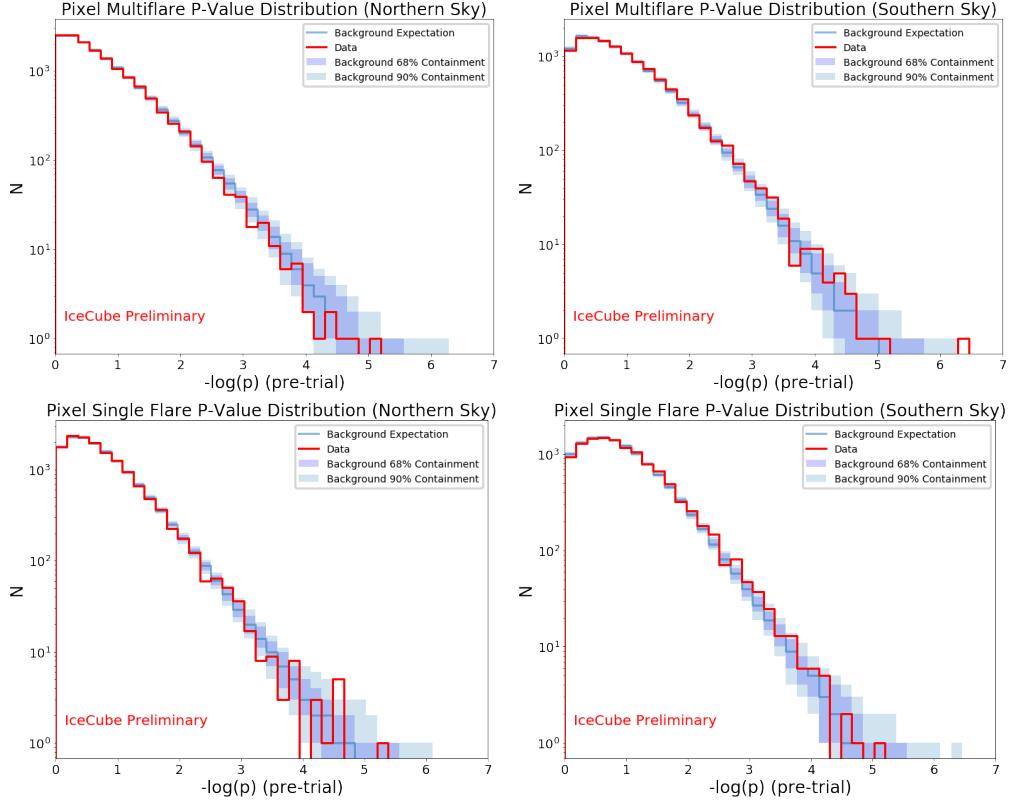


Figure 6.5: The local p-value distributions for the local multi-flare pixel significance (top) and the local single-flare pixel significance (bottom), for both data (red) and background maps (blue bands). In all cases, the observed distribution is consistent with the background expectation.

1234 6.3.2 Locations of External Interest

1235 Though this analysis was designed to be spatially untriggered (scanning over the entire sky,
 1236 rather than the locations of a select few sources), the neutrino "flare curves" produced are a
 1237 potentially powerful tool for exploring the historical behavior of source candidates identified
 1238 though other methods. The skymap produced here is similar to the TXS 0506+056 followup
 1239 analysis [3], except that this analysis has now been performed at almost every location in
 1240 the sky. This allows for easy followup of external triggers, as identifying potential transient
 1241 neutrino emission associated with a source candidate is as easy as examining the flare curve
 1242 at that location in the sky.

1243 To demonstrate the use of the all-sky flare curve map produced here, we examine the
 1244 flare curves of several time-integrated source candidates identified by [4] in order to explore

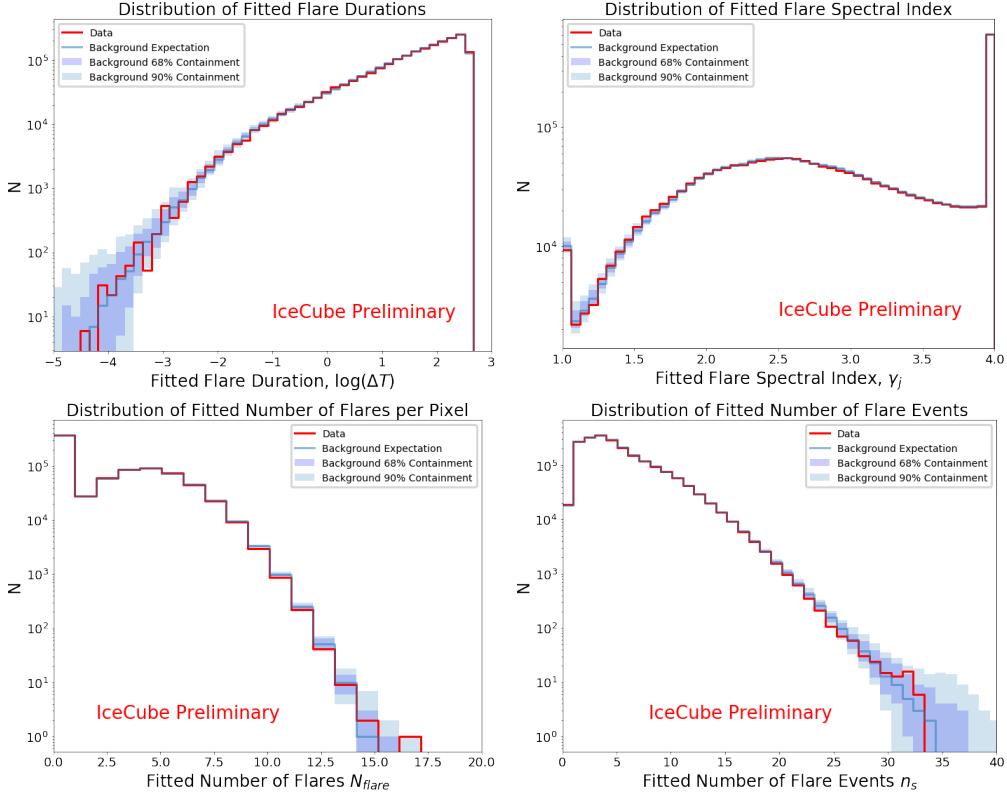


Figure 6.6: Distributions of fitted flare parameters: flare duration (top, left), spectral index (top, right), number of flares fit at a particular candidate location

1245 the potential of temporally clustered neutrino emission from these locations.

1246 6.3.2.1 NGC 1068

1247 The most significant location identified by the all-sky time-integrated analysis is spatially
 1248 coincident with the Seyfert II galaxy NGC 1068. This source was also tested as part of
 1249 a catalog of 110 candidate neutrino emitters, and was the brightest object in the catalog,
 1250 with an associated post-trial significance of 2.9σ [4]. These significances all arise from time-
 1251 integrated tests that do not take into account the arrival time of the contributing neutrino
 1252 events. It is then potentially interesting to explore the temporal structure of this excess with
 1253 the flare curves that were produced as part of the multi-flare skymap.

1254 The flare curve generated at the location of NGC 1068 can be seen in figure 6.8. Though
 1255 this location is significant under the time-integrated analysis, the multi-flare p-value is only
 1256 $p = 0.016$ (pre-trial), with no individual flare candidate having a local significance greater

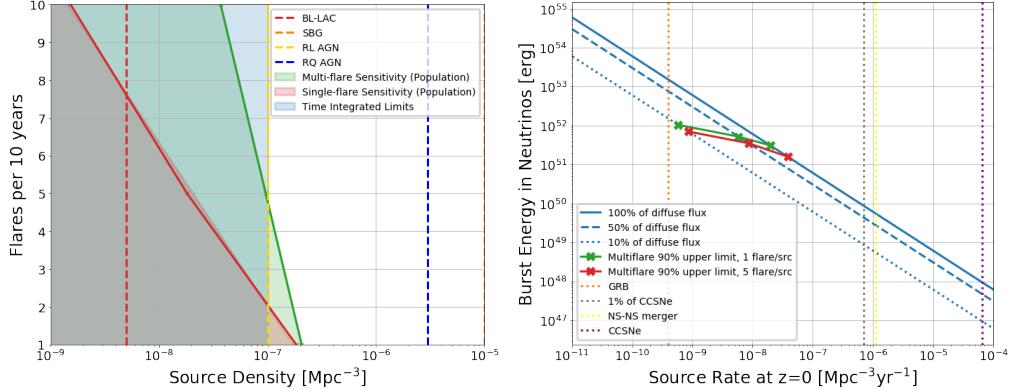


Figure 6.7: Left: 90% upper limits on neutrino source density and per-source flare rate obtained from the all-sky multi-flare analysis. In this plot, transient sources are assumed to compose 100% of the astrophysical neutrino flux, with the FIRESONG simulation package [50] being used to distribute the flux among sources according to the Hopkins and Beacom (2006) source evolution model [51]. Flares are simulated to have a duration of 20 days, and a spectral index of $\gamma = 2.28$. All flares on the same source are assumed to have the same intrinsic intensity, equal to the time integrated source flux divided by the number of flares. Right: The 90% upper limits on the source rate (source density \times flare rate) as a function of neutrino burst energy. Blue lines correspond to combinations of source rate and neutrino burst energy that reproduce specific fractions of the diffuse astrophysical neutrino flux. In both plots, the parameters associated with several candidate source populations are shown for comparison..

1257 than $p = 0.072$. Note however, that these results are not in tension with the time-integrated
 1258 significance, as the time-integrated analysis tests only for an excess of events over the entire
 1259 livetime of the sample. By contrast, a source with high multi-flare significance requires that
 1260 events not only be clustered spatially, but temporally as well. As such, sources with high
 1261 time-integrated significance do not necessarily have a correspondingly high multi-flare signif-
 1262 icance, particularly if there is not significant temporal clustering of events that contribute
 1263 to the time-integrated result.

1264 Nonetheless, we have here provided a description of the historical behavior of the neutrino
 1265 emission associated with NGC 1068. Despite the time-integrated excess observed in previous
 1266 analyses, there does not appear to be significant temporal clustering of events at this location.

1267 **6.3.2.2 Other Time-Integrated Candidates: PKS 1424+240 and GB6**

1268 **J1542+6129**

1269 In addition to identifying NGC 1068 as a potential source candidate, the population analysis
 1270 component of the time integrated analysis [4] also identified an excess of 3.3σ associated
 1271 with the combination of the sources NGC 1068, TXS 0506+056, PKS 1424+240, and GB6
 1272 J1542+6129. Figure 6.9 shows the flare curves that were obtained at the locations of PKS
 1273 1424+240 and GB6 J1542+6129.

1274 Similar to NGC 1068, PKS 1424+240 does not display significant temporal clustering
 1275 of neutrino events, and has a local multi-flare p-value of $p = 0.085$. By contrast, GB6
 1276 J1542+612 seems to have a somewhat significant flare candidate beginning at MJD=57564.878
 1277 and ending at MJD=57944.512. The pre-trial significance of this flare candidate alone is
 1278 $p = 0.00173$ (2.9σ). Combining all the flares candidates at the location of GB6 J1542+612
 1279 using the multi-flare test statistics results in a local multi-flare significance of $p = 0.0098$
 1280 (2.3σ). While this is not particularly significant (especially when the all-sky trial factor is
 1281 considered), it does inform us that the period between MJD=57564.878 and MJD=57944.512
 1282 was likely a large contributor to the time-integrated significance for this particular source. If
 1283 there are further multi-messenger signals from this location in the future, the neutrino flare
 1284 candidates at this location may become of interest.

1285 **6.3.2.3 Follow-up of the ANTARES Source Candidate J0242+1101**

1286 *Author note: Due to the timing of the request for this follow-up, this section was added 1
 1287 week before the submission of this thesis. Results are very preliminary, but I've included it
 1288 because I think it's a nice example of how these multi-flare fits can be used to cross check
 1289 results from other neutrino telescopes*

1290 The ANTARES telescope [52] is a water cherenkov telescope, similar to IceCube, but
 1291 located in the Mediterranean sea instead of the at the South Pole. It is significantly smaller
 1292 than the IceCube detector, consisting of only 12 lines of 25 optical modules each. The lo-

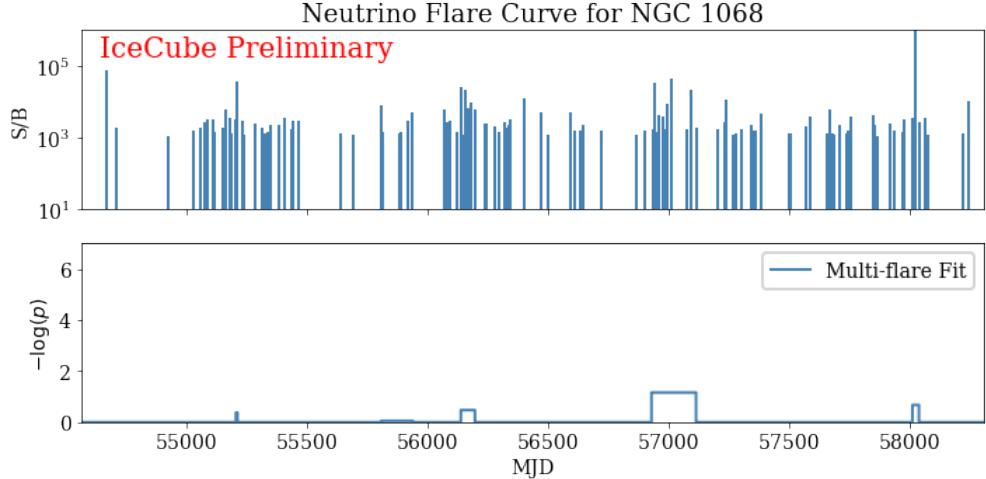


Figure 6.8: The flare curve at the location of NGC 1068, a source candidate that was identified as potentially interesting in the time-integrated analysis [4]. While this is the brightest spot in the time-integrated analysis, the multi-flare analysis does not reveal any significant temporal clustering of events at this location, resulting in a local multi-flare significance of $p = 0.016$

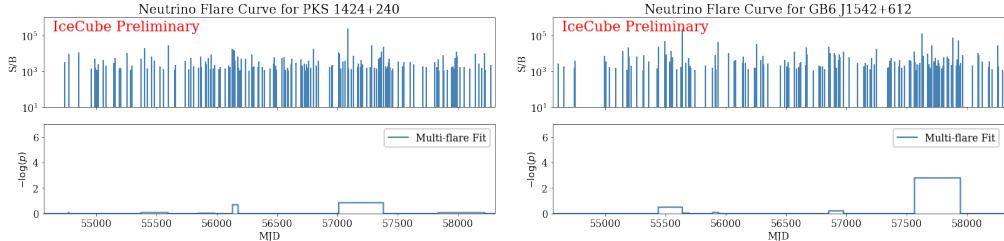


Figure 6.9: The flare curves at the locations of PKS 1424+240 and GB6 J1542+612. While PKS 1424+240 does not display any significant temporal clustering (having a local multi-flare p-value of $p = 0.085$), GB6 J1542+612 has a potentially interesting flare candidate beginning at MJD=57564.878 and ending at MJD=57944.512. This flare candidate has a local flare significance of $p = 0.00173$ (2.9σ), contributing to the overall pre-trial multi-flare significance for GB6 J1542+612 of $p = 0.0098$ (2.3σ).

1293 cation of ANTARES in the northern hemisphere makes it ideal for studying astrophysical
 1294 neutrino events from the southern sky, as the Earth absorbs the atmospheric muon back-
 1295 ground in this region, similar to how IceCube is more sensitive to events from the northern
 1296 sky.

1297 Though ANTARES does not fit for a significant astrophysical diffuse flux [53], the col-
 1298 laboration has still performed searches for point sources of astrophysical neutrinos. Notably,
 1299 at the most recent International Cosmic Ray Conference (2021)¹, the ANTARES collabora-

1300 tion presented a temporally untriggered search for correlations between radio-bright blazars
 1301 and neutrino flare candidates in ANTARES data [54]. The ANTARES collaboration noted
 1302 that one of their most significant candidates, J0242+1101, overlapped with the detection of
 1303 a high-energy IceCube event, and contacted the IceCube collaboration for follow-up.

1304 It should be noted that the ANTARES results on their own are not statistically signifi-
 1305 cant: the most significant source candidate has a post-trial p-value of only 40%, and only 7
 1306 source candidates have pre-trial significances greater than 3σ (equivalent to the background
 1307 expectation when testing 2774 sources). Nonetheless, for the sake of completeness, the Ice-
 1308 Cube collaboration assembled a followup analysis, of which the multi-flare fits shown below
 1309 were a part.

1310 Since multi-flare fits were created for the entire sky as part of the multi-flare skymap
 1311 analysis described above, follow-up is potentially as easy as pulling the appropriate flare curve
 1312 from the stored skymap file containing flare fits at every location. However, for the sake of
 1313 completeness, in this particular case the multi-flare analysis was re-run at the exact location
 1314 of the ANTARES source candidate (J0242+1101, RA=40.62°, Dec=11.01°), in addition
 1315 to modifying the flare candidate seeding procedure to ensure that flare candidates with
 1316 durations up to $\Delta t = 1000$ days were tested.

1317 The results of the multi-flare followup for J0242+1101 can be seen in figure 6.10. No
 1318 significant flares were fit, with the largest flare candidate in IceCube data having a local
 1319 significance of only $p = 0.31$. There was also not an excess of flares observed, with the pre-
 1320 trial multi-flare significance of this location being $p = 0.52$. The 50 TeV IceCube event that
 1321 was noted by ANTARES in [54] does not appear to be significant in the timing distribution
 1322 of Icecube events, as the flare search algorithm was unsuccesful in pairing that event up with
 1323 other IceCube events to fit a significant flare.

¹<https://icrc2021.desy.de/>

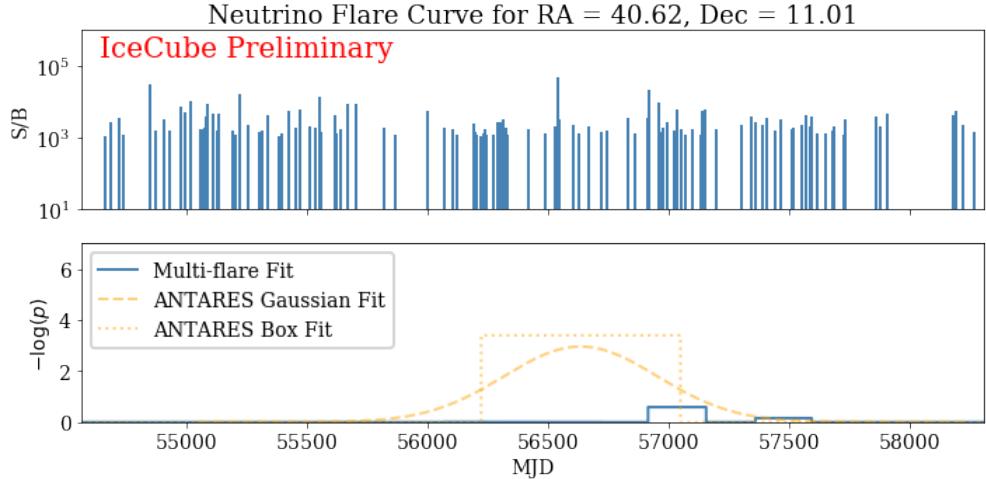


Figure 6.10: The flare curve at the location of J0242+1101, a source candidate that was identified as potentially interesting by the ANTARES collaboration. The results of the untriggered flare fits performed by the ANTARES collaboration are shown in orange, while the blue shows the IceCube data (top-panel), and corresponding multi-flare fit (bottom panel). There does not appear to be a flare as strong as the reported ANTARES flare in IceCube data, with the IceCube multi-flare followup presented here returning a p-value of only $p = 0.52$.

1324 6.3.2.4 TXS 0506+056

1325 In addition to being part of the excess that was identified in the all-sky time-integrated
 1326 analysis, TXS 0506+056 is notable for both the multi-messenger coincidence sparked by the
 1327 high energy IceCube alert IC-170922A [2], as well as the archival analysis that revealed a
 1328 3.5σ neutrino flare that occurred in 2014, prior to the 2017 high energy alert event [3].

1329 The all-sky multi-flare analysis presented in this work includes the location of TXS
 1330 0506+056, and we can consequently generate a flare curve for this source candidate. The
 1331 flare curve observed in the all-sky multi-flare analysis can be seen in figure 6.11. TXS
 1332 0506+056 has a local multi-flare significance of $p = 3.37 \times 10^{-4}$ (3.4σ), with the main
 1333 contributor being the 2014 neutrino flare candidate, beginning on MJD=56927.86 and ending
 1334 on MJD=57072.99.

1335 Readers who have been paying close attention to the content of this thesis (a group which
 1336 at this point probably includes my thesis committee and like, no one else) may notice that
 1337 the flare curve shown in figure 6.11 seems to be in disagreement with the published result

1338 with regards to the 2014 TXS flare . While the result shown in [3] reports a significance for
 1339 the 2014 flare candidate of 3.5σ , the all-sky multi-flare analysis presented here observes a
 1340 local significance for this flare of only $p = 0.0054$ (2.5σ).

1341 The difference in significance between the two results can be explained by the different
 1342 event selections that were used in each analysis. Descriptions of these event samples may
 1343 be found in section 3 of this thesis. While [2] used the PointSourceTracks v002p03 event
 1344 sample, the all-sky multi-flare analysis presented here uses the PointSourceTracks v003p02
 1345 event sample. The most relevant difference between the two event samples in this case are
 1346 the pre-cuts: PointSourceTracks v003p02 introduces a cut prior to the event selection BDT
 1347 that requires events to have a reconstructed track length greater than 200 meters. This
 1348 pre-cut does not exist in PointSourceTracks v002p03.

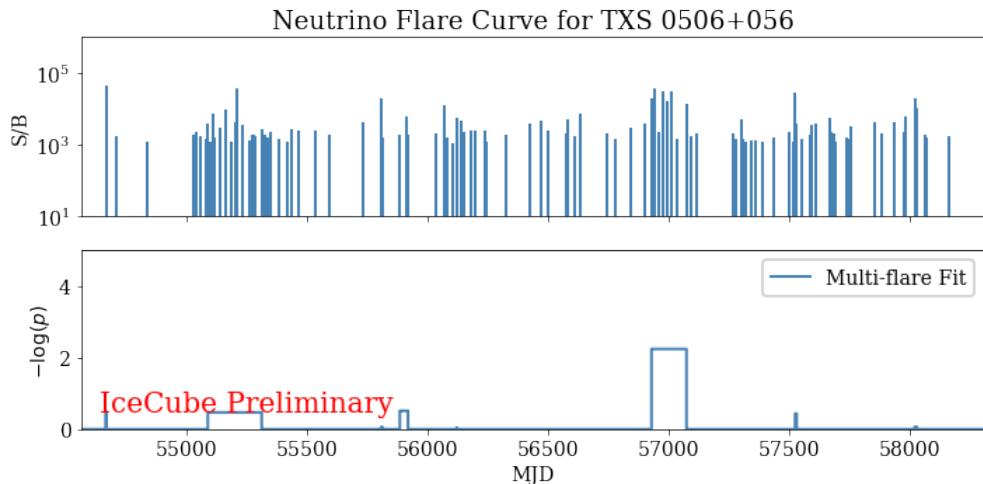


Figure 6.11: The flare curve obtained from the all-sky multi-flare analysis for TXS 0506+056. The pre-trial multi-flare significance of this location is $p = 3.37 \times 10^{-4}$ (3.4σ). Interestingly, the 2014 flare candidate is not seen at the original significance of 3.5σ that was reported in [2]. While the 2014 flare candidate is present, this analysis obtains an associated local significance of only $p = 0.0054$ (2.5σ).

1349 As a result of the track length pre-cut that was introduced in PointSourceTracks v003p02,
 1350 two events (referred to by their event IDs: 40914587 and 56262988) that were major contrib-
 1351 utors to the 2014 TXS 0506+056 neutrino flare candidate were removed. These events have
 1352 a cascade-like topology, though in PointSourceTracks v002p03, they are reconstructed as if
 1353 they were tracks, leading to potentially inaccurate descriptions of the event direction and

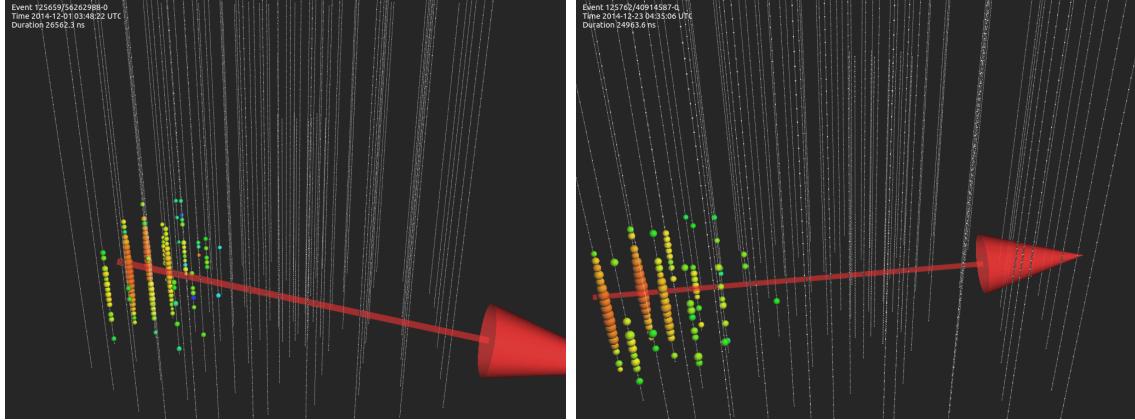


Figure 6.12: The two cascade-like events (event IDs: 40914587 and 56262988) that contributed to the 2014 TXS 0506+056 neutrino flare candidate in PointSourceTracks v002p03, but are not present in PointSourceTracks v003p02. The red arrows show the directional reconstruction associated with the track reconstruction that was used, however given the cascade topology of these events, this reconstruction is likely to be inaccurate.

1354 energy. Since these events are cascade-like, they have a short reconstructed track length,
 1355 and as such these events do not exist in PointSourceTracks v003p02. It is additionally worth
 1356 noting that even as cascades, these events are poorly localized, as they are only partially
 1357 contained in the detector. Attempts at reconstructing these events as cascades using the
 1358 methods described in [34] failed to produce any kind of useful directional estimate.

1359 Manually removing the two cascade events from the PointSourceTracks v002p03 data
 1360 sample and recomputing the significance of the 2014 TXS 0506+056 neutrino flare results in
 1361 a drop in significance comparable to what was seen when using PointSourceTracks v003p02,
 1362 as can be seen in table 6.2. The drop in significance cannot be otherwise adequately explained
 1363 by changes to the angular reconstruction or other differences between the two versions of
 1364 the data sample. The time-integrated sensitivity of the two samples is comparable, and
 1365 favors PointSourceTracks v3 at the declination of TXS 0506+056. As such, the drop in
 1366 significance of the 2014 TXS 0506+056 flare candidate is unlikely to have been caused by
 1367 PointSourceTracks v003p02 being an overall less sensitive sample.

1368 It should be noted that the result published in [3] is not incorrect, despite using an
 1369 older data sample that contains unforeseen background events. Since the methods used for
 1370 the original untriggered flare analysis use a data-driven background estimation, the cascade

Untriggered Flare Cross-check Results

Sample	p (pre-trial)	T_{start}	T_{stop}	n_s	γ
PSTracks v2 [3]	7.0e-5	56937.81	57096.22	14.39	2.20
PSTracks v2 w/o cascades	1.17e-3	56937.81	57112.65	12.22	2.26
PSTracks v3 (multi-flare skymap)	5.4e-3	56927.86	57072.99	11.87	2.22

Table 6.2: The results of repeating the untriggered flare analysis preformed in [3], but using PSTracks v3 in place of PSTracks v2, the dataset that was originally used. The apparent drop in significance when using PSTracks v3 can be explained by cascade-like events present in v2 that have been removed from v3.

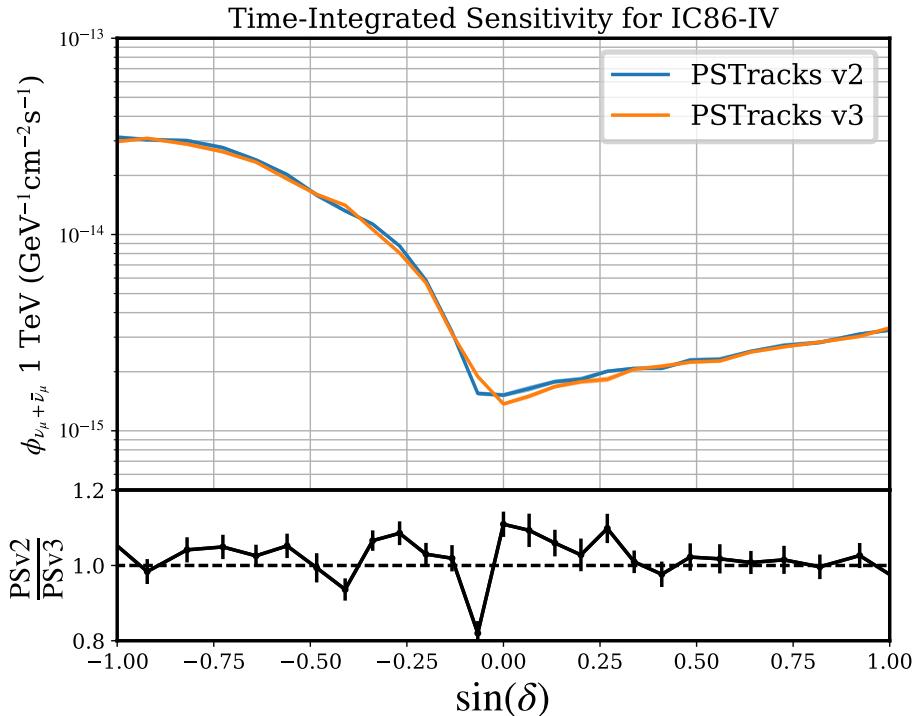


Figure 6.13: A comparison of the time integrated sensitivities of PointSourceTracks v002p03 and PointSourceTracks v003p02. At the declination of TXS 0506+056 ($\delta = 5.7^\circ$), PointSourceTracks v003p02 has a slightly better sensitivity.

1371 background in PointSourceTracks v002p03 is accounted for in the original 3.5σ significance
 1372 that was reported for the 2014 neutrino flare candidate.

1373 Though the cascade background is handled differently in the PointSourceTracks v002p03
 1374 analysis versus the PointSourceTracks v003p02 analysis, the two results are not inconsistent
 1375 with one another. Figure 6.14 shows the results of comparing the results of simulated
 1376 flares comparable to the 2014 TXS flare injected into both PointSourceTracks v002p03 and

1377 PointsourceTracks v003p02. As the background test statistic distributions for both samples
 1378 are similar, comparing the test statistic values obtained using each sample is a good proxy for
 1379 comparing the flare local significance. For this test, trials of simulated signal at the location
 1380 of TXS 0506+056 were injected into background maps generated using PointSourceTracks
 1381 v002p03. The injected signal was chosen to replicate the best fit parameters in the original
 1382 PointSourceTracks v002p03 untriggered flare analysis [3]: 13 events with a spectral index
 1383 of $\gamma = 2.2$, injected over a time window of length $\Delta t = 158$ days. A test statistic for each
 1384 of these PointSourceTracks v002p03 signal trials was then calculated. The same simulated
 1385 signal events were then injected into background trials generated with PointsourceTracks
 1386 v003p02, but any events that do not pass the PointSourceTracks v003p02 event selection
 1387 were removed. A new PointSourceTracks v003p02 test statistic is then calculated for each
 1388 of these trials with injected signal.

1389 Figure 6.14 shows the results of this crosscheck, showing the distribution of test statistics
 1390 differences between the v2 and v3 test statistics. While on average the test statistic associ-
 1391 ated with a signal of this type is higher in PointSourceTracks v003p02, the distribution is
 1392 rather wide, and it is not uncommon for the PointSourceTracks v003p02 result to be lower
 1393 significance than the PointSourceTracks v002p03 result. A drop in significance comparable
 1394 or greater than what was seen in the observed v2 and v3 results ($TS_{v3} - TS_{v2} = 11.84$)
 1395 occurs in 8% of the simulated trials.

1396 While there appears to be a drop in significance for the 2014 TXS 0506+056 untrig-
 1397 gered neutrino flare when using to the newer PointSourceTracks v003p02 data sample, it
 1398 is important to realize that the updated v3 result is not inconsistent with the previous v2
 1399 result. Additionally, the 2014 flare composes only part of the ensemble of results that make
 1400 TXS 0506+056 an interesting source candidate, as it is the combination of the 2014 untrig-
 1401 gered neutrino flare with the 2017 high energy alert event and multi-messenger observations
 1402 that suggest TXS 0506+056 as a source of astrophysical neutrinos. The significance of the
 1403 2014 TXS 0506+056 flare will likely continue to be monitored as the IceCube collaboration
 1404 improves its data samples and reconstruction techniques, hopefully converging on a more

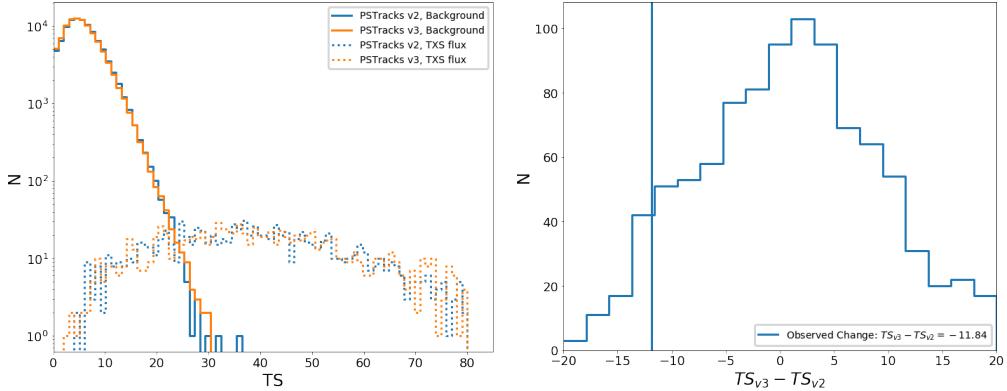


Figure 6.14: Simulated trials comparing the significance of a flare comparable to the 2014 TXS 0506+056 flare in both PointSourceTracks v002p03 and PointSourceTracks v003p02. Left: the test statistic distributions for both datasets in the background (solid) and injected signal (dotted) cases. The PointSourceTracks v003p02 signal is identical to the PointSourceTracks v002p03 signal, except with events that do not pass the v3 selection removed. Right: A comparison of the v2 and v3 test statistics for each simulated signal trial, with the observed test statistic difference plotted as a vertical line. A drop in significance at least as large as what was seen in observed data occurs in 8% of simulated trials.

1405 precise measurement to the flare significance.

1406

Chapter 7

1407

Outlook

1408 In this work, we have introduced a novel method for fitting ensembles of neutrino flare
1409 candidates at a particular source location. This method was applied to two catalogs of
1410 source candidates motivated by TXS 0506+056: a catalog of northern-sky 3LAC blazars, and
1411 a “self-triggered” catalog of high energy IceCube events. Though neither catalog displayed
1412 a significant excess of neutrino flares, this allowed us to further constrain potential transient
1413 neutrino emission from blazars.

1414 The multi-flare algorithm introduced here was also applied on an all-sky scale. No signif-
1415 icant population of neutrino flares was observed, though flare curves were produced at every
1416 location in the sky, providing a potentially useful tool for future multi-messenger searches
1417 wishing to incorporate information on the historical neutrino emission from source candidate
1418 locations, similar to what was done in [3].

1419 The non-detection of a significant population of transient neutrino sources is somewhat
1420 interesting from the perspective of constraining astrophysical neutrino source populations.
1421 Whatever the sources of astrophysical neutrinos may be, they cannot produce neutrino flares
1422 that are so bright or numerous that the data becomes inconsistent with the background
1423 expectation on an all-sky scale. This places limits on the source density, flare rate, and
1424 neutrino burst energy that are not associated with any particular source class, serving as a
1425 “universal limit” of sorts on the behavior of potential astrophysical neutrino transients.

1426 It should be noted, however, that a non-detection of neutrino sources on an all-sky scale
1427 is not entirely unexpected. The all-sky trial factor is quite large, and the most significant
1428 neutrino source candidates to date (TXS 0506+056 and NGC 1068) were both originally
1429 identified in spatially triggered analyses, where the candidate location was already identified
1430 as potentially interesting by other astrophysical messengers. Future, spatially triggered
1431 searches may be sensitive to individual sources that have been obscured by the all-sky trial
1432 factor. Ideally, the neutrino flare curves introduced in this work would aid with this, as the
1433 historical neutrino information provides an additional piece to the multi-messenger puzzle.

1434 Though the first applications of the multi-flare algorithm did not yield any significant dis-
1435 covery of astrophysical neutrino sources, the future of neutrino astronomy is still quite bright.
1436 With two interesting source candidates already identified, the answer to the question of the
1437 source of astrophysical neutrinos (and consequently, the source of high energy cosmic rays) is
1438 potentially beginning to become more clear. The upcoming years bring improvements to the
1439 IceCube detector calibration and event reconstruction, providing a more accurate descrip-
1440 tion of existing data, as well as the construction of IceCube Gen2 which will boast increased
1441 effective area at high energies, thereby reducing the atmospheric neutrino background. Ad-
1442 ditionally, upcoming radio-based neutrino experiments such as RNO-G, BEACON, IceCube
1443 Gen2-Radio, PUEO, and POEMMA will provide a view into the ultra-high energy neutrino
1444 regime, beyond what the current iteration of IceCube is realistically capable of measuring.
1445 As the identification of candidate neutrino emission from TXS 0506+056 was triggered by a
1446 single high energy neutrino event, the expansion of neutrino detectors into the high energy
1447 regime could lead the way to identifying future neutrino sources.

1448 **Chapter 8**

1449 **Appendix: Statistics Background**

1450 Much of the material presented in this thesis requires an understanding of a few key areas
1451 of statistics. This material is typically covered in almost any introductory level statistics
1452 course, but is generally not part of the standard undergraduate physics curriculum. For
1453 this reason, I've collected some of the key concepts in this section, to potentially serve as a
1454 starting point for people who may not already be familiar with the statistical underpinnings
1455 of recent source searches in neutrino astronomy. An interactive ipython notebook showing
1456 examples of these concepts can also be found as part of the materials for the 2019-2021
1457 IceCube bootcamps¹.

1458 **8.1 Estimators**

1459 An estimator is a procedure applied to the data sample which gives a numerical value for a
1460 property of the parent population, or, as appropriate, a property or parameter of the parent
1461 distribution function.

1462 This is a general definition: The sample of interest may have been drawn from a large
1463 parent population, for which some property is to be inferred. Alternatively, it may have
1464 been generated from a distribution function, arising from a basic law being investigated.

¹<https://events.icecube.wisc.edu/event/105/contributions/1054/>

1465 This distribution function also has parameters and properties, which one may also wish to
 1466 calculate (e.g. σ for a 1 dimensional gaussian distribution).

1467 Estimators are generally not classified as "right" or "wrong", rather we tend to classify
 1468 estimators as "good" or "bad" based on their properties. Some properties that are often
 1469 relevant when describing estimators:

- 1470 • An estimator is *consistent* if it tends to the true value as the number of data values
 1471 tends to infinity.
- 1472 • An estimator is *unbiased* if its expectation value is equal to the true value.
- 1473 • An estimator is *efficient* if its variance is small.

1474 Estimators can be any number of the above, but good estimators should fulfill all three
 1475 descriptions [55].

1476 8.2 Likelihoods

1477 One class of estimators that is commonly used in neutrino astronomy are *likelihood-based*
 1478 *estimators*. For a set of data, $x_1, x_2, x_3 \dots x_N$ produced from some underlying distribution
 1479 with parameter a , the probability of obtaining that particular data set is the product of
 1480 the individual probabilities of obtaining each of the x_i . This product is referred to as the
 1481 *likelihood* (eq. 8.1 and 8.2):

$$L(x_1, x_2, x_3 \dots x_N, a) = P(x_1; a)P(x_2; a) \dots P(x_N; a) \quad (8.1)$$

$$L(x_1, x_2, x_3 \dots x_N, a) = \prod_{i=1}^N P(x_i; a) \quad (8.2)$$

1482 The likelihood is typically used to fit the model parameter a , given some observed data
 1483 $x_1, x_2, x_3 \dots x_N$ by finding the value of a which maximizes the likelihood function. Note that
 1484 a can be a vector of values in the case that we have a model with multiple free parameters.

1485 For computational reasons, the process of maximizing the likelihood is oftentimes done
 1486 by minimizing the negative log-likelihood instead. Taking the logarithm of the likelihood
 1487 function turns the product into a sum, making the derivative more intuitive and easier to
 1488 work with numerically, and the switch from maximization of the likelihood to minimizing
 1489 the negative likelihood is purely convention.

1490 8.3 Confidence Intervals

1491 Suppose we want to know the value of a parameter X and have estimated it from the data,
 1492 giving result x . We know the resolution of our underlying distribution, and thus the variance,
 1493 $V(x)$ and the standard deviation σ . Our goal is then to create a "confidence interval": An
 1494 interval within which there is a C % chance that a single observation will lie within the
 1495 interval, where C is our "confidence level". Formally, the confidence interval (x_-, x_+) for a
 1496 given confidence level C obeys the requirement that (eq. 8.3):

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x)dx = C \quad (8.3)$$

1497 There are a few possible ways to construct this interval:

1498 1. The **symmetric interval**: x_- and x_+ are equidistant from the mean, μ , i.e $x_+ - \mu =$
 1499 $\mu - x_-$

1500 2. The **central interval**: The probability above and below the interval are equal: $\int_{-\infty}^{x_-} P(x)dx =$
 1501 $\int_{x_+}^{\infty} P(x)dx = (1 - C)/2$

1502 Common values for C are 68% (1σ), 90% (1.64σ), and 99% (2.58σ). There is a trade
 1503 off between a narrow interval and low confidence. You can say with great confidence that a
 1504 value will lie within a very wide interval, but if one wants to tie it down more precisely the
 1505 confidence lessens.

1506 8.4 Hypothesis Testing

1507 Many problems in science can be reduced to attempting to distinguish between 2 distinct
 1508 possibilities, where the observed data has different properties under each. In many cases,
 1509 we are attempting to distinguish between a "null hypothesis", which is used to refer to a
 1510 "default" model (e.g. the absence of neutrino point sources), and an "alternative hypothesis",
 1511 which represents the addition of some new physics, model parameter, or disagreement with
 1512 the "default" case (e.g the presence of neutrino point sources). Statistical hypothesis testing
 1513 is a framework for comparing these two hypotheses, given some set of observed data.

1514 We can roughly break down the process of hypothesis testing into the following steps:

- 1515 1. State relevant null and alternative hypotheses
- 1516 2. State the test statistic
- 1517 3. Derive the distribution of the test statistic under the null hypothesis
- 1518 4. Select significance level (α) at which the null hypothesis will be rejected
- 1519 5. Compute an observed test statistic from the data
- 1520 6. Calculate a p-value and decide whether to reject the appropriate hypothesis

1521 8.4.1 State relevant null and alternative hypotheses

1522 When constructing an analysis, we should be careful to state the null and alternative hy-
 1523 potheses. Typically, the null hypothesis (H_o) refers to a statement or default position that
 1524 there is no relationship between two phenomena, that there is no signal, or that our cur-
 1525 rent models describe the data well. The alternative (H_a) hypothesis usually describes the
 1526 opposite: that there is a relationship between two phenomena, that there is a signal, or that
 1527 our models to NOT describe the data well. Here are some example pairs of hypotheses one
 1528 might encounter when working with IceCube data:

1529 • H_o : Neutrino event arrival directions are not significantly correlated with the positions
 1530 of blazars identified in Fermi's 3FGL catalog

1531 • H_a : Neutrino event arrival directions are correlated with the positions of blazars iden-
 1532 tified in Fermi's 3FGL catalog

1533 • H_o : A 3-flavor oscillation model describes the IceCube data well

1534 • H_a : IceCube data is inconsistent with a 3-flavor model

1535 It is important to be careful when formulating null and alternative hypotheses so as to
 1536 not overreach with the claims being made should the alternative hypothesis be accepted.

1537 In general, it's best to keep in mind the specifics of how a particular analysis is being
 1538 constructed, and what the property of the data being measured actually is. For example, in
 1539 the first pair of hypotheses above, it would be improper for our alternative hypothesis to be
 1540 "blazars are a source of astrophysical neutrinos" if we were only examining the positions of
 1541 the 3FGL catalog, which covers only a subset of all observed blazars.

1542 **8.4.2 State a test statistic**

1543 A *test statistic* is a quantity derived from the sample, intended to be a numerical summary
 1544 of a dataset that reduces the data to one value that can be used for hypothesis testing. In
 1545 general, the test statistic is selected in such a way as to quantify, within observed data,
 1546 behaviors that would distinguish the null and alternative hypotheses. These are very similar
 1547 to estimators, and we can oftentimes use estimators as test statistics.

1548 An important property of the test statistic is that its sampling distribution under the
 1549 null hypothesis must be calculable either exactly or approximately. There are many types of
 1550 test statistics that you can use, and what the exact form of the test statistic is will usually be
 1551 determined by your analysis. One example of a test statistic is the chi-squared test statistic
 1552 intended to compare counts in an oscillation analysis (eq. 8.4):

$$\chi^2 = \sum_{i \in bins} \frac{(n_i^{\nu+\mu_{atm}} - n_i^{data})^2}{(\sigma_i^{data})^2 + (\sigma_{\nu+\mu_{atm},i})^2} \quad (8.4)$$

1553 Where χ^2 is the test statistic, i refers to bins in zenith and azimuth, $n_i^{\nu+\mu_{atm}}$ and n_i^{data} are
 1554 the number of events in a particular bin in both simulation and data, and $\sigma_{\nu+\mu_{atm},i}$ and σ_i^{data}
 1555 are similar the per-bin standard deviations in both simulation and data. This test statistic
 1556 is used to measure the agreement of the data with a "nominal" model that can be simulated
 1557 to obtain the values $n_i^{\nu+\mu_{atm}}$ and $\sigma_{\nu+\mu_{atm},i}$.

1558 Another example of a commonly used test statistic in neutrino astronomy is the point
 1559 source likelihood (eq. 8.5):

$$TS = -2 \log\left(\frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s)}\right) \quad (8.5)$$

1560 This is a likelihood ratio test statistic, where \mathcal{L} is a likelihood function that describes
 1561 how clustered events are near a particular location. In its simplest form, this likelihood has
 1562 1 parameter, n_s , that describes the number of excess events seen near the location being
 1563 examined.

1564 The likelihood ratio is a very powerful test statistic. Given two hypotheses H_o and $H_a(a)$
 1565 where $H_o = H_a(\tilde{a})$ (they are *nested* models), then the Neyman-Pearson lemma states that
 1566 the likelihood ratio test has the strongest statistical power (i.e. correctly reject H_o when H_1
 1567 is true). This is the case for many point source searches, where the null hypothesis ("There
 1568 are zero signal events, $n_s = 0$) is nested in the alternative hypothesis ("There are a nonzero
 1569 number of signal events, $n_s > 0$ ").

1570 Note that test statistics are essentially estimators of the alternative hypothesis: a "more
 1571 extreme" test statistic should correspond to the data being better described by the alter-
 1572 native hypothesis. In fact, most test statistics are in fact based on estimators of some
 1573 statistical property of the data. The point source test statistic, for example, is based around
 1574 a likelihood estimator of the number of clustered neutrino events.

1575 **8.4.3 Select a significance level at which the null hypothesis will**
 1576 **be rejected**

1577 Since in testing our hypothesis, we have two options for our result (accept/reject the null
 1578 hypothesis), and there are two options for the "actual" state of our null hypothesis (either
 1579 it is true or it isn't), there are four possible outcomes describing whether we were correct or
 1580 not:

	H_o rejected	Accept (Fail to reject) H_o	
1581	H_o False	Correct	Type II error (false negative)
	H_o True	Type II error (false positive)	Correct

1582 If you reject a true null hypothesis, then this is a type II error. If you accept a false
 1583 null hypothesis, then this is a type I error. "Type I" and "type II" are needlessly confusing
 1584 names, so let's instead refer to these with the more intuitive names "false positive" and
 1585 "false negative".

1586 Both types of errors are bound to happen sometimes, but we're typically more concerned
 1587 with false positives: We don't want to make any claims of discovery when in fact it was
 1588 simply a background fluctuation. We can control the rate at which this happens with the
 1589 "significance" of the test (α). The *significance* refers to the probability of obtaining a false
 1590 positive.

1591 Given a hypothesis test with some test statistic (where we can calculate the distribution
 1592 of this test statistic under the null hypothesis, see above!), we can divide the range of our
 1593 test statistic into the acceptance region, containing test statistics less extreme than some
 1594 pre-defined significance threshold, and the rejection region, containing test statistics more
 1595 extreme than this threshold. If our observed test statistic falls in the acceptance region, we
 1596 will accept the null hypothesis to be true. If our observed test statistic falls into the rejection
 1597 region, we will reject the null hypothesis.

1598 The probability of a false positive is then simply the integral of our null hypothesis test
 1599 statistic distribution over the rejection region. This is the "significance" of the test. In
 1600 principle, we can set the significance of our test to be whatever we want, simply by changing
 1601 the acceptance and rejection regions. In practice, we typically want to set a high bar for
 1602 announcing a result (this corresponds to a low significance). In particle physics, the threshold
 1603 for rejecting the null hypothesis is typically set such that the significance is 2.87×10^{-7} (the
 1604 one-sided five sigma threshold). In other disciplines, the convention is different, with social
 1605 scientists and biologists typically rejecting the null hypothesis at the .01 or .05 significance
 1606 level.

1607 You might sometimes hear significance referred to in units of "sigma" (e.g. " 5σ " or " 3σ ").
 1608 This can be obtained by the process shown in figure 8.1, by mapping the significance to a
 1609 normal distribution. This simply provides a standardized way of describing how "extreme"
 1610 the observed significance is: more "sigmas" corresponds to a result that is less likely to be
 1611 seen under the null hypothesis.

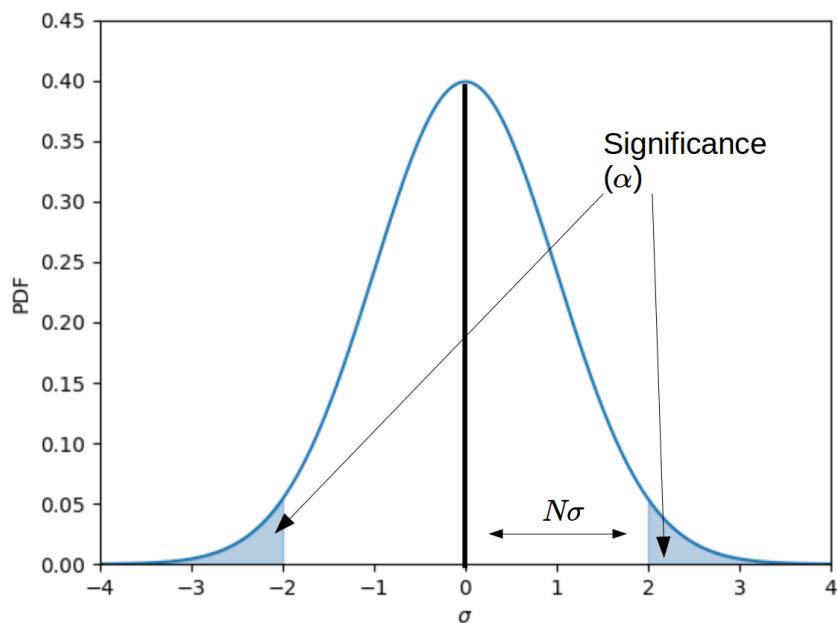


Figure 8.1: The mapping of significance into units of sigma using a normal distribution, for a two tailed test.

1612 **8.4.4 Compute the TS from observed data**

1613 This step is fairly straightforward, as the test statistic has already been defined. Simply
 1614 run your test statistic algorithm using the observed data generated from your experimental
 1615 apparatus.

1616 **8.4.5 Decide whether to accept or reject the null hypothesis**

1617 There are two different, but equivalent constructions for deciding whether to reject the null
 1618 hypothesis or not. The first construction involves pre-defining an *acceptance region* prior
 1619 to calculating the test statistic associated with the observed data. The acceptance region
 1620 corresponds to the range of test statistic values for which the null hypothesis cannot be
 1621 rejected. This is typically chosen to correspond to a particular significance (e.g. 5%, 1%,
 1622 0.01% etc.). If the observed test statistic lies outside of the acceptance region corresponding
 1623 to $X\%$ significance, then we can say that the null hypothesis is rejected at the $X\%$ signif-
 1624 icance level. Note that since we had to pick our significance threshold (and consequently
 1625 define an acceptance region) prior to computing an observed test statistic, we can make no
 1626 further claims on how extreme the result is: if our acceptance region is $TS > 12$, it does not
 1627 matter if we observe a $TS = 12.1$ or if we observe a $TS = 99.0$. In both instances we will
 1628 reject the null hypothesis as the significance level we initially chose.

1629 An alternative, but mathematically equivalent, construction involves the calculation of a
 1630 *p-value*. A p-value is the probability, under the null hypothesis, of obtaining a test statistic
 1631 at least as extreme as the one we have observed from data. If our p-value is smaller than a
 1632 pre-determined significance threshold, then we can reject the null hypothesis. The p-value
 1633 can be calculated by simply integrating our null hypothesis test statistic distribution over
 1634 the range of values more extreme than what was observed. For the one-tailed case where a
 1635 larger test statistic is correlated with stronger evidence of the alternative hypothesis, this is
 1636 (eq. 8.6):

$$p = \int_{TS_{obs}}^{\infty} PDF_{H_0} \quad (8.6)$$

1637 Where TS_{obs} is our observed test statistic and PDF_{H_0} is the distribution of test statistics
 1638 under the null hypothesis. It is important to note that the above equation is only valid for
 1639 the positive, one-tailed case (the alternative hypothesis is only associated with test statistic
 1640 values on one side of the distribution). A more general formula could be expressed as (eq.
 1641 8.7):

$$p = \int_{R|TS_{obs}} PDF_{H_0} \quad (8.7)$$

1642 Where here $R|TS_{obs}$ is the rejection region of the test statistic distribution associated
 1643 with a particular observed test statistic (i.e. it is the region of the background test statistic
 1644 distribution containing values more extreme than what was observed). This distinction is
 1645 particularly important when conducting two-tailed tests, where test statistics that are either
 1646 above an upper bound, or below a lower bound may be cause for rejecting the null hypothesis.

1647 8.5 Trial Factors

1648 Each time you obtain the answer of an analysis from real data, you have done one experi-
 1649 mental trial. It is common to perform multiple trials when working with a specific analysis
 1650 technique.

1651 • In the neutrino sources group, we might test the probability that one location is con-
 1652 sistent with background expectation. If we were to apply this framework to a list of
 1653 30 source candidate locations, then this would be 30 trials.

1654 • Repeating an analysis with new cuts. We might count the number of neutrinos with
 1655 $E > 1$ TeV in 1 year, then count the number of neutrinos with $E > 10$ TeV in 1 year.
 1656 This would be 2 trials.

- 1657 • Repeating with different signal hypotheses. You might look for emission from blazars
 1658 using both the hypothesis of steady emission and a flare hypothesis (2 trials)

1659 Trials can be both independent, or correlated:

- 1660 • **Independent:** Search for a new source/signal in 1 year of data, repeat the search in
 1661 the next year of data (separately). This would be 2 independent trials, as the results
 1662 of the first year will not affect the results of the second year
- 1663 • **Correlated:** Search for a new source/signal in 1 year of data, repeat the search with
 1664 2 years of data, including the first year. This would be 2 trials, but they would be
 1665 correlated, since the results of the first 2 years of data will be affected by the results
 1666 of the first year alone.

1667 Accounting for trials is important, as given enough trials even the null hypothesis can
 1668 produce a significant result. A simple and conservative way to account for trials is to assume
 1669 that trials are uncorrelated and apply the Sidak correction (eq. 8.8):

$$P_{post} = 1 - (1 - P_{pre})^N \quad (8.8)$$

1670 Where P_{post} is the trial-corrected p-value, P_{pre} is the smallest p-value observed across
 1671 all trials performed, and N is the number of trials. This formula is intended to describe
 1672 uncorrelated trials, however it can also be used to obtain a conservative trial correction
 1673 for correlated trials as well. More sophisticated methods of accounting for correlated trials
 1674 typically involve simulation of the null hypothesis, and are oftentimes specific to a particular
 1675 experiment.

1676 When reporting a trial corrected p-value, it is additionally important to specifically state
 1677 which trials are being accounted for. Accounting for trials across all sources in a catalog
 1678 will result in a different p-value than accounting for all analyses performed by the IceCube
 1679 collaboration, yet both are "post-trial" p-values.

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