

1 Every Flare, Everywhere: Untriggered Searches for Astrophysical Neutrino Transients Using Data

2 From the IceCube Detector

3 *by*

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20 Abstract

21 Recent results from the IceCube collaboration regarding the blazar TXS 0506+056 suggest
22 the presence of neutrino flares that are not temporally coincident with a significant corre-
23 sponding gamma ray flare. Such flares are particularly difficult to identify, as their presence
24 must be inferred from the temporal distribution of neutrino data alone. While previously
25 existing statistical methods fit for only the largest neutrino flare candidate at a particular
26 location, this work introduces an extension to fit for the contributions from multiple flare
27 candidates at once, thereby increasing the sensitivity to sub-threshold neutrino flares. The
28 need for a multiple flare fit is further motivated by the increasing duration of the IceCube
29 data available for analysis. In this work, this "multi-flare" method is applied to data from
30 the IceCube Neutrino Observatory using two different source catalogs to explore emission
31 from candidates similar to TXS 0506+056. This method is additionally used to create a
32 "multi-flare" skymap, describing the temporal variability of IceCube data at every point in
33 the neutrino sky. These results can be used to place further constraints on potential pop-
34 ulations of transient neutrino sources, serving as a complement to existing time-integrated
35 and time-dependent methods.

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57 **Contents**

58	Abstract	i
59	Acknowledgements	ii
60	1 Introduction	1
61	2 Astrophysical Messengers	4
62	2.1 Cosmic Rays	4
63	2.1.1 Energy Spectrum	4
64	2.1.2 Acceleration Mechanism	5
65	2.2 Photons	7
66	2.2.1 Leptonic Radiative Processes	8
67	2.2.2 Hadronic Radiative Processes	9
68	2.2.3 Gamma Rays as an Astrophysical Messenger	11
69	2.3 Neutrinos	12
70	2.3.1 Production Mechanisms	12
71	2.3.2 Observed Neutrino Spectrum	13
72	2.3.3 Neutrinos as Astrophysical Messengers	14
73	2.3.4 AGN as Neutrino Source Candidates	15
74	2.3.5 Galactic Neutrino Sources	19
75	2.3.6 Constraints on Neutrino Source Populations from Diffuse Measurements	20
76	3 The IceCube Detector	22

77	3.1 Detection Mechanism	22
78	3.2 The Physical Components of the IceCube Detector	25
79	3.3 Event Types	28
80	3.3.1 Tracks	29
81	3.3.2 Cascades	30
82	3.3.3 Double Cascades	30
83	3.4 Triggering	31
84	3.5 Directional Reconstruction	31
85	3.5.1 LineFit	32
86	3.5.2 SplineMPE	32
87	3.5.3 Paraboloid	33
88	3.6 Energy Reconstruction	34
89	3.7 IceCube Event Samples Overview	35
90	3.7.1 PointSourceTracks v002p03	36
91	3.7.2 PointSourceTracks v003p02	37
92	3.7.3 NorthernTracks v002p06	38
93	4 Review and Improvements to Statistical Methods in Neutrino Astronomy	45
94	4.1 Clustering Analysis Outline for an Arbitrary Test Statistic	46
95	4.2 Unbinned Time Integrated Methods	48
96	4.3 Unbinned Time-Dependent Methods	52
97	4.3.1 Optimizing for a Single Flare	52
98	4.3.2 Improving the Search for Temporal Clustering: Fitting Ensembles of Multiple Flares	55
100	5 Applications of the Multi-flare Algorithm to Source Catalogs	60
101	5.1 Self-Triggered Catalog	61
102	5.2 Fermi 3LAC Blazars	64

103	6 An All-Sky Multi-flare Analysis	76
104	6.1 Motivation	76
105	6.2 Analysis Construction	77
106	6.3 Results	79
107	6.3.1 Hotspots and Populations Analysis	79
108	6.3.2 Locations of External Interest	83
109	7 Outlook	95
110	8 Appendix: Statistics Background	97
111	8.1 Estimators	97
112	8.2 Likelihoods	98
113	8.3 Confidence Intervals	99
114	8.4 Hypothesis Testing	100
115	8.4.1 State relevant null and alternative hypotheses	100
116	8.4.2 State a test statistic	101
117	8.4.3 Select a significance level at which the null hypothesis will be rejected	103
118	8.4.4 Compute the TS from observed data	105
119	8.4.5 Decide whether to accept or reject the null hypothesis	105
120	8.5 Trial Factors	106
121	Bibliography	108

₁₂₂ **Chapter 1**

₁₂₃ **Introduction**

₁₂₄ Though cosmic rays were first discovered in 1912, their origins have to date remained a
₁₂₅ mystery. Since cosmic rays are charged particles and their paths through the universe are
₁₂₆ bent by magnetic fields, identifying the sources of cosmic rays is not as simple as tracing
₁₂₇ back their arrival directions at Earth. The discovery of the astrophysical neutrino flux in
₁₂₈ 2013 [1] provides a new avenue for investigating this problem, as the sources of astrophysical
₁₂₉ neutrinos are suspected to be the same as high energy cosmic rays, but neutrinos are not
₁₃₀ charged, and thus their arrival directions point directly back to their source. Identifying
₁₃₁ the sources of astrophysical neutrinos (and consequently cosmic rays) would provide an
₁₃₂ entirely new view of the universe, potentially allowing us to examine regions opaque to more
₁₃₃ traditional astrophysical messengers such as photons.

₁₃₄ With the IceCube Neutrino Observatory having collected over 10 years of data, the
₁₃₅ first hints of astrophysical neutrino sources are beginning to be visible: a high energy alert
₁₃₆ event from the direction of the blazar TXS 0506+056 sparked multi-messenger followup that
₁₃₇ suggests neutrino emission from that location [2][3], and recent studies of time integrated
₁₃₈ neutrino emission from candidate blazars reveal a 3.5σ excess of astrophysical neutrino events
₁₃₉ from the direction of the Seyfert II galaxy NGC 1068 [4].

₁₄₀ These first hints of astrophysical neutrino sources raise several further questions: Could
₁₄₁ there be more sources like TXS 0506+056 or NGC 1068? How many? Is neutrino emission

142 time-dependent, or steady? Perhaps most importantly, if there are more sources, how can
143 we identify them? Though neutrinos are an excellent astrophysical messenger due to the
144 correlation of their arrival directions with their source of origin, neutrino astronomy is made
145 significantly more difficult by the existence of the atmospheric neutrino flux, which produces
146 a large, irreducible background for searches of astrophysical neutrino sources. Because of
147 this, high energy neutrino astronomy relies either on triggering off a multi-messenger signal
148 (such as a blazar that was also detected in gamma rays), or leveraging knowledge about the
149 pattern of neutrino emission (for example, the arrival directions of astrophysical neutrinos
150 should be clustered near their sources of origin, while the atmospheric background should
151 be isotropically distributed).

152 This work will focus specifically on exploring the possibility of time-dependent neutrino
153 emission, motivated in part by the archival followup that was performed by IceCube at the
154 location of the blazar TXS 0506+056 [3]. A novel method for fitting decorrelated ensembles of
155 neutrino flare candidates is introduced and subsequently applied to source catalogs assembled
156 according properties of the TXS 0506+056 result that could potentially define a class of
157 astrophysical neutrino sources. This new method of fitting ensembles of neutrino flares is
158 intended to fill a methodological gap that exists in the field, as it provides a framework for
159 combining ("stacking") neutrino flare candidates.

160 This method is additionally applied to the entire neutrino sky, allowing for a more gen-
161 eral examination of potential clustering of neutrino data in both space and time. If neutrino
162 data is clustered beyond what is expected from the background hypothesis (isotropic arrival
163 directions distributed uniformly in time), then that would be evidence of a population of
164 astrophysical neutrino sources. Even in the case of a non-discovery, the method introduced
165 here produces neutrino "light-curves" (here referred to as "flare curves") which are of po-
166 tential use for future multi-messenger searches that may wish to incorporate information on
167 historical neutrino emission from a particular source candidate.

	Time Independent	Untriggered Time-Dependent
Single-Source Searches	Time-integrated skymap analysis [4]	Single flare analysis [3]
Source Stacking	Blazar catalog analysis [5]	This work!

Figure 1.1: Examples of some of the types of astrophysical neutrino source searches that have been performed using IceCube data. Time-integrated analyses search for an excess of events over the entire data sample livetime, ignoring any information about potential temporal clustering. By contrast, untriggered, time-dependent searches attempt to fit for temporal neutrino clusters ("flares") without using a multi-messenger lightcurve as a template. Single source searches report the most significant result from a small number of source candidate locations, corrected by a trial factor, while source stacking analyses combine information from many source locations under the assumption that each source candidate is a weak emitter, thereby increasing the search sensitivity to low individual source flux.

¹⁶⁸ Chapter 2

¹⁶⁹ Astrophysical Messengers

¹⁷⁰ This work will focus primarily on cosmic rays, photons, and neutrinos, however other mes-
¹⁷¹ sengers, such as gravitational waves, can provide unique information as well.

¹⁷² 2.1 Cosmic Rays

¹⁷³ 2.1.1 Energy Spectrum

¹⁷⁴ Cosmic rays are high energy protons and atomic nuclei, originating from outside the Earth's
¹⁷⁵ atmosphere. It is difficult to do traditional astronomy using cosmic rays (in the sense of using
¹⁷⁶ their arrival directions to identify sources) due to the fact that cosmic rays are charged, and
¹⁷⁷ consequently their paths are bent by magnetic fields in the universe. However, the cosmic
¹⁷⁸ ray spectrum is well studied, with many measurements having been made over the past
¹⁷⁹ century since their initial discovery. In general, the spectrum can be modeled as a power
¹⁸⁰ law (eq. 2.1):

$$N(E)dE \propto E^{-x}dE \quad (2.1)$$

¹⁸¹ where N describes the number of cosmic ray particles, E is their energy, and x is an energy-
¹⁸² dependent spectral index. The spectrum has several notable features, shown in figure 2.1.

At lower energies (below 10^{16} eV), the spectrum is well described by an $E^{-2.7}$ flux. At 10^{16} eV, the spectrum softens to $E^{-3.1}$, commonly referred to as the "knee". At even higher energies (around $10^{18.5}$ eV), the spectrum hardens again. This region is referred to as the "ankle", and is thought to correspond to the point where extragalactic cosmic rays begin to dominate the flux.

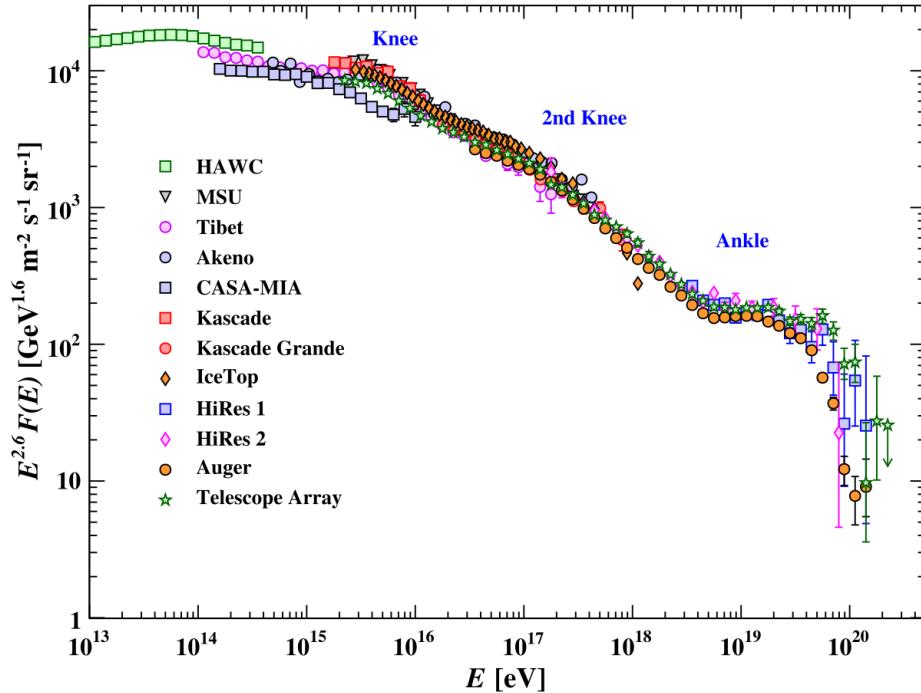


Figure 2.1: A plot of the cosmic ray spectrum assembled from observations from a variety of experiments. Major features visible are the knee ($\approx 10^{16}$ eV), and the ankle ($\approx 10^{19}$ eV).
[5]

2.1.2 Acceleration Mechanism

Though the exact origin of cosmic rays is still unknown, we can infer how these particles are being accelerated based on our knowledge of how charged particles behave.

Suppose a particle encounters a moving cloud of plasma. If it is large enough, the cloud will act as a massive scatterer. The overall collision is the result of a large number of individual scatterings inside the cloud, and the outgoing angle of the original particle is

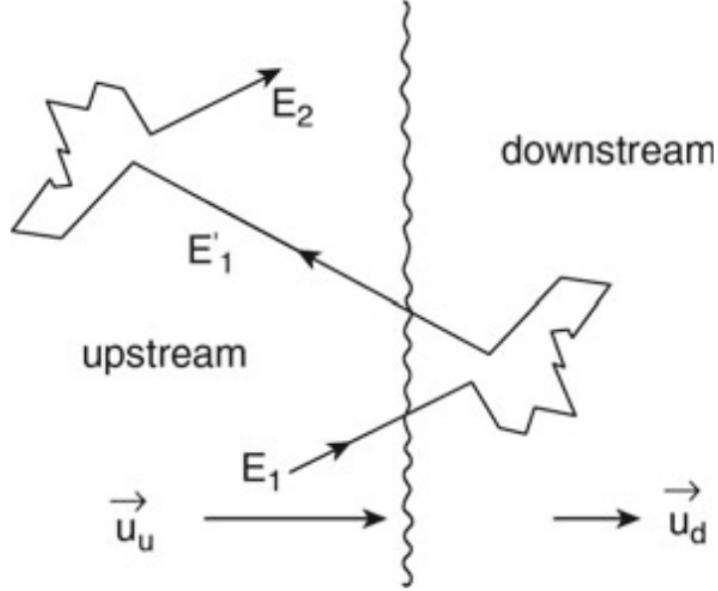


Figure 2.2: Fermi acceleration of a plane shock front[6].

194 essentially random. On average the energy of the particle after a collision with this boundary
 195 will increase by a factor (eq. 2.2):

$$\langle \frac{\Delta E}{E} \rangle \approx \frac{4}{3}\beta^2 \quad (2.2)$$

196 Where $\beta = V/c$, corresponding to the boundary velocity. This is *second order Fermi*
 197 *acceleration*, and is generally insufficient to explain the cosmic ray spectrum. If we instead
 198 assume a plane shock front, as in figure 2.4 , then the post-collision velocity of the particle
 199 is no longer randomly distributed, and we instead obtain the expression (eq. 2.3):

$$\langle \frac{\Delta E}{E} \rangle \approx \frac{4}{3}\beta \quad (2.3)$$

200 This describes *first order Fermi acceleration*. From here, we can calculate the expected
 201 flux measured at Earth to be (eq. 2.4):

$$\frac{dN}{dE} \propto \left(\frac{E}{E_0}\right)^{-\Gamma} \quad (2.4)$$

202 where $\Gamma = \alpha + 1$, $\alpha \approx \frac{3P_e}{4\beta}$, and P_e is the probability that a particle may escape the
 203 shock region (proportional to the velocity, V). Note that this is a power law with almost
 204 constant spectral index, as both β and P_e are proportional to V , similar to what is seen in
 205 the observed cosmic ray spectrum [6].

206 We can additionally derive constraints on the maximum energy to which a particle can be
 207 accelerated based off the magnetic field intensity, B , and the size of the accelerating region,
 208 R (eq. 2.5):

$$E_{max} = eBR \approx 10^{21} \left[\frac{R}{1 \text{ pc}} \right] \left[\frac{B}{1 \text{ Gauss}} \right] \text{ eV} \quad (2.5)$$

209 This is known as *Hillas' condition*, and can inform us about the properties of cosmic
 210 ray accelerators. We can plot this condition to show sources which satisfy this requirement
 211 for a particular energy, producing what is commonly referred to as Hillas' plot (Figure 2.3).
 212 Sources must exist above the diagonal thresholds in order to be valid accelerators of cosmic
 213 rays of certain energies. For example, sources capable of accelerating protons to 100 EeV
 214 energies must exist above the dashed line in figure 2.3. Sources below are either too small,
 215 or have magenetic fields that are too weak.

216 2.2 Photons

217 Perhaps the oldest and most well-studied astrophysical messenger, photons have the useful
 218 property of being electrically neutral. Unlike cosmic rays, their paths are not bent by
 219 magnetic fields, and the arrival directions of photons can be used to identify their origin,
 220 particularly at high energies due to their good angular resolution. Photons additionally span
 221 a large energy range, and can be used to explore a wide variety of astrophysical production
 222 processes. For the purposes of this work, we are most interested in the highest energy
 223 photons (gamma rays, with energies greater than 100 keV), which can be produced through
 224 either leptonic or hadronic processes.

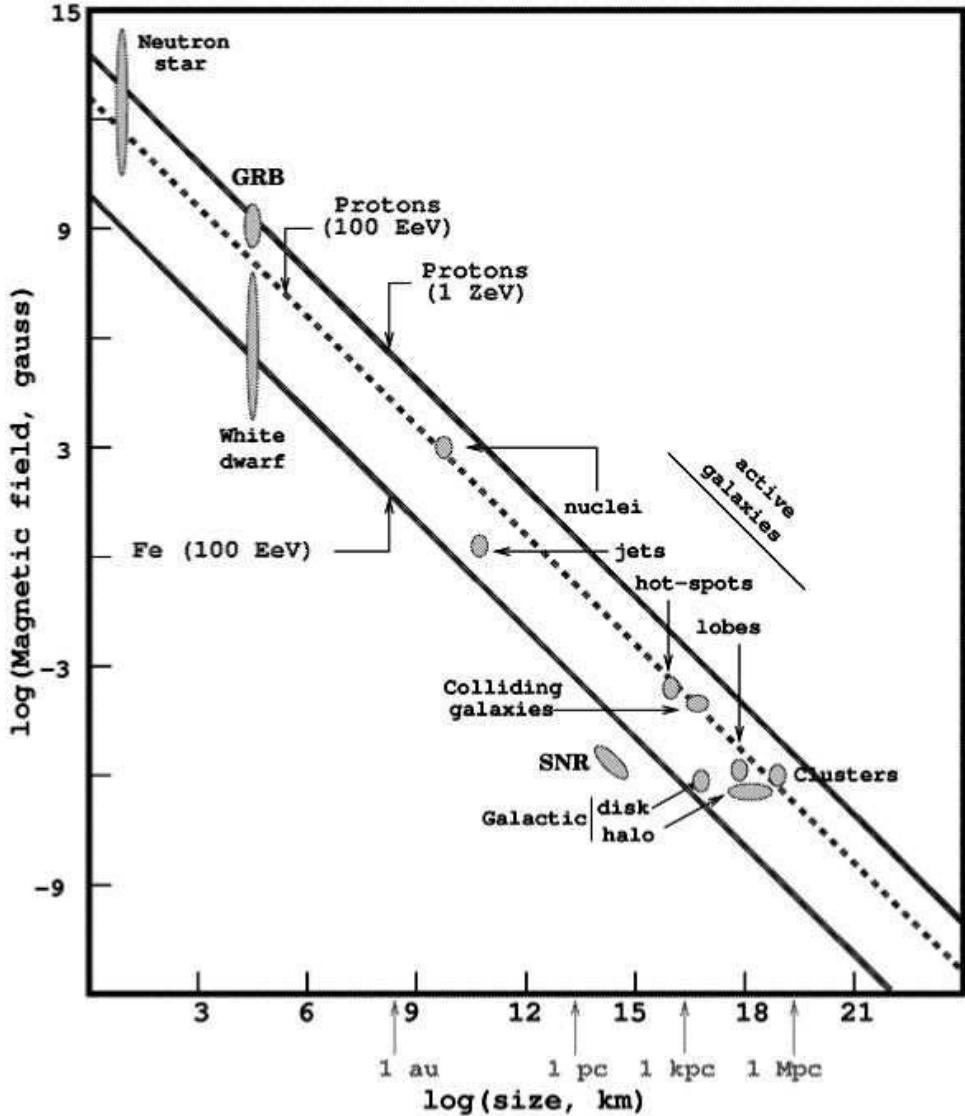


Figure 2.3: A Hillas plot, showing source populations capable of accelerating cosmic rays to a given energy, given their size and magnetic field strength [7].

225 2.2.1 Leptonic Radiative Processes

226 While photons cannot be directly accelerated by electric and magnetic fields, there are a
227 variety of mechanisms by which processes involving leptons can produce high energy photons.

- 228 • **Synchrotron radiation:** Radially accelerated relativistic particles will release syn-
229 chrotron radiation. This typically occurs as charged particles (such as electrons) spiral
230 through magnetic fields. The power loss, (dE/dT) , due to synchrotron radiation for a

231 particle of mass M and charge Z can be expressed as (eq. 2.6):

$$-\frac{dE}{dT} \approx 2.6\left(\frac{Zm_e}{M}\right)^4\left(\frac{E}{1 \text{ keV}}\right)^2\left(\frac{B}{1 \text{ G}}\right)^2 \text{ keV s}^{-1} \quad (2.6)$$

232 Where m_e is the electron mass, E is the particle energy, and B is the magnetic field
 233 strength. From equation 2.6 it can be seen that synchrotron emission is significantly
 234 more relevant for electrons than protons, due to the $1/M^4$ term.

235 • **Compton Scattering and Inverse Compton Scattering:** Photons may scatter
 236 off an electron at rest, and in doing so experience a wavelength shift according to (2.7):

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\hbar\omega}{m_e c^2}(1 - \cos \alpha) \quad (2.7)$$

238 Where λ' and λ correspond to the final and initial photon wavelengths, and α is the
 239 deflection angle of the photon. If a low energy photon collides with a high energy
 240 electron, the energy of the photon may exit with more energy than it began with,
 241 providing an effective mechanism for increasing the photon energy.

242 A combination of these two processes can produce the two peak structure typically seen
 243 in the spectrum of observed gamma ray sources. The lower energy peak is due to synchrotron
 244 radiation, while the higher energy peak corresponds to low energy photons being scattered
 245 to higher energies via inverse compton scattering.

246 2.2.2 Hadronic Radiative Processes

247 While leptonic processes can produce high energy gamma rays, hadronic processes will pro-
 248 duce gamma rays in addition to neutrinos and cosmic rays. Protons accelerated to high
 249 energies in an astrophysical source may interact with other protons via (eq. 2.8):



250 Where X is the appropriate nucleus or hadron depending on the pions produced, and on
 251 average there is expected to be equal production of neutral and charged pions. Additionally,

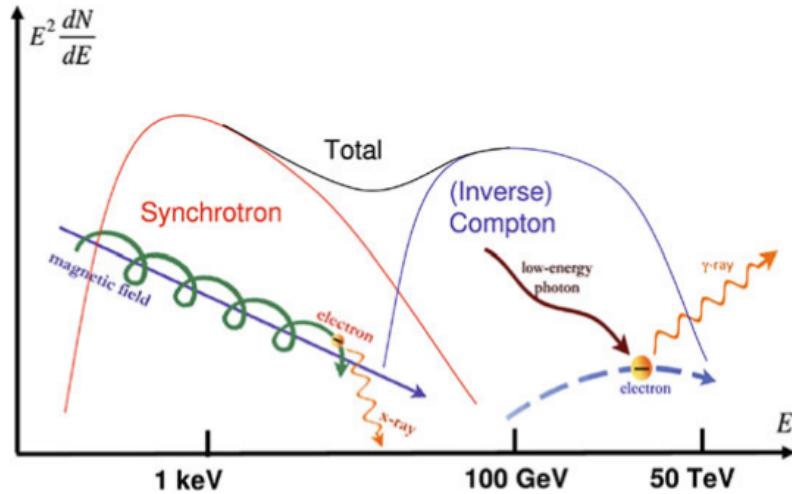
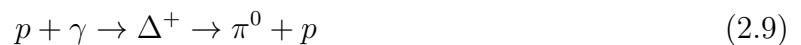
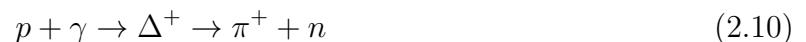


Figure 2.4: A cartoon plot of the gamma ray spectrum for a high energy gamma ray source. The two peak structure arises from the two production mechanisms of gamma rays. At lower energies, photons are produced by synchrotron radiation of electrons spiraling around magnetic field lines. The higher energy peak is then formed by the lower energy photons being scattered off energetic electrons via inverse compton scattering [8].

252 protons may interact with photons, providing an additional path to pion production (eq. 2.9
 253 and 2.10):



254



255 These resultant pions can subsequently decay, producing child particles depending on the
 256 particular pion type. Neutral pions will decay into gamma rays ($\pi^0 \rightarrow \gamma + \gamma$), and charged
 257 pions will decay to a lepton and their associated (anti)neutrino (most often muons and muon
 258 neutrinos: $\pi^+ \rightarrow \nu_\mu + \mu^+$, $\pi^- \rightarrow \bar{\nu}_\mu + \mu^-$). As such the sources of cosmic rays are expected
 259 to additionally produce a high energy gamma ray flux.

260 **2.2.3 Gamma Rays as an Astrophysical Messenger**

261 Gamma rays are a robust astrophysical messenger. As previously mentioned, photons are
 262 uncharged and their arrival directions can consequently be used to identify their origin.
 263 Additionally, the gamma ray energy can be used to infer information about the primary
 264 particle energy for a known production mechanism. Gamma rays are also well studied, and
 265 modern detectors boast high event rates: Fermi-LAT, for example, observes enough events to
 266 be capable of studying the temporal variability of sources on the timescale of approximately
 267 a day.

268 Gamma rays are not without their limitations, however. Note that while at the high-
 269 est energies gamma rays may be produced by both leptonic and hadronic processes, only
 270 hadronic processes are expected to additionally produce neutrinos and cosmic rays. For this
 271 reason, high energy gamma rays alone cannot be used to identify the source of high energy
 272 cosmic rays.

273 Additionally, high energy gamma rays may also interact electromagnetically with the
 274 ambient radiation in the universe, undergoing pair production (eq. 2.11):

$$\gamma_{VHE} + \gamma_{CMB} \rightarrow e^+ + e^- \quad (2.11)$$

275 This interaction can occur between high energy gamma rays and photons from the CMB,
 276 leading to an event horizon for gamma rays of a particular energy. Figure 2.5 shows this
 277 horizon as a function of distance and photon energy. The photon event horizon prevents us
 278 from studying the highest energy photons from many AGN, and there is a significant region
 279 of parameter space below the highest observed cosmic ray energy that cannot be examined
 280 with photons alone.

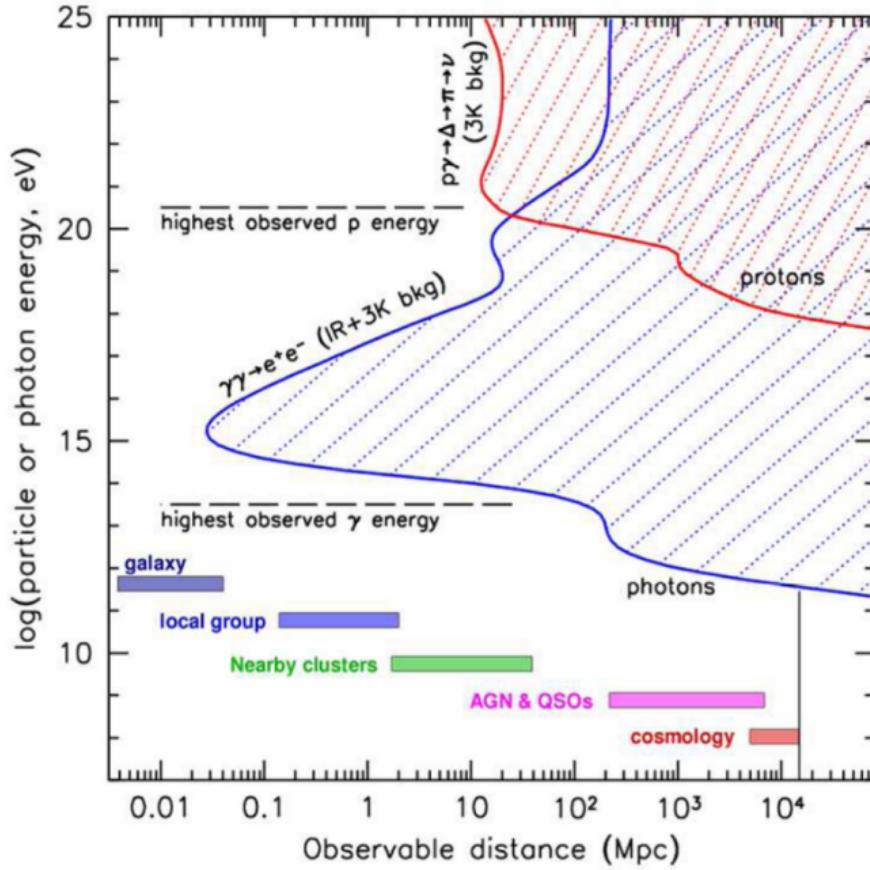


Figure 2.5: A plot of the photon and cosmic ray event horizons, showing the regions/energy scales visible using observations originating from photons (blue) and cosmic rays (red). Pair production of high energy photons with the ambient radiation in the universe prevents photon observations at long distances and high energies, including much of the energy scale below the highest observed cosmic ray energy. Also note that the photon event horizon can prevent us from studying the highest energy photons from AGN [9].

281 2.3 Neutrinos

282 2.3.1 Production Mechanisms

283 Neutrinos in astrophysical sources are produced via charged pion decay from pions pro-
 284 duced from cosmic ray interactions with either photons or protons. The charge of the pion
 285 determines whether more neutrinos or anti-neutrinos are produced (eq. 2.12 and 2.13):

$$\pi^+ \rightarrow \nu_\mu + \mu^+ \rightarrow \nu_\mu + e^+ + \nu_e + \bar{\nu}_\mu \quad (2.12)$$

$$\pi^- \rightarrow \nu_\mu + \mu^- \rightarrow \bar{\nu}_\mu + e^- + \bar{\nu}_e + \nu_\mu \quad (2.13)$$

286 In either case, note that the expected flavor ratio of the neutrinos and anti-neutrinos
 287 produced at the source is $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. Combining this initial flavor ratio
 288 with neutrino oscillations over cosmic distances produces an expected observed astrophysical
 289 neutrino flavor ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$.

290 Neutrinos are also produced by cosmic rays interacting with the Earth's atmosphere. This
 291 is referred to as the atmospheric neutrino flux, and has contributions both from pion decay
 292 ($\pi^+ \rightarrow \nu_\mu + \mu^+$, $\pi^- \rightarrow \bar{\nu}_\mu + \mu^-$), as well as kaon decay ($K_L^0 \rightarrow \pi^\pm + e^\mp + \nu_e$, $K_L^0 \rightarrow \pi^\pm + \mu^\mp + \nu_\mu$,
 293 $K^+ \rightarrow \mu^+ + \nu_\mu$, $K^+ \rightarrow \pi^0 + e^+ + \nu_e$, $K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$). Below 100 TeV, the resultant
 294 spectrum closely follows that of cosmic rays, with a spectral index of $\gamma_{atmos} = 2.7$. At higher
 295 energies the shower mesons are able to travel further in the atmosphere, giving them an
 296 increased opportunity to interact and steepening the spectral index to $\gamma_{atmos} = 3.7$. The
 297 atmospheric neutrino flux exists as an irreducible background for many studies of the origin
 298 of astrophysical neutrinos.

299 2.3.2 Observed Neutrino Spectrum

300 Both the astrophysical and atmospheric neutrino fluxes have been observed in data from
 301 the IceCube detector, with the astrophysical neutrino spectral index being measured to be
 302 $\gamma_{astro} \approx 2.28$ [10]. While astrophysical and atmospheric neutrinos are largely indistinguish-
 303 able on an individual event basis, the difference in spectra between the two populations
 304 ($\gamma_{astro} \approx 2.28$, $\gamma_{atmos} \approx 3.7$) allows us to distinguish between the two on a statistical ba-
 305 sis. Higher energy events are more likely to be astrophysical in origin, and an equal share
 306 of astrophysical and atmospheric neutrinos can be expected at ≈ 200 TeV. Currently, the
 307 source(s) of the astrophysical neutrino flux have yet to be identified. For this reason, the
 308 observed astrophysical neutrino flux is commonly referred to as the "diffuse" astrophysical
 309 neutrino flux (as opposed to a "point source" flux, where specific astrophysical objects that

310 are producing neutrino are identified).

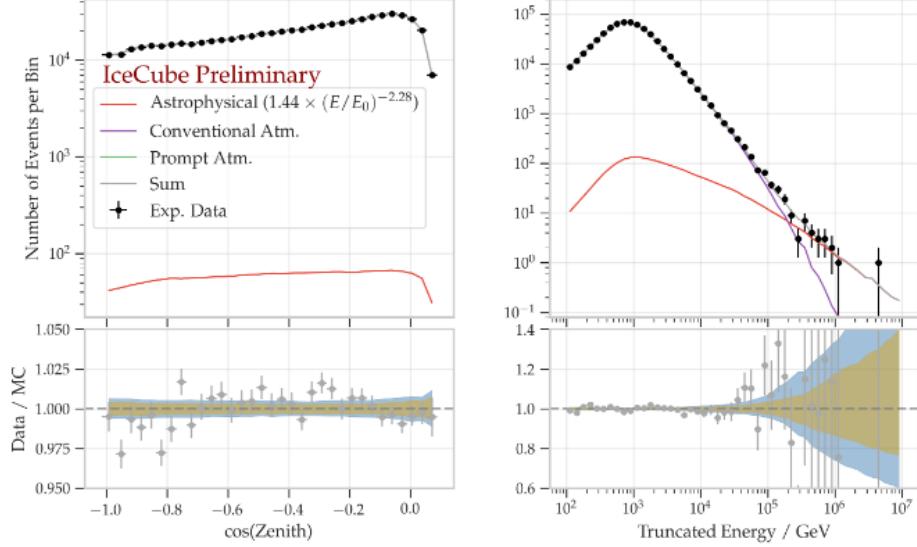


Figure 2.6: Observation of the astrophysical neutrino flux in through-going tracks detected by the IceCube Neutrino Observatory. The effect is most visible in the plot of the observed energy spectrum on the right, showing the atmospheric contribution (blue) and the astrophysical contribution (red). The excess of events at high energies is unable to be explained without including a non-zero astrophysical contribution [10].

311 2.3.3 Neutrinos as Astrophysical Messengers

312 As noted in the previous section, hadronic processes in astrophysical sources can produce
 313 neutrinos via pion decay. Like gamma rays, neutrinos are electrically neutral, and thus
 314 their paths are not bent by magnetic fields. Unlike gamma rays, however, neutrinos do not
 315 interact electromagnetically, instead interacting only via the weak force. Because of this,
 316 the mean free path of neutrinos traveling through the universe is significantly greater than
 317 that of photons, allowing us to use neutrinos to probe regions normally opaque to gamma
 318 rays.

319 Astrophysical neutrinos are not without their drawbacks, however. The fact that neutrinos interact only weakly necessitates the construction of large detectors in order to observe
 320 an appreciable number of astrophysical events. Such detectors also observe a large, irre-

ducible background of atmospheric neutrinos, particularly at lower (TeV) energies. For this reason, multi-messenger approaches that combine information from neutrinos and other astrophysical messengers are commonly used to attempt to identify the sources of astrophysical neutrinos.

2.3.4 AGN as Neutrino Source Candidates

Since neutrinos can be used as an indicator of hadronic processes in astrophysical sources, identifying the source of the astrophysical neutrino flux is a promising route to also identifying the source of ultra-high energy cosmic rays. Though the source of the astrophysical neutrino flux has yet to be fully determined, in recent years there have been promising hints that extragalactic AGN are at least partially responsible.

An active galactic nucleus (AGN) is a compact region at the center of a galaxy displaying elevated electromagnetic emission, such that the additional luminosity cannot be attributed to stars. The luminosity is expected to originate from the accretion disk surrounding a supermassive black hole at the center of the galaxy, where conditions are thought to be ideal for high-energy particle acceleration and production.

The angle at which the AGN is viewed relative to its jet determines its exact classification, as can be seen in figure 2.7. If the AGN is viewed "down" the jet (i.e. the jet is directed at Earth), the AGN is seen to be particularly luminous and is referred to as a blazar. Blazars can be further divided into two subclasses: flat spectrum radio quasars (FSRQ), which have luminous broad emission lines and continuous thermal emission, and BL-Lacs, which have weak or absent broad emission lines. AGN viewed off-axis fall into a variety of other classifications, with further sub-classification being defined by the spectrum of the AGN in various wavelength bands.

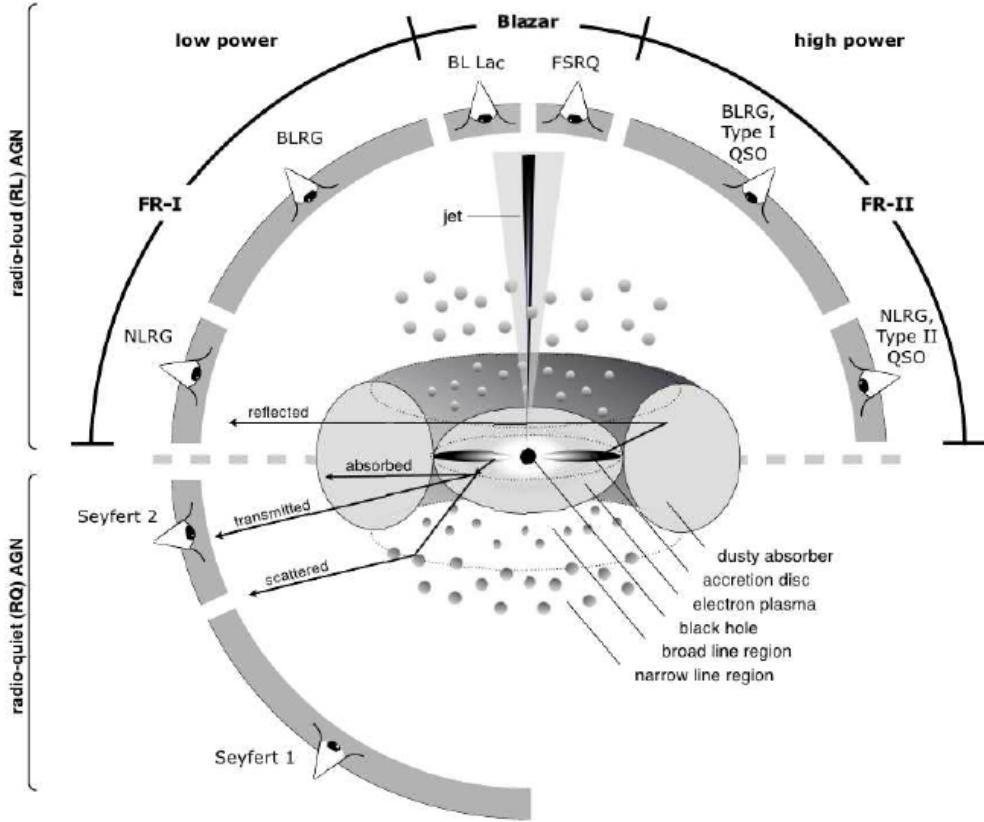


Figure 2.7: The unified AGN model, showing the various classifications of AGN for different observation angles. If the jet is directed at Earth, the object is viewed as a blazar, while other viewing angles can obscure the jet, leading to alternative classification (e.g. as a Seyfert galaxy) [11].

345 2.3.4.1 TXS 0506+056

346 On September 22, 2017, the IceCube Neutrino Observatory detected a high energy neu-
 347 trino, IceCube-170922A, with an energy of approximately 290 TeV. The arrival direction of
 348 IceCube-170922A was consistent with the location of the blazar TXS 0506+056, and sparked
 349 multi-messenger followup from a variety of other telescopes and detectors. Notably, it was
 350 determined that TXS 0506+056 was flaring in gamma rays at the time IC-170922A was
 351 observed. The significance of the positional correlation of a high energy IceCube event and
 352 a flaring blazar was calculated to be 3.0σ , providing the first hint that blazars may be a
 353 source of high energy astrophysical neutrinos [2].

354 In addition to examining the multi-messenger correlation of a high energy IceCube event
 355 and a flaring blazar, an analysis of archival IceCube data was performed at the coordinates

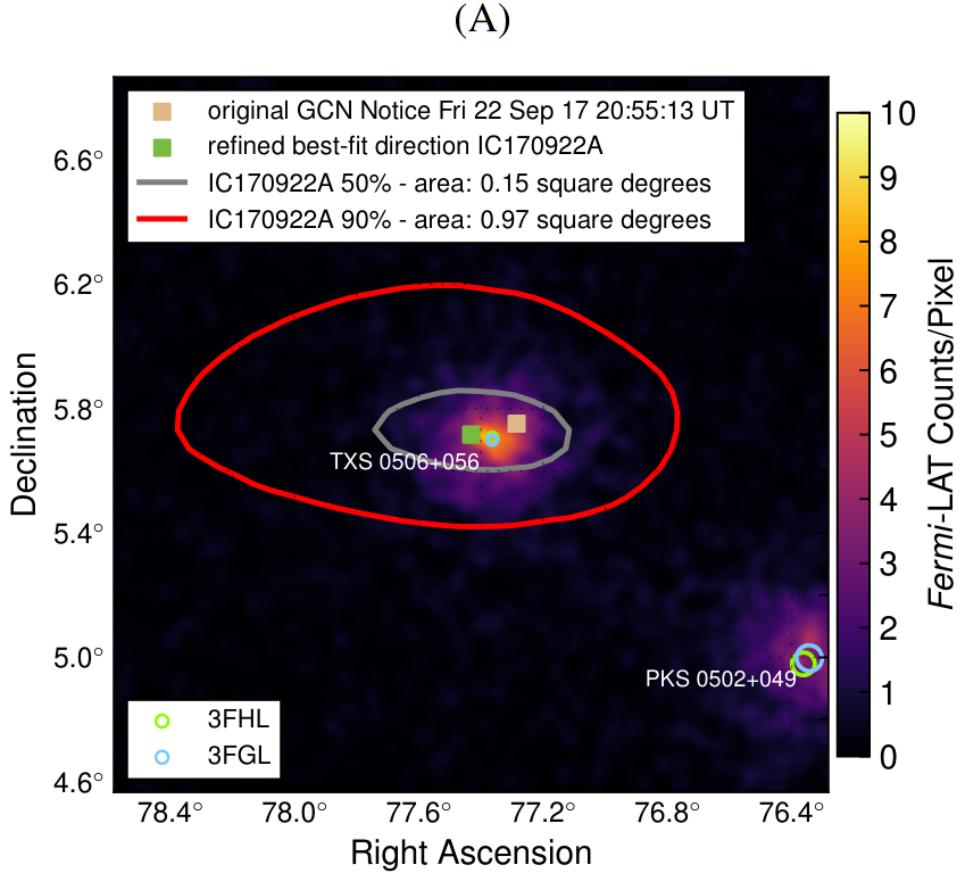


Figure 2.8: The Fermi-LAT skymap near the location of the observed high energy IceCube event IC170922A, with the 50% and 90% error contours associated with IC170922A plotted in grey and red, respectively. The significance associated with the spatial coincidence of the high energy event and TXS 0506+056 was found to be 3.0σ [2].

of IC170922A, examining the historical behavior of neutrino emission from this location of the sky over the previous nine years. In addition to the original high energy event observed in 2017, this archival analysis also revealed a 158 day period of elevated neutrino emission beginning in September 2014. The significance of this archival "neutrino flare" was calculated to be 3.5σ . Unlike the 2017 high energy event, the 2014 neutrino flare did not correspond to a flaring period for TXS 0506+056 in gamma rays. [3]

The combination of a 3.0σ multimessenger result with the 3.5σ "untriggered" neutrino flare in 2014 suggest a significance of TXS 0506+056 as a neutrino source of at least 3σ . This makes for a strong argument for extragalactic blazars as interesting source candidates

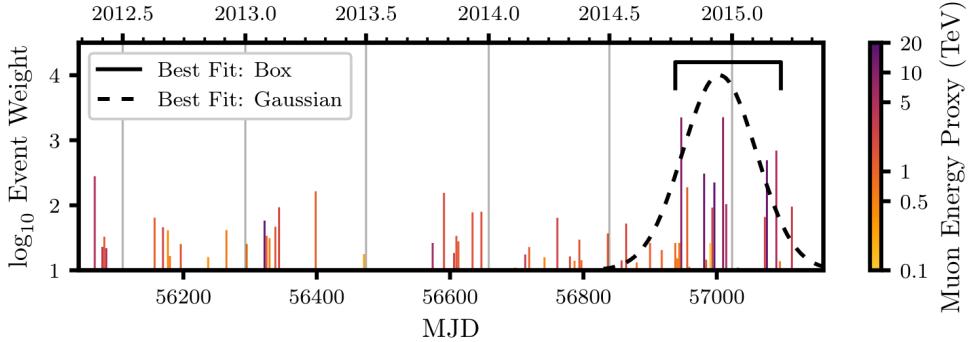


Figure 2.9: The result of the archival analysis of IceCube data near the location of TXS 0506+056, with the 3.5σ flare shown. This flare is separate from the high energy event observed in 2017, and there was no corresponding flare observed in gamma rays during this time period [3].

365 from the perspective of neutrino astronomy.

366 2.3.4.2 NGC 1068

367 In addition to the multimessenger and archival results associated with TXS 0506+056, the
 368 results of the 10-year time integrated IceCube analysis [4] also seem to suggest AGN as
 369 candidates for astrophysical neutrino emission. In this all-sky, untriggered, time integrated
 370 analysis, the most significant point in the northern sky appears to be spatially coincident
 371 with the Seyfert II galaxy NGC 1068. Notably, NGC 1068 was also included in an associated
 372 time integrated catalog analysis. In this catalog analysis, the pre-trial significance of time-
 373 integrated neutrino emission from NGC 1068 was 1.8×10^{-5} , corresponding to a significance
 374 of 2.9σ . At a 14.4 Mpc distance, NGC 1068 is the most luminous Seyfert II galaxy detected
 375 by Fermi-LAT, and NGC 1068 had additionally been hypothesized as a candidate cosmic
 376 ray accelerator prior to this particular analysis [12][13][14].

377 It should additionally be noted that the catalog analysis mentioned above identified
 378 three other objects which, together with NGC 1068, collectively form a 3.3σ excess over
 379 the background expectation. These objects are the Seyfert II galaxy NGC 1068, the blazar
 380 TXS 0506+056, and the BL Lacs PKS 1424+240 and GB6 J1542+6129 [4], providing further
 381 indication of AGN as potential neutrino emitters.

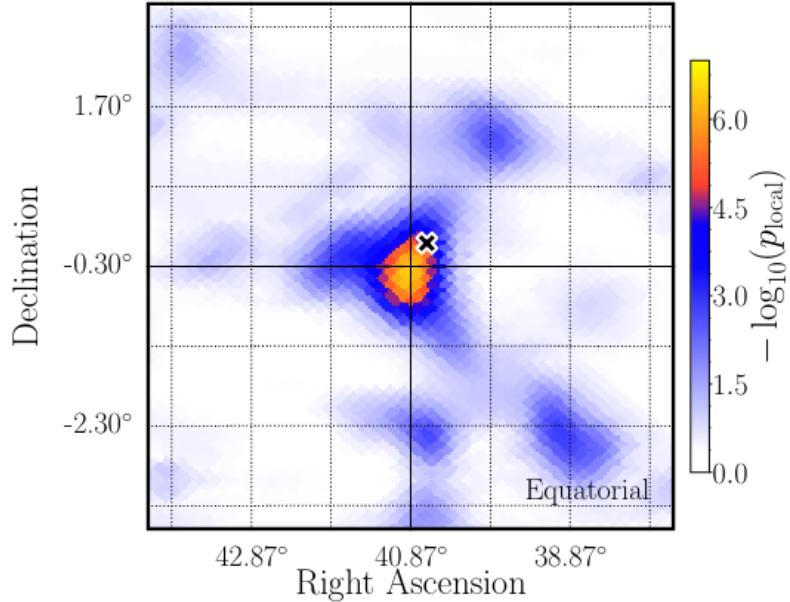


Figure 2.10: The results of the untriggered time integrated IceCube point source search analysis, showing the most significant spot in the northern sky, with the location of NGC 1068 plotted as the black "X" [4].

382 2.3.5 Galactic Neutrino Sources

383 Though this work will primarily focus on searches for extra-galactic point sources of the
 384 astrophysical neutrino flux, potential neutrino emission from sources within our galaxy is
 385 an active area of study as well. Notably, one of the first confirmed sources of lower energy
 386 astrophysical neutrinos was Supernova 1987A [15], a supernova occurring within the Milky
 387 Way. Other galactic objects that are considered candidates for neutrino emission include
 388 supernova remnants [16], as well as pulsar wind nebula [17], both of which are known to
 389 produce gamma rays.

390 The majority of the galactic plane lies in the southern sky. Since neutrino telescopes make
 391 use of the earth to block out atmospheric muon backgrounds (see subsequent chapter), this
 392 means that to best study the galactic plane, a neutrino observatory located in the northern
 393 hemisphere would be required. While there are several such telescopes either planned or
 394 under construction [18][19], they have yet to reach effective volumes comparable to the

395 existing IceCube observatory (located in the southern hemisphere).

396 **2.3.6 Constraints on Neutrino Source Populations from Diffuse
397 Measurements**

398 It is important to keep in mind that whatever the sources of astrophysical neutrinos may
399 be, they must combine to reproduce the measured diffuse astrophysical neutrino flux. The
400 current best fit astrophysical neutrino spectrum, assuming a single power law in energy, is
401 given as (eq. 2.14):

$$\frac{d\phi_{\nu+\bar{\nu}}}{dE} = 1.44^{+0.25}_{-0.24} \left(\frac{E}{100\text{TeV}}\right)^{-2.28^{+0.08}_{-0.09}} 10^{-18} \text{GeV}^{-1}\text{cm}^{-1}\text{s}^{-1}\text{sr}^{-1} \quad (2.14)$$

402 This places constraints on the density and luminosity of potential astrophysical neutrino
403 source populations. If a candidate source population has sources that are too numerous
404 and/or too bright, then that source population would produce a higher flux of astrophysical
405 neutrinos than has been observed, and an explanation of the nondetection of the additional
406 flux is required. Similarly, if sources are too sparse and/or too dim, then such a source class
407 is incapable of explaining the entirety of the measured diffuse flux.

408 These constraints can be summarized in figure 2.11, showing the band of the measured
409 diffuse flux in source density/luminosity space, as well as several commonly discussed source
410 populations. It should be noted that in this plot, the luminosity values for various source
411 populations are derived from the electromagnetic luminosity L_γ . In principle, the true ratio
412 L_ν/L_γ remains unknown, and consequently the positions of sources along the luminosity axis
413 may shift according to this value.

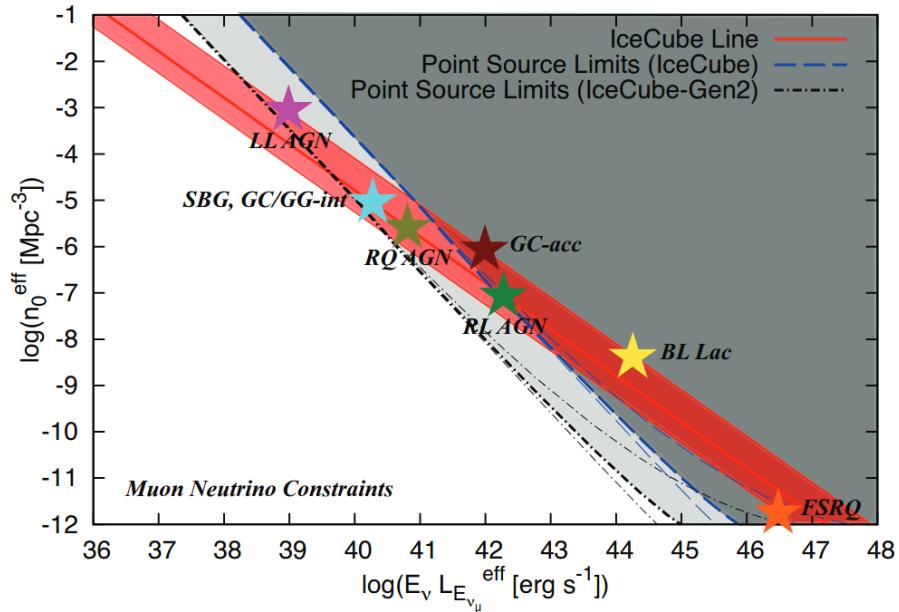


Figure 2.11: Constraints on potential populations of sources of the high energy astrophysical neutrino flux. The red band represents combinations of source density and luminosity that are consistent with the measured astrophysical neutrino flux. Source populations that lie above this band are too bright and/or too numerous, while source below this band are too dim/too sparse. Also shown are limits derived from the non-observation of neutrino sources in time-integrated analyses performed by the IceCube collaboration as of 2016, as well as a projected set of limits that would be associated with a non-observation from IceCube-Gen2 [20].

⁴¹⁴ Chapter 3

⁴¹⁵ The IceCube Detector

⁴¹⁶ The IceCube Detector is a large (cubic kilometer scale) water cherenkov detector situated
⁴¹⁷ underneath the ice at the south pole. Its large volume makes it ideal for studying high energy
⁴¹⁸ (TeV and higher) neutrino events originating from either the atmosphere or astrophysical
⁴¹⁹ sources.

⁴²⁰ 3.1 Detection Mechanism

⁴²¹ The cross section for neutrino interaction in Earth can be seen in figure 3.1. These cross
⁴²² sections are small, but the Earth is also quite large. We can calculate the mean free path
⁴²³ for neutrinos traveling through Earth as (eq. 3.1):

$$\lambda = \frac{1}{\sigma_\nu \rho_{Earth}} \quad (3.1)$$

⁴²⁴ Using the neutrino cross section near 1 PeV ($\sigma_\nu \approx 1.15 \times 10^{-33} \text{ cm}^2$) [21], and the density
⁴²⁵ of the Earth ($\approx 5.5 \text{ g/cm}^3$, corresponding to a nucleon density of $\rho_{Earth} = 3.3 \times 10^{24} \text{ cm}^{-3}$),
⁴²⁶ we obtain an estimate of the neutrino mean free path of $\approx 2600 \text{ km}$. Since the diameter
⁴²⁷ of the Earth is approximately $1.3 \times 10^4 \text{ km}$, we can conclude that the Earth is opaque to
⁴²⁸ high energy neutrinos. We can detect the neutrinos that interacted in the Earth by way of
⁴²⁹ identifying their interaction products.

430 For the purposes of neutrino point source searches, we primarily focus on neutrinos
 431 interacting either through charged current (CC) or neutral current (NC) interactions. In
 432 both cases, the energy of the neutrinos observed by the IceCube detector is high enough
 433 that neutrinos interact through deep inelastic scattering with nucleons in the antarctic ice.
 434 In neutral current (NC) interactions, the process is mediated by a neutral Z boson, as shown
 435 on the left in figure 3.2. In this case, the product of the interaction is a neutrino, meaning
 436 the only visible signature of this interaction is the production of a hadronic particle cascade.

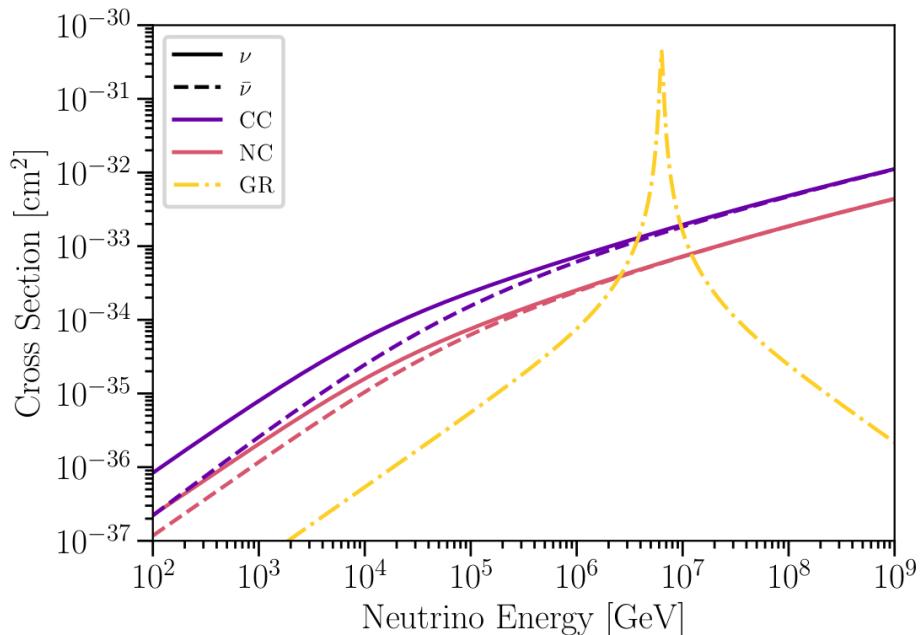


Figure 3.1: The cross sections of various neutrino interactions in the Earth. The yellow curve corresponds to Glashow Resonance [22], whereby an anti-electron neutrino combines with an atomic electron to produce a W+. These events are relatively rare in IceCube data, and are subsequently not used in the context of astrophysical neutrino source searches.

437 Charged current interactions are mediated by a charged W boson, and can be summarized
 438 as (eq. 3.2 and 3.3):

$$\nu_\ell + n \rightarrow \ell^- + p \quad (3.2)$$

439

$$\bar{\nu}_\ell + p \rightarrow \ell^+ + n \quad (3.3)$$

440 The Feynman diagrams for this process can be seen on the right in figure 3.2. Notably,
 441 charged current interactions produce an outgoing lepton ℓ in addition to a hadronic cascade.
 442 This is particularly useful in the case that the lepton produced is a muon, as muons can
 443 travel a sizeable distance (hundreds, or even thousands of meters) before decaying. If the
 444 lepton receives enough energy, it will subsequently produce cherenkov radiation, as it will
 445 be traveling faster than the local speed of light in ice. This radiation is emitted at an angle
 446 θ_c relative to the direction of travel, where θ_c is given by (eq. 3.4):

$$\cos(\theta_c) = \frac{1}{n\beta} \quad (3.4)$$

447 Where n is the index of refraction of the medium through which the particle is traveling
 448 (in this case ice), $\beta = \frac{v}{c}$, and v is the velocity of the traveling particle (the lepton, in this
 449 case). For ice, $n = 1.31$, and this emission angle is approximately 41 degrees. The number of
 450 photons expected per unit track length is given by the Frank-Tamm formula [23] (eq. 3.5):

$$\frac{dN}{dx d\lambda} = \frac{2\pi z\alpha}{\lambda^2} \sin^2(\theta_c) \quad (3.5)$$

451 Where z is the charge of the ionizing particle, λ is the wavelength of the emitted radiation,
 452 α is the fine structure constant ($\approx \frac{1}{137}$), and θ_c is the cherenkov angle given by eq. 3.4. Peak
 453 emission is found in the optical portion of the light spectrum, between 350 and 600 nm
 454 corresponding to a characteristic blue hue.

455 To summarize, neutrinos traveling through the antarctic ice will occasionally interact,
 456 producing child particles that will emit cherenkov radiation. We can then build an array
 457 of photon detectors to detect these photons, and subsequently infer information about the
 458 original incident particles.

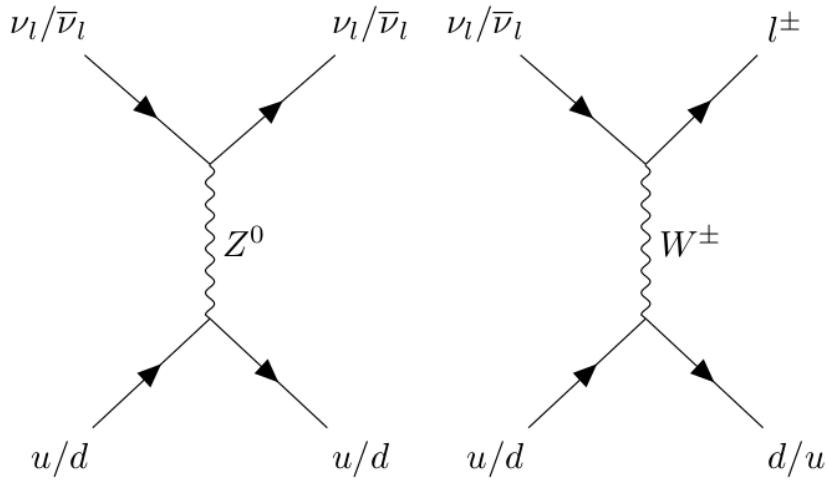


Figure 3.2: The Feynman diagrams corresponding to neutral current (left) and charged current (right) neutrino interactions. Charged current interactions can produce a detectable outgoing lepton in addition to a hadronic cascade, while neutral current interactions only produce a hadronic cascade.

459 **3.2 The Physical Components of the IceCube
460 Detector**

461 As outlined in the previous section, the strategy for detecting neutrinos with a water
462 cherenkov detector is not to directly detect the neutrinos themselves, but rather to detect
463 the cherenkov photons from the outgoing particles resulting from the neutrino interactions.
464 The IceCube detector accomplishes this through the use of a large number of photomultiplier
465 tubes (PMTs). At the very highest level, a PMT is a device that converts photons to an
466 electrical signal that can then be read out by a set of associated electronics. The details of
467 general PMT design and operation can be found in [24].

468 In IceCube, PMTs are housed in a single unit referred to as a digital optical module
469 ("DOM"), seen in figure 3.3. Each module contains the PMT and its associated electronics,
470 as well as a set of LED flashers that can be used to calibrate the detector once the DOMs have
471 been lowered into the ice. DOMs are arranged onto 86 vertical strings, 78 of which contain
472 60 DOMs spaced 17 meters apart along the length of the string. These strings are arranged

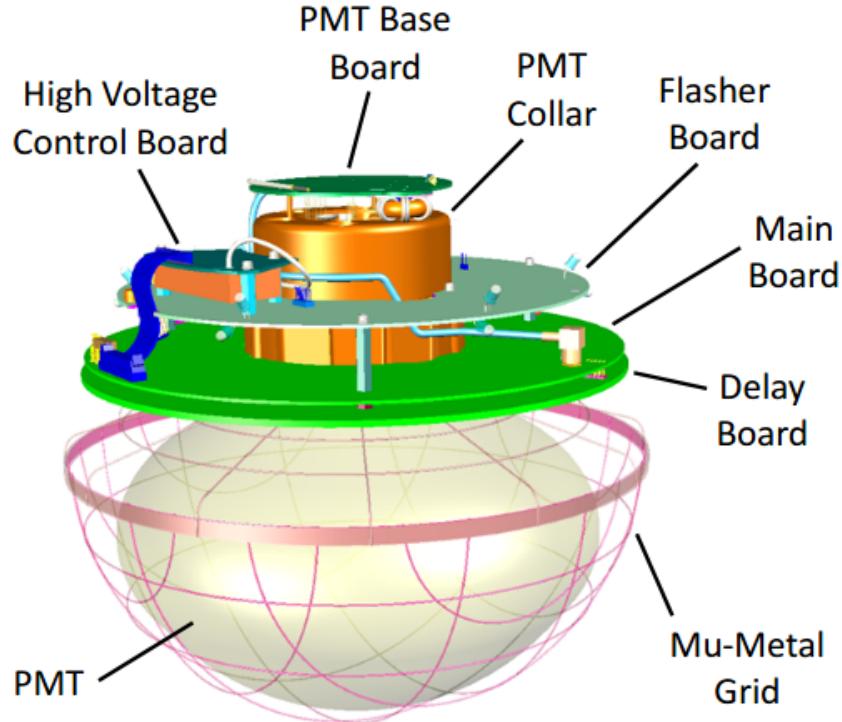


Figure 3.3: A diagram of a single IceCube digital optical module (DOM). The entire apparatus is encased in a pressurized glass sphere for protection (not shown). The PMT faces downward for optimal collection of photons from upgoing neutrino events. In addition to the PMT, each DOM also contains digitization electronics and an array of LED flashers that can be used for calibration [25].

473 into a hexagonal grid pattern under the south pole ice with a separation between strings
 474 of approximately 125 meters, resulting in a cubic kilometer of instrumented ice extending
 475 between 1450 and 2450m below the surface of the south pole ice sheet. The remaining 8
 476 strings form the DeepCore sub-array, a more densely instrumented region near the center of
 477 the detector, primarily used for studying lower (\approx 100s of GeV) scale events. PMTs that are
 478 part of the DeepCore portion of the detector have a higher quantum efficiency than those
 479 on the other 78 strings [26].

480 A diagram of the IceCube detector can be seen in figure 3.4. Due to the weakly interacting

481 nature of neutrinos combined with a power law spectrum, a large detector volume is critical
 482 to detecting the astrophysical neutrino flux. Prior to IceCube's construction, calculations
 483 based on the likely linkage of neutrinos to the observed high energy cosmic ray flux indicated
 484 that a cubic-kilometer scale detector would be necessary to observe an appreciable event
 485 rate of astrophysical neutrinos [27][28]. The size and design of IceCube are well suited for
 486 detecting astrophysical neutrino events in the TeV+ energy range, as the large instrumented
 487 volume ensures a significant number of interactions in this energy range, and the detector's
 488 nanosecond scale timing resolution makes event reconstruction possible for events producing
 489 cherenkov radiation over the scale of ≈ 100 s of meters.

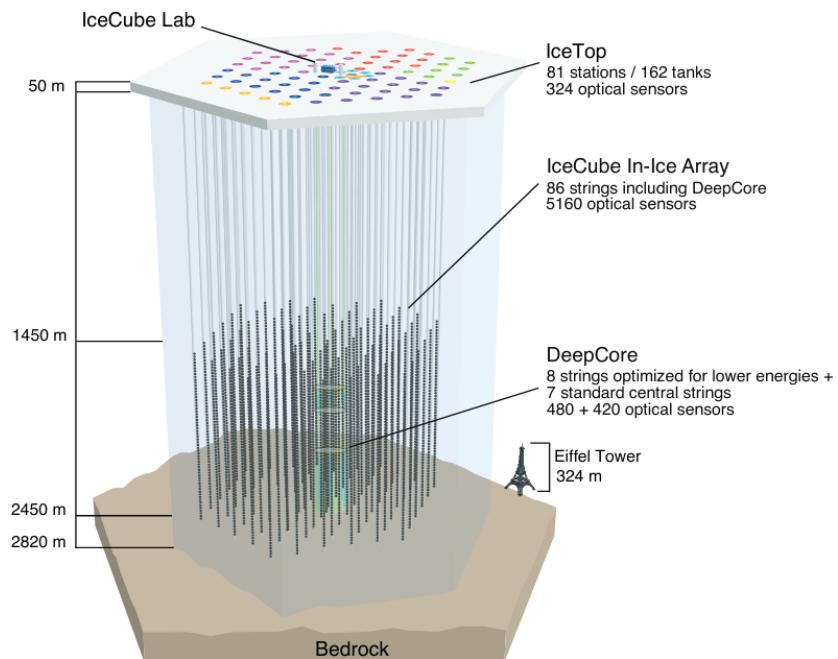


Figure 3.4: A diagram of the IceCube detector. The detector consists of 86 strings of DOMs lowered into holes drilled in the pattern of a hexagonal grid defined by a separation of 125 meters. Each string is formed by 60 DOMs connected by a cable, used for powering the DOMs and communication with the IceCube Lab (ICL) on the surface. The instrumented detector volume begins at a depth of 1450 meters, and extends down to 2450 meters, resulting in approximately a cubic kilometer of instrumented ice [25].

490 As the IceCube detector is situated in the south pole ice, a proper description of the
 491 optical properties of the ice is key for being able to accurately reconstruct events. While the

ice is generally clear, it does contain several impurities and features that make it optically nonuniform. The scattering and absorbtion of light are known to vary as a function of depth, as seen in figure 3.5. The large peak near a depth of 2000 meters is thought to correspond to a layer of dust deposited on the south pole ice sheet many years in the past, and is commonly referred to as "the dust layer" (physicists are not always the most creative with names).

In addition to varying as function of depth, the optical properties of the ice also vary as a function of azimuth. The largest effects along this axis are the ice tilt and ice anisotropy. The ice tilt refers to the layers of ice with similar optical properties being not perfectly horizontal, and results in a variation in scattering and absorption as light travels in different directions through the ice sheet. The ice anisotropy is similar, but has an observed additional axis of symmetry, leading to variations that are twice as frequent as a function of azimuth. Notably, while this effect was original observed in calibration data using LED flashers, it is visible in data from observed atmospheric muons, as seen in figure 3.6.

The final major optical nonuniformity of the detector ice is the hole ice. During the construction of the IceCube detector, holes were drilled in the south pole ice, and strings of DOMs were lowered down the appropriate depth. The water in these holes then refroze with different optical properties than the surrounding ice. The column of refrozen ice is referred to as the "hole ice", typically containing an inner region where air bubbles have been trapped referred to as the "bubble column". The hole ice has significantly shorter scattering distances, and is an area of active study within the IceCube collaboration.

3.3 Event Types

IceCube events can be generally be categorized into three different types: tracks, cascades, and double cascades. While neutral current events exclusively correspond to cascades, charged current events can produce any of the three, dependent on the variety of the outgoing lepton. This gives IceCube the ability to identify the flavor of the incident neutrino, and this fact can be leveraged to do a large amount of interesting physics. However, for the

518 purposes of this work, we are primarily interested in tracks, due to their excellent pointing
 519 resolution as discussed below.

520 **3.3.1 Tracks**

521 From equation 3.2 we can see that charged current interactions produce an outgoing lepton,
 522 which will in turn produce cherenkov radiation if it is energetic enough. In particular, if the
 523 outgoing particle is a muon, it will travel a significant distance before decaying or exiting the
 524 detector. As the muon produces cherenkov radiation as it travels through the ice, the pattern
 525 of DOMs which detect this radiation will resemble a line, as shown in figure 3.7. These events
 526 are referred to as "tracks". If the neutrino interaction occurred inside the IceCube detector,
 527 then the track will appear to start within the detector, and the event can be referred to as
 528 a "starting track". If instead the neutrino interaction occurred outside the detector, and
 529 the resultant muon traveled through the detector, then the event is instead referred to as a
 530 "through-going track" (again, physicists are not the most creative). Through-going tracks
 531 can result from astrophysical neutrinos, as well as atmospheric neutrinos and atmospheric
 532 muons (muons resulting from cosmic ray interactions in the atmosphere). Starting tracks,
 533 however, can only be produced by muons originating from atmospheric or astrophysical
 534 neutrinos. For the purposes of this work, we will focus primarily on through-going tracks,
 535 however starting tracks are also scientifically interesting, and can be used for point-source
 536 and diffuse analyses of the neutrino flux as well [30].

537 Through-going tracks are notable for their good angular resolution (up to 0.25 degrees
 538 at high energies), particularly at high energies (> 1 TeV), as seen in figure 4.1. This makes
 539 them an excellent candidate for attempting to do astronomy, as they can be expected to
 540 point back to their point of origin with reasonable accuracy. Energy reconstruction of these
 541 events is somewhat more challenging, however, due to a combination of stochastic energy
 542 losses of the muon producing the cherenkov photons, and the fact that the event itself is
 543 often not entirely contained in the detector. The energy resolution of tracks in IceCube is

544 approximately a factor of 2 at 10 TeV, though this increases for higher energy events [31].

545 3.3.2 Cascades

546 Neutral current neutrino interactions will always produce a hadronic cascade and an outgoing
 547 neutrino which is not detectable. Charged current interactions will produce a hadronic
 548 cascade in addition to an outgoing lepton. If the outgoing lepton is a muon (or potentially an
 549 extremely energetic tau), the track morphology described above is produced. If the incident
 550 neutrino is a ν_e , resulting in an outgoing electron, which in turn results in an electromagnetic
 551 shower that has an extension of ≈ 10 meters. The cascade morphology produced in this case
 552 is indistinguishable from the hadronic cascade produced by charged current interactions.
 553 While cascades tend to have relatively poor ($\approx 5^\circ$) pointing resolution [34] due to their
 554 spherical symmetry, they have excellent energy resolution, as even for high energy events, the
 555 entirety of the cascade is often contained within the detector, making energy reconstruction
 556 as simple as counting the total number of cherenkov photons produced. The energy resolution
 557 of cascades is approximately 15%, potentially even lower at higher energies [5].

558 3.3.3 Double Cascades

559 If the outgoing lepton from a CC interaction is a tau, then the tau will decay with an
 560 average decay length of 50 meters/PeV, potentially observable above energies of 200 TeV. If
 561 the initial tau is high enough energy, this can produce a signature of two spatially separated
 562 hadronic cascades in the detector. If the tau is lower energy, the two cascades may not be
 563 spatially resolvable, though may still be able to be identified by examining the waveforms
 564 observed by hit DOMs. This type of event was only recently observed in IceCube data
 565 [35]. Notably, tau events of these energies are not expected to be produced by atmospheric
 566 processes, and consequently this observation provides further evidence of an astrophysical
 567 neutrino flux.

568 3.4 Triggering

569 IceCube DOMs are subject to dark noise in the detector originating from PMTs emitting
 570 electrons from the cathode in the absence of an external photoelectron. Dark noise below 0°
 571 celsius is primarily caused by radioactivity in the PMT and DOM glass. This noise should
 572 not be correlated between nearby DOMs, and can be dramatically reduced by imposing the
 573 triggering requirement that neighboring DOMs also experience a signal with $1 \mu s$. These
 574 hits are classified as "Hard Local Coincidence" (HLC). An eight-channel simple majority
 575 trigger (SMT8) is used to trigger a readout window where DOM information is written to
 576 file. This trigger requires eight DOMs to have HLC hits within a $5 \mu s$ window.

577 Events are further filtered through several additional algorithms to pare down events
 578 based on temporal and spatial coincidence of DOM hits. The result is a "Level 2" event rate
 579 of approximately 2.5 kHz. These events can be additionally filtered to select specifically for
 580 high quality track-like events (originating mostly from atmospheric muons, but also from
 581 atmospheric and astrophysical muon neutrinos as well), reducing the event rate to a few
 582 Hz. The most relevant cuts that are used to achieve this are cuts on zenith and total
 583 charge deposited, which together form the IceCube "MuonFilter" [36]. This level of data
 584 is often referred to within the collaboration as "Muon Level 3", and primarily consists of
 585 through-going track-like events.

586 3.5 Directional Reconstruction

587 In this section we discuss algorithms for reconstructing the direction of through-going track
 588 events observed by the IceCube detector. Different algorithms may be used for cascades,
 589 though understanding these is unimportant for the remaining content of this thesis. Those
 590 who are interested in the reconstruction of cascade and double cascade events can refer to
 591 [5].

592 **3.5.1 LineFit**

593 An initial guess for the reconstructed direction of the track can be obtained by ignoring the
 594 geometry of the cherenkov cone and the optical properties of the ice, and assuming a plane
 595 wave of light traveling along a straight line in the detector. The location of each DOM that
 596 observes photons (\mathbf{r}_i) can then be written as a line (eq. 3.6):

$$\mathbf{r}_i = \mathbf{r} + \mathbf{v} \cdot t_i \quad (3.6)$$

597 Where t_i is the time that the i th DOM observes photons, \mathbf{r} is the vertex of the track,
 598 and \mathbf{v} is the velocity of light in the ice. A χ^2 fit can then be preformed to determine the
 599 free parameters \mathbf{r} and \mathbf{v} (eq. 3.7):

$$\chi^2 \equiv \sum_{i=1}^{N_{tot}} (\mathbf{r}_i - \mathbf{r} - \mathbf{v} \cdot t_i)^2 \quad (3.7)$$

600 This can be solved analytically, resulting in fitted vectors for \mathbf{r} and \mathbf{v} (eq. 3.8 and 3.9):

$$\mathbf{r} = \langle \mathbf{r}_i \rangle - \mathbf{v} \cdot \langle t_i \rangle \quad (3.8)$$

$$\mathbf{v} = \frac{\langle \mathbf{r}_i \cdot t_i \rangle - \langle \mathbf{r}_i \rangle \cdot \langle t_i \rangle}{\langle t_i^2 \rangle - \langle t_i \rangle^2} \quad (3.9)$$

601 This corresponds to a vertex (\mathbf{r}) and a direction (\mathbf{v}) describing the path of the particle.
 602 Further discussion on this topic can be found in [37].

603 **3.5.2 SplineMPE**

604 An improved reconstruction of the particle direction can be obtained by iterating on the
 605 initial LineFit reconstruction described above. The LineFit result is treated as a seed for
 606 a likelihood based reconstruction that takes the cherenkov angle and ice properties into
 607 account. A description of the specific likelihood components may be found in section 3 of
 608 [37]. In the simplest implementation of this likelihood method, only information from the

609 first photoelectron observed by a particular DOM is used. This is referred to as a single
 610 photoelectron fit, or "SPEFit". For events that produce multiple hits on a single DOM,
 611 this information can be included in the likelihood as well, producing a multi-photoelectron
 612 (MPE) fit. Since photons arriving after the first are likely to have experienced at least some
 613 scattering, a proper description of the ice properties is necessary for an accurate MPE fit.
 614 This is incorporated into the likelihood described in [37] via the use of tabulated timing
 615 and light yield distributions for various DOM/photon configurations given an ice model
 616 developed from fits to LED flasher data [38]. The information in these tables is stored via
 617 a multi-dimensional spline, allowing these tables to be used directly as PDFs in the MPE
 618 likelihood, hence the name for this variety of reconstruction: "SplineMPE".

619 3.5.3 Paraboloid

620 There is some inherent uncertainty associated with the observation and reconstruction of a
 621 particular event. For an event originating from a particular true position, the reconstructed
 622 position is expected to be drawn from distribution centered on the true position. Properly
 623 describing this distribution is key to characterizing the per-event uncertainty associated
 624 with the directional reconstruction, and is important for obtaining accurate results in a
 625 point source analysis.

626 A semi-analytic description of the angular error associated with a particular event can
 627 be obtained from the likelihood map associated with the direction reconstruction outlined
 628 in the previous section. An ellipse can be fit around the global minimum of the likelihood
 629 map, describing a 1σ containment region. The ellipse is parameterized by two variables, σ_1
 630 and σ_2 describing the scales of the two axes of the ellipse, and an average circularized error
 631 can be computed as (eq. 3.10)[39]:

$$\sigma_{evt} = \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{2}} \quad (3.10)$$

632 This description can be further improved by comparing this error with the "true error"

633 obtained from examining the difference between the simulated event direction and the cor-
 634 responding reconstructed direction. By multiplying the above circularized error by the ratio
 635 of the reconstructed error to the "true" error, we remove potential bias due to effects not ac-
 636 counted for by the directional reconstruction algorithm. For example, one such effect would
 637 be the angle between the incident neutrino and the outgoing muon. Since the reconstruc-
 638 tion algorithm only reconstructs the direction of the track associated with the muon, the
 639 direction of the neutrino is still technically unknown. While at high energies the muon and
 640 it's parent neutrino can be assumed to be colinear, this angle becomes a significant source
 641 of uncertainty below 1 TeV.

642 3.6 Energy Reconstruction

643 While the direction of track event in IceCube can be reconstructed with good precision
 644 using the methods described previously, reconstructing the energy is more difficult. The
 645 primary reason for this is simply a lack of information: since many tracks originate from
 646 events neutrino interactions outside of the instrumented volume, the entirety of the muon
 647 track is not contained within the detector. This prevents the detector from behaving as
 648 a calorimeter, and estimates must be made to extrapolate the initial event energy based
 649 on the portion of the track observed. Similar to the directional reconstruction, an energy
 650 reconstruction of a particular event can be obtained using a likelihood based approach based
 651 on the individual DOM observations.

652 At its core, this likelihood approach assumes that the number of detected photoelectrons
 653 (k) is expected to be poisson distributed with a mean directly proportional to the energy:
 654 $\lambda = E\Lambda$ (eq. 3.11), where Λ is the number of photons the event produces per unit energy:

$$\mathcal{L} = \frac{(E\Lambda)^k}{k!} e^{-E\Lambda} \quad (3.11)$$

655 Maximizing this likelihood over all j DOMs that see photoelectrons results in the relation
 656 (eq. 3.12):

$$E = \sum k_j / \sum \Lambda_j \quad (3.12)$$

657 Which is largely a statement that the event energy is proportional to the number of
 658 observed photons, scaled by some factor that is the sum of the light yield scaling functions
 659 Λ_j . These functions depend on a variety of factors including the geometry and ice optical
 660 properties. Much like in the case of the directional reconstruction, these functions are often
 661 obtained from tabulated monte carlo data, smoothed with a multi-dimensional spline.

662 This likelihood can then be applied to calculate the energy loss rate ($\langle dE/dx \rangle$) of a par-
 663 ticular muon. Above 1 TeV, this energy loss rate is roughly proportional to the muon energy.
 664 A robust estimate of this rate can be obtained by fitting segmented energy losses along the
 665 muon track using the methodology described in [40]. Once a list of segmented energy losses
 666 is calculated, there are several different approaches to converting to a reconstructed energy.

667 • **MuEX** takes the average of the segmented energy losses and uses that as an estimate
 668 of the energy

669 • **TruncatedEnergy** first removes the largest 40% of energy losses before calculating
 670 an average, ideally reducing the variance in the calculation of $\langle dE/dx \rangle$.

671 A comparison of these approaches can be found in [40], and both methods provide approx-
 672 imately 35% precision in $\log_{10} E$ for an initial muon energy of 10^4 GeV, improving slightly
 673 to approximately 30% in $\log_{10} E$ at higher energies.

674 3.7 IceCube Event Samples Overview

675 As the Icecube detector is capable of doing a wide variety of science, there exists a plethora
 676 of event samples used within the collaboration. Even within the context of searches for point
 677 sources of astrophysical neutrinos, there exist several event samples of IceCube events, each
 678 with their own selection criterion and set of reconstructed parameters. In many cases, the

679 distinguishing features between event selections used in point source analyses are purity and
 680 event rate. An ideal point source event sample would have both a high purity of astrophysical
 681 events in addition to a high event rate. Unfortunately, this is not possible with the current
 682 iteration of the IceCube data, as we observe relatively few events above 100 TeV (where
 683 the highest purity of astrophysical events can be achieved), and below 100 TeV there is a
 684 significant irreducible background of atmospheric neutrinos.

685 For this reason, point source event samples in the IceCube collaboration generally fol-
 686 low two different philosophies. "Low statistics/high purity" samples contain relatively few
 687 events, but the events in these samples have a high probability of being astrophysical in
 688 origin [30] [41]. By contrast "high statistics/low purity" samples contain a higher number
 689 of events, including additional astrophysical events. However in doing so, these samples also
 690 include significantly more atmospheric (background) events [10] [4]. In the context of point
 691 source searches, these samples typically rely on advanced statistical methods in their analy-
 692 sis pipeline to distinguish between clustered astrophysical signal and isotropic atmospheric
 693 background.

694 Even within the category of high statistics/low purity samples used for neutrino source
 695 searches, there exist multiple samples that are regularly used within the IceCube collabora-
 696 tion. The following sections will briefly outline three such samples that are relevant to the
 697 analyses presented later in this work. A comparison of the three samples outlined here can
 698 be seen in table ?? and figure 3.9

699 3.7.1 PointSourceTracks v002p03

700 Starting with Muon level 3 data, this sample applies an additional BDT to attempt to select
 701 for well-reconstructed muon neutrino interactions from mis-reconstructed atmospheric muon
 702 background. The variables used in this BDT were associated with a track-like event topology,
 703 and the BDT was trained using both background data, as well as simulation of both an $E^{-2.0}$
 704 and $E^{-2.7}$ signal. The result is a high-statistics sample of mostly track-like neutrino events

705 in the northern sky, consisting of both atmospheric and astrophysical neutrinos [33]. Events
 706 in this sample use a SplineMPE reconstruction (“plain” settings, corresponding to a balance
 707 of computation time and precision) for the event direction, and a MuEX reconstruction for
 708 the event energy.

709 This sample covers seven years of IceCube data (2008-2015), and was used for the histor-
 710 ical untriggered flare analysis of TXS0506+056 [3], where a 3.5σ neutrino flare was identified
 711 in 2014.

712 **3.7.2 PointSourceTracks v003p02**

713 Though this sample is oftentimes treated as a “successor” to PointSourceTracks v002p03, it
 714 is in many ways a completely new event selection. This sample covers 10 years (2008-2018)
 715 of IceCube data. Like v002p03, this sample also starts with Muon level 3 data, however here
 716 the BDT is trained to additionally reject cascade-like events in addition to accepting track-
 717 like events. Additionally, an improved directional reconstruction is used: the SplineMPE
 718 reconstruction algorithm is applied twice, with the second application using the results of the
 719 first as a seed, and additionally including the energy estimation. This results in improved
 720 angular resolution [42].

721 This sample additionally requires that events pass some precuts prior the the application
 722 of the BDT. Events are only selected if they satisfy the conditions below:

- 723 • The length of empty track (the portion of the reconstructed track that does not have
 724 any associated DOM hits) must be $\leq 400\text{m}$
- 725 • The reconstructed track length must be at least 200m
- 726 • The number of hit DOMs must be ≥ 12
- 727 • The number of DOMs that observe direct (non-scattered) photons must be ≥ 6

- 728 • $\cos(\theta_{geo}^2) < 0.2$, where θ_{geo} is the angle between independent reconstructions calculated
 729 for the first and second half of the track only. For a high quality track, both the first
 730 half reconstruction and the second half reconstruction should lie almost parallel.

731 This sample also has different requirements for events observed from the southern sky.

732 The minimum track length and maximum empty track length pre-cuts are applied as in
 733 the northern sky, in addition to an additional cut that the initial estimated uncertainty

734 ($\sigma_{paraboloid}$) must be less than 5 degrees. A cut on the likelihood reconstructions is also
 735 enforced, requiring $R \log(\mathcal{L}) > 9$, where $R \log(\mathcal{L})$ is the "reduced log likelihood", where the

736 maximum likelihood value, \mathcal{L} is divided by the number of degrees of freedom, n_{dof} ($n_{dof} = 5$
 737 in this particular case). Events in the southern sky are also required to have hits on more

738 than 5 different IceCube strings if the number of direct DOM hits was fewer than 12 [42].

739 This sample is notable as it was the sample used for the 10-year all-sky IceCube time

740 integrated analysis, where NGC 1068 was identified as having a significance of 3σ [4]. This

741 sample has also been publicly released for use outside the IceCube collaboration [31].

742 **3.7.3 NorthernTracks v002p06**

743 The NorthernTracks sample is a high statistics, low purity sample of through-going tracks

744 that was originally developed for the purpose of performing a diffuse fit of the atmospheric

745 and astrophysical neutrino spectrum. As such, this sample has relatively good data/MC

746 agreement (as this is essential to the diffuse fit analysis pipeline), but is restricted to only

747 events in the northern sky. Additionally, this sample does not make use of data from

748 DeepCore DOMs, a decision that was made in order to homogenize the detector. This

749 sample covers 8 years of livetime (2009-2017) has been used for point source analyses as well

750 as diffuse studies of the neutrino spectrum [32].

751 The precuts for this sample in the northern sky are identical to PointSourceTracks

752 v003p02, however this selection then uses two separate BDTs to further filter events: one

753 BDT to select for track-like events, and a separate BDT to reject cascade-like events. In

754 practice the application of these two BDTs is similar, but not perfectly identical, to the
755 application of the single BDT in PointSourceTracks v003p02. Also of note is that the BDTs
756 used in the NorthernTracks selection are trained exclusively using simulation, for both signal
757 and background.

758 This sample uses a SplineMPE directional reconstruction (with "max" settings, priori-
759 tizing precision over computation speed), and a TruncatedEnergy estimator as the energy
760 reconstruction.

	PSTracks v2	PSTracks v3	NorthernTracks
Pre-cuts	No	Yes	Yes
BDT 1	Selects tracks	Selects tracks and rejects cascades	Selects tracks
BDT 2	None	None	Rejects cascades
Signal training set	Simulation	Simulation	Simulation
Background training set	Data	Data	Simulation
Direction reconstruction	SplineMPE ("plain")	SplineMPE ("plain") $\times 2$	SplineMPE ("max")
Angular error estimator	Paraboloid	Paraboloid	Paraboloid
Energy estimator	MuEX	MuEX	TruncatedEnergy
DeepCore included?	Yes	Yes	No
Livetime	7 years (2008-2015)	10 years (2008-2018)	8 years (2009-2017)
Declination Range	$-90^\circ < \delta < 90^\circ$	$-90^\circ < \delta < 90^\circ$	$-5^\circ < \delta < 90^\circ$
Events	711,878	1,134,451	493,252

Table 3.1: A comparison of several similar high statistics/low purity event samples used by the IceCube collaboration

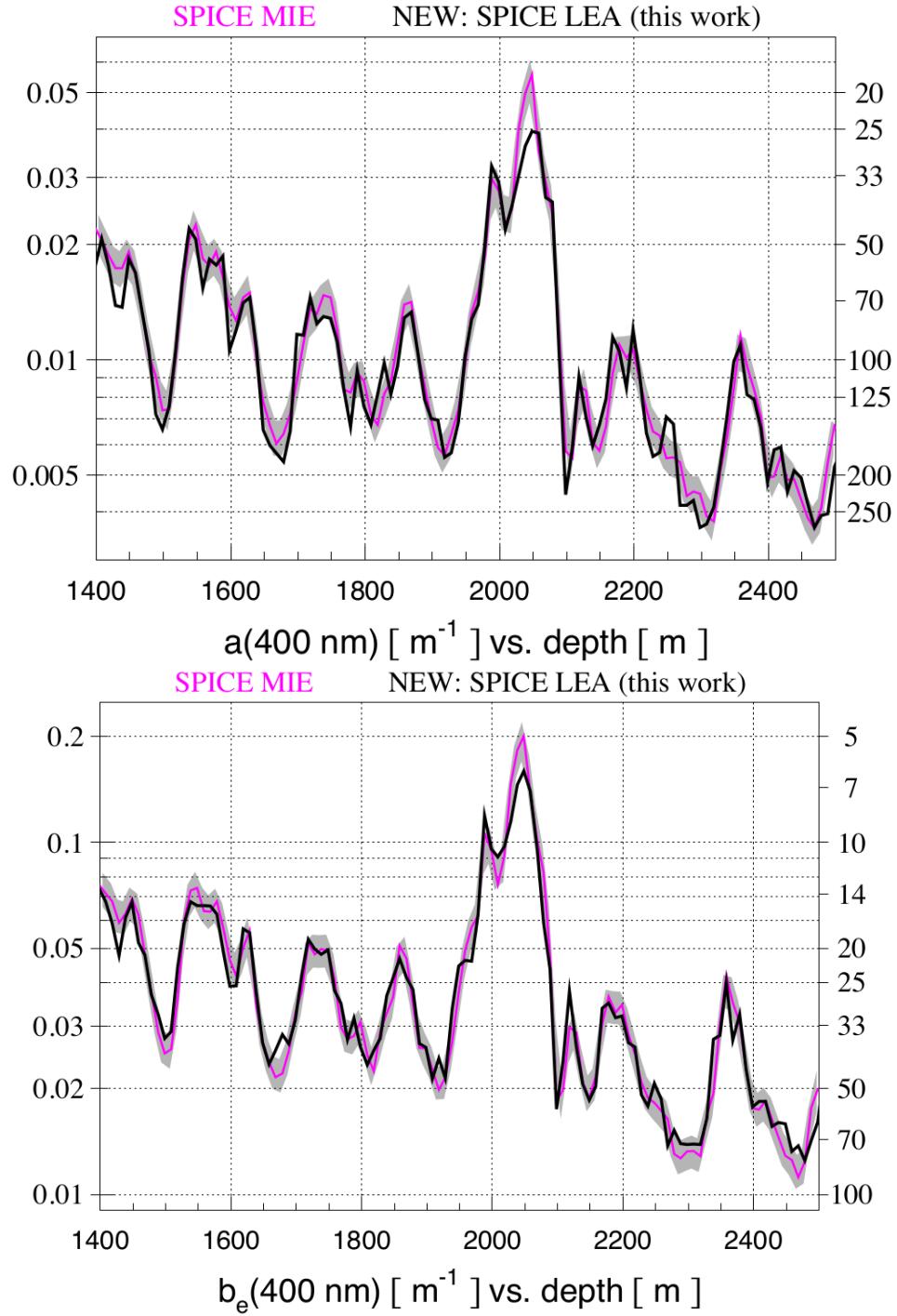


Figure 3.5: Plots of the absorption (top) and scattering (bottom) of 400 nm light as a function of depth in the south pole ice. The peak near a depth of 2000 meters is referred to as "the dust layer", and is thought to correspond to a layer of dust deposited at the south pole at some point in the Earth's history. [29]

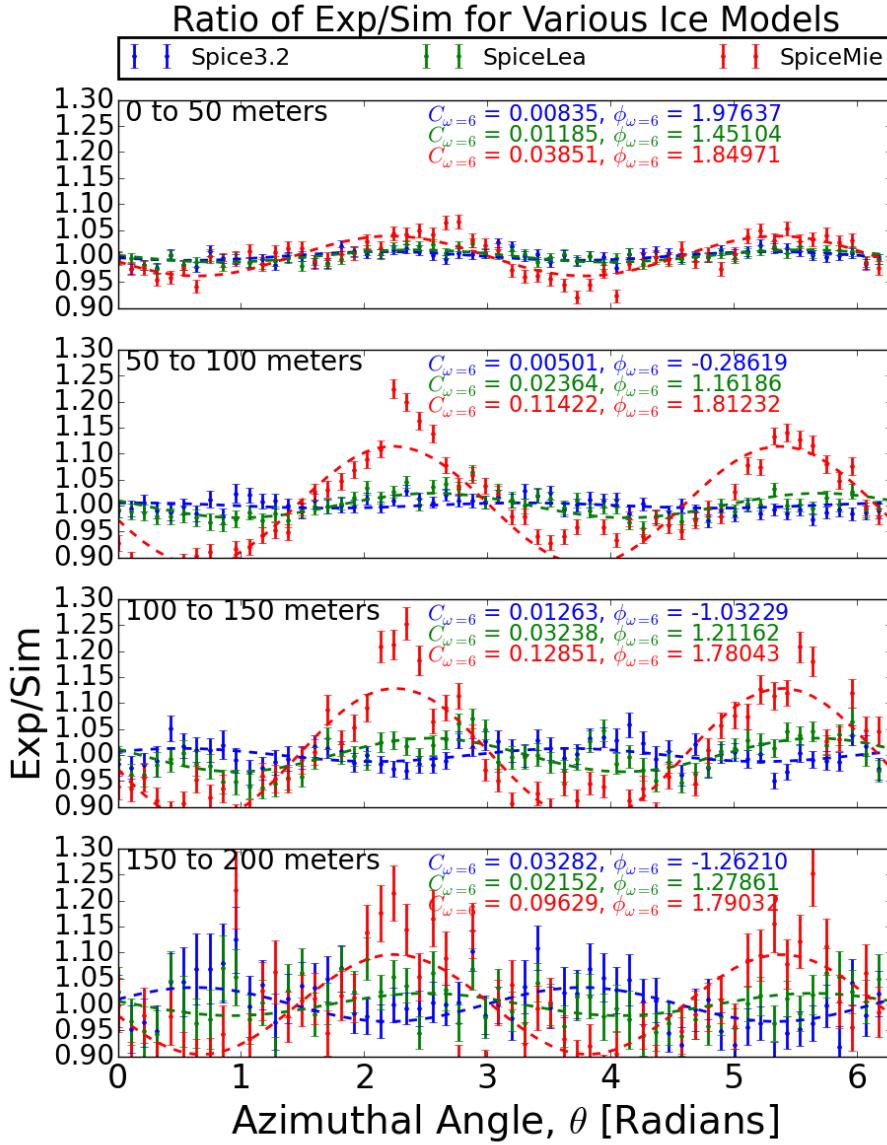


Figure 3.6: Plots of the ratio of light seen from atmospheric muons in experiment and simulation using various ice models (SpiceMie, SpiceLea, and Spice3.2), as a function of azimuthal angle between the muon track and the observing DOM. SpiceMie does not account for the ice anisotropy, resulting in a sinusoidal shape that grows with distance. SpiceLea and Spice3.2 both account for the anisotropy, and consequently the sinusoidal shape is reduced in amplitude when using these ice models.

IceCube Event Types

Early → Late

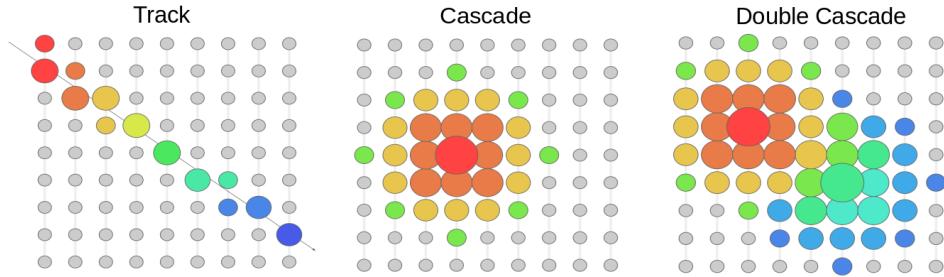



Figure 3.7: Cartoons in the x/z plane depicting the various event types seen by the IceCube detector. Grey circles are intended to represent individual DOMs, while colored circles correspond to DOMs that saw photoelectrons. The size of the circle corresponds to the amount of charge seen, while the color denotes the timing. Of most relevance to this work are tracks (left), as their long lever arm provides good pointing resolution.

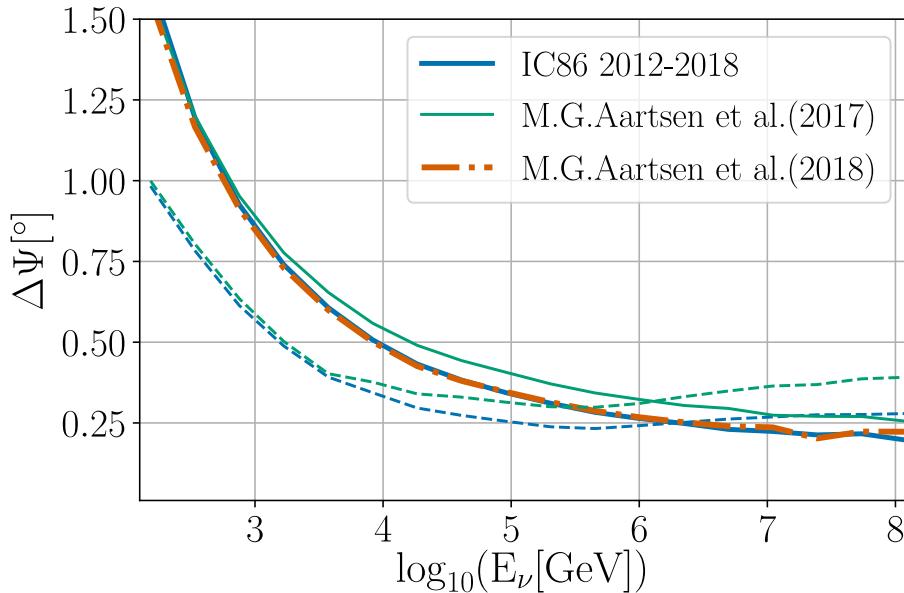


Figure 3.8: The median angle between simulated neutrino and reconstructed muon directions as a function of energy for various high-statistics IceCube samples composed of track events. The dark blue line corresponds to PSTracks v003p02, the light blue line corresponds to PSTracks v002p03, and the orange dashed line corresponds to NorthernTracks v002p06 (see section 1.7 for a description of these data samples). The solid lines describe events in the northern hemisphere, while the dashed lines are events in the southern hemisphere [4][32][33].

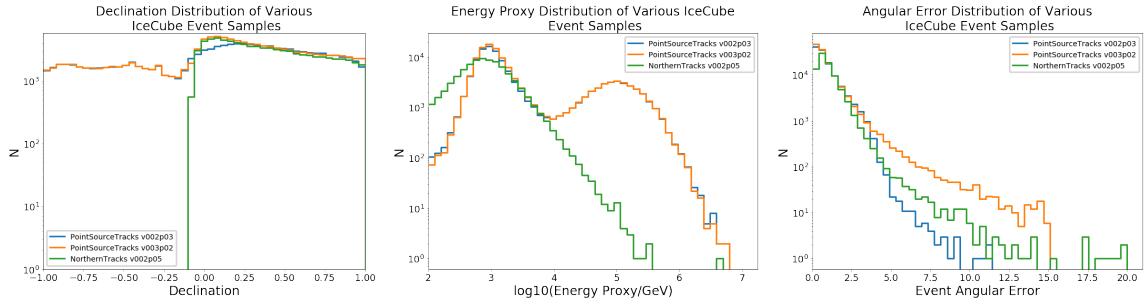


Figure 3.9: Distributions of the event variables in the three samples discussed in this section. The NorthernTracks sample does not contain events in the southern sky, as can be seen in the declination distribution on the left. This additionally has the effect of altering the overall energy proxy distribution for the sample, as the higher-energy peak in the energy proxy distribution for the PointSourceTracks samples is composed primarily of high energy southern sky events.

761 **Chapter 4**

762 **Review and Improvements to**
763 **Statistical Methods in Neutrino**
764 **Astronomy**

765 Point source searches in neutrino astronomy are essentially weighted clustering analyses:
766 each event is determined by a set of coordinates (right ascension, declination, angular error,
767 and time), and a weight (energy), and the task is to determine if high weight points are
768 clumped beyond what is expected purely from statistical fluctuations. Most attempts to
769 do this make use of a similar approach, using data-driven background estimation and a
770 likelihood-based estimator of clustering. This section outlines the general framework of such
771 an analysis, as well as several variants of the likelihood-based clustering estimator that is
772 commonly used. In the simplest case, only spatial clustering is examined, and temporal
773 information is excluded entirely ("time-integrated" analyses), however it can be additionally
774 interesting to explore the possibility of temporal clustering of events as well, increasing the
775 dimensionality of the clustering problem by one. Later in this section we introduce an
776 improved method for fitting ensembles of spatial and temporal clusters of events ("flares")
777 to characterize the data, which is an improvement over existing methods which only make
778 use of information from the most significant flare. This fills in the methodological gap that

779 can be seen in table 1.1.

780 4.1 Clustering Analysis Outline for an Arbitrary Test 781 Statistic

782 There are many ways of constructing a test statistic in neutrino astronomy, in addition to
783 a variety of types of clustering to look for (for example, clustering near various source cata-
784 logs, clustering in space and time, clustering with weights according to source/event energy
785 weights, clustering about an extended spatial feature such as the galactic plane). However,
786 almost all clustering analyses in this area share several features in their construction:

- 787 1. A test statistic is formulated, which ideally tests the degree of clustering of astrophys-
788 ical events near a particular source candidate location. This test statistic is typically
789 a likelihood-based test statistic of a form similar to that which is described in the
790 following section, however this test statistic is often modified to reflect the specifics of
791 the clustering hypothesis being tested.
- 792 2. A background test statistic distribution is calculated using maps generated from data
793 that has the right ascension values of events randomized. This destroys any cluster-
794 ing that may have already been present, providing an excellent representation of the
795 null hypothesis (a purely diffuse astrophysical neutrino flux with no spatial or tem-
796 poral clustering). As this background estimation is data-driven, it is robust against
797 unknown backgrounds that may be present in the sample. Scrambling the data does
798 not affect the sample event content, and consequently the underlying distributions of
799 event declination, energy, and arrival time are unchanged.
- 800 3. Alternative hypothesis test statistic distributions may be obtained from simulations of
801 signal events. If the analysis is not sensitive to the total astrophysical neutrino flux,
802 simulated events can simply be added to the background maps generated above. If the

analysis is sensitive to the total astrophysical flux, then more sophisticated methods (such as moving existing events instead of injecting new events) may be required. Once maps containing signal have been generated, they can be used both to characterize the performance of the analysis, as well as compute upper limits in the event of a null result.

4. The performance of the test statistic being used is typically evaluated using the *sensitivity* and/or *discovery potential*. The *sensitivity* refers to the amount of signal that needs to be injected before 90% of the test statistic distribution is greater than the median of the background test statistic distribution. Similarly the *discovery potential* refers to the amount of signal that needs to be injected before 50% of the test statistic distribution is greater than the $N\sigma$ significance threshold in the background test statistic distribution (N is typically either 3 or 5, but can be other values. This is typically clarified by stating that the value reported is the "N-sigma discovery potential"). "Amount of signal" is intentionally ambiguous here, as different analyses are sensitive to different types of signal. The sensitivity and discovery potential can be cast in terms of any number of variables that may be relevant to a particular analysis. For example, in the time-integrated case, these values are often reported as a flux, however time-dependent analyses may report a fluence, and analyses sensitive to source populations may even report their sensitivity as a curve in the space of source density and source luminosity.

5. The test statistic is calculated for the observed data sample, and this observed test statistic is compared to the distribution generated in step 2 to calculate a p-value. This p-value is then used to either accept/reject the null hypothesis of no clustering.

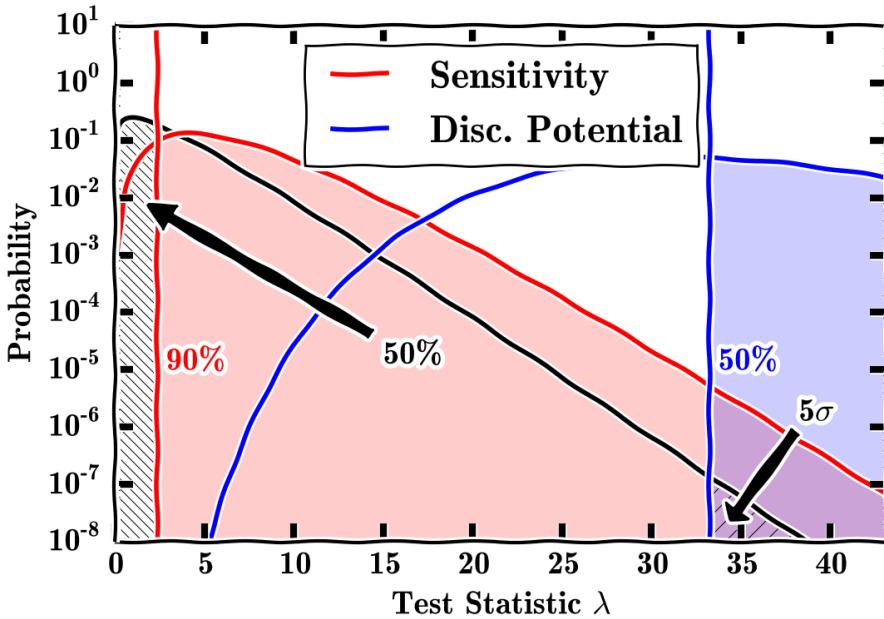


Figure 4.1: A toy plot showing the definitions of sensitivity and discovery potential. [43]

826 4.2 Unbinned Time Integrated Methods

827 Astrophysical point sources may be distinguished from the atmospheric neutrino background
 828 by way of clustering: if astrophysical sources are present in the data, events should be clus-
 829 tered near one another, while atmospheric events are expected to be isotropically distributed.
 830 Additionally, since high energy neutrino events are more likely to be astrophysical in origin,
 831 clustering of these events can be an even stronger indicator of the effect of neutrino point
 832 sources in the data. In short, we would like to construct a test statistic that reflects these
 833 observations. This can be done by way of a likelihood-based test statistic, using a likelihood
 834 composed of signal and background PDFs that describe the spatial and energy properties of
 835 events relative to a source candidate location. For a neutrino sample composed of N total
 836 events, the likelihood of the data for some number of signal (clustered astrophysical) events,
 837 n_s is (eq. 4.1)[44]:

$$\mathcal{L}(n_s, \gamma) = \prod_{i=1}^N \left[\frac{n_s}{N} S_i + \left(1 - \frac{n_s}{N}\right) B_i \right] \quad (4.1)$$

838 Where here, S_i and B_i are PDFs that describe the spatial and energy distributions

839 of signal and background events relative to the source candidate location. S_i and B_i are
 840 themselves composed of spatial and energy components (eq. 4.2 and 4.3).

$$S_i = R(r_i) \times \mathcal{E}(E_i, \delta_i | \gamma) \quad (4.2)$$

841

$$B_i = \frac{1}{\Omega} \times \mathcal{E}(E_i, \delta_i | Atm_\nu) \quad (4.3)$$

842 Note that the source spectral index, γ , enters as a free parameter in the energy portion
 843 of the signal PDF.

844 For the background PDF, the construction is relatively straightforward: background
 845 events originate from all directions equally, so the only effect that produces anisotropies is
 846 the detector acceptance. Since the IceCube detector acceptance does not vary as a function
 847 of right ascension, we can define B_i for a particular declination band to simply be $1/\Omega$,
 848 where Ω is the solid angle of a declination band centered on the source candidate declination.
 849 Similarly, the background energy PDF can be obtained by measuring the energy distribution
 850 of observed events as a function of declination. 2D histograms of event counts as a function
 851 of declination and event energy are assembled, and these distributions are then splined to
 852 create PDFs that can be used to evaluate B_i for events at an arbitrary declination and
 853 energy.

854 Unlike background events, signal events are expected to be clustered near the source
 855 candidate location. For this reason, the spatial component of the signal PDF, $R(r_i)$, is
 856 assumed to be a 2-D gaussian centered on the source candidate location (eq. 4.4):

$$R(r_i) = \frac{1}{2\pi\sigma_i^2} e^{-\frac{r_i^2}{2\sigma_i^2}} \quad (4.4)$$

857 Where r_i is the angular distance between the i th event and the source candidate location
 858 and σ_i is the event angular error.

859 The energy component of S_i is obtained from simulation: Simulated events are weighted
 860 according to a particular astrophysical spectral index, creating a 2D PDF describing the sum

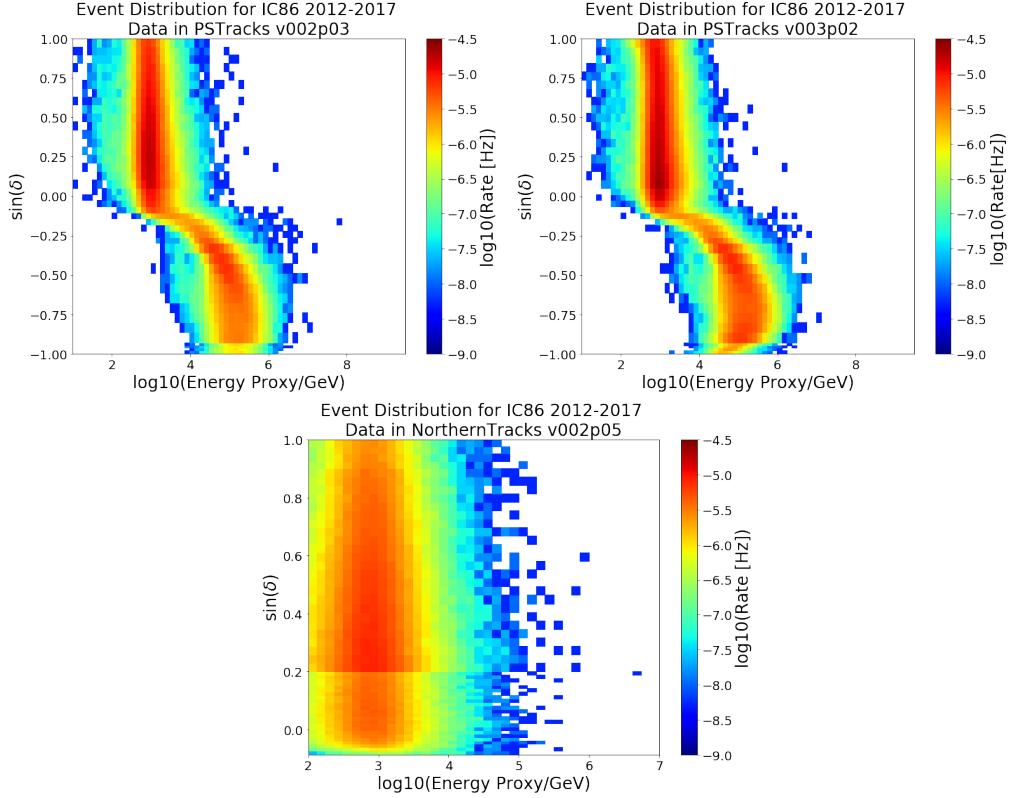


Figure 4.2: 2D histograms of declination and energy proxy for the 3 IceCube track event samples discussed in this document: PointSourceTracks v002p03 (left), PointSourceTracks v003p02 (right) and NorthernTracks v005p02 (bottom). Splines of these distributions are used as background PDFs in the clustering likelihood described in equation 4.1[31]. The NorthernTracks dataset is restricted to the northern sky, and additionally the splines associated with this dataset use finer binning near the horizon, hence the seeming discontinuity near $\sin(\delta) = 0.2$

861 of simulated event weights as a function of declination and energy. These maps are then
 862 divided by the background PDF described above to create a map describing the "signalness"
 863 of events at a particular energy and declination, given a spectral index. As these maps are
 864 created for a range of spectral index values (typically ranging between $\gamma = 1$ and $\gamma = 4$),
 865 a 3-D PDF of event energy, declination, and spectral index hypothesis results from this
 866 process. This is precisely the function $\mathcal{E}(E_i, \delta_i | \gamma)$ that enters the likelihood, and can be used
 867 to fit for γ , given events observed at declination δ with energy E_i .

868 The likelihood described in eq. 4.1 can be maximized as a function of s and γ , resulting
 869 in best fit parameters \hat{n}_s and $\hat{\gamma}$. We can then compute a test statistic for a given data sample
 870 from the likelihood ratio (4.5):

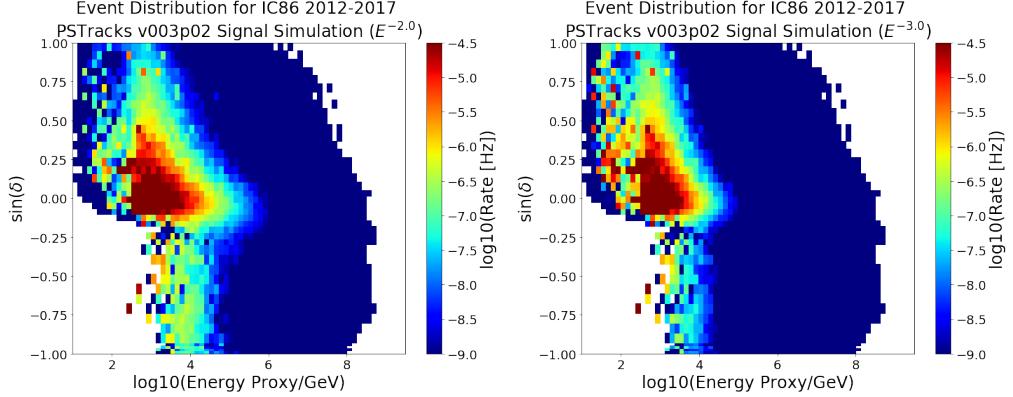


Figure 4.3: Distributions of simulated signal events in PointSourceTracks v003p02 for $E^{-2.0}$ (left) and $E^{-3.0}$ (right) spectra. The ratio of these distributions to the background expectation is then used to calculate the combined spatial and energy 2D PDF ratio ($S(r_i, E_i)/B(r_i, E_i)$) for a particular spectral index hypothesis. Similar distributions can be made for spectral indices ranging from $\gamma = 1.0$ to $\gamma = 4.0$, allowing for the signal spectral index to be fit by the likelihood.

$$TS = -2 \log \frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s, \hat{\gamma})} \quad (4.5)$$

871 This test statistic can then be used in the generalized framework described in the sections
 872 above to test the hypothesis of spatial clustering of astrophysical neutrino events near a
 873 particular source location.

874 This framework can be applied to multiple locations at once, and the results combined by
 875 summing the individual test statistics calculated at each location (note that this is equivalent
 876 to calculating the product of the likelihoods). This process is often referred to as "source
 877 stacking", and can be used to improve sensitivity to dim sources, provided there are multiple
 878 emitters in the source candidate list.

879 Other methods of combining information across multiple source locations exist as well. In
 880 particular, the binomial test is a popular way of doing this. Given a list of source candidates
 881 and their associated p-values, we can search for the most significant combination of source
 882 candidates by employing the binomial test-statistic (eq. 4.6)

$$p(k) = \sum_{i=k}^{N_{eff}} \binom{N_{eff}}{i} p_k^i (1-p_k)^{N_{eff}-i} \quad (4.6)$$

883 Where $p(k)$ is the p-value associated with combining the results of the k most significant
 884 sources, N_{eff} is the effective number of trials (often simply equal to the total number of
 885 source candidates), and p_k is the p-value of the k th most significant source. The minimum
 886 of $p(k)$ can then be computed and treated as a test statistic, to obtain a p-value associated
 887 with the best-fit number of sources (k_{min}) in a particular catalog.

888 4.3 Unbinned Time-Dependent Methods

889 In addition to being clustered in space, astrophysical neutrino events may also be clustered in
 890 time (a neutrino "flare"). Accounting for this temporal clustering may allow us to identify
 891 sources that are insignificant under a corresponding time integrated analysis. Studying
 892 the temporal variation of source candidates can also inform us of the specifics of the source
 893 dynamics of particle production, as the time scale of observed flares is related to the physical
 894 scale of the astrophysical objects in which particle production is occurring. This method
 895 was used in [3] to identify the 2014 neutrino flare candidate with a significance of 3.5σ .

896 4.3.1 Optimizing for a Single Flare

897 We can modify our time-integrated likelihood to account for potential temporal clustering
 898 as well by simply appending a temporal PDF to the existing PDFs that describe the spatial
 899 and energy components of the analysis (eq. 4.7, 4.8):

$$S_i = R(r_i) \times \mathcal{E}(E_i, \delta_i | \gamma) \times \mathcal{T}(t_i) \quad (4.7)$$

$$B_i = \frac{1}{\Omega} \times \mathcal{E}(E_i, \delta_i | Atm_\nu) \times \frac{1}{\Delta T} \quad (4.8)$$

900 Where ΔT is the full livetime of the sample, and the associated temporal background
 901 PDF is $1/\Delta T$, as the background event rate should be constant. In practice, this is assembled
 902 in a data-driven manner by measuring the data sample event rate in each of the 8-hour

903 segments ("runs") that are used to segment the data. In this way, seasonal variations of the
 904 detector event rate are accounted for by the likelihood, as the flare candidates are always
 905 being compared to the local event rate in time.

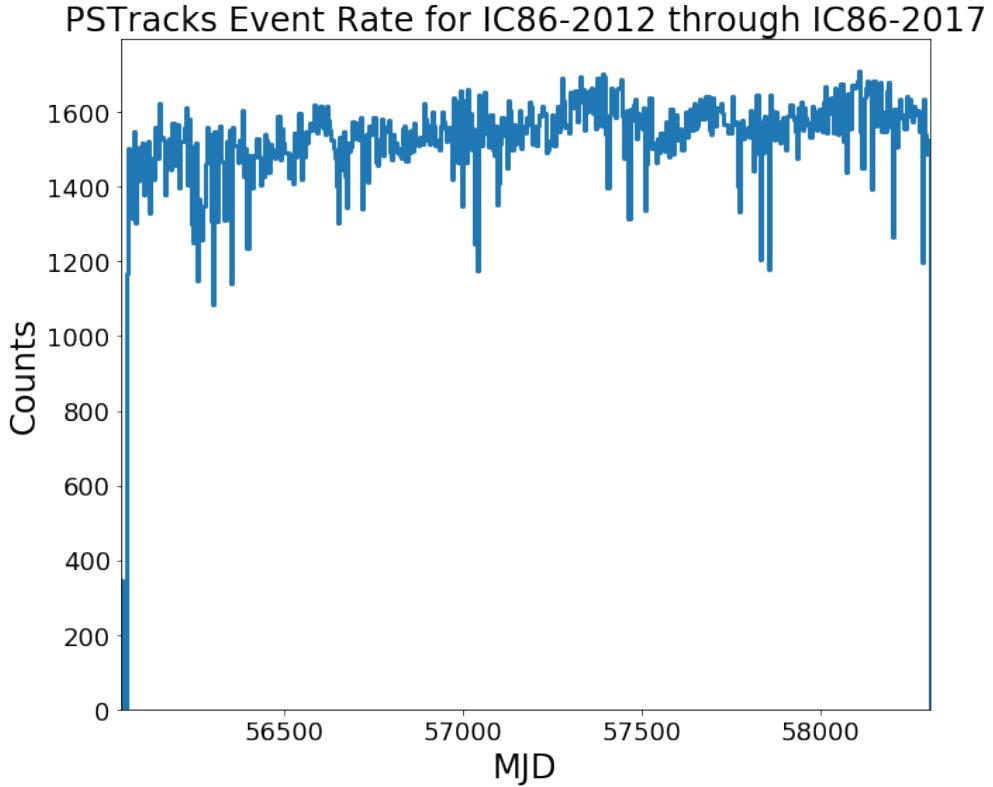


Figure 4.4: The event rate of the IC86 seasons of the PointSourceTracks v3 data sample. This event rate can be used to generate a background temporal PDF in the time-dependent clustering likelihood.

906 The temporal contribution to the signal PDF, $\mathcal{T}(t_i)$, can take various forms depending
 907 on the particular shape that one assumes for the time profile of flare candidates, but the
 908 most basic temporal profile can be used is a box (eq. 4.9):

$$\mathcal{T}(t_i|t_0, \Delta t) = \begin{cases} 0 & t_i < t_0 \\ \frac{1}{\Delta t} & t_0 \leq t_i \leq t_0 + \Delta t \\ 0 & t_i > t_0 + \Delta t \end{cases} \quad (4.9)$$

909 This box-shaped temporal PDF essentially acts as a mask, filtering for events that occur
 910 within the time window between t_0 and $t_0 + \Delta t$. The events within this time window then

contribute to the likelihood in a similar manner to the time-integrated case. Note that by adding this temporal PDF to the likelihood, we have also introduced 2 new parameters that the likelihood can be maximized with respect to: t_0 and ΔT , which describe the flare start time and duration, respectively.

Similar to the time-integrated case, a likelihood ratio test statistic can be assembled from this likelihood, however this test statistic can be further improved in the time-dependent case by accounting for the duration of flare candidates that were scanned over (eq. 4.10):

$$TS = -2 \log \left[\frac{\Delta T}{\hat{\Delta}t} \frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s, \hat{\gamma}, \hat{t}_0, \hat{\Delta}t)} \right] \quad (4.10)$$

Where the factor $\frac{\Delta T}{\hat{\Delta}t}$ can be interpreted as a trial factor that accounts for the fact that there are significantly more short flare candidates than long ones (e.g. there is only 1 potential flare candidate with $\Delta t = \Delta T$, but there are many flares that can be made with $\Delta t = \Delta T/100$). This factor normalizes the test statistic scale between short and long flares [44].

In practice, this process is extremely computationally intensive due to the number of likelihood minimizations that need to be performed. This can be mitigated by seeding flares with events that already have a high probability of being signal in origin, based off their energy and arrival direction. An ensemble of seed events can be defined to the set of events with S_i/B_i greater than some threshold value, where S_i and B_i refer to the signal and background PDF components of the time integrated likelihood (describing only the spatial and energy properties of contributing events). Typically this threshold is set to be relatively low ($S_i/B_i > 1$), however higher values can also be used if computational requirements are particularly stringent (e.g. if running over the entire sky).

Many analyses compute the test statistic in eq. 4.10 for a variety of flare candidates, and then use the flare with the maximum test statistic as the "best-fit" flare. The test statistic associated with this "best-fit" flare is then compared to a distribution of "best-fit" flares obtained from the background case (generated from data scrambled in right ascension),

resulting in a p-value that describes the strength of the clustering hypothesis relative the null hypothesis of no clustering.

In simulated signal trials, the likelihood-based flare test statistic is able to reconstruct the injected number of signal events in each flare with reasonable accuracy, and the per-flare spectral index fits show similar agreement as well.

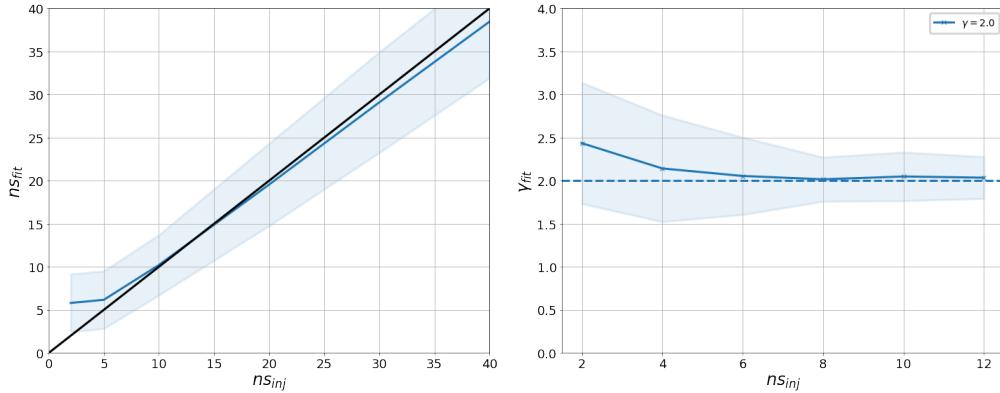


Figure 4.5: The best fit number of signal events in a flare, and the best fit spectral index versus the true number of signal events injected in simulated trials. On average, for flares with $>\approx 5$ events, the time-dependent likelihood is able to accurately estimate the true number of signal events.

4.3.2 Improving the Search for Temporal Clustering: Fitting Ensembles of Multiple Flares

As the IceCube detector collects more data, it becomes increasingly useful to be able to test the hypothesis of multiple neutrino flares that may have occurred at a particular location. Fitting only the largest flare is advantageous if the source flare rate is expected to be low enough that sources only flare once over the data-taking period, however if sources flare multiple times the single-flare analysis potentially misses a significant amount of useful information. By incorporating information from multiple flares at a particular location, an analysis can be sensitive to a smaller individual flare intensity. This is analogous to spatial stacking analyses, as can be seen in table 1.1, but we have additionally improved our algorithm to stack *flares* instead of just spatial source candidates. In this section we discuss

952 the construction of this method, and the following sections describe the results of the first
 953 applications of this method to various source catalogs.

954 The process of fitting multiple flares at a particular source candidate location is similar
 955 to the single-flare maximization procedure described above. Flare candidates are seeded by
 956 events with a high S_i/B_i ratio near the candidate location. For each flare candidate, the
 957 test statistic described in equation 4.10 is calculated. Unlike the single flare case, these
 958 flare candidates are then ordered by decreasing test statistic value, and any flares that
 959 overlap in time with a different flare that itself has a higher test statistic are removed.
 960 Additionally, any flares with a test statistic < 0 are removed. What remains is a temporally
 961 decorrelated ensemble of flare candidates, each with $TS > 0$. This ensemble functions as
 962 a neutrino "flare curve" (similar to light curves produced by photon-based telescopes), that
 963 describes the temporal variability of the signal originating from a source candidate location.
 964 A "multi-flare" test statistic describing the significance of the temporal variability of this
 965 source candidate can then be calculated as the sum of the individual flare candidate test
 966 statistics (eq. 4.11):

$$\widetilde{TS} = \sum_{j=0}^{N_{flares}} TS_j \quad (4.11)$$

967 Where here, j is an index that refers to the individual flares that compose the neutrino
 968 flare curve. This test statistic can then be compared to an ensemble of similar test statis-
 969 tics generated from right-ascension scrambled (background) data to obtain a final p-value
 970 describing the significance of the set of flares that were fit at a particular source candidate
 971 location.

972 An estimation of the total number of signal events can be obtained in a similar manner
 973 (eq. 4.12), where the total number of signal events associated with a particular source
 974 candidate is just the sum of the best-fit number of n_s in each contributing flare candidate:

$$\widetilde{n}_s = \sum_{j=0}^{N_{flares}} \hat{n}_{sj} \quad (4.12)$$

975 Note that this similar to time-integrated source stacking mentioned above, however in-
 976 stead of stacking test statistics associated with spatially distinct locations, we are instead
 977 stacking spatially and temporally distinct *flares*.

978 The multi-flare algorithm is, in some sense, inclusive of the single flare algorithm: by
 979 fitting all the flares at a source candidate location, we have obviously also fit for the largest
 980 flare. It is thus trivial to obtain the single-flare significance once the multi-flare result is
 981 obtained. In order to do this, simply compare the highest flare candidate test statistic that
 982 was obtained with a distribution of single-flare test statistics obtained in a similar manner
 983 from background (right-ascension scrambled) data.

984 In addition to calculating the local significance of the largest flare, the local significance
 985 of the other flare candidates composing the flare curve fit by the multi-flare algorithm can
 986 also be obtained in a similar manner. We can define the "local significance" of a particular
 987 flare (not necessarily the largest) to be the fraction of flares in the background distribution
 988 with $TS_j > TS_{j,observed}$. Note however, that the calculation of multi-flare significance is done
 989 in the space of TS_j , not the space of the corresponding local significances p_j . This means
 990 that the multi-flare significance is *not* simply the product of the component flare p_j 's, as each
 991 flare is not entirely statistically independent from the others (e.g. once the largest flare is
 992 fit, the remaining available livetime in which other flares can be fit is reduced by an amount
 993 equal to the duration of the largest flare.)

994 As mentioned above, the multi-flare algorithms is particularly useful in the case of several
 995 similarly sized flares. In this case, while the single flare algorithms will identify the correct
 996 number of events in the largest flare, the estimation of the total number of signal events
 997 associated with the source candidate will be incorrect (as there is a non-negligible portion
 998 of events that belong to flares that were not identified by the single flare algorithm). By
 999 contrast, the multiflare algorithm improves the estimation of n_s in the case of multiple
 1000 similarly sized flares, as signal events in all flare candidates (not just the largest one) are
 1001 able to contribute. This improvement is shown in figure 4.7.

1002 As an example of a case where the multi-flare algorithm is advantageous, consider the

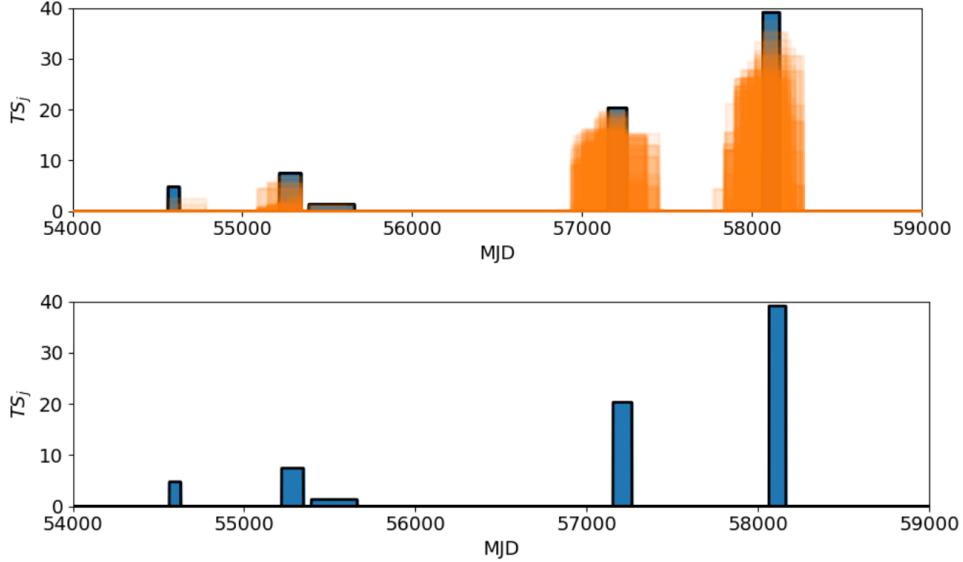


Figure 4.6: A graphical example of the progression of the multi-flare decorrelation algorithm. Colored boxes represent flare candidates that were tested, seeded by events with high S_i/B_i ratios. The height of each box corresponds to the individual flare candidate test statistic, TS_j . Orange flares overlap with another flare with a higher test statistic, and are consequently removed, leaving only the blue flares. The test statistics of the blue flares are then summed, and can be used for hypothesis testing of flaring neutrino emission at this source candidate location.

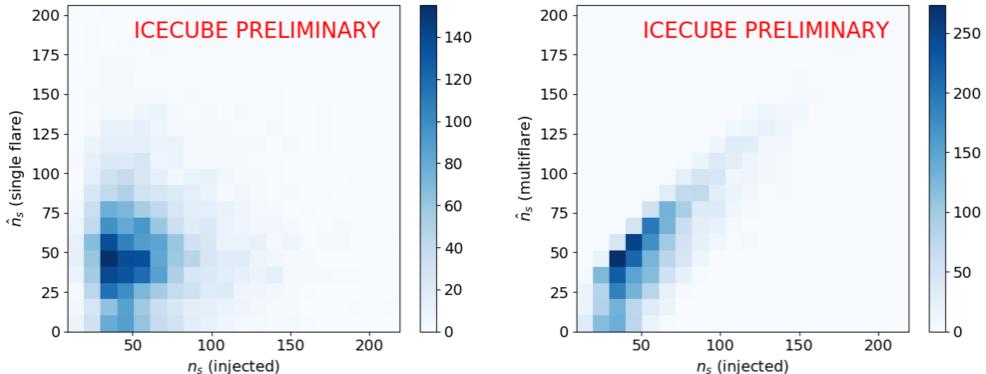


Figure 4.7: Left: the injected vs. fit number of signal events when attempting to use the single flare algorithm to describe an ensemble of neutrino flares. Right: The same plot, but using the multiflare estimation of n_s that accounts for signal events in all flare candidates.

1003 case shown in 4.8, which has 3 flares injected at MJD=55246.3, 55807.4, and 57632.7 at
 1004 a test location of $(\text{RA}, \text{Dec}) = (77.45^\circ, 5.61^\circ)$. The single flare method fits only the flare
 1005 at MJD = 55246.3 (with $\text{TS} = 22.66$), while the multiflare method fits all flares together,
 1006 with a multi-flare test statistic of $\widetilde{\text{TS}} = 60.52$. While the largest individual flare only has

1007 a significance of 3.29σ , by combining information from all the flares that were fit at this
 1008 location, we arrive at a multi-flare significance of 4.7σ .

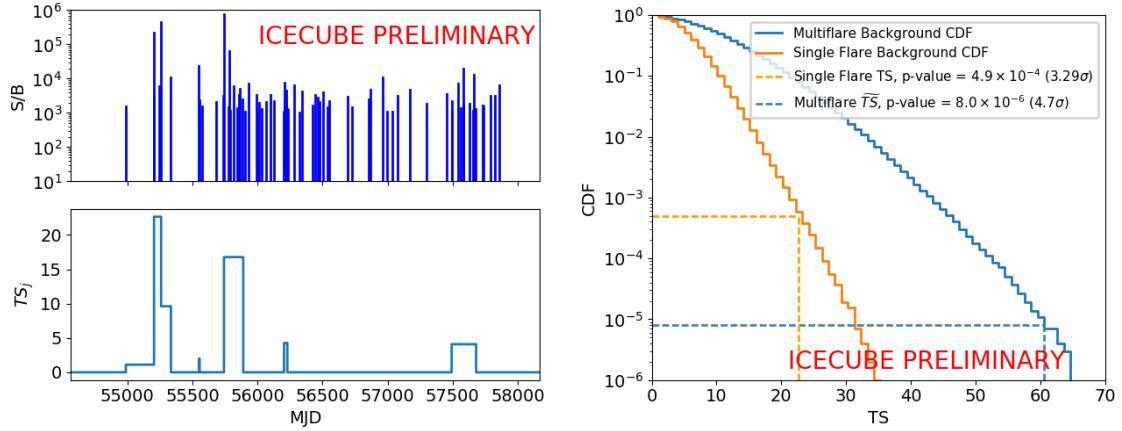


Figure 4.8: Left, top: Events that pass the S/B threshold cut for generating windows, for a source located at $(\text{ra}, \text{dec}) = (77.45^\circ, 5.61^\circ)$. There are 3 injected flares, centered at $\text{MJD}=55246.3$, 55807.4 , and 57632.7 . Left, bottom: The single flare method fits only the flare at $\text{MJD} = 55246.3$ (with $\text{TS} = 22.66$), while the multiflare method fits all flares together, with a global test statistic of $\bar{\text{TS}} = 60.52$. Right: The background test statistic distributions for a single source, located at declination = 5.61° . The background multiflare test statistic distribution is shown in blue, while the single flare method is shown in orange. The vertical lines represent the test statistics associated with the flare curve on the left. The single flare p-value for this source is 4.9×10^{-4} (3.29σ), while the multiflare p-value is 8.0×10^{-6} (4.7σ).

1009 **Chapter 5**

1010 **Applications of the Multi-flare
1011 Algorithm to Source Catalogs**

1012 The multi-flare algorithm introduced in the previous section may be applied to an ensemble
1013 of sources that share common features. This is common practice in neutrino astronomy, as
1014 examining emission from a catalog allows us to explore the possibility of neutrino emission
1015 from a class of sources, rather than a specific individual source. Here, we explore two
1016 catalogs designed to explore source features related to those associated with the analysis of
1017 TXS 0506+056. Namely, the fact that the analysis was initially triggered by a high-energy
1018 neutrino event, and also the fact that TXS 0506+056 is a blazar. For the former, we assemble
1019 a catalog of high energy IceCube events to treat as "sources" (a "self-triggered" catalog
1020 analysis), while for the latter we use the pre-existing catalog of Fermi 3LAC blazars [45].

1021 In both cases, the multi-flare algorithm is applied at each source candidate location,
1022 resulting in a multi-flare test statistic (and corresponding pre-trial p-value) associated with
1023 each source candidate. To test for potential sub-populations of strong emitters within the
1024 catalog, we additionally calculate a best-fit number of multi-flare sources via iteratively
1025 summing the sources with the largest test statistics (5.1). For a given data set, the sources are
1026 ordered by their multiflare test statistic, \widetilde{TS} . A p-value for $k = 1, 2, 3, \dots N_{srcs}$ is calculated,
1027 and subsequently the k that produces the minimum p-value is selected (k_{best}).

$$\widetilde{TS}_{all} = \sum_{m=0}^{k_{best}} \widetilde{TS}_m \quad (5.1)$$

1028 An additional trial factor is then calculated by applying this procedure to maps of data
 1029 with randomized right ascension values to assemble a distribution of \widetilde{TS}_{all} representative of
 1030 the null hypothesis. A final p-value can then be obtained by comparing an observed \widetilde{TS}_{all}
 1031 with this null hypothesis distribution.

1032 For all the analyses detailed in this section, the NorthernTracks v002p05 sample was used.
 1033 This sample is described earlier in this document (section 2.5).

1034 5.1 Self-Triggered Catalog

1035 For this catalog, we select the locations of all the events in the NorthernTracks v002p05 data
 1036 sample that have a reconstructed energy proxy greater than 200 TeV. This energy threshold
 1037 is chosen as it is roughly the point where 50% or more of neutrino events are expected to
 1038 be astrophysical in origin. We can consequently expect that this catalog contains a non-
 1039 negligible number of astrophysical neutrino sources, and can test the hypothesis of additional
 1040 time dependent neutrino emission from these locations. Events that have been selected as
 1041 source candidates for the catalog are subsequently removed from the sample, and do not
 1042 otherwise contribute to the calculation of a multi-flare test statistic.

1043 This catalog contains 32 source candidates, all in the northern sky. Notably, this cat-
 1044 alog does not include IC170922A (the alert event that triggered the followup analysis on
 1045 TXS 0506+056), as the data sample does not extend past the 2015 season, and IC170922A
 1046 occurred in the 2017 season.

1047 Applying the multi-flare algorithm described above to this catalog of the locations of
 1048 32 high energy neutrino events results in a best-fit number of sources of $k_{best} = 4$, with
 1049 an associated p-value of $p = 0.017$ (2.13σ), shown in figure 5.5. Detailed results for all 32
 1050 source candidates, including fitted number of events and pre-trial p-values, can be viewed in
 1051 table 5.1.

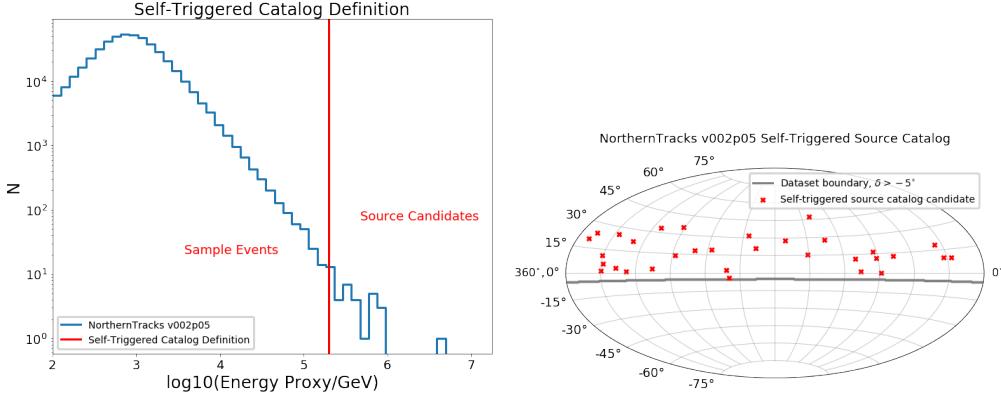


Figure 5.1: Left: The self-triggered catalog definition: Events with energy proxies to the right of the vertical red line (200 TeV) are treated as source candidates. Events to the left of the red line are then investigated for clustering around the locations of the source candidate events. Right: The self-triggered catalog assembled from the NorthernTracks v002p05 data sample, consisting of all events in the sample with reconstructed energy proxy greater than 200 TeV. The locations of these events are treated as source candidates for the purposes of applying the multi-flare algorithm, however the events themselves are removed from the sample prior to calculating a test statistic.

1052 As mentioned in previous sections, a significant advantage of the multi-flare algorithm is

1053 the generation of neutrino "light curves" (or "flare curves") that show the historical activity

1054 of a source candidate. The flare curves associated with the 4 most significant sources in the

1055 self-triggered catalog can be seen in figure 5.3.

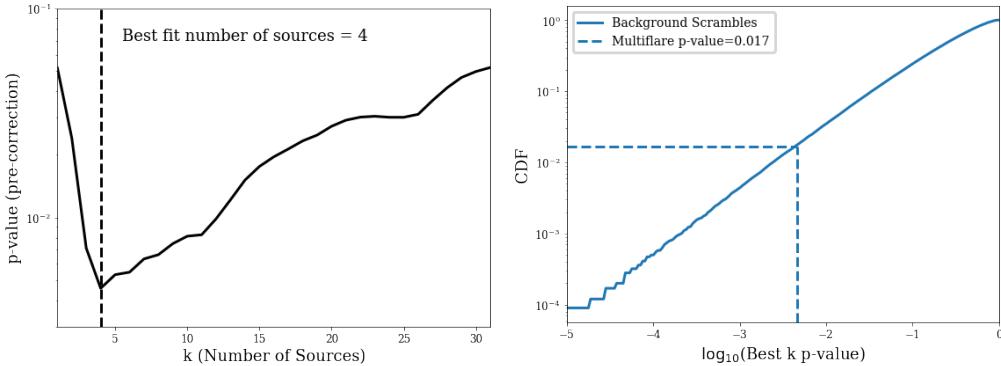


Figure 5.2: Left: The local significance of stacking the k highest test statistic sources, as a function of k . The best fit value of $k = 4$ can be seen as the minimum of this curve. Right: The trial corrected result for stacking the top 4 sources together, shown as a vertical line superimposed on the background distribution obtained by applying the algorithm to right-ascension scrambled data.

1056 Though not statistically significant, these results have several interesting features. Though

RA	Dec	\hat{n}_s	p (pre-trial)
36.69	18.32	47.21	0.00197
272.14	35.66	30.71	0.00729
170.19	27.85	45.75	0.00834
93.26	16.33	30.02	0.02667

Table 5.1: The top 4 most significant multi-flare source candidates in the self-triggered catalog composed of high energy IceCube neutrino events. Collectively, these sources form an excess with a significance of $p = 0.017$.

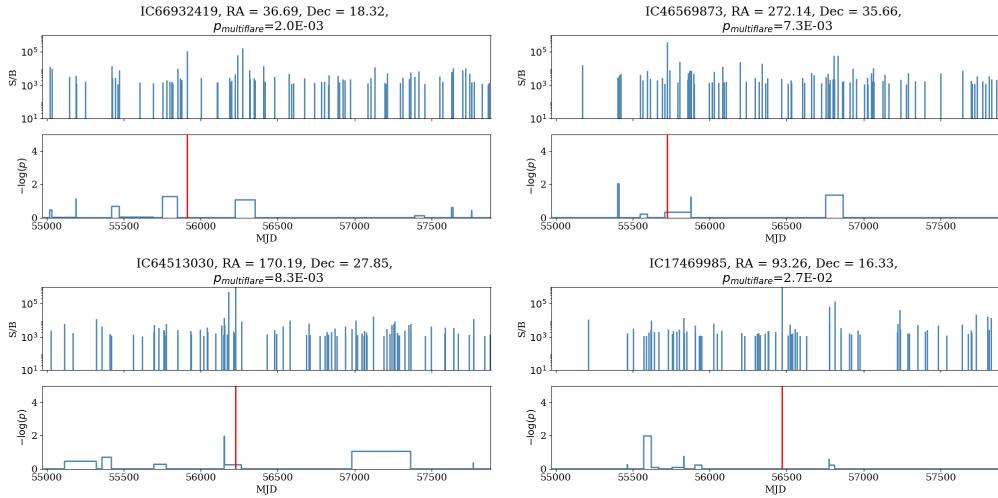


Figure 5.3: The neutrino "flare curves" associated with the 4 most significant multiflare source candidates in the self-triggered catalog. The top panel of each subplot shows the event weights, calculated by taking the ratio of the spatial and energy components of the signal and background PDFs described in equation 4.2 and 4.3, while the bottom panels show the fitted ensemble of decorrelated flares, with the local per-flare p-value plotted on the y-axis. The vertical red line denotes the arrival time of the high energy event used to define the source candidate location. This event is removed from the sample prior to applying the multiflare-algorithm, and consequently these events do not contribute to the flare curves shown in this figure.

1057 the high-energy "seed" events are removed prior to the calculation of a flare curve, 7 of the
 1058 flare curves generated include flares that would have included a high energy seed event.
 1059 Though not statistically significant, this is certainly above average, as only 11% of back-
 1060 ground trials have 7 or more flare curves with seed event/flare correlations.

1061 It is also potentially interesting to investigate the distributions of the parameters fit by
 1062 the multi-flare likelihood. By comparing the observed distributions with those expected from
 1063 background scrambles, we can check for inconsistencies of our data with the null hypothesis.

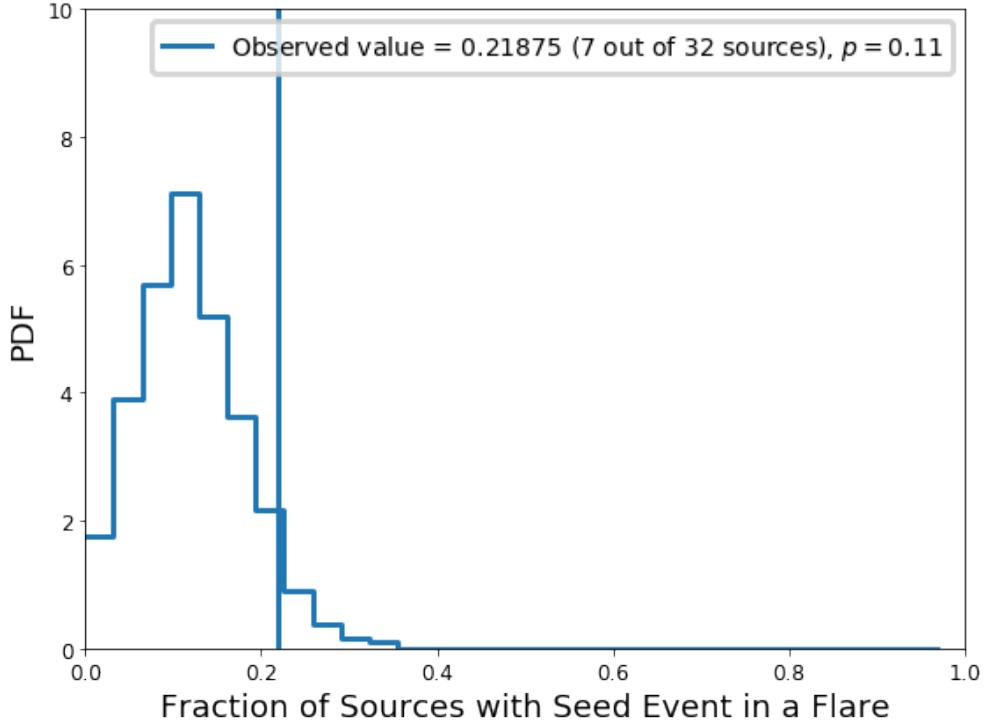


Figure 5.4: The distribution of number of flare curves that have a temporal correlation between the high energy seed event, and a fitted flare, obtained from sets of data where the right ascension of events has been randomized. The observed value (7 out of 32 sources, or 21.9%) is shown as a blue vertical line. 11% of trials have more than 7 correlations between seed events and fitted flares.

1064 5.2 Fermi 3LAC Blazars

1065 The 2014 TXS 0506+056 neutrino flare was notable not only for its association with a high
 1066 energy IceCube alert, but also for its spatial coincidence with the blazar TXS 0506+056. In
 1067 searching for additional neutrino sources, it is then not unreasonable to assemble a search
 1068 for neutrino flares associated with a large catalog of blazars. For this purpose, we use the
 1069 Fermi 3LAC catalog [45] to define a set of source candidates. Previous analyses searching
 1070 for spatial coincidence with earlier iterations of this catalog have been performed, but were
 1071 unable to identify any statistically significant excess [46]. Here, we instead specifically search
 1072 for transient emission using multi-flare algorithm described above.

1073 The Fermi 3LAC catalog is a catalog of AGNs detected by Fermi-LAT, consisting of
 1074 gamma ray sources in the third Fermi-LAT catalog (3FGL) [47] between 100 MeV and

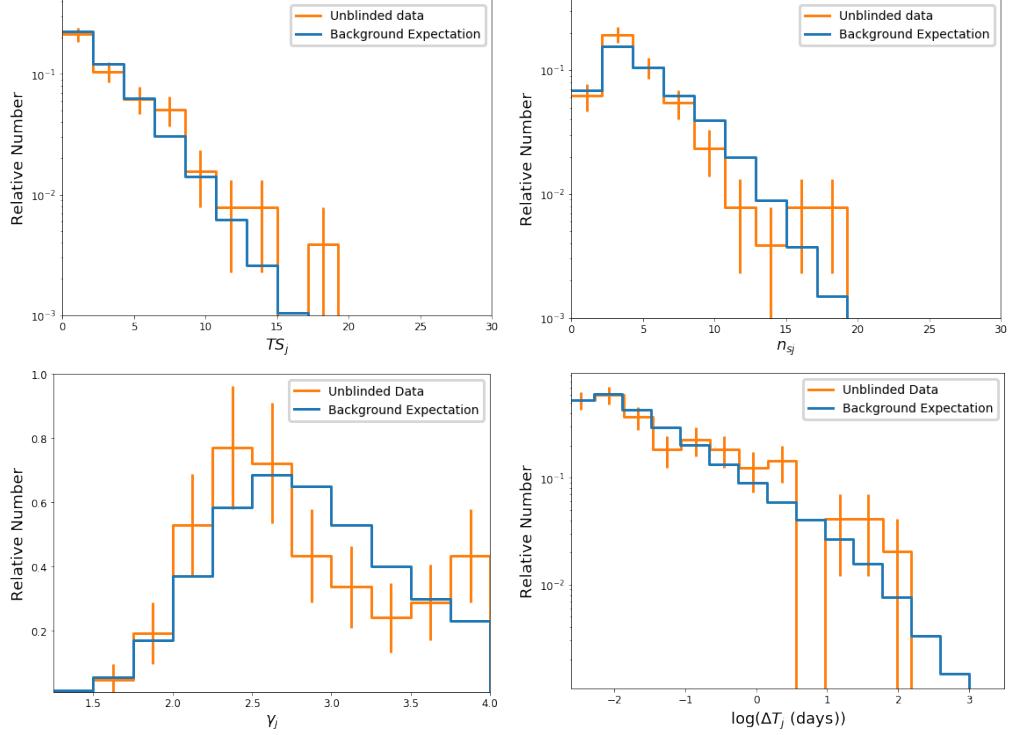


Figure 5.5: Distributions of fitted flare parameters for flares associated with the self-triggered catalog in both background scrambles (blue) and unblinded data (orange). While there is potentially some deformation in the distribution of fitted spectral indices (bottom left), a 2-sample K-S test comparing the observed and background distributions only returns a p-value of $p = 0.26$, indicating that the blue and the orange distributions are not significantly inconsistent with one another.

1075 300 GeV with a Fermi test statistic greater than 25 between August 4, 2008, and July 31,
 1076 2012 [45]. The catalog contains 1591 objects, the majority (98%) of which are blazars,
 1077 roughly evenly split between FSRQs and BL Lacs. Notably, TXS 0506+056 is a member of
 1078 this catalog.

1079 In constructing this analysis, we select for blazars at declinations greater than -5° , as
 1080 the IceCube data sample (NorthernTracks v002p05) does not extend below this point. We
 1081 impose no further cuts, and all sources are weighted equally when calculating a multi-flare
 1082 test statistic. The final catalog to be used for the multi-flare analysis consists of 1023 blazars,
 1083 the locations of which are shown in figure 5.6.

1084 The results of applying the multi-flare algorithm to this catalog of 3LAC blazars can be
 1085 summarized in figure 5.8. The optimization procedure for the most significant combination

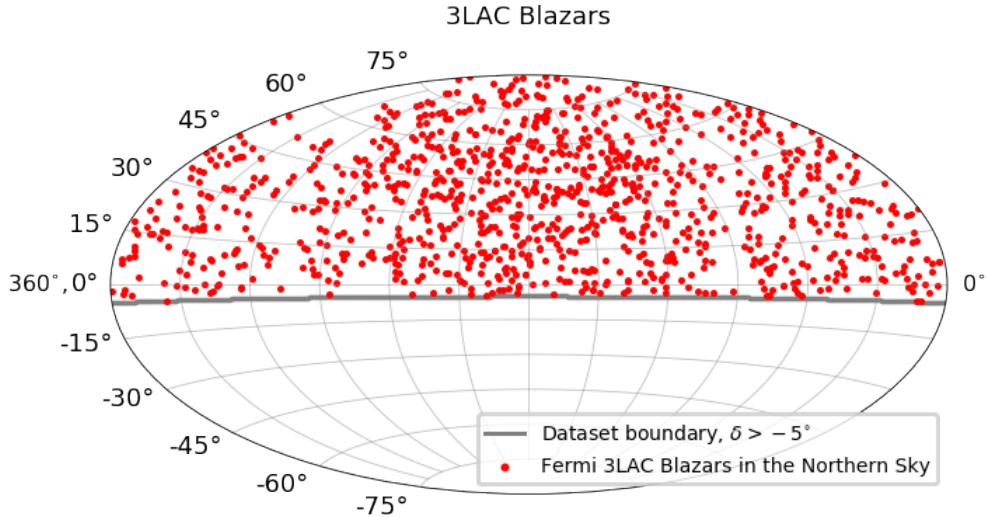


Figure 5.6: The locations of the 3LAC blazars that compose the catalog used for a multi-flare analysis. Blazars with $\delta < 5^\circ$ are not considered, as the IceCube NorthernTracks v002p05 data sample does not extend into this region. There are 1023 blazars that are considered as source candidates for the multi-flare analysis.

1086 of sources returned a best fit number of sources of $k = 125$, with an associated post-trial
 1087 p-value of $p = 0.06$. A list of these source candidates can be seen in table 5.2. As the
 1088 significance of this excess is consistent with the null hypothesis, we do not claim discovery
 1089 of neutrino flares associated with 3LAC blazars.

1090 Interestingly, despite the presence of the 2014 neutrino flare identified in [3], TXS 0506+056
 1091 is not the most significant source candidate, having a pre-trial p-value of only $p = 9.24 \times 10^{-3}$.
 1092 This is not unexpected, considering that there does not appear to much activity (in terms
 1093 of neutrino flares) at this location beyond the 2014 flare itself. As the multi-flare algorithm
 1094 performs will return a high significance when there are multiple, moderately significant flares
 1095 to stack together, it is unsurprising that there are other sources that have a higher multi-flare
 1096 significance. As an example, the source candidate with the highest multi-flare significance is
 1097 1RXS J154604.6+081912. Rather than having a single large flare, the multi-flare algorithm
 1098 fits multiple flares at this source candidate location, which combined have a significance of
 1099 $p = 8.1 \times 10^{-5}$, despite none of the individual flares having a local significance much greater
 1100 than $p = 0.01$.

1101 Similar to the self-triggered catalog, we can additionally examine the distributions of

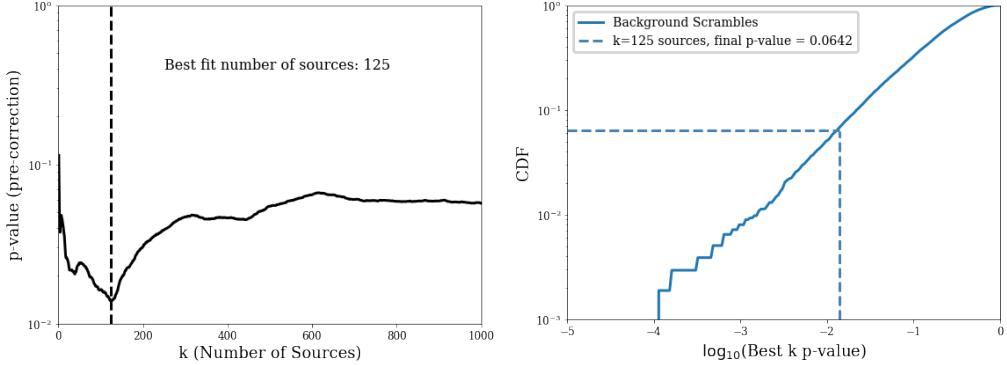


Figure 5.7: Left: The local significance of stacking the k highest test statistic sources in the 3LAC blazar catalog, as a function of k . The best fit value of $k = 125$ can be seen as the minimum of this curve. Right: The trial corrected result for stacking the top 125 sources together, shown compared the background distribution obtained by applying the algorithm to right-ascension scrambled data. As the final p-value associated with the top 125 blazar sources is only $p = 0.06$, there was no significant excess of neutrino flares observed to be associated with 3LAC blazars.

1102 the flare parameters that were fit for all the sources in the 3LAC catalog. No significant
 1103 deviations from the background expectation are observed in any of the flare parameter
 1104 distributions, as seen in figure 5.8.

1105 As with the self-triggered catalog, we also obtain neutrino flare curves describing the
 1106 historical variability of each source candidate. Figure 5.9 shows the flare curves of several
 1107 3LAC source candidates of note, including TXS 0506+056. While none of these sources are
 1108 significant enough to claim discovery, flare curves like the ones shown here are a potentially
 1109 valuable tool for multi-messenger analyses in the future that may seek to correlate neutrino
 1110 emission with other astrophysical messengers.

1111 Though the application of the multi-flare algorithm to this catalog of 3LAC blazars did
 1112 not result in a significant detection, it does allow us to constrain the behavior of neutrino
 1113 flares associated with 3LAC blazars. Given that this method stacks flares together, the lack
 1114 of a significant results suggest that there is an upper limit to how bright and numerous
 1115 blazar neutrino flares may be. We express these limits in terms of the flare rate (how many
 1116 flares occurred over the lifetime of the data sample), and the per-flare E^2 flux. This allows
 1117 us to compare to previously calculated limits obtained from a time-integrated analysis [46].

1118 These upper limits can be seen in figure ??

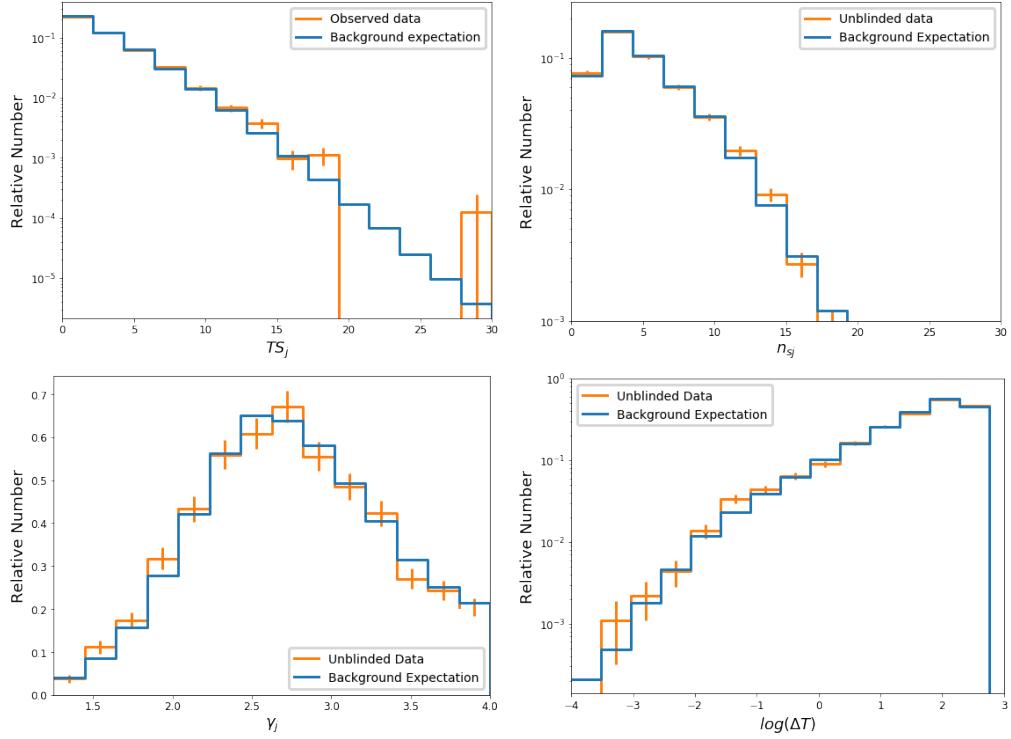


Figure 5.8: Distributions of fitted flare parameters for flares associated with the 3LAC blazar catalog in both background scrambles (blue) and unblinded data (orange). All distributions of fit parameters from observed data appear to be consistent with the background expectation. As this catalog includes TXS 0506+056, the 2014 neutrino flare originally discovered in the IC170922A follow-up analysis [3] is visible as the rightmost entry in the histogram of flare TS_j values in the plot in the upper left, having a value of $TS_j = 27.9$. There are no other flares that were fit in this catalog with $TS_j > 20$.

Table 5.2: The top 125 multiflare source candidates in the 3LAC blazar catalog.

Source Candidate Name	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1RXS J154604.6+081912	bll	236.52	8.32	53.49	8.11e-5
RBS 1467	bll	227.18	27.15	55.11	3.05e-4
GB6 J0723+2859	fsrq	110.98	28.99	40.48	4.58e-4
RBS 1558	bll	241.59	56.51	28.88	1.92e-3
PMN J2324+0801	bll	351.19	8.04	28.15	3.11e-3

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
B2 2214+24B	bll	334.25	24.36	36.90	3.79e-3
GB6 J0850+4855	bll	132.50	48.92	38.18	4.75e-3
4C +20.25	fsrq	171.49	20.10	27.83	5.02e-3
MG2 J094148+2728	fsrq	145.45	27.48	60.58	5.12e-3
TXS 2241+406	bll	341.05	40.95	22.61	5.23e-3
TXS 0213+619	bcu III	34.26	62.19	30.15	5.32e-3
GB6 J0100+0745	bll	15.09	7.76	37.33	6.45e-3
RX J0850.5+3455	bll	132.65	34.92	20.42	7.16e-3
B2 1436+37B	fsrq	219.72	37.18	49.47	7.83e-3
MG1 J165034+0824	fsrq	252.66	8.41	39.19	7.83e-3
PKS 0256+075	fsrq	44.86	7.79	42.53	9.04e-3
TXS 0506+056	bll	77.36	5.69	20.71	9.24e-3
1ES 1421+582	bll	215.66	58.03	28.49	0.0100
TXS 0518+211	bll	80.44	21.21	48.49	0.0118
W Comae	bll	185.38	28.23	30.32	0.0123
NVSS J141828+354250	bcu II	214.62	35.71	40.74	0.0128
PKS 1532+01	fsrq	233.72	1.52	30.78	0.0131
PKS 1424+240	bll	216.75	23.80	52.78	0.0148
B3 2319+444	fsrq	350.58	44.76	41.93	0.0160
PKS 0039+230	fsrq	10.52	23.33	48.89	0.0168
B3 2238+410	bll	340.28	41.34	21.24	0.0175
TXS 2315+189	bcu II	349.60	19.25	31.98	0.0197
B3 2322+396	bll	351.32	39.96	30.07	0.0199
NVSS J080637+774607	bcu II	121.66	77.77	52.32	0.0215
MG1 J010908+1816	bll	17.28	18.27	26.59	0.0220

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1H 0323+342	nlsy1	51.17	34.18	10.09	0.0221
NVSS J131921+775823	bcu II	199.84	77.97	16.99	0.0224
RX J1351.3+1115	bll	207.84	11.25	27.93	0.0228
RGB J1808+468	bll	272.00	46.83	29.19	0.0252
RX J1149.5+2439	bll	177.38	24.66	33.52	0.0266
TXS 2157+102	bll	330.03	10.50	47.04	0.0271
S5 1357+76	fsrq	209.48	76.72	35.44	0.0274
S5 1803+784	bll	270.19	78.47	27.73	0.0287
GB6 J1439+4958	bll	219.95	49.97	32.32	0.0291
B2 2234+28A	bll	339.09	28.48	25.36	0.0298
GB6 J0929+5013	bll	142.31	50.23	20.09	0.0306
RGB J2054+002	bll	313.74	0.26	26.79	0.0315
1ES 1028+511	bll	157.83	50.89	35.27	0.0333
GB6 J0331+6307	bcu II	52.97	63.14	26.86	0.0335
PKS 2320-035	fsrq	350.88	-3.28	15.57	0.0350
GB6 J0934+3926	bll	143.53	39.44	32.46	0.0352
B2 1811+31	bll	273.40	31.74	35.37	0.0356
GB6 J0937+5008	fsrq	144.30	50.15	27.81	0.0357
TXS 1015+057	fsrq	154.62	5.51	19.30	0.0361
TXS 1614+473	fsrq	243.92	47.19	16.66	0.0374
4C +04.42	fsrq	185.59	4.22	19.22	0.0376
MG2 J110606+2812	fsrq	166.53	28.21	41.99	0.0396
RX J1246.9+4423	bll	191.75	44.39	22.30	0.0396
B2 2308+34	fsrq	347.77	34.42	41.08	0.0408
TXS 2106-030	bll	317.19	-2.84	16.86	0.0410

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
NVSS J125820+612049	bll	194.59	61.35	36.36	0.0425
SBS 0812+578	bll	124.09	57.65	41.15	0.0426
WN B1609.6+8517	bcu II	240.13	85.16	37.14	0.0434
PMN J2227+0037	bll	336.99	0.62	28.56	0.0440
1RXS J234332.5+343957	bll	355.89	34.66	25.27	0.0440
GB6 J0529+0934	bcu II	82.26	9.58	32.69	0.0450
MG2 J131037+2447	bcu III	197.66	24.81	27.71	0.0458
3C 454.3	fsrq	343.49	16.15	37.56	0.0459
GB6 J0929+7304	bcu II	142.43	73.07	25.62	0.0459
ZS 0214+083	bll	34.32	8.62	36.08	0.0485
3C 264	rdg	176.27	19.61	36.47	0.0485
RXS J094620.5+010459	bll	146.58	1.08	19.50	0.0486
NRAO 512	fsrq	250.12	39.78	21.52	0.0490
1H 0323+022	bll	51.56	2.42	43.87	0.0494
GB6 J1027+7428	bcu II	156.85	74.47	31.92	0.0495
RX J0805.4+7534	bll	121.36	75.57	30.25	0.0505
4C +73.07	bcu II	142.25	72.95	22.60	0.0514
B3 1222+438	bll	186.21	43.59	28.13	0.0516
MG1 J120448+0408	fsrq	181.22	4.14	33.72	0.0525
GB6 J0148+5202	bcu III	27.08	52.03	19.63	0.0556
7C 1823+6856	bll	275.89	68.96	21.57	0.0561
MG2 J180948+2910	bll	272.44	29.17	34.89	0.0591
NVSS J022304+682154	bcu II	35.77	68.37	31.49	0.0591
OM 280	bll	177.58	24.30	42.49	0.0626
PKS 1717+177	bll	259.80	17.75	24.64	0.0627

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
1RXS J133021.4+444117	bll	202.59	44.69	22.24	0.0650
S4 1726+45	fsrq	261.87	45.51	34.78	0.0664
RX J1702.6+3115	bll	255.66	31.26	25.34	0.0664
SBS 1410+530	bcu I	212.96	52.82	24.04	0.0666
MG1 J154628+1817	bll	236.60	18.29	40.11	0.0668
MG2 J190411+3627	bcu II	286.05	36.45	34.19	0.0713
TXS 2331+073	fsrq	353.55	7.61	27.29	0.0731
PKS 2047+098	bcu II	312.44	10.05	19.18	0.0734
GB6 J1542+6129	bll	235.74	61.50	34.32	0.0736
S5 0159+723	bll	30.89	72.55	30.54	0.0765
TXS 2032+117	fsrq	308.65	11.91	21.51	0.0777
S5 1027+74	bcu I	157.84	74.70	30.89	0.0777
RGB J1426+340	bll	216.53	34.07	45.49	0.0778
B3 0350+465	bcu III	58.63	46.72	22.88	0.0778
RGB J1742+597	bll	265.63	59.75	37.12	0.0781
1RXS J125117.4+103914	bll	192.82	10.65	18.18	0.0782
MG3 J184126+2910	bcu II	280.34	29.16	37.55	0.0787
TXS 1645+635	fsrq	251.49	63.50	36.91	0.0793
B3 1058+413	bcu III	165.35	41.06	44.91	0.0799
87GB 152947.5+574636	bll	232.74	57.61	21.01	0.0802
GB6 J0229+6706	bcu III	37.34	67.11	11.03	0.0806
MG1 J125348+0326	bll	193.45	3.44	27.69	0.0806
GB6 J0342+3858	fsrq	55.57	38.99	33.59	0.0806
PMN J0148+0129	bll	27.14	1.48	18.43	0.0818
87GB 164812.2+524023	bll	252.35	52.59	33.97	0.0827

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Table 5.2 – *Continued from previous page*

Source Candidate	Source Class	RA (deg)	Dec (deg)	\hat{n}_s	p (pre-trial)
GB6 J0024+0349	fsrq	6.19	3.82	31.06	0.0830
B3 1307+433	bll	197.36	43.08	22.38	0.0839
NVSS J092542+595812	bll	141.43	59.97	17.27	0.0913
TXS 1549+089	bll	238.01	8.85	30.66	0.0915
RX J0202.9-0223	bcu II	30.72	-2.39	11.81	0.0922
PKS 1203+04	ssrq	181.58	4.10	39.00	0.0926
3C 221	rdg	143.78	39.70	28.81	0.0931
RBS 0909	bll	162.86	39.72	30.20	0.0942
NVSS J121500+500216	bll	183.75	50.04	36.18	0.0956
RX J1101.3+4108	bll	165.35	41.15	39.73	0.0962
OX 131	fsrq	320.25	19.02	28.54	0.0970
PKS 0017+200	bll	4.91	20.36	25.43	0.0973
1ES 0647+250	bll	102.69	25.05	33.94	0.0990
TXS 0237+655	bcu II	40.34	65.72	22.76	0.102
NVSS J224753+441317	bll	341.97	44.22	29.52	0.102
OI 280	fsrq	117.72	12.52	22.89	0.102
B2 1348+30B	fsrq	207.72	30.58	27.45	0.102
TXS 1833+137	bcu III	278.90	13.81	23.62	0.105
RX J1027.4+6317	bll	156.85	63.30	31.32	0.105
PKS 2354-021	bll	359.35	-1.87	5.92	0.105

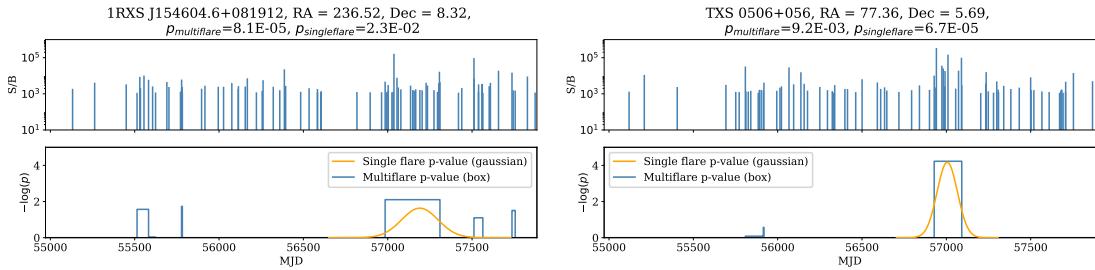


Figure 5.9: Neutrino flare curves for several sources of note in the multi-flare 3LAC blazar catalog analysis. The flare curves generated by the multi-flare algorithm are shown in blue, while the results of a corresponding single-flare analysis (that only fits the largest flare on each source) are shown in orange. Left: 1RXS J154604.6+081912, the most significant 3LAC blazar in the multi-flare analysis, having a pre-trial multi-flare p-value of $p = 8.11 \times 10^{-5}$. Note that for this particular source, the multi-flare significance driven by a set of five moderately significant flares, none of which are particularly significant on their own. The significance of this source in the single-flare analysis was only $p = 0.02$. Right: The flare curve for TXS 0506+056, showing the sizeable 2014 neutrino flare originally observed in [3]. The multi-flare p-value for TXS 0506+056 is only $p = 9.24 \times 10^{-3}$, as other than the 2014 neutrino flare, there are no other particularly significant neutrino flare candidates at this location.

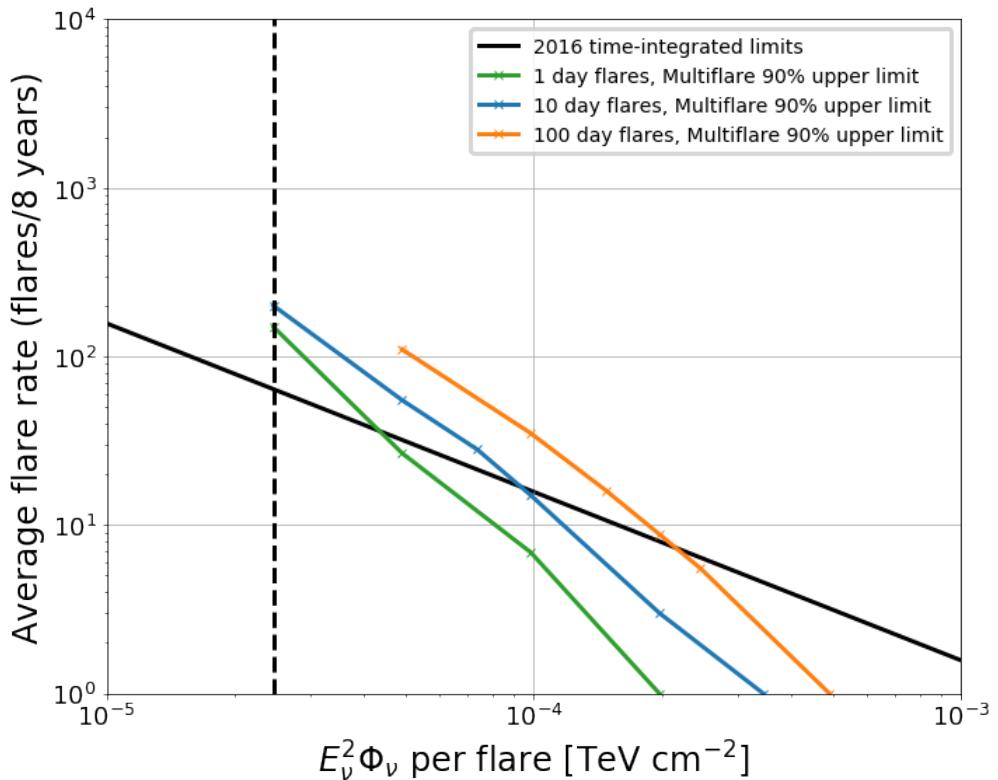


Figure 5.10: The 90% upper limits associated with the non-detection of a multi-flare signal using the 3LAC blazar catalog. The solid black line represents the time integrated limits obtained from [46]. Colored curves represent limits associated with various flare durations, as the multi-flare method imposes more stringent limits in the case that flares are shorter (for a fixed E^2 flux per flare). Note that the parameter space that is realistically accessible to a time-dependent analysis is somewhat compressed: points below the x-axis do not produce more than 1 flare in the data sample, on average, and points to the left of the vertical dashed line have an average flare size of < 1 event.

1120 Chapter 6

1121 An All-Sky Multi-flare Analysis

1122 6.1 Motivation

1123 While the previous section outlined the application of the multi-flare algorithm to a pair
1124 of source catalogs, an additional obvious application of the method is to simply apply the
1125 algorithm across the entire neutrino sky. While catalog searches have the advantage of a
1126 greatly reduced trial factor, they also rely on having a mostly-correct guess of the underlying
1127 source population. By contrast, an all-sky search could potentially reveal sources without
1128 requiring any prior knowledge of what the specific sources could be, and could additionally
1129 even identify astrophysical neutrino sources with no multi-messenger counterpart. Given the
1130 results of a temporally untriggered neutrino flare associated with TXS 0506+056 [3], it is
1131 only natural to ask whether other such flares exist in IceCube’s astrophysical neutrino data.
1132 An all-sky multi-flare analysis would be an appropriate approach of detecting a population
1133 of TXS-like flares, should they exist.

1134 Similar to the FAVA analysis produced by the Fermi collaboration [48], an IceCube
1135 multi-flare skymap provides a description of the temporal variability of every point in the
1136 sky. Even in the absence of a statistically significant population of neutrino flares, these
1137 neutrino “flare curves” may be of use for multi-messenger analyses in the future, similar to
1138 what was done for TXS 0506+056 [3][2].

1139 6.2 Analysis Construction

1140 Conceptually, the construction of an all-sky multi-flare analysis is fairly straightforward: a
 1141 grid of pixels with is defined over the entire sky using the HEALPY software package [49].
 1142 Using a HEALPY grid with a `Nside=256` results in a grid of 786,432 pixels, each with a
 1143 radius of 0.12° . Pixels with declinations $\delta > 85^\circ$ or $\delta < -85^\circ$ are excluded, as the data-
 1144 driven method of estimating background using data scrambled in right ascension does not
 1145 perform well in this region due to the small statistics of declination bands near the poles.
 1146 The multi-flare algorithm is then applied to the central location of each pixel, resulting in a
 1147 map of multi-flare test statistics, each associated with an individual pixel.

1148 The significance of each pixel can then be calculated by comparing the observed multi-
 1149 flare test statistic to a distribution of similar test statistics obtained by applying the above
 1150 procedure to maps of IceCube events that have had their right ascension locations random-
 1151 ized, providing a description of the null hypothesis. Pixels are divided into 40 declination
 1152 bands, and for each declination band a chi-squared distribution is fit to the distribution of
 1153 test statistics in that declination range. A pre-trial local significance for each pixel is calcu-
 1154 lated by comparing a particular pixel test statistic to the chi-squared distribution that was
 1155 fit in the corresponding declination band. Once this has been done for every pixel, a map of
 1156 multi-flare p-values has been obtained, describing the local multi-flare significance of every
 1157 pixel in the sky.

1158 Once a p-value map has been generated, several basic tests can be conducted. The most
 1159 obvious test is to simply check if the most significant pixel is more significant than is expected
 1160 from the background case (a "hotspot" test). A distribution of most significant multi-flare
 1161 p-values is created by applying the procedure above to a set of background maps. The most
 1162 significant pixel in the observed multi-flare map can then be compared to this distribution to
 1163 obtain a final hotspot significance that has been trial-corrected for the all sky trial factor. As
 1164 the event selection is different in the northern and southern skies, this process is conducted
 1165 separately for declinations $\delta > -5^\circ$ and $\delta < -5^\circ$

1166 Populations of multi-flare sources may also be tested for using a binomial test. A set of
 1167 ”spatially independent” local hotspots may be obtained by defining a list of all pixels that
 1168 are at least 1° away from a more significant pixel. The binomial test statistic p-value of the
 1169 population test is then defined as (eq. 6.1):

$$p(k) = \sum_{i=k}^{N_{eff}} \binom{N_{eff}}{i} p_k^i (1 - p_k)^{N_{eff}-i} \quad (6.1)$$

1170 Here, $p(k)$ is correlated with the significance of observing k hot spots with a p-value of p_k
 1171 or less, and N_{eff} is the effective number of trials associated with the list of hot spots, chosen
 1172 to produce proper containment of the final binomial p-values (e.g. a final binomial p-value
 1173 of $p = 0.1$ or less should only occur in 10% of background trials). In this case, $N_{eff} = N_{pixels}$
 1174 produces proper containment. Hot spots are ordered by decreasing significance, and k is
 1175 varied to identify the most significant combination. The $p(k)$ associated with the best fit k
 1176 is then compared to a distribution of $p(k)$ ’s obtained in a similar manner from data scrambled
 1177 in right ascension, resulting in a final post-trial binomial p-value. Like with the study of
 1178 the most significant pixel, this process is conducted separately in the northern and southern
 1179 skies.

1180 Since the multi-flare algorithm fits for every flare candidate at a particular pixel, it is
 1181 trivial to extract the single-flare skymap results from this process as well. The procedure
 1182 is almost identical to that which was outlined above, except instead of using the multi-flare
 1183 test statistic at each pixel, the test statistic of the flare with the highest test statistic at each
 1184 pixel is used. The hotspot and population tests then proceed as normal.

1185 This particular analysis uses the PSTracks003p02 data set.

1186 **6.3 Results**

1187 **6.3.1 Hotspots and Populations Analysis**

1188 The multi-flare pre-trial p-value map can be seen in figure 6.1, with the locations of the
 1189 brightest multi-flare pixels marked. The most significant locations identified by the multi-
 1190 flare algorithm have pre-trial p-values of $p = 9.2 \times 10^{-6}$, located at (RA, Dec)=($145.02^\circ, 36.42^\circ$)
 1191 and $p = 3.5 \times 10^{-7}$, located at (RA, Dec)=($126.21^\circ, -24.81^\circ$). These pre-trial p-values can be
 1192 corrected to account for the all-sky trial factor using the process described above, resulting
 1193 in post-trial p-values of $p = 0.69$ for the northern sky hot spot and $p = 0.06$ for the southern
 1194 sky hot spot.

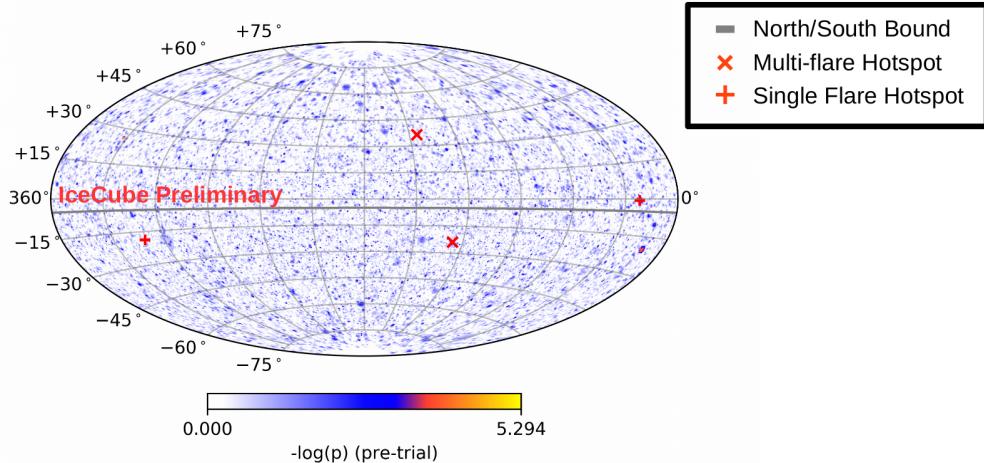


Figure 6.1: The p-value map produced by applying the multi-flare algorithm to the entire neutrino sky between $-85^\circ < \delta < 85^\circ$. The locations of the most significant multi-flare pixels in each hemisphere are shown as red "x's", while the locations of the most significant individual flares in each hemisphere are shown as red "+"s". The gray line denotes the boundary between what is considered the "northern" and "southern" sky by the IceCube data sample that was used.

1195 The flare curves for the multi-flare hotspots can be seen in figure 6.3. Note that the
 1196 multi-flare test statistic is a measure of the activity of a source integrated over the entire
 1197 livetime of the data sample. While no individual flares that were fit at either the northern
 1198 or southern hotspot are particularly significant on their own, the combination of many

1199 moderately significant flares is what makes these pixels the brightest multi-flare pixels in
 1200 their respective hemispheres.

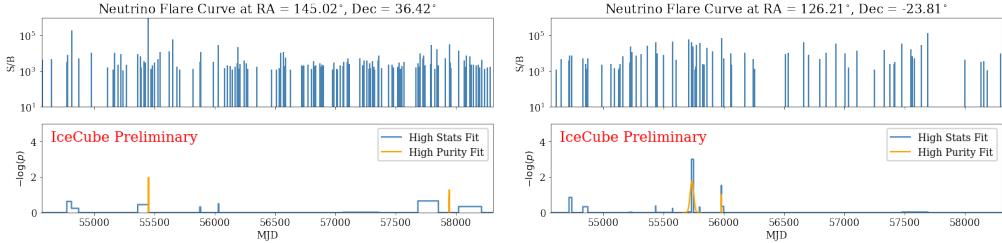


Figure 6.2: The flare curves returned by the multi-flare algorithm for the most significant pixel in the northern sky (left), and southern sky (right). For comparison, a complementary high-purity fit that uses a gaussian hypothesis and more stringent cuts on flare decorrelation is shown in orange. In both the northern and southern sky, the multi-flare significance is fueled not by individual flares with high local significance, but rather by a large number of moderately significant flares.

1201 Since the application of the high-statistics multi-flare analysis involves fitting every pos-
 1202 sible flare in the data, it is trivial to additionally calculate the significance of the largest in-
 1203 dividual flare candidate that was fit in both the northern and southern sky. We find that the
 1204 most significant flare candidate in the northern sky is located at $(\text{RA}, \text{Dec}) = (21.97^\circ, -0.60^\circ)$
 1205 (recall that the "northern sky" refers to declinations between -5° and 85°), and has a pre-
 1206 trial significance of $p = 5.08 \times 10^{-6}$ ($p = 0.82$ post-trial). The most significant flare candidate
 1207 in the southern sky is located at $(\text{RA}, \text{Dec}) = (311.66^\circ, -18.84^\circ)$, and has a pre-trial signifi-
 1208 cance of $p = 6.8 \times 10^{-6}$ ($p = 0.53$ post-trial).

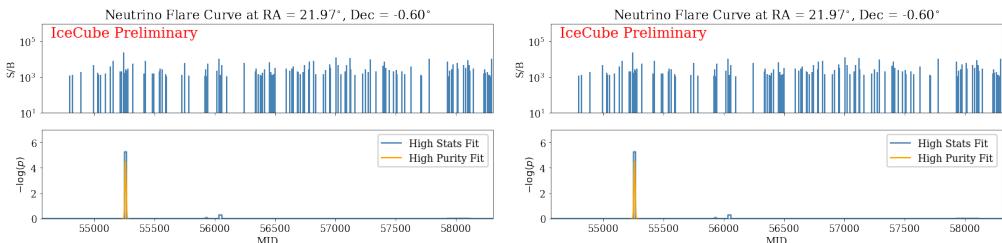


Figure 6.3: The flare curves returned by the multi-flare algorithm for the most significant flares identified in the northern sky (left), and southern sky (right). For comparison, a complementary high-purity fit that uses a gaussian hypothesis and more stringent cuts on flare decorrelation is shown in orange. Since neither of these locations appear to display significant activity beyond the large individual flares, these do not constitute particularly significant locations from a multi-flare perspective.

1209 In addition to examining the most significant pixels that were seen, it is also informative
 1210 to conduct a population analysis on the set of spatially independent ($> 1^\circ$ separation)
 1211 hotspots using the binomial test framework described above. In both the northern and
 1212 southern sky, the binomial test was conducted for the set of spatially independent hotspots,
 1213 defined by both the local multi-flare and single-flare significance. In all cases, the best-fit
 1214 combination of hotspots was $k = 1$, tagging only the most significant pixel, and none of the
 1215 associated binomial p-values were significant (results may be viewed in table 6.1).

Analysis	Search	Hemisphere	Pre-trial p-value	Post-trial p-value
Multi-flare	Hottest spot	North	9.2×10^{-6}	0.69
		South	3.5×10^{-7}	0.06
	Population test	North	0.98	0.98
		South	0.12	0.12
Single Flare	Hottest spot	North	5.08×10^{-6}	0.82
		South	6.70×10^{-6}	0.53
	Population test	North	0.88	0.88
		South	0.91	0.91

Table 6.1: Summary of the tests that were performed on the unblinded multi-flare skymap. Both hotspot and population analyses were conducted using both the local multi-flare significance (summing contributions from all flare candidates at a particular location) and the local single-flare significance (taking only the largest flare fit at each location). There were no significant transient neutrino sources identified by any of the tests conducted.

1216 The results of the population analysis are unsurprising, given the distribution of per-
 1217 pixel local p-values observed from the data. In the presence of a significant population of
 1218 flaring neutrino sources, some deformation of the local p-value distribution should be visible,
 1219 however as can be seen in figure 6.5, the observed distribution is entirely consistent with
 1220 what is expected from background maps.

1221 Similar to the p-value distributions, the distributions of fitted flare parameters are also
 1222 consistent with the background expectation, as can be seen in figure 6.6. There does not
 1223 appear to be a significant sub-population of transient sources with a particular flare duration,
 1224 spectral index, flare rate, or flare flux.

1225 The observed diffuse astrophysical neutrino flux [10] places restrictions on how bright

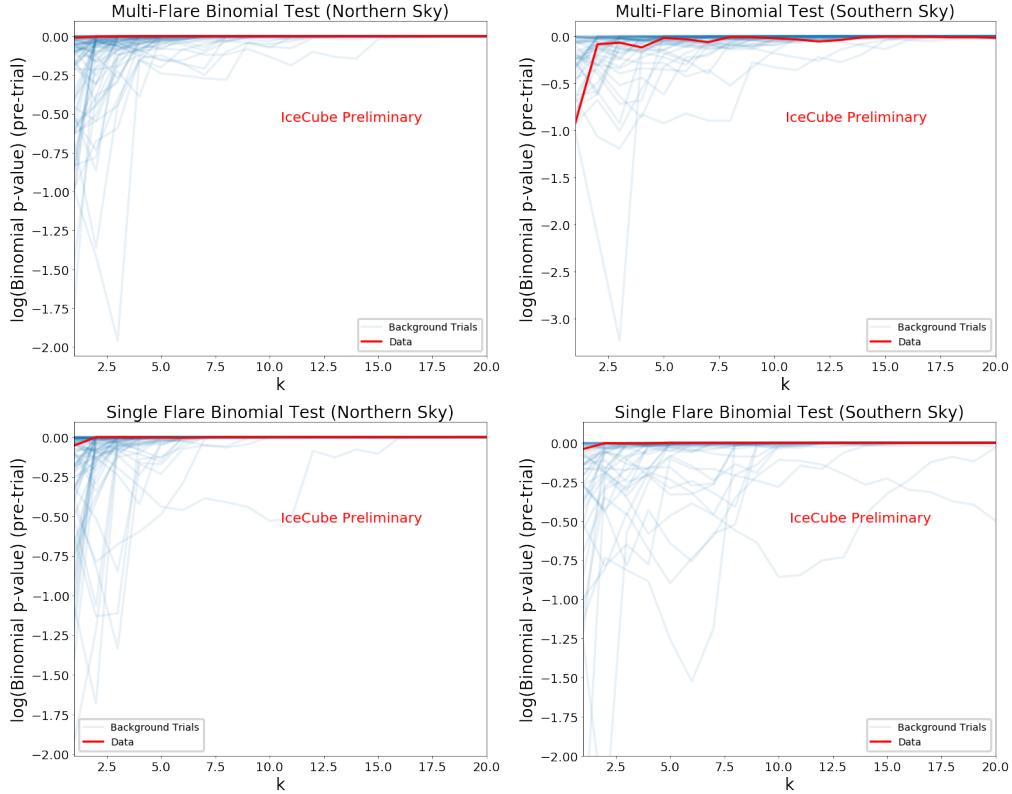


Figure 6.4: The local significance of the binomial test for various values of k , the fitted number of hotspots, for both single and multi-flare binomial tests conducted in both the northern and southern hemispheres. Unblinded data is shown in red, while results obtained from background maps are shown in blue. In all cases using unblinded data, the observed binomial test curve has a global minimum at $k = 1$, with an associated p-value that is consistent with the background expectation.

1226 and numerous neutrino flares may be. The non-detection of a population of transient astro-
 1227 physical neutrino sources can be used to further constrain this space. These limits can be
 1228 seen in figure 6.7, which shows 90% upper limits on source flare rate and burst energy in
 1229 neutrinos, as well as limits that can be placed on the per-source flare rate. Note that these
 1230 limits are a statistical statement about the data itself: If the "true" parameters describing
 1231 a population of transient neutrino sources lies above these lines, the data would have had a
 1232 larger population of significant flares than what was observed.

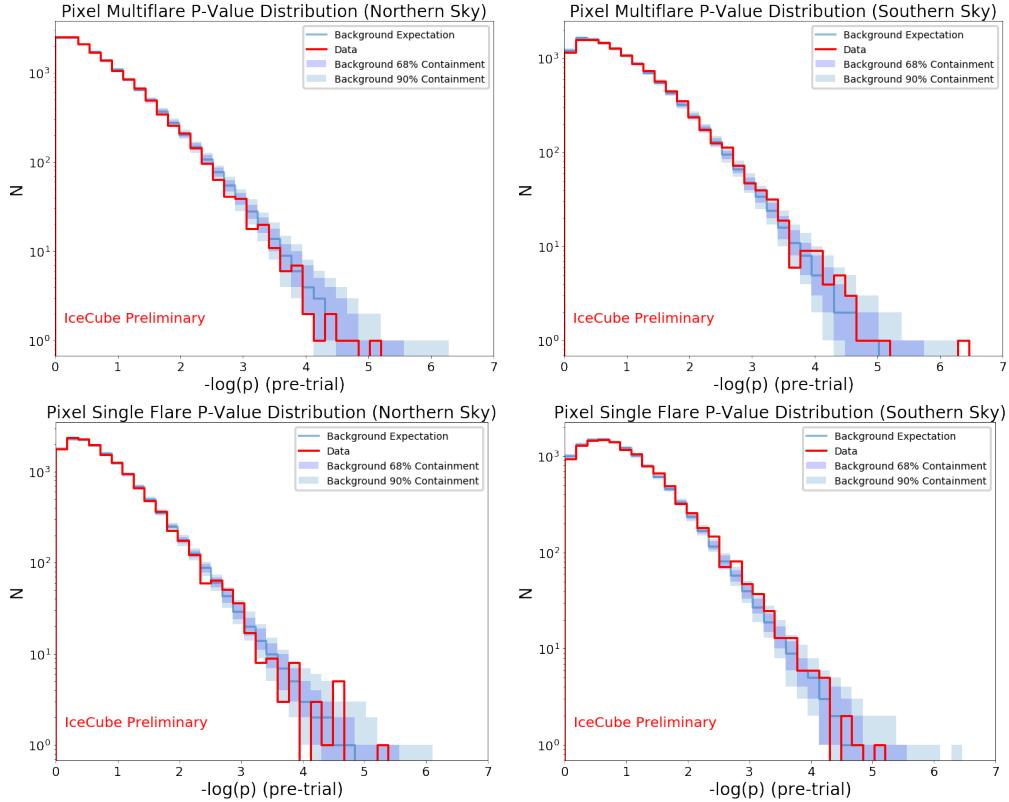


Figure 6.5: The local p-value distributions for the local multi-flare pixel significance (top) and the local single-flare pixel significance (bottom), for both data (red) and background maps (blue bands). In all cases, the observed distribution is consistent with the background expectation.

1233 6.3.2 Locations of External Interest

1234 Though this analysis was designed to be spatially untriggered (scanning over the entire sky,
 1235 rather than the locations of a select few sources), the neutrino "flare curves" produced are a
 1236 potentially powerful tool for exploring the historical behavior of source candidates identified
 1237 though other methods. The skymap produced here is similar to the TXS 0506+056 followup
 1238 analysis [3], except that this analysis has now been performed at almost every location in
 1239 the sky. This allows for easy followup of external triggers, as identifying potential transient
 1240 neutrino emission associated with a source candidate is as easy as examining the flare curve
 1241 at that location in the sky.

1242 To demonstrate the use of the all-sky flare curve map produced here, we examine the
 1243 flare curves of several time-integrated source candidates identified by [4] in order to explore

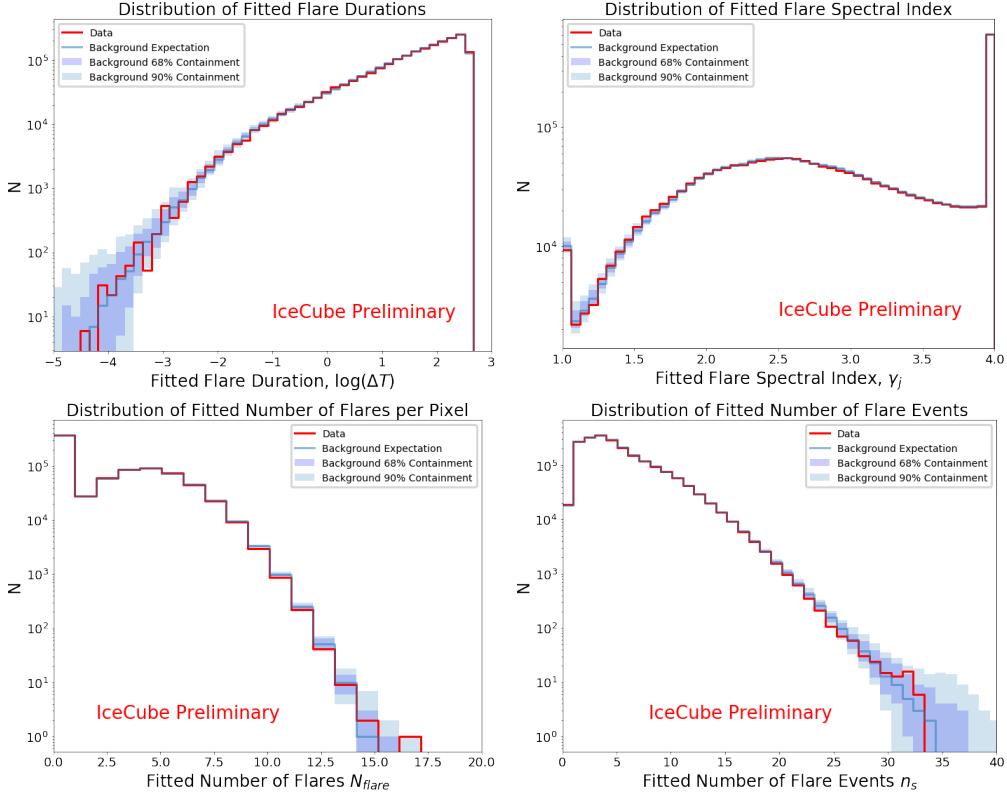


Figure 6.6: Distributions of fitted flare parameters: flare duration (top, left), spectral index (top, right), number of flares fit at a particular candidate location

1244 the potential of temporally clustered neutrino emission from these locations.

1245 6.3.2.1 NGC 1068

1246 The most significant location identified by the all-sky time-integrated analysis is spatially
 1247 coincident with the Seyfert II galaxy NGC 1068. This source was also tested as part of
 1248 a catalog of 110 candidate neutrino emitters, and was the brightest object in the catalog,
 1249 with an associated post-trial significance of 2.9σ [4]. These significances all arise from time-
 1250 integrated tests that do not take into account the arrival time of the contributing neutrino
 1251 events. It is then potentially interesting to explore the temporal structure of this excess with
 1252 the flare curves that were produced as part of the multi-flare skymap.

1253 The flare curve generated at the location of NGC 1068 can be seen in figure 6.8. Though
 1254 this location is significant under the time-integrated analysis, the multi-flare p-value is only
 1255 $p = 0.016$ (pre-trial), with no individual flare candidate having a local significance greater

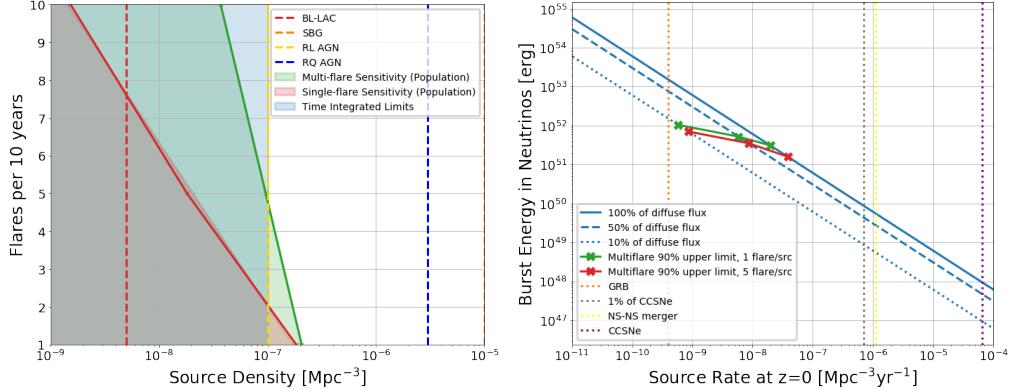


Figure 6.7: Left: 90% upper limits on neutrino source density and per-source flare rate obtained from the all-sky multi-flare analysis. In this plot, transient sources are assumed to compose 100% of the astrophysical neutrino flux, with the FIRESONG simulation package [50] being used to distribute the flux among sources according to the Hopkins and Beacom (2006) source evolution model [51]. Flares are simulated to have a duration of 20 days, and a spectral index of $\gamma = 2.28$. All flares on the same source are assumed to have the same intrinsic intensity, equal to the time integrated source flux divided by the number of flares. Right: The 90% upper limits on the source rate (source density \times flare rate) as a function of neutrino burst energy. Blue lines correspond to combinations of source rate and neutrino burst energy that reproduce specific fractions of the diffuse astrophysical neutrino flux. In both plots, the parameters associated with several candidate source populations are shown for comparison..

1256 than $p = 0.072$. Note however, that these results are not in tension with the time-integrated
 1257 significance, as the time-integrated analysis tests only for an excess of events over the entire
 1258 lifetime of the sample. By contrast, a source with high multi-flare significance requires that
 1259 events not only be clustered spatially, but temporally as well. As such, sources with high
 1260 time-integrated significance do not necessarily have a correspondingly high multi-flare signif-
 1261 icance, particularly if there is not significant temporal clustering of events that contribute
 1262 to the time-integrated result.

1263 Nonetheless, we have here provided a description of the historical behavior of the neutrino
 1264 emission associated with NGC 1068. Despite the time-integrated excess observed in previous
 1265 analyses, there does not appear to be significant temporal clustering of events at this location.

1266 **6.3.2.2 Other Time-Integrated Candidates: PKS 1424+240 and GB6**

1267 **J1542+6129**

1268 In addition to identifying NGC 1068 as a potential source candidate, the population analysis
 1269 component of the time integrated analysis [4] also identified an excess of 3.3σ associated
 1270 with the combination of the sources NGC 1068, TXS 0506+056, PKS 1424+240, and GB6
 1271 J1542+6129. Figure 6.9 shows the flare curves that were obtained at the locations of PKS
 1272 1424+240 and GB6 J1542+6129.

1273 Similar to NGC 1068, PKS 1424+240 does not display significant temporal clustering
 1274 of neutrino events, and has a local multi-flare p-value of $p = 0.085$. By contrast, GB6
 1275 J1542+612 seems to have a somewhat significant flare candidate beginning at MJD=57564.878
 1276 and ending at MJD=57944.512. The pre-trial significance of this flare candidate alone is
 1277 $p = 0.00173$ (2.9σ). Combining all the flares candidates at the location of GB6 J1542+612
 1278 using the multi-flare test statistics results in a local multi-flare significance of $p = 0.0098$
 1279 (2.3σ). While this is not particularly significant (especially when the all-sky trial factor is
 1280 considered), it does inform us that the period between MJD=57564.878 and MJD=57944.512
 1281 was likely a large contributor to the time-integrated significance for this particular source. If
 1282 there are further multi-messenger signals from this location in the future, the neutrino flare
 1283 candidates at this location may become of interest.

1284 **6.3.2.3 Follow-up of the ANTARES Source Candidate J0242+1101**

1285 *Author note: Due to the timing of the request for this follow-up, this section was added 1
 1286 week before the submission of this thesis. Results are very preliminary, but I've included it
 1287 because I think it's a nice example of how these multi-flare fits can be used to cross check
 1288 results from other neutrino telescopes*

1289 The ANTARES telescope [52] is a water cherenkov telescope, similar to IceCube, but
 1290 located in the Mediterranean sea instead of the at the South Pole. It is significantly smaller
 1291 than the IceCube detector, consisting of only 12 lines of 25 optical modules each. The lo-

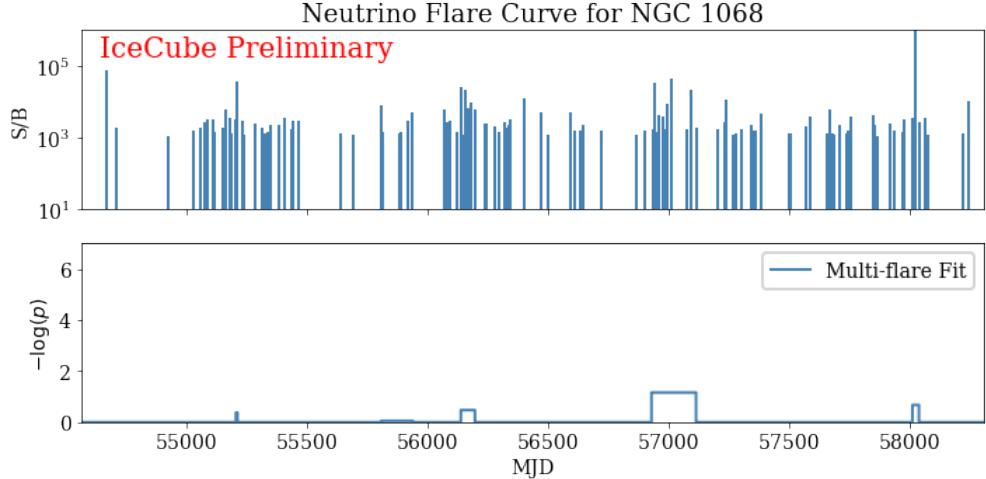


Figure 6.8: The flare curve at the location of NGC 1068, a source candidate that was identified as potentially interesting in the time-integrated analysis [4]. While this is the brightest spot in the time-integrated analysis, the multi-flare analysis does not reveal any significant temporal clustering of events at this location, resulting in a local multi-flare significance of $p = 0.016$

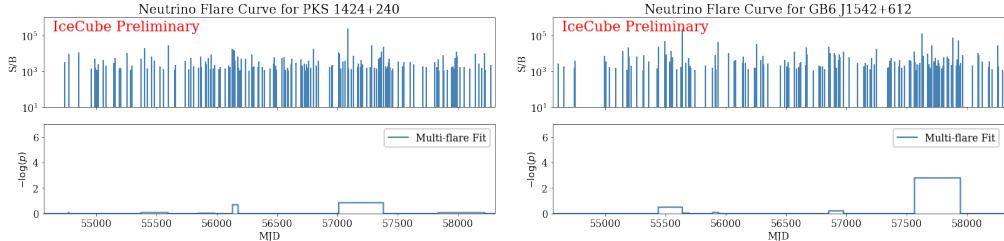


Figure 6.9: The flare curves at the locations of PKS 1424+240 and GB6 J1542+612. While PKS 1424+240 does not display any significant temporal clustering (having a local multi-flare p-value of $p = 0.085$), GB6 J1542+612 has a potentially interesting flare candidate beginning at MJD=57564.878 and ending at MJD=57944.512. This flare candidate has a local flare significance of $p = 0.00173$ (2.9σ), contributing to the overall pre-trial multi-flare significance for GB6 J1542+612 of $p = 0.0098$ (2.3σ).

1292 cation of ANTARES in the northern hemisphere makes it ideal for studying astrophysical
 1293 neutrino events from the southern sky, as the Earth absorbs the atmospheric muon back-
 1294 ground in this region, similar to how IceCube is more sensitive to events from the northern
 1295 sky.

1296 Though ANTARES does not fit for a significant astrophysical diffuse flux [53], the col-
 1297 laboration has still performed searches for point sources of astrophysical neutrinos. Notably,
 1298 at the most recent International Cosmic Ray Conference (2021)¹, the ANTARES collabora-

1299 tion presented a temporally untriggered search for correlations between radio-bright blazars
 1300 and neutrino flare candidates in ANTARES data [54]. The ANTARES collaboration noted
 1301 that one of their most significant candidates, J0242+1101, overlapped with the detection of
 1302 a high-energy IceCube event, and contacted the IceCube collaboration for follow-up.

1303 It should be noted that the ANTARES results on their own are not statistically signifi-
 1304 cant: the most significant source candidate has a post-trial p-value of only 40%, and only 7
 1305 source candidates have pre-trial significances greater than 3σ (equivalent to the background
 1306 expectation when testing 2774 sources). Nonetheless, for the sake of completeness, the Ice-
 1307 Cube collaboration assembled a followup analysis, of which the multi-flare fits shown below
 1308 were a part.

1309 Since multi-flare fits were created for the entire sky as part of the multi-flare skymap
 1310 analysis described above, follow-up is potentially as easy as pulling the appropriate flare curve
 1311 from the stored skymap file containing flare fits at every location. However, for the sake of
 1312 completeness, in this particular case the multi-flare analysis was re-run at the exact location
 1313 of the ANTARES source candidate (J0242+1101, RA=40.62°, Dec=11.01°), in addition
 1314 to modifying the flare candidate seeding procedure to ensure that flare candidates with
 1315 durations up to $\Delta t = 1000$ days were tested.

1316 The results of the multi-flare followup for J0242+1101 can be seen in figure 6.10. No
 1317 significant flares were fit, with the largest flare candidate in IceCube data having a local
 1318 significance of only $p = 0.31$. There was also not an excess of flares observed, with the pre-
 1319 trial multi-flare significance of this location being $p = 0.52$. The 50 TeV IceCube event that
 1320 was noted by ANTARES in [54] does not appear to be significant in the timing distribution
 1321 of Icecube events, as the flare search algorithm as unsucessful in pairing that event up with
 1322 other IceCube events to fit a significant flare.

¹<https://icrc2021.desy.de/>

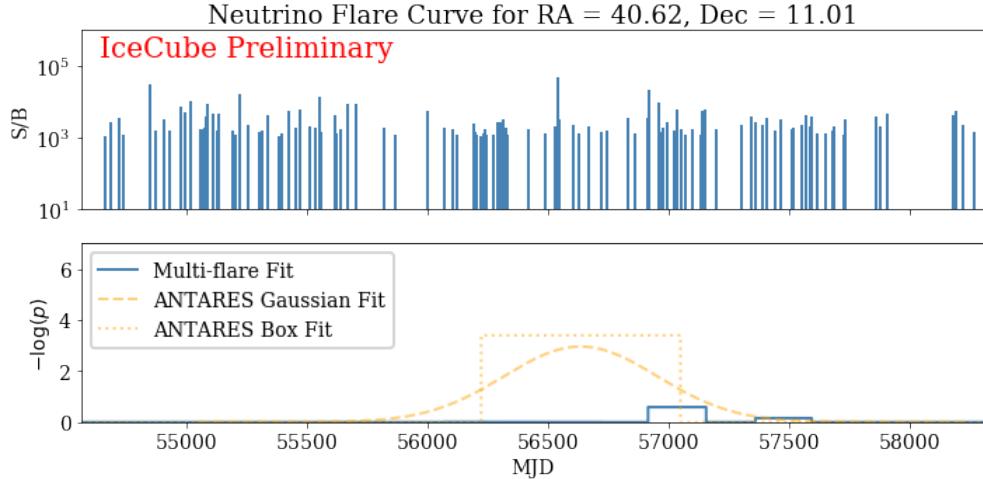


Figure 6.10: The flare curve at the location of J0242+1101, a source candidate that was identified as potentially interesting by the ANTARES collaboration. The results of the untriggered flare fits performed by the ANTARES collaboration are shown in orange, while the blue shows the IceCube data (top-panel), and corresponding multi-flare fit (bottom panel). There does not appear to be a flare as strong as the reported ANTARES flare in IceCube data, with the IceCube multi-flare followup presented here returning a p-value of only $p = 0.52$.

1323 6.3.2.4 TXS 0506+056

1324 In addition to being part of the excess that was identified in the all-sky time-integrated
 1325 analysis, TXS 0506+056 is notable for both the multi-messenger coincidence sparked by the
 1326 high energy IceCube alert IC-170922A [2], as well as the archival analysis that revealed a
 1327 3.5σ neutrino flare that occurred in 2014, prior to the 2017 high energy alert event [3].

1328 The all-sky multi-flare analysis presented in this work includes the location of TXS
 1329 0506+056, and we can consequently generate a flare curve for this source candidate. The
 1330 flare curve observed in the all-sky multi-flare analysis can be seen in figure 6.11. TXS
 1331 0506+056 has a local multi-flare significance of $p = 3.37 \times 10^{-4}$ (3.4σ), with the main
 1332 contributor being the 2014 neutrino flare candidate, beginning on MJD=56927.86 and ending
 1333 on MJD=57072.99.

1334 Readers who have been paying close attention to the content of this thesis (a group which
 1335 at this point probably includes my thesis committee and like, no one else) may notice that
 1336 the flare curve shown in figure 6.11 seems to be in disagreement with the published result

1337 with regards to the 2014 TXS flare . While the result shown in [3] reports a significance for
 1338 the 2014 flare candidate of 3.5σ , the all-sky multi-flare analysis presented here observes a
 1339 local significance for this flare of only $p = 0.0054$ (2.5σ).

1340 The difference in significance between the two results can be explained by the different
 1341 event selections that were used in each analysis. Descriptions of these event samples may
 1342 be found in section 3 of this thesis. While [2] used the PointSourceTracks v002p03 event
 1343 sample, the all-sky multi-flare analysis presented here uses the PointSourceTracks v003p02
 1344 event sample. The most relevant difference between the two event samples in this case are
 1345 the pre-cuts: PointSourceTracks v003p02 introduces a cut prior to the event selection BDT
 1346 that requires events to have a reconstructed track length greater than 200 meters. This
 1347 pre-cut does not exist in PointSourceTracks v002p03.

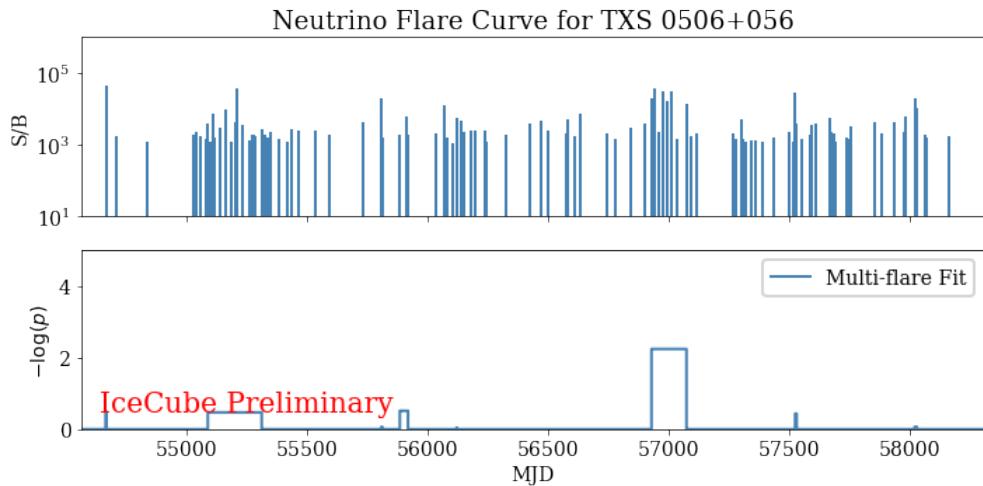


Figure 6.11: The flare curve obtained from the all-sky multi-flare analysis for TXS 0506+056. The pre-trial multi-flare significance of this location is $p = 3.37 \times 10^{-4}$ (3.4σ). Interestingly, the 2014 flare candidate is not seen at the original significance of 3.5σ that was reported in [2]. While the 2014 flare candidate is present, this analysis obtains an associated local significance of only $p = 0.0054$ (2.5σ).

1348 As a result of the track length pre-cut that was introduced in PointSourceTracks v003p02,
 1349 two events (referred to by their event IDs: 40914587 and 56262988) that were major contrib-
 1350 utors to the 2014 TXS 0506+056 neutrino flare candidate were removed. These events have
 1351 a cascade-like topology, though in PointSourceTracks v002p03, they are reconstructed as if
 1352 they were tracks, leading to potentially inaccurate descriptions of the event direction and

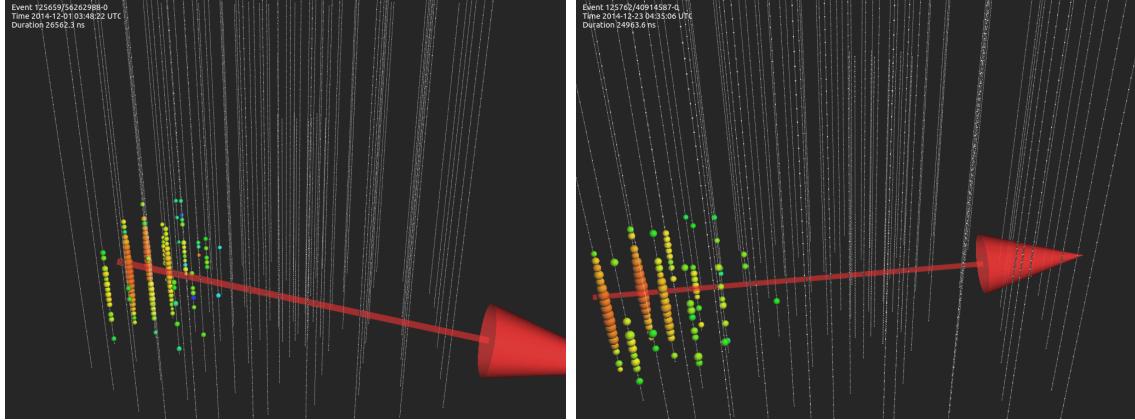


Figure 6.12: The two cascade-like events (event IDs: 40914587 and 56262988) that contributed to the 2014 TXS 0506+056 neutrino flare candidate in PointSourceTracks v002p03, but are not present in PointSourceTracks v003p02. The red arrows show the directional reconstruction associated with the track reconstruction that was used, however given the cascade topology of these events, this reconstruction is likely to be inaccurate.

1353 energy. Since these events are cascade-like, they have a short reconstructed track length,
 1354 and as such these events do not exist in PointSourceTracks v003p02. It is additionally worth
 1355 noting that even as cascades, these events are poorly localized, as they are only partially
 1356 contained in the detector. Attempts at reconstructing these events as cascades using the
 1357 methods described in [34] failed to produce any kind of useful directional estimate.

1358 Manually removing the two cascade events from the PointSourceTracks v002p03 data
 1359 sample and recomputing the significance of the 2014 TXS 0506+056 neutrino flare results in
 1360 a drop in significance comparable to what was seen when using PointSourceTracks v003p02,
 1361 as can be seen in table 6.2. The drop in significance cannot be otherwise adequately explained
 1362 by changes to the angular reconstruction or other differences between the two versions of
 1363 the data sample. The time-integrated sensitivity of the two samples is comparable, and
 1364 favors PointSourceTracks v3 at the declination of TXS 0506+056. As such, the drop in
 1365 significance of the 2014 TXS 0506+056 flare candidate is unlikely to have been caused by
 1366 PointSourceTracks v003p02 being an overall less sensitive sample.

1367 It should be noted that the result published in [3] is not incorrect, despite using an
 1368 older data sample that contains unforeseen background events. Since the methods used for
 1369 the original untriggered flare analysis use a data-driven background estimation, the cascade

Untriggered Flare Cross-check Results

Sample	p (pre-trial)	T_{start}	T_{stop}	n_s	γ
PSTracks v2 [3]	7.0e-5	56937.81	57096.22	14.39	2.20
PSTracks v2 w/o cascades	1.17e-3	56937.81	57112.65	12.22	2.26
PSTracks v3 (multi-flare skymap)	5.4e-3	56927.86	57072.99	11.87	2.22

Table 6.2: The results of repeating the untriggered flare analysis preformed in [3], but using PSTracks v3 in place of PSTracks v2, the dataset that was originally used. The apparent drop in significance when using PSTracks v3 can be explained by cascade-like events present in v2 that have been removed from v3.

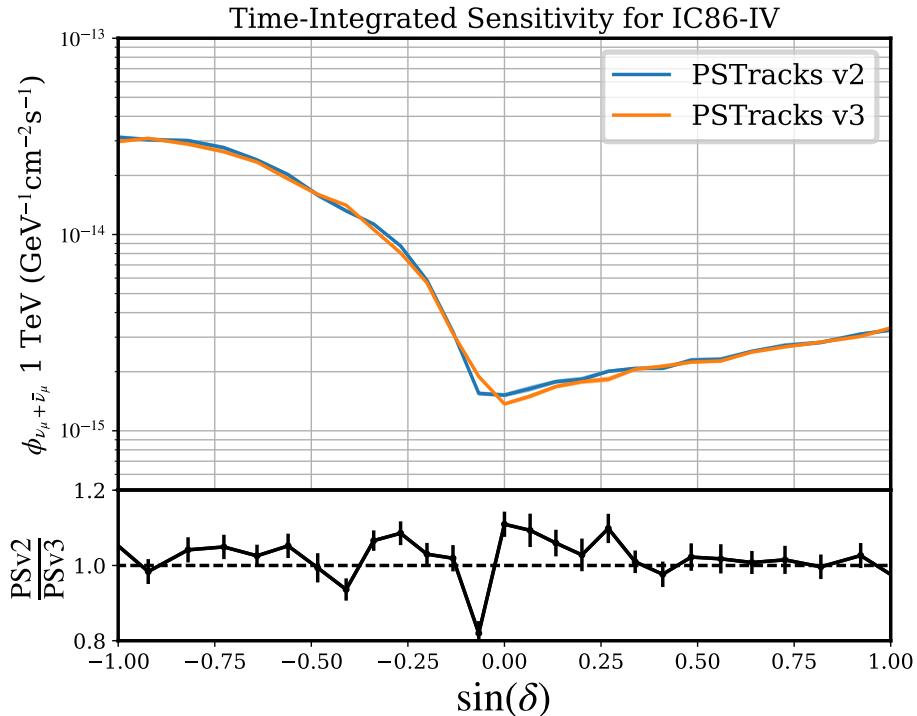


Figure 6.13: A comparison of the time integrated sensitivities of PointSourceTracks v002p03 and PointSourceTracks v003p02. At the declination of TXS 0506+056 ($\delta = 5.7^\circ$), PointSourceTracks v003p02 has a slightly better sensitivity.

1370 background in PointSourceTracks v002p03 is accounted for in the original 3.5σ significance
 1371 that was reported for the 2014 neutrino flare candidate.

1372 Though the cascade background is handled differently in the PointSourceTracks v002p03
 1373 analysis versus the PointSourceTracks v003p02 analysis, the two results are not inconsistent
 1374 with one another. Figure 6.14 shows the results of comparing the results of simulated
 1375 flares comparable to the 2014 TXS flare injected into both PointSourceTracks v002p03 and

1376 PointsourceTracks v003p02. As the background test statistic distributions for both samples
 1377 are similar, comparing the test statistic values obtained using each sample is a good proxy for
 1378 comparing the flare local significance. For this test, trials of simulated signal at the location
 1379 of TXS 0506+056 were injected into background maps generated using PointSourceTracks
 1380 v002p03. The injected signal was chosen to replicate the best fit parameters in the original
 1381 PointSourceTracks v002p03 untriggered flare analysis [3]: 13 events with a spectral index
 1382 of $\gamma = 2.2$, injected over a time window of length $\Delta t = 158$ days. A test statistic for each
 1383 of these PointSourceTracks v002p03 signal trials was then calculated. The same simulated
 1384 signal events were then injected into background trials generated with PointsourceTracks
 1385 v003p02, but any events that do not pass the PointSourceTracks v003p02 event selection
 1386 were removed. A new PointSourceTracks v003p02 test statistic is then calculated for each
 1387 of these trials with injected signal.

1388 Figure 6.14 shows the results of this crosscheck, showing the distribution of test statistics
 1389 differences between the v2 and v3 test statistics. While on average the test statistic associ-
 1390 ated with a signal of this type is higher in PointSourceTracks v003p02, the distribution is
 1391 rather wide, and it is not uncommon for the PointSourceTracks v003p02 result to be lower
 1392 significance than the PointSourceTracks v002p03 result. A drop in significance comparable
 1393 or greater than what was seen in the observed v2 and v3 results ($TS_{v3} - TS_{v2} = 11.84$)
 1394 occurs in 8% of the simulated trials.

1395 While there appears to be a drop in significance for the 2014 TXS 0506+056 untrig-
 1396 gered neutrino flare when using to the newer PointSourceTracks v003p02 data sample, it
 1397 is important to realize that the updated v3 result is not inconsistent with the previous v2
 1398 result. Additionally, the 2014 flare composes only part of the ensemble of results that make
 1399 TXS 0506+056 an interesting source candidate, as it is the combination of the 2014 untrig-
 1400 gered neutrino flare with the 2017 high energy alert event and multi-messenger observations
 1401 that suggest TXS 0506+056 as a source of astrophysical neutrinos. The significance of the
 1402 2014 TXS 0506+056 flare will likely continue to be monitored as the IceCube collaboration
 1403 improves its data samples and reconstruction techniques, hopefully converging on a more

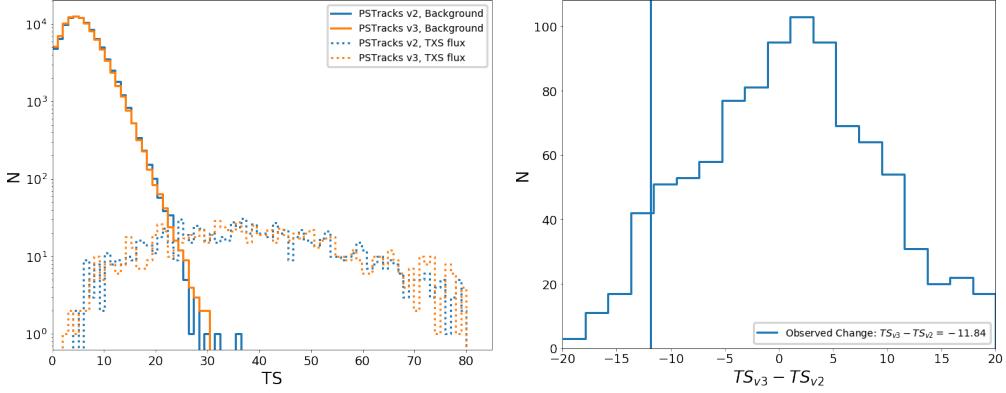


Figure 6.14: Simulated trials comparing the significance of a flare comparable to the 2014 TXS 0506+056 flare in both PointSourceTracks v002p03 and PointSourceTracks v003p02. Left: the test statistic distributions for both datasets in the background (solid) and injected signal (dotted) cases. The PointSourceTracks v003p02 signal is identical to the PointSourceTracks v002p03 signal, except with events that do not pass the v3 selection removed. Right: A comparison of the v2 and v3 test statistics for each simulated signal trial, with the observed test statistic difference plotted as a vertical line. A drop in significance at least as large as what was seen in observed data occurs in 8% of simulated trials.

1404 precise measurement to the flare significance.

1405

Chapter 7

1406

Outlook

1407 In this work, we have introduced a novel method for fitting ensembles of neutrino flare
1408 candidates at a particular source location. This method was applied to two catalogs of
1409 source candidates motivated by TXS 0506+056: a catalog of northern-sky 3LAC blazars, and
1410 a “self-triggered” catalog of high energy IceCube events. Though neither catalog displayed
1411 a significant excess of neutrino flares, this allowed us to further constrain potential transient
1412 neutrino emission from blazars.

1413 The multi-flare algorithm introduced here was also applied on an all-sky scale. No signif-
1414 icant population of neutrino flares was observed, though flare curves were produced at every
1415 location in the sky, providing a potentially useful tool for future multi-messenger searches
1416 wishing to incorporate information on the historical neutrino emission from source candidate
1417 locations, similar to what was done in [3].

1418 The non-detection of a significant population of transient neutrino sources is somewhat
1419 interesting from the perspective of constraining astrophysical neutrino source populations.
1420 Whatever the sources of astrophysical neutrinos may be, they cannot produce neutrino flares
1421 that are so bright or numerous that the data becomes inconsistent with the background
1422 expectation on an all-sky scale. This places limits on the source density, flare rate, and
1423 neutrino burst energy that are not associated with any particular source class, serving as a
1424 “universal limit” of sorts on the behavior of potential astrophysical neutrino transients.

1425 It should be noted, however, that a non-detection of neutrino sources on an all-sky scale
1426 is not entirely unexpected. The all-sky trial factor is quite large, and the most significant
1427 neutrino source candidates to date (TXS 0506+056 and NGC 1068) were both originally
1428 identified in spatially triggered analyses, where the candidate location was already identified
1429 as potentially interesting by other astrophysical messengers. Future, spatially triggered
1430 searches may be sensitive to individual sources that have been obscured by the all-sky trial
1431 factor. Ideally, the neutrino flare curves introduced in this work would aid with this, as the
1432 historical neutrino information provides an additional piece to the multi-messenger puzzle.

1433 Though the first applications of the multi-flare algorithm did not yield any significant dis-
1434 covery of astrophysical neutrino sources, the future of neutrino astronomy is still quite bright.
1435 With two interesting source candidates already identified, the answer to the question of the
1436 source of astrophysical neutrinos (and consequently, the source of high energy cosmic rays) is
1437 potentially beginning to become more clear. The upcoming years bring improvements to the
1438 IceCube detector calibration and event reconstruction, providing a more accurate descrip-
1439 tion of existing data, as well as the construction of IceCube Gen2 which will boast increased
1440 effective area at high energies, thereby reducing the atmospheric neutrino background. Ad-
1441 ditionally, upcoming radio-based neutrino experiments such as RNO-G, BEACON, IceCube
1442 Gen2-Radio, PUEO, and POEMMA will provide a view into the ultra-high energy neutrino
1443 regime, beyond what the current iteration of IceCube is realistically capable of measuring.
1444 As the identification of candidate neutrino emission from TXS 0506+056 was triggered by a
1445 single high energy neutrino event, the expansion of neutrino detectors into the high energy
1446 regime could lead the way to identifying future neutrino sources.

¹⁴⁴⁷ Chapter 8

¹⁴⁴⁸ Appendix: Statistics Background

¹⁴⁴⁹ Much of the material presented in this thesis requires an understanding of a few key areas
¹⁴⁵⁰ of statistics. This material is typically covered in almost any introductory level statistics
¹⁴⁵¹ course, but is generally not part of the standard undergraduate physics curriculum. For
¹⁴⁵² this reason, I've collected some of the key concepts in this section, to potentially serve as a
¹⁴⁵³ starting point for people who may not already be familiar with the statistical underpinnings
¹⁴⁵⁴ of recent source searches in neutrino astronomy. An interactive ipython notebook showing
¹⁴⁵⁵ examples of these concepts can also be found as part of the materials for the 2019-2021
¹⁴⁵⁶ IceCube bootcamps¹.

¹⁴⁵⁷ 8.1 Estimators

¹⁴⁵⁸ An estimator is a procedure applied to the data sample which gives a numerical value for a
¹⁴⁵⁹ property of the parent population, or, as appropriate, a property or parameter of the parent
¹⁴⁶⁰ distribution function.

¹⁴⁶¹ This is a general definition: The sample of interest may have been drawn from a large
¹⁴⁶² parent population, for which some property is to be inferred. Alternatively, it may have
¹⁴⁶³ been generated from a distribution function, arising from a basic law being investigated.

¹<https://events.icecube.wisc.edu/event/105/contributions/1054/>

1464 This distribution function also has parameters and properties, which one may also wish to
 1465 calculate (e.g. σ for a 1 dimensional gaussian distribution).

1466 Estimators are generally not classified as "right" or "wrong", rather we tend to classify
 1467 estimators as "good" or "bad" based on their properties. Some properties that are often
 1468 relevant when describing estimators:

1469 • An estimator is *consistent* if it tends to the true value as the number of data values
 1470 tends to infinity.

1471 • An estimator is *unbiased* if its expectation value is equal to the true value.

1472 • An estimator is *efficient* if its variance is small.

1473 Estimators can be any number of the above, but good estimators should fulfill all three
 1474 descriptions [55].

1475 8.2 Likelihoods

1476 One class of estimators that is commonly used in neutrino astronomy are *likelihood-based*
 1477 *estimators*. For a set of data, $x_1, x_2, x_3 \dots x_N$ produced from some underlying distribution
 1478 with parameter a , the probability of obtaining that particular data set is the product of
 1479 the individual probabilities of obtaining each of the x_i . This product is referred to as the
 1480 *likelihood* (eq. 8.1 and 8.2):

$$L(x_1, x_2, x_3 \dots x_N, a) = P(x_1; a)P(x_2; a) \dots P(x_N; a) \quad (8.1)$$

$$L(x_1, x_2, x_3 \dots x_N, a) = \prod_{i=1}^N P(x_i; a) \quad (8.2)$$

1481 The likelihood is typically used to fit the model parameter a , given some observed data
 1482 $x_1, x_2, x_3 \dots x_N$ by finding the value of a which maximizes the likelihood function. Note that
 1483 a can be a vector of values in the case that we have a model with multiple free parameters.

1484 For computational reasons, the process of maximizing the likelihood is oftentimes done
 1485 by minimizing the negative log-likelihood instead. Taking the logarithm of the likelihood
 1486 function turns the product into a sum, making the derivative more intuitive and easier to
 1487 work with numerically, and the switch from maximization of the likelihood to minimizing
 1488 the negative likelihood is purely convention.

1489 8.3 Confidence Intervals

1490 Suppose we want to know the value of a parameter X and have estimated it from the data,
 1491 giving result x . We know the resolution of our underlying distribution, and thus the variance,
 1492 $V(x)$ and the standard deviation σ . Our goal is then to create a "confidence interval": An
 1493 interval within which there is a C % chance that a single observation will lie within the
 1494 interval, where C is our "confidence level". Formally, the confidence interval (x_-, x_+) for a
 1495 given confidence level C obeys the requirement that (eq. 8.3):

$$Prob(x_- \leq x \leq x_+) = \int_{x_-}^{x_+} P(x)dx = C \quad (8.3)$$

1496 There are a few possible ways to construct this interval:

1497 1. The **symmetric interval**: x_- and x_+ are equidistant from the mean, μ , i.e $x_+ - \mu =$
 1498 $\mu - x_-$

1499 2. The **central interval**: The probability above and below the interval are equal: $\int_{-\infty}^{x_-} P(x)dx =$
 1500 $\int_{x_+}^{\infty} P(x)dx = (1 - C)/2$

1501 Common values for C are 68% (1σ), 90% (1.64σ), and 99% (2.58σ). There is a trade
 1502 off between a narrow interval and low confidence. You can say with great confidence that a
 1503 value will lie within a very wide interval, but if one wants to tie it down more precisely the
 1504 confidence lessens.

1505 8.4 Hypothesis Testing

1506 Many problems in science can be reduced to attempting to distinguish between 2 distinct
 1507 possibilities, where the observed data has different properties under each. In many cases,
 1508 we are attempting to distinguish between a "null hypothesis", which is used to refer to a
 1509 "default" model (e.g. the absence of neutrino point sources), and an "alternative hypothesis",
 1510 which represents the addition of some new physics, model parameter, or disagreement with
 1511 the "default" case (e.g the presence of neutrino point sources). Statistical hypothesis testing
 1512 is a framework for comparing these two hypotheses, given some set of observed data.

1513 We can roughly break down the process of hypothesis testing into the following steps:

- 1514 1. State relevant null and alternative hypotheses
- 1515 2. State the test statistic
- 1516 3. Derive the distribution of the test statistic under the null hypothesis
- 1517 4. Select significance level (α) at which the null hypothesis will be rejected
- 1518 5. Compute an observed test statistic from the data
- 1519 6. Calculate a p-value and decide whether to reject the appropriate hypothesis

1520 8.4.1 State relevant null and alternative hypotheses

1521 When constructing an analysis, we should be careful to state the null and alternative hy-
 1522 potheses. Typically, the null hypothesis (H_o) refers to a statement or default position that
 1523 there is no relationship between two phenomena, that there is no signal, or that our cur-
 1524 rent models describe the data well. The alternative (H_a) hypothesis usually describes the
 1525 opposite: that there is a relationship between two phenomena, that there is a signal, or that
 1526 our models to NOT describe the data well. Here are some example pairs of hypotheses one
 1527 might encounter when working with IceCube data:

1528 • H_o : Neutrino event arrival directions are not significantly correlated with the positions
 1529 of blazars identified in Fermi's 3FGL catalog

1530 • H_a : Neutrino event arrival directions are correlated with the positions of blazars iden-
 1531 tified in Fermi's 3FGL catalog

1532 • H_o : A 3-flavor oscillation model describes the IceCube data well

1533 • H_a : IceCube data is inconsistent with a 3-flavor model

1534 It is important to be careful when formulating null and alternative hypotheses so as to
 1535 not overreach with the claims being made should the alternative hypothesis be accepted.

1536 In general, it's best to keep in mind the specifics of how a particular analysis is being
 1537 constructed, and what the property of the data being measured actually is. For example, in
 1538 the first pair of hypotheses above, it would be improper for our alternative hypothesis to be
 1539 "blazars are a source of astrophysical neutrinos" if we were only examining the positions of
 1540 the 3FGL catalog, which covers only a subset of all observed blazars.

1541 **8.4.2 State a test statistic**

1542 A *test statistic* is a quantity derived from the sample, intended to be a numerical summary
 1543 of a dataset that reduces the data to one value that can be used for hypothesis testing. In
 1544 general, the test statistic is selected in such a way as to quantify, within observed data,
 1545 behaviors that would distinguish the null and alternative hypotheses. These are very similar
 1546 to estimators, and we can oftentimes use estimators as test statistics.

1547 An important property of the test statistic is that its sampling distribution under the
 1548 null hypothesis must be calculable either exactly or approximately. There are many types of
 1549 test statistics that you can use, and what the exact form of the test statistic is will usually be
 1550 determined by your analysis. One example of a test statistic is the chi-squared test statistic
 1551 intended to compare counts in an oscillation analysis (eq. 8.4):

$$\chi^2 = \sum_{i \in bins} \frac{(n_i^{\nu+\mu_{atm}} - n_i^{data})^2}{(\sigma_i^{data})^2 + (\sigma_{\nu+\mu_{atm},i})^2} \quad (8.4)$$

1552 Where χ^2 is the test statistic, i refers to bins in zenith and azimuth, $n_i^{\nu+\mu_{atm}}$ and n_i^{data} are
 1553 the number of events in a particular bin in both simulation and data, and $\sigma_{\nu+\mu_{atm},i}$ and σ_i^{data}
 1554 are similar the per-bin standard deviations in both simulation and data. This test statistic
 1555 is used to measure the agreement of the data with a "nominal" model that can be simulated
 1556 to obtain the values $n_i^{\nu+\mu_{atm}}$ and $\sigma_{\nu+\mu_{atm},i}$.

1557 Another example of a commonly used test statistic in neutrino astronomy is the point
 1558 source likelihood (eq. 8.5):

$$TS = -2 \log\left(\frac{\mathcal{L}(n_s = 0)}{\mathcal{L}(\hat{n}_s)}\right) \quad (8.5)$$

1559 This is a likelihood ratio test statistic, where \mathcal{L} is a likelihood function that describes
 1560 how clustered events are near a particular location. In its simplest form, this likelihood has
 1561 1 parameter, n_s , that describes the number of excess events seen near the location being
 1562 examined.

1563 The likelihood ratio is a very powerful test statistic. Given two hypotheses H_o and $H_a(a)$
 1564 where $H_o = H_a(\tilde{a})$ (they are *nested* models), then the Neyman-Pearson lemma states that
 1565 the likelihood ratio test has the strongest statistical power (i.e. correctly reject H_o when H_1
 1566 is true). This is the case for many point source searches, where the null hypothesis ("There
 1567 are zero signal events, $n_s = 0$) is nested in the alternative hypothesis ("There are a nonzero
 1568 number of signal events, $n_s > 0$ ").

1569 Note that test statistics are essentially estimators of the alternative hypothesis: a "more
 1570 extreme" test statistic should correspond to the data being better described by the alter-
 1571 native hypothesis. In fact, most test statistics are in fact based on estimators of some
 1572 statistical property of the data. The point source test statistic, for example, is based around
 1573 a likelihood estimator of the number of clustered neutrino events.

1574 **8.4.3 Select a significance level at which the null hypothesis will**
 1575 **be rejected**

1576 Since in testing our hypothesis, we have two options for our result (accept/reject the null
 1577 hypothesis), and there are two options for the "actual" state of our null hypothesis (either
 1578 it is true or it isn't), there are four possible outcomes describing whether we were correct or
 1579 not:

	H_o rejected	Accept (Fail to reject) H_o	
1580	H_o False	Correct	Type II error (false negative)
	H_o True	Type II error (false positive)	Correct

1581 If you reject a true null hypothesis, then this is a type II error. If you accept a false
 1582 null hypothesis, then this is a type I error. "Type I" and "type II" are needlessly confusing
 1583 names, so let's instead refer to these with the more intuitive names "false positive" and
 1584 "false negative".

1585 Both types of errors are bound to happen sometimes, but we're typically more concerned
 1586 with false positives: We don't want to make any claims of discovery when in fact it was
 1587 simply a background fluctuation. We can control the rate at which this happens with the
 1588 "significance" of the test (α). The *significance* refers to the probability of obtaining a false
 1589 positive.

1590 Given a hypothesis test with some test statistic (where we can calculate the distribution
 1591 of this test statistic under the null hypothesis, see above!), we can divide the range of our
 1592 test statistic into the acceptance region, containing test statistics less extreme than some
 1593 pre-defined significance threshold, and the rejection region, containing test statistics more
 1594 extreme than this threshold. If our observed test statistic falls in the acceptance region, we
 1595 will accept the null hypothesis to be true. If our observed test statistic falls into the rejection
 1596 region, we will reject the null hypothesis.

1597 The probability of a false positive is then simply the integral of our null hypothesis test
 1598 statistic distribution over the rejection region. This is the "significance" of the test. In
 1599 principle, we can set the significance of our test to be whatever we want, simply by changing
 1600 the acceptance and rejection regions. In practice, we typically want to set a high bar for
 1601 announcing a result (this corresponds to a low significance). In particle physics, the threshold
 1602 for rejecting the null hypothesis is typically set such that the significance is 2.87×10^{-7} (the
 1603 one-sided five sigma threshold). In other disciplines, the convention is different, with social
 1604 scientists and biologists typically rejecting the null hypothesis at the .01 or .05 significance
 1605 level.

1606 You might sometimes hear significance referred to in units of "sigma" (e.g. " 5σ " or " 3σ ").
 1607 This can be obtained by the process shown in figure 8.1, by mapping the significance to a
 1608 normal distribution. This simply provides a standardized way of describing how "extreme"
 1609 the observed significance is: more "sigmas" corresponds to a result that is less likely to be
 1610 seen under the null hypothesis.

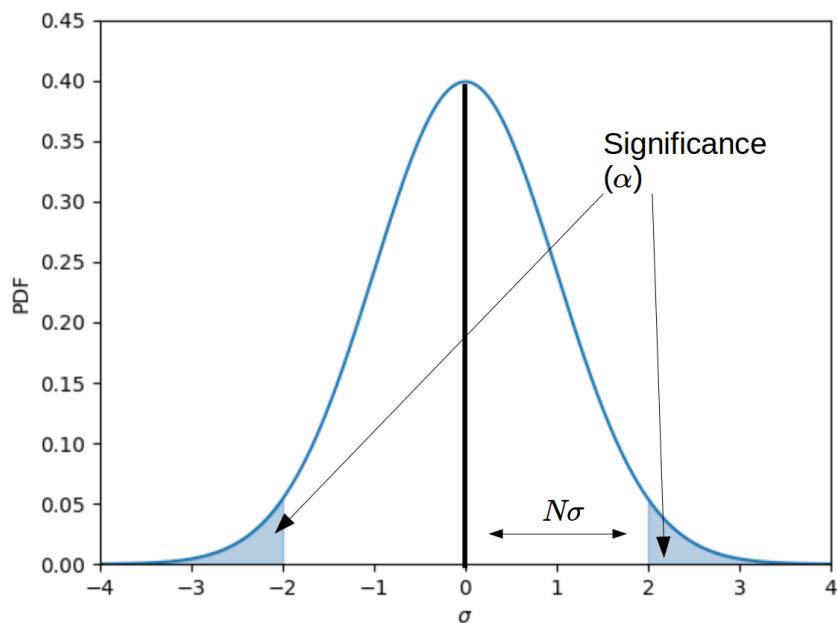


Figure 8.1: The mapping of significance into units of sigma using a normal distribution, for a two tailed test.

1611 **8.4.4 Compute the TS from observed data**

1612 This step is fairly straightforward, as the test statistic has already been defined. Simply
 1613 run your test statistic algorithm using the observed data generated from your experimental
 1614 apparatus.

1615 **8.4.5 Decide whether to accept or reject the null hypothesis**

1616 There are two different, but equivalent constructions for deciding whether to reject the null
 1617 hypothesis or not. The first construction involves pre-defining an *acceptance region* prior
 1618 to calculating the test statistic associated with the observed data. The acceptance region
 1619 corresponds to the range of test statistic values for which the null hypothesis cannot be
 1620 rejected. This is typically chosen to correspond to a particular significance (e.g. 5%, 1%,
 1621 0.01% etc.). If the observed test statistic lies outside of the acceptance region corresponding
 1622 to $X\%$ significance, then we can say that the null hypothesis is rejected at the $X\%$ signif-
 1623 icance level. Note that since we had to pick our significance threshold (and consequently
 1624 define an acceptance region) prior to computing an observed test statistic, we can make no
 1625 further claims on how extreme the result is: if our acceptance region is $TS > 12$, it does not
 1626 matter if we observe a $TS = 12.1$ or if we observe a $TS = 99.0$. In both instances we will
 1627 reject the null hypothesis as the significance level we initially chose.

1628 An alternative, but mathematically equivalent, construction involves the calculation of a
 1629 *p-value*. A p-value is the probability, under the null hypothesis, of obtaining a test statistic
 1630 at least as extreme as the one we have observed from data. If our p-value is smaller than a
 1631 pre-determined significance threshold, then we can reject the null hypothesis. The p-value
 1632 can be calculated by simply integrating our null hypothesis test statistic distribution over
 1633 the range of values more extreme than what was observed. For the one-tailed case where a
 1634 larger test statistic is correlated with stronger evidence of the alternative hypothesis, this is
 1635 (eq. 8.6):

$$p = \int_{TS_{obs}}^{\infty} PDF_{H_0} \quad (8.6)$$

1636 Where TS_{obs} is our observed test statistic and PDF_{H_0} is the distribution of test statistics
 1637 under the null hypothesis. It is important to note that the above equation is only valid for
 1638 the positive, one-tailed case (the alternative hypothesis is only associated with test statistic
 1639 values on one side of the distribution). A more general formula could be expressed as (eq.
 1640 8.7):

$$p = \int_{R|TS_{obs}} PDF_{H_0} \quad (8.7)$$

1641 Where here $R|TS_{obs}$ is the rejection region of the test statistic distribution associated
 1642 with a particular observed test statistic (i.e. it is the region of the background test statistic
 1643 distribution containing values more extreme than what was observed). This distinction is
 1644 particularly important when conducting two-tailed tests, where test statistics that are either
 1645 above an upper bound, or below a lower bound may be cause for rejecting the null hypothesis.

1646 8.5 Trial Factors

1647 Each time you obtain the answer of an analysis from real data, you have done one experi-
 1648 mental trial. It is common to perform multiple trials when working with a specific analysis
 1649 technique.

1650 • In the neutrino sources group, we might test the probability that one location is con-
 1651 sistent with background expectation. If we were to apply this framework to a list of
 1652 30 source candidate locations, then this would be 30 trials.

1653 • Repeating an analysis with new cuts. We might count the number of neutrinos with
 1654 $E > 1$ TeV in 1 year, then count the number of neutrinos with $E > 10$ TeV in 1 year.
 1655 This would be 2 trials.

- 1656 • Repeating with different signal hypotheses. You might look for emission from blazars
 1657 using both the hypothesis of steady emission and a flare hypothesis (2 trials)

1658 Trials can be both independent, or correlated:

- 1659 • **Independent:** Search for a new source/signal in 1 year of data, repeat the search in
 1660 the next year of data (separately). This would be 2 independent trials, as the results
 1661 of the first year will not affect the results of the second year
- 1662 • **Correlated:** Search for a new source/signal in 1 year of data, repeat the search with
 1663 2 years of data, including the first year. This would be 2 trials, but they would be
 1664 correlated, since the results of the first 2 years of data will be affected by the results
 1665 of the first year alone.

1666 Accounting for trials is important, as given enough trials even the null hypothesis can
 1667 produce a significant result. A simple and conservative way to account for trials is to assume
 1668 that trials are uncorrelated and apply the Sidak correction (eq. 8.8):

$$P_{post} = 1 - (1 - P_{pre})^N \quad (8.8)$$

1669 Where P_{post} is the trial-corrected p-value, P_{pre} is the smallest p-value observed across
 1670 all trials performed, and N is the number of trials. This formula is intended to describe
 1671 uncorrelated trials, however it can also be used to obtain a conservative trial correction
 1672 for correlated trials as well. More sophisticated methods of accounting for correlated trials
 1673 typically involve simulation of the null hypothesis, and are oftentimes specific to a particular
 1674 experiment.

1675 When reporting a trial corrected p-value, it is additionally important to specifically state
 1676 which trials are being accounted for. Accounting for trials across all sources in a catalog
 1677 will result in a different p-value than accounting for all analyses performed by the IceCube
 1678 collaboration, yet both are "post-trial" p-values.

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