Laboratorio 3

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Ejercicio #1

$n = [\sigma(\sigma(\sigma(0)))] = 3$
$m = [\sigma(\sigma(\sigma(\sigma(0))))] = 4$

$n \oplus m =$
$\sigma(\sigma(\sigma(0 \oplus m)))$
$\sigma(\sigma(\sigma(m)))$
$\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0)))))))$

Ejercicio #2

$$a \otimes b := \begin{cases} 0 & \text{si } a = o \\ 0 & \text{si } b = o \\ a & \text{si } b = 1 \\ b & \text{si } a = 1 \\ \sum_0^b a & \text{si } b = \sigma(i) \\ \sum_0^a b & \text{si } a = \sigma(i) \end{cases}$$

Caso base
b = 1
$a = \sigma(n)$
$a \otimes b$
$\sigma(n) \otimes \sigma(0)$
$\sigma(n)$

Caso inductivo
$b = \sigma(\sigma(0))$
$a = \sigma(n) = \sigma(\sigma(\sigma(0)))$

$a \otimes b$
$\sigma(n) \otimes \sigma(\sigma(0))$
$\sigma(n) \oplus \sigma(n)$
$\sigma(\sigma(\sigma(0))) \oplus \sigma(\sigma(\sigma(0)))$
$\sigma(\sigma(\sigma(\sigma(\sigma(\sigma(0))))))$

Ejercicio #3

- $\sigma(\sigma(\sigma(0))) \otimes 0$

  $a = \sigma(\sigma(\sigma(0)))$
  $b = 0$

  $a \otimes b =$
  $\sigma(\sigma(\sigma(0))) \otimes 0 = 0$

- $\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$

  $a = \sigma(\sigma(\sigma(0)))$
  $b = \sigma(0)$

  $a \otimes b =$
  $\sigma(\sigma(\sigma(0))) \otimes \sigma(0)$
  $\sigma(\sigma(\sigma(0)))$

- $\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$

  $a = \sigma(\sigma(\sigma(0)))$
  $b = \sigma(\sigma(0))$

  $\sigma(\sigma(\sigma(0))) \otimes \sigma(\sigma(0))$
  $\sigma(\sigma(\sigma(0))) \oplus \sigma(\sigma(\sigma(0)))$
  $\sigma(\sigma(\sigma(\sigma(\sigma(0))))))$

Ejercicio #4

- $a \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$

  Caso base

  $a = 0$
  $0 \oplus \sigma(\sigma(0)) = \sigma(\sigma(a))$
  $\sigma(\sigma(0)) = \sigma(\sigma(a))$

  Caso inductivo

  $a = \sigma(a)$
  $\sigma(a) \oplus \sigma(\sigma(0)) = \sigma(\sigma(\sigma(a)))$
  $\sigma(\sigma(0 \oplus \sigma(a))) = \sigma(\sigma(\sigma(a)))$ Por definicion de suma
  $\sigma(\sigma(\sigma(a))) = \sigma(\sigma(\sigma(a)))$ Por propiedad de suma

- $a \otimes b = b \otimes a$

$$a \otimes c := \begin{cases} 0 & \text{si } a = o \\ 0 & \text{si } c = o \\ a & \text{si } c = 1 \\ c & \text{si } a = 1 \\ (a \otimes c) \oplus a & \text{si } c = \sigma(c) \end{cases}$$

Caso base
$a = 0$
$a \otimes b = b \otimes a$
$0 \otimes b = b \otimes 0$
$0 = 0$

Caso inductivo
$a = \sigma(a)$
$a \otimes b = b \otimes a$

$a \otimes \sigma(b) = (a \otimes b) \otimes a$ Por definicion de la multiplicacion
$(a \otimes b) \oplus b = (b \otimes a) \oplus b$ Por hipotesis inductiva
$(a \otimes b) \oplus b = (a \otimes b) \oplus b$

- $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

Caso base

$c = 0$
$a \otimes (b \otimes 0) = (a \otimes b) \otimes 0$
$a \otimes 0 = 0$
$0 = 0$
Caso inducctivo

$a \otimes (b \otimes c) = (a \otimes b) \otimes c$ Hipotesis inductiva
$c = \sigma(c)$
$a \otimes (b \otimes \sigma(c)) = (a \otimes b) \otimes \sigma(c)$ Por definicion de multiplicacion
$a \otimes (b \oplus \sigma(c)) = (a \otimes b) \otimes c \oplus (a \otimes b)$ Por hipotesis inductiva
$a \otimes (b \oplus \sigma(c)) = a \otimes (b \otimes c) \oplus a \otimes b$ La suma es conmutativa
$a \otimes (b \oplus \sigma(c)) = (a \otimes b) \oplus a \otimes (b \otimes c)$ Por factor comun
$a \otimes (b \oplus \sigma(c)) = a \otimes (b \oplus (b \otimes c))$ Por definicion de multiplicacion
$a \otimes (b \oplus \sigma(c)) = a \otimes (b \oplus \sigma(c))$

- $(a \oplus b) \otimes c = (a \otimes c) \oplus (b \otimes c)$

Caso base
$c = 0$
$(a \oplus b) = n$
$n \otimes 0 = a \otimes 0 \oplus b \otimes 0$
$0 = 0 \oplus 0$
$0 = 0$

Caso inductivo
$(a \oplus b) \otimes c = a \otimes c \oplus b \otimes c$
$c = \sigma(c)$
$(a \oplus b) \otimes \sigma(c) = a \otimes c \oplus b \otimes c$ Por definicion de multiplicacion
$(a \oplus b) \otimes c \oplus (a \oplus b) = (a \otimes c) \oplus a \oplus (b \otimes c) \oplus b$ La suma es conmutativa
$(a \oplus b) \otimes c \oplus (a \oplus b) = (a \otimes c) \oplus (b \otimes c) \oplus a \oplus b$ Por factor comun
$(a \oplus b) \otimes c \oplus (a \oplus b) = (a \oplus b) \otimes c \oplus a \oplus b$ La suma es comutativa
$(a \oplus b) \otimes c \oplus (a \oplus b) = (a \oplus b) \otimes c \oplus (a \oplus b)$