Faculty of Engineering & Applied Science

MECE 2860U - Fluid Mechanics

Project 1

Group 45

Study of Fluid Statics through the Concept of Buoyancy: A Comprehensive Analysis of Hot Air Balloon

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1. Introduction:

In 246 BC Archimedes was assigned to construct the biggest boat ever made. The plan proposed was named “Syracusia” ; the dimensions were 55 m long, 14 m wide, and 13 m high. Its size was originally estimated as at least 150 tons cargo, or 300 to 400 tons displacement at full load. It was utterly impossible for its time to create such a behemoth structure like that to combat the initial design problems; it was the job of Archimedes to pioneer the field of Naval Architecture and Marine Engineering to construct this mega structure.

One day when he was bathing he discovered something and ran naked yelling “EUREKA” throughout the royal palace. What he discovered was that the buoyant force on an object submerged in a fluid is equal to the weight of the fluid displaced by the object; this phenomenon is also called ‘Archimedes Principle’ [1]. Mathematically, Archimedes' principle can be expressed as:

$$\mathbf{F(b)=\rho \cdot V \cdot g}$$

Where:

- **F(b) =is the buoyant force,**
- **ρ (rho) is the density of the fluid,**
- **V is the volume of the fluid displaced by the object, and**
- **g is the acceleration due to gravity.**

According to this equation, the buoyant force is equivalent to the fluid's weight that the object has displaced. Based on their densities and volumes, it aids in the explanation of why things float or sink in fluids.

Our group decided to go ahead with:

Study of Fluid Statics through the Concept of Buoyancy: A Comprehensive Analysis of Hot Air Balloons

A hot air balloon consists of three parts: envelope, basket, and burner system, as it can be seen in figure 1. By using the concept Archimedes principle it propagates through air currents. A large envelope filled with heated air powers a hot air balloon. The balloon rises as a result of the air's reduced density and increased buoyancy as it warms. By modifying the temperature inside the envelope, pilots can control altitude. Wind currents are essential for navigation, and landing is accomplished by cooling the air to decrease buoyancy. Typically the majority of hot air balloons are Obovoid in shape as shown in figure intended to collect the most hot air possible at the top, where it is most effective at producing lift. In addition to supporting the basket, burners, fuel, and passengers safely, this places the least amount of strain on the fabric and structure [2]. The buoyant force experienced by the balloon is equivalent to the reduced density of air also expressed as.

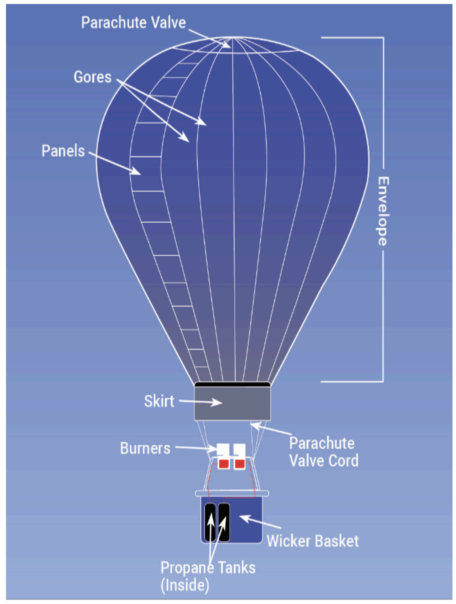


Figure 1: Example of a Hot air Balloon Components [3]

For the purpose of this project our group is going to select a hot air balloon with specific size, shape and dimensions and analyse fluid statics forces acting upon it.

2. Problem Analysis:

The centre of gravity and forces acting on a hot-air balloon at an altitude 1000 metres above sea level will be analysed in order to calculate and analyse the stability and generated lift on the balloon. Due to the varying sizes of hot-air balloons and varying air densities inside the hot-air balloon, approximate values will be used. The dimensions of the approximate hot-air balloon's envelope is 15 metres in diameter and 20 metres in height. The approximate density of the air inside the envelope will be taken as 0.9486 kg/m^3. This is the density for dry air at 99 °C [4]. The volume of the air in the envelope will be calculated based on the dimensions of the balloon. The basket of the balloon will also be accounted for in the Center of Gravity (COG) calculation. The basket will be approximated to a cube with an edge length of 1.2m. We chose 1.2m as the cube edge as it is just a bit more than half the height of an average human (1.6-1.8m). The mass of the basket will be approximated to 300 kg [5] including all contents. The density of outside air at 1000m was found to be 1.112 kg/m^3 from standard atmospheric air properties tables [6]. The weight of the envelope will also be approximated to 100 kg [7].

3. Calculations:

Buoyancy Force

Based on Archemides principle we know that Buoyancy force is equal to the mass of the fluid and the acceleration of gravity, so $F_b = m_f*g$, and since we know $m_f = V*\rho$, mass of fluid is equal to its volume times its density, taking into account the difference between the pressure of the cold air and warm air, since we need this difference to get a resulting force, we take $\Delta\rho = \rho_c - \rho_h$ where ρ_c is the density of the cold air, and ρ_h is the density of the hot air. Then the equation becomes:

$$F_b = V \times (\Delta\rho) \times g$$

Taking into to consideration:

$G = 9.81 \frac{m}{s^2}$

$\rho_c = 1.112 \frac{kg}{m^3}$

$\rho_h = 0.9486 \frac{kg}{m^3}$

$V = 2319.378951 \text{ m}^3$

Thus our Buoyancy force is:

$$\left|F_b\right| = 2319.378951m^3 \times (1.112 \frac{kg}{m^3} - 0.9486 \frac{kg}{m^3}) \times 9.81\frac{m}{s^2}$$

$$\left|F_b\right| = 3.72\ kN$$

Weight Force

Using the mass of the basket 300kg, the mass of the heated air, 2200.2kg, and mass of the envelope, 100kg, we can find the weight force, using F = ma, where a is gravity. F = 2800.2kg*9.81m/s^2 = 27.5kN

Center of Gravity

For the centre of gravity, assuming the centre of the bottom of the basket to be at point (0, 0, 0), a few assumptions were made to simplify calculations. Firstly, for simplicity the balloon is assumed to be the shape of a hemisphere on top of a cone, while the basket is assumed to be a cube. As stated above, the balloon is assumed to have a radius of 7.5 metres and a height of 20 metres. This means the radius of the hemisphere and cone are both equal to the radius of the balloon, 7.5 m. The height of the sphere is 7.5 m / 2 = 3.75 m, while the cone's height is 20 m - 3.75 m = 16.25 m. It is also assumed that the mass of each part is evenly distributed throughout it, and that therefore due to the symmetry of the balloon the centroid is located in the horizontal centre (this assumption includes the basket with people inside).

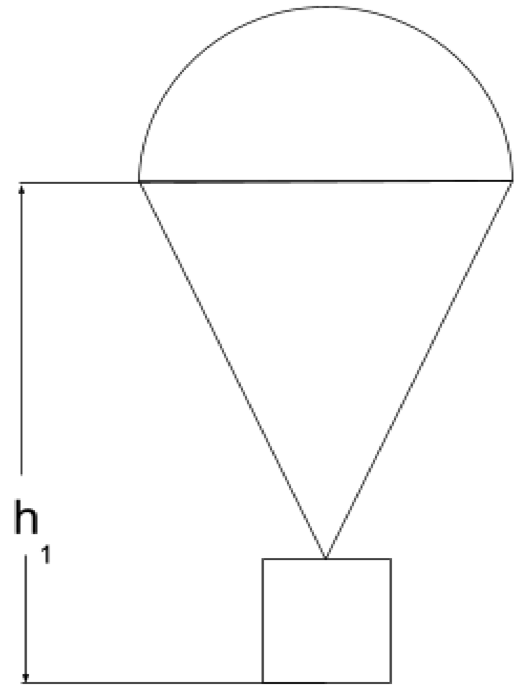
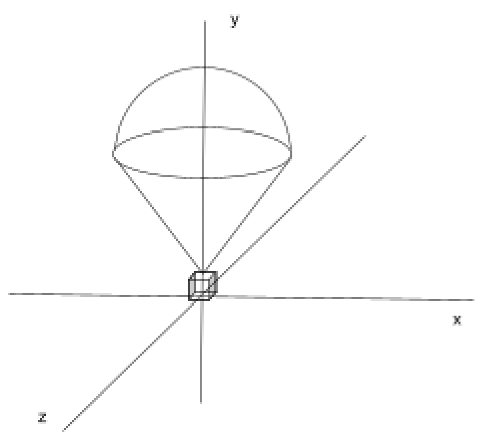


Figure 2: A 2-dimensional representation of assumed balloon shape, with dimension h_1 labelled.



Figures 3: a 3-dimensional representation of assumed shape of hot air balloon

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W}$$

$\bar{y}$ = centre of gravity of composite body
$\tilde{y}$ = centre of gravity of individual parts
W = weight of composite parts

$h_1 = h_{cube} + h_{cone}$
$= 12.5\text{ m} + 1.2\text{ m} = 13.7\text{ m}$

Part	$\tilde{y}$	$m = \cancel{V}\rho$
Hemisphere	$h_1 + \frac{3}{8}r \quad = 13.7\text{ m} + \frac{3}{8}*7.5\text{ m} = 16.5125\text{ m}$	Both equal the mass of the envelope + hot air. Mass of envelope is assumed to be 100 kg, mass of hot air is calculated below
Cone	$h_1 - \frac{h_{cone}}{4} = 13.7\text{m} - \frac{16.25}{4}\text{m} = 9.6375\text{ m}$	
Cube	$0.5h_{cube} \quad = 0.5 * 1.2\text{ m} \quad = 0.6\text{ m}$	300 kg (assumed)

Table 1: Centroid Equations

For envelope centroid without mass:

$$\cancel{V}_{hemisphere} = \frac{2}{3}\pi r^3 = \frac{1125}{4}\pi\text{ m}^3$$

$$\cancel{V}_{cone} = \frac{1}{2}\pi r^2 h = 1435.806017\text{ m}^3$$

$$\cancel{V}_{balloon} = \frac{1125}{4}\pi + 1435.806017\text{ m}^3 = 2319.378951\text{ m}^3$$

$$\bar{y} = \frac{\Sigma\tilde{y}V}{\Sigma V} \text{ (where V represents volume } \cancel{V}) = \frac{16.5125*\frac{1125}{4}\pi+9.6375*1435.806017}{\frac{1125}{4}\pi+1435.806017}$$

$$\overline{y_{balloon}} = 12.25654762\text{ m}$$

$$m_{hot\ air} = \rho_{hot\ air} * \cancel{V}_{balloon} = 0.9486\text{kg/m}^3 * 2319.378951 = 2200.162873\text{ kg}$$

$$\bar{y} = \frac{\Sigma\tilde{y}W}{\Sigma W} = \frac{\Sigma\tilde{y}m}{\Sigma m} = \frac{0.6\ m * 300\ kg + 12.25654762\ m * (2200.162873+100)\ kg}{300\ kg + 2200.162873\ kg + 100\ kg} = 10.91164561$$

m

$$\approx 10.91\text{ m}$$

(Gravity factors out to mass because gravity acts the same on all parts, non-rounded values used in calculations above)

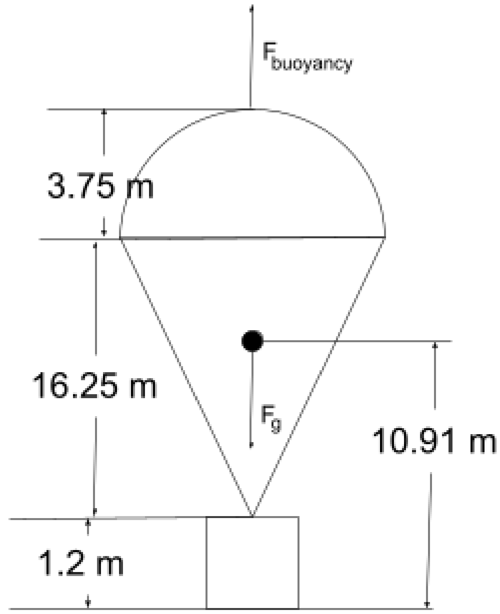


Figure 3: Diagram for Center of Gravity

The centroid of the balloon is calculated to be located roughly 12.26 m above the bottom of the basket, and the mass of the balloon is calculated to be (100 + 2200.162873) kg from the envelope and hot air, respectively. The balloon's volume is equal to $2319.378951\ m^3$.

For the basket, it is assumed to be a cube and therefore the volume is $(1.2\ m)^3$, 1.728 m^3. The basket is assumed to have a mass of 300 kg.

Finally, using the formula for the centre of gravity of composite shapes, the balloon's centre of gravity is roughly 10.91 m above the base of the basket. Horizontally the centre of gravity can be assumed to be centred due to the symmetry of the balloon and because the exact locations of many of the separate parts such as the riders, burner and fuel are neglected due to variation between different balloons.

As depicted in the image above, the force of buoyancy acts above the force of gravity. This illustrates mathematically why both boats and air balloons rely on buoyancy yet air balloons don't risk tipping the same way boats do. The buoyancy force is not pushing the base of the object upwards, and gravity is not pulling the top of the object downwards.

Generating lift

Increasing the air temperature inside the envelope makes it less dense than the surrounding (ambient) air. The balloon floats because of the buoyant force exerted on it.

4. Conclusions:

The comprehensive analysis of the forces acting on a hot-air balloon at an altitude of 1000 metres above sea level has provided us with valuable insights into the stability and lift generated by the balloon. The application of Archimedes' principle, coupled with the consideration of the centre of gravity, has allowed us to calculate these forces with a high degree of accuracy.

The buoyancy force, calculated based on the volume and density of the air inside the balloon's envelope, was found to be 3.72 kN. This force is a direct result of the difference in pressure between the hot air inside the balloon and the colder outside air.

The weight force, which takes into account the mass of the basket, the heated air, and the envelope, was calculated to be 27.5 kN. This force acts downwards and is balanced by the upward buoyancy force to keep the balloon afloat.

The net force experienced by the balloon is a resulting -23.87kN (upwards being positive) or 23.87kN downwards, the resulting acceleration is 8.5 m/s^2 downwards. This results determines that the balloon at this position will be descending. This high acceleration might be attributed to including the volume of the cone part as seen in figure 2, in our calculations causing the mass of the heated air to be much higher.

The centre of gravity was calculated by considering the balloon as a combination of a hemisphere and a cone, and the basket as a cube. The symmetry of the balloon and the even distribution of mass throughout each part allowed us to locate the centroid in the horizontal centre.

This study has not only deepened our understanding of fluid statics and their application in hot air balloons but also highlighted the importance of effective teamwork in solving complex engineering problems. The project has demonstrated the practical implementation of theoretical concepts, reinforcing the value of both scholarly and practical knowledge in engineering.

The calculations and analyses conducted in this study are based on approximate values due to the varying sizes of hot-air balloons and varying air densities inside the hot-air balloon. Despite these approximations, the results provide a clear and practical understanding of the forces acting on a hot-air balloon and its stability at a given altitude. This study serves as a testament to the power of combining connections in solving intricate engineering problems.

Overall, this project has been a valuable learning experience, providing a clear perspective on fluid statics, the principles of buoyancy, and the practical application of these concepts in the real world.

References:

[1] J. M. Turfa and A. G. Steinmayer, "The Syracusia as a giant cargo vessel," International Journal of Nautical Archaeology, vol. 28, no. 2, pp. 105–125, May 1999. doi:10.1111/j.1095-9270.1999.tb00826.x

[2] "Anatomy of a special shape," MJ Ballooning, https://www.mjballooning.com/blog/anatomy-of-a-special-shape (accessed Feb. 5, 2024).

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[4] "Hot Air Balloon," Academic Accelerator, https://academic-accelerator.com/encyclopedia/hot-air-balloon (accessed Feb. 5, 2024).

[5] G. Southworth, "Average hot air balloon and basket weight (with 31 examples)," OutdoorTroop, https://outdoortroop.com/average-hot-air-balloon-and-basket-weight-with-31-examples/ (accessed Feb. 5, 2024).

[6] "U.S. standard atmosphere vs. altitude," The Engineering ToolBox, https://www.engineeringtoolbox.com/standard-atmosphere-d_604.html (accessed Feb. 4, 2024).

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