Transient temperature analysis in Pool Boiling of a cylindrical Zirconium rod in 71.2 bar and 1 bar pressure

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Abstract-This study investigates the temporal evolution of the temperature of Zirconium cladding in water Pool Boiling conditions under a constant heat source generated by decay heat at two different pressures: 71.2 bar and 1 bar. The study aimed to determine whether the Pool Boiling heat transfer mechanism was sufficient to effectively remove the heat from the cladding without a significant increase in temperature. It was shown that in both pressure conditions starting from a small difference between the wall temperature and the saturated pool liquid temperature, a steady state condition was reached in the Natural Convection/Nucleate Boiling region for 1 bar pressure and for the 71.2 bar pressure condition a steady state was reached in the Nucleate Boiling region. Additionally, it was interesting to observe the evolution in time of the cladding temperature when the initial temperature was set at the Film boiling region. The results show that the Film boiling heat transfer mechanism effectively removed heat from the cladding wall in 1 bar pressure conditions, meanwhile, in the 71.2 bar condition, the Film Boiling region wasn't reached. These findings were obtained using MATLAB code developed by the author of this paper at the following link.MATALB code

I. INTRODUCTION

In Nuclear reactor design, it is essential to ensure that in one of the many accidental conditions, such as the immediate cease of water circulation of the pumps, the cladding temperature does not reach critical conditions. During the shutdown of a reactor, a non-negligible amount of heat is produced due to decay products inside the core. This heat must be removed. In standard operating shutdown procedures, water pumps are activated to maintain an adequate amount of heat removal through water circulation. Without water circulation, a Pool Boiling model can adequately describe the heat transfer mechanism on the heated cladding surface. When considering a fuel rod bundle, the prediction scheme required to model the heat transfer is rather complex since classical Pool Boiling is characterized by the fact that the heated surface can freely communicate with the surrounding pool of liquid because of the close spacing of the rods, circulation is limited, and thus, an overall inferior thermal performance is expected. The transient analysis is more complicated than a standard fuel rod. It is in the interest of this paper to investigate a more simplified model that is a single vertical cylindrical Zirconium rod in Pool Boiling conditions. The pressure was set at 71.2 bar to simulate heat removal through Pool Boiling in nominal BWR pressure conditions. The cladding was taken at a temperature slightly above the saturated temperature at 71.2 bars. Even though this situation is not realistic regarding nuclear reactor accidents, this simple model aims to investigate the standard temperature evolution in time of a fuel rod. For this reason, the Film Boiling heat transfer model was implemented. When we have an immediate shutdown of the water pumps, the cladding temperature is already in the Film Boiling region and can only evolve over time. The modeling at 1 bar was used as a reference model for comparison purposes. The four fundamental modes of heat transfer in Pool Boling are: *Natural Convection*, *Nucleate Boiling*, *Transition Boiling* and *Film Boiling*. The **Theoretical framework** section is the guideline used for the following work, and it is divided into five sections. Section *one* delivers a basic mathematical introduction to the transient equation used to describe the evolution in time of the temperature, this equation will be repeatedly used in different regions of the Pool Boiling curve with slight modification in order to account for the different mechanism of heat transfer. Section *two* describes the Natural Convection model of heat transfer up to the Onset of Nucleate Boiling, section *three* describes the Onset of Nucleate Boiling, section *four* describes the heat transfer in Nucleate Boiling up to steady state and section *five* describes the Film Boiling heat transfer mechanism for the rod starting at the temperature at the Leidenfrost point. Results for 1 bar and 71.2 bar conditions are shown in the **Results and Discussion** section. (All variables are specified at the end of the paper in the **Table Index**).

II. THEORETICAL FRAMEWORK

The differential equation that describes the evolution in time of the internal energy of a system is given by:

$$\frac{dU}{dt} = Q_{in} - Q_{out} \tag{1}$$

We can rewrite the above differential equation for the wall temperature of the Zirconium cladding

$$\frac{dT_{wall}}{dt} = \frac{Q_{in}}{M_{rod}c_{Zr}} - \frac{A_w \alpha_{conv}(T_{wall} - T_{sat})}{M_{rod}c_{Zr}} \tag{2}$$

The solution to equation (1) (with the proper boundary conditions) fully describes the evolution in time of my system. It contains a source term given by the decay heat and a removal term given by the convection term. However, this equation assumes that the cladding temperature $T(r,t) = T(t)$, which is not always the case. We must therefore check that the $Bi \leq 0.1$ condition is met in every instance in time.

Because of the very complex nature of the α_{conv}, analytically, the differential equation is not solvable; we shall therefore use Numerical Methods to find an approximate solution. The Zirconium thermal conductivity is calculated using the following formula:
$k_{Zr} = 8.8527 + 7.0820 \cdot 10^{-3} \cdot T + 2.5329 \cdot 10^{-6} \cdot T^2 + 2.9918 \cdot 10^3 \cdot T^{-1}$.(Reference [2])
This formula calculates for each increase in temperature the correct Zirconium thermal conductivity.

A. Natural Convection

When a stagnant single-phase fluid is in contact with a heated surface, it will exchange heat through *Natural Convection*. Natural convection is a type of heat transfer that occurs due to the density differences in a fluid caused by temperature variation. The motion of the fluid is driven by the buoyancy forces generated by these temperature differences. When the fluid is heated, the temperature increases, and the fluid near the heat source becomes less dense and rises due to the buoyancy force. As the fluid rises, it moves away from the heat source and cools, as it cools, it becomes denser and sinks back down toward the heat source. This process creates a continuous circulation pattern. This type of heat transfer is very sensitive to the geometry of the surface and the way we cool our surface. In our discussion we treat the cooling of a vertical cylindrical fuel rod, therefore the characteristic length L_c is the length of the rod. The relationship that describes *Natural Convection* is given by the adimensional equation:

$$Nu = C(Ra)^n; \tag{3}$$

Ra is the *Rayleigh number*, and is defined as the product between the *Grashoff number*(Gr) and the *Prandtl number*(Pr). The value of the terms n and C depend on the type of flow pattern that is established. In particular, given a vertical surface:
$n = 0.25$; $C_{nc} = 0.75$ for *Laminar flow*
$n = 0.33$; $C_{nc} = 0.15$ for *Turbolent flow*
By explicating the terms of equation (3) to yield the heat transfer coefficient we obtain α_{conv}. In order to calculate the dependency of the T_{wall} with t, equation (2) must be solved with the appropriate expression for the α_{conv}. Defined as:

$$\alpha_{nc} = C_{nc}\frac{k_l}{L}\left(\frac{g\beta L^3\rho_l^2}{\mu_l^2}\frac{c_{p,l}\mu_l}{k_l}\right)^n (T_{wall} - T_{sat})^n \tag{4}$$

In order to verify the type of flow pattern we must calculate in each iteration the Ra number. In particular:
$Ra > 10^3$ and $Ra < 10^9$ for *Laminar flow*
$Ra > 10^9$ for *Turbolent flow*
For each iteration in time the Ra is computed and verified with the *Laminar flow* condition, since in this paper the values chosen for n and C are chosen to be based on *Laminar* flow for the liquid. The behavior of the T_{wall} vs t is only needed up to the *Oneset of Nucleate Boiling* temperature.

B. Onset of Nucleate Boling

When a surface is heated, tiny bubbles of vapor form at nucleation sites. These bubbles then rise to the surface and release the vapor into the surrounding medium. These nucleation sites are typically microscopic irregularities or impurities on the heated surface, such as scratches or rough spots. As heat is applied, these sites become hot enough to initiate boiling and small bubbles of vapor form at these locations. In order to describe the physical point where bubble formation starts, we will use as a first approach the *Davis Henderson Model*. According to this model, the way to predict the *Onset of Nucleate Boiling* is to determine the *critical radius*. This radius is the radius for which the curvature of the bubble at the irregularity is exactly 90°. From a more analytical point of view, it is the point where the *Bubble Equilibrium curve* is tangent to the *Temperature profile of the liquid*, which is assumed to be linear up to the *conduction thickness* δ_t since it obeys the *Fourier law of conduction*. The limits of this model as we will see reside in that fact that the rc critical radius is independent from the "shape of the cavity", leading to erroneous conclusions to the *dimensions* of the cavity. The ΔT predicted by the model is a *Lower bound for nucleation*, meaning that nucleation does not occur below this temperature. To solve for the T_{onb}, we must know the mode of heat transfer up to the *Onset of Nucleate Boiling*, that is *Natural Convection*. According to the *Davis Henderson Model* the heat flux removed at the onset, is:

$$q''_{onbDH} = \frac{(T_{onb} - T_{sat})^2 h_{lg} k_l}{8\sigma T_{sat} v_g} \tag{5}$$

We substitute the heat flux of equation (5) into equation (6) with the heat transfer coefficient equal to equation (4) and solve for T_{onb}

$$q''_{nc} = \alpha_{nc}(T_{onb} - T_{sat}) \tag{6}$$

The equation obtained for the T_{onb} is particularly complicated, but if we group the constants, we arrive at a simple form of this type where we can solve for the T_{onb}:

$$(T_{onb} - T_{sat})^{n-1} = \frac{k_2}{k_1} \tag{7}$$

Once we calculate the T_{onb} we will call this temperature T_{onbDH}, that is the temperature at the *Onset of Nucleate Boiling* for the DH model. We can calculate using equation (5) the q''_{onb}, that is the Heat flux exchanged at the Onset. Once calculated all of these quantities we have to determine the *critical radius*. If rc is greater than the *critical radius* for water on metallic surfaces, we call this radius $r_{c,max}$, then the *Davis Henderson Model* is to be discarded. We calculate the r_c and compare it with $r_{c,max}$. Using this equation:

$$r_c = \sqrt{\frac{2\sigma T_{sat} v_g k_l}{q''_{onb} h_{lg}}} \tag{8}$$

If $r_c \geq r_{c,max}$, (we will see that in *Results and Discussion* section this is the case for both pressures) we must use a different equation that is;

$$T_{sat} + \frac{2\sigma T_{sat} v_g}{h_{lg} r_{c,max}} = T_{onb} - \frac{q''_s}{k_l} r_{c,max} \tag{9}$$

By substituting in q''_s of equation (9) the expression of equation (6) with α_{nc} equal to equation (4), we can solve using a numerical method the non-linear equation:

$$(T_{onb} - T_{sat}) - \frac{r_{c,max}k_1}{k_l}(T_{onb} - T_{in})^{n+1} - k3 = 0 \tag{10}$$

By solving this non linear equation we will obtain the true temperature at the *Onset of Nucleate Boiling*. Solving the problem by knowing directly the value of the $r_{c,max}$, means that the *Bubble Equilibrium curve* does not intersect tangentially the *Temperature profile of the liquid*, but intersect the curve in two distinct points $r_{c,min}$ and $r_{c,max}$. In particular at the $r_{c,max}$, we compute a ΔT that is sure to activate all nucleation cavities, since the roughness is not uniform on the surface, and that cavities require different temperatures to be activated.

C. Nucleate Boiling

Once we have reached the *Onset of Nucleate Boiling*, we know that all nucleation cavities have been activated and therefore *Nucleate Boiling* starts. This region of the Pool Boiling curve is comprised of two parts. At low superheating we have a *Region of isolated bubbles*, at high superheating we have a *Region of bubble columns and vapour jets*.
Nucleate Boiling is the preferred way to exchange large amounts heat with small variations between the T_{wall} and T_{sat}. If the amount of heat exchanged increases to very large values it can lead to a phenomenon known as *Boiling crisis* given by a specific value of the heat flux, known as the *Critical heat flux*. This point results in a rapid reduction in heat transfer efficiency and potentially may damage the heated surface. This critical point is explained by hydrodynamic instability theory. We can calculate the critical heat flux using the *Kutateladze Model* given by:

$$q''_{chf} = K\rho_g^{1/2}h_{lg}[\sigma g(\rho_{lg})]^{1/4} \tag{11}$$

Where $K = 0.16$ [reference(1) pg105]. Once we have determined the critical heat flux we can determine the range of validity where *Nucleate Boiling* models can be applied. In this paper, two models have been used. The *Rosenhow Model* and the *Mostinski Model*. The differential equation with the *Rosenhow Model* is the following:

$$\frac{dT_{wall}}{dt} = \frac{Q_{in}}{M_{rod}c_s} - \frac{\mu_l h_{lg}[g(\frac{\rho_{lg}}{\sigma})]^{1/2}[\frac{c_{p,l}}{C_{s,f}h_{fg}Pr_l^n}]^3(T_{wall} - T_{sat})^3}{M_{rod}c_s} A_w \tag{12}$$

We will solve this differential equation using the *Forward Euler method*. The value for $C_{s,f} = 0.013$ will be used because of the absence of information regarding the $C_{s,f}$ for the Zirconium Water combination. For the *Mostinski Model* the equations implemented are the following:

$$A^\star = 0.101p_c^{0.69}; F_{pr} = 1.8p_r^{0.17} + 4p_r^{1.2} + 10p_r^{10}; \tag{13}$$

$$\frac{dT_{wall}}{dt} = \frac{Q_{in}}{M_{rod}c_{Zr}} - A_w\frac{A^\star F_{pr}(T_{wall} - T_{sat})^{1/0.3}}{M_{rod}c_{Zr}} \tag{14}$$

In both models, it is necessary to calculate the *Onset of Nucleate Boiling* with the following equations:

$$T_{onb-Rosenhow} = (\frac{q''_{onb}}{R_{constant}})^{1/3} + T_{sat} \tag{15}$$

$$T_{onb-Mostinski} = \frac{(q''_{onb})^{0.3}}{A^\star F_{pr}} + T_{sat} \tag{16}$$

q''_{onb} is the heat flux at the *Onset of Nucleate Boiling* determined in the previous section. It is necessary to calculate these points because they give important insight regarding the way boiling occurs and is predicted in accordance with the different models. It will be shown in the *Results and Discussion* section that in different pressure conditions, these values can be either greater or lower than the T_{onb} predicted in the *Natural Convection region*. In the case of lower T_{onb} it means that boiling requires less ΔT to be maintained respect to the ΔT to be onsetted. In the case of higher T_{onb} it means that boiling requires a greater ΔT to be maintained than to be onsetted. It is important to notice that because of the very high heat transfer coefficient in the *Nucleate Boiling region* the Bi number defined as $Bi = \frac{\alpha_{conv}L_c}{k_{Zr}}$, may be larger than 0.1. When this condition is met the T is not only a function of *time* but also of *position*. To solve the PDE associated to the heat equation a *Finite difference method* is needed. Because of the cylindrical geometry and the heat characteristics of the problem, it is better to work with a *Cylindrical heat equation* with the following simplifications:
1) Heat source generated is isotropic and uniform along ϕ and the z direction, we can therefore remove the dependency on ϕ and z;
2) The Heat removed through boiling is the same for all values of z, we have uniform heat removal along the rod.
The following heat equation becomes:

$$c_{Zr}\rho_{Zr}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(k_{Zr}r\frac{\partial T}{\partial r}) + q_{rod} \tag{17}$$

The discretized heat equation uses the following finite differences approximations:

$$\begin{cases} \frac{dT}{dr} \simeq \frac{T(i+1)-T(i-1)}{2\Delta r} \\ \\ \frac{d^2T}{dr^2} \simeq \frac{T(i+1)-2T(i)+T(i-1)}{\Delta r^2} \end{cases} \tag{18}$$

With the following Boundary conditions

$$\begin{cases} -k_{Zr}\frac{dT}{dr}|_{r=R} = \alpha_{conv,nb}(T_{wall} - T_{sat}) \\ -k_{Zr}\frac{dT}{dr}|_{r=0} = 0 \\ T(r,0) = T_{Bi_{stop}} \end{cases} \tag{19}$$

This numerical method is needed to have a correct understanding of the *Transient* behavior of the temperature. What will be shown in the *Results and Discussion* section is that the temperature reached in steady state conditions is in the safe range for the cladding.

D. Film Boiling

The last region of interest in this paper is the *Film Boiling region*. This region is a stage in the process of boiling where a continuous layer of vapor, or film forms on the heated surface. This film acts as an insulator, reducing the heat transfer rate between the surface and the liquid. The formation of the layer can lead to a rapid increase in surface temperature and is therefore a dangerous region. Heat transfer is dominated by conduction, convection, and radiation.
It is possible to notice that given a specific heat flux input, the same q''_s is achieved in different region of the Pool Boiling curve with different ΔT, including the *Film Boiling region*.

Given equation (2), the solution to the problem varies considerably when a specific initial temperature is computed to kickstart the numerical solution. This means that when we insert the initial temperature of the wall, we must already know to which part of the Pool Boiling curve we are referring. What will be shown is that following the temperature behavior of the wall starting from a $\Delta T \simeq 0$, the *Film Boiling region* is not reached. But for the reasons mentioned above regarding the possibility to have Film Boiling that we must consider this region in our analysis. The cladding temperature after an immediate shutdown of the water pumps can already be at a temperature where Film Boiling is the only way to remove heat. Before exploring this part of the Pool Boiling curve we must know the minimum value of the heat flux and temperature necessary to kickstart the Film boiling convection. This point of the curve is known as the *Leidenfrost point*. The ΔT required is given by the *Berenson Temperature* that is:

$$\Delta T_{Berenson} = 0.127 \frac{\rho_g h_{lg}}{k_g} g^{2/3} (\frac{\sigma}{g \rho_{lg}})^{1/2} (\frac{\mu_g}{g \rho_{lg}})^{2/3} \tag{20}$$

If we wish to calculate the heat flux at the *Leidenfrost point* we compute the following equation

$$q''_{FBmin} = 0.09 \rho_g h_{lg} (\frac{g \sigma \rho_{lg}}{\rho_l^2 + \rho_g^2 + 2\rho_l \rho_g})^{1/4} \tag{21}$$

A typical film boiling heat transfer coefficient is of the form:

$$\alpha_{film} = C_1 [\frac{k_l^3 g \rho_g (\rho_l - \rho_v) h'_{fg}}{L_{film} \mu_g (T_{wall} - T_{sat})}]^{\frac{1}{4}} \tag{22}$$

$$h'_{fg} = h_{fg} [1 + \frac{C_2 c_{p,g} (T_{wall} - T_{sat})}{h_{fg}}] \tag{23}$$

For a vertical fuel rod of radius R (Bailet et al., 1973, [Reference 1 pg 106])
$C_1 = 0.4$
$C_1 = 0.68$
$L_{film} = R$
By using equations (22) and (23) in equation (2) we calculate T_{wall} vs t

III. INPUT DATA 1 BAR

Here we will show the data input used for the analysis for **1 bar pressure**. (Dimensions and Heat data are taken from [Reference 3])

- L =3.588 [m]
- D = 0.0112 [m]
- Q_{core} = 3500 [MWth] (Used in Matlab code)
- N_{rods} = 74 (Used in Matlab code)
- $N_{assemblies}$ = 764 (Used in Matlab code)
- Q_{rod} = 2476.3 [Wth]
- ρ_{Zr} = 6511 [Kg/m3]
- c_{Zr} = 270 [J/Kg K]
- V_{clad} = 0.00035349 [m3]
- M_{rod} = 2.3016 [Kg]
- Cs = 621.4266 [J/K]
- A_w = 0.1262 [m3]
- T_{wall_0} = 373.1600 [K]
- P = 1 [bar]
- T_{sat} = 373.15 [K]
- μ_l = 0.0002814 [Pa s]
- μ_g = 0.00001228 [Pa s]
- ρ_l = 958.6369 [Kg/m3]
- ρ_g = 0.5903 [Kg/m3]
- g = 9.81 [m/s2]
- k_l = 0.68 [W/(m K)]
- $c_{p,l}$ = 4.2161 [kJ/(Kg K)]
- $c_{p,g}$ = 2.0759 [kJ/(Kg K)]
- σ= 0.0590 [N/m]
- h_g = 2674.9 [kJ/Kg]
- h_l = 417.4365 [kJ/Kg]
- v_g = 1.6940 [m3/Kg]
- β = 7.51E-04 [1/K]

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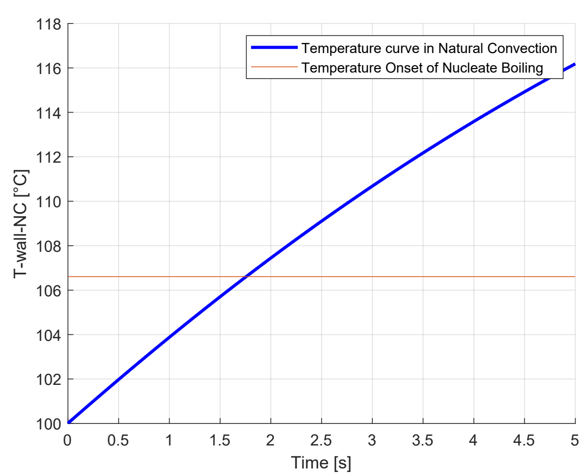
IV. RESULTS AND DISCUSSION 1 BAR

Every model described in the *THEORETICAL FRAMEWORK* section is used in the analysis of the paper with exactly the same order. - 1 Bar pressure results:

A. 1 bar pressure Natural Convection-Onset of Nucleate Boiling

Implementing the *Natural Convection model* and solving the differential equation numerically using *Euler forward method* with a T_{wall_0} = 100.01 [$°C$] (373.16 [K]), we plot the *Temperature* vs *time*: (*Ra* is in the *Laminar flow*)



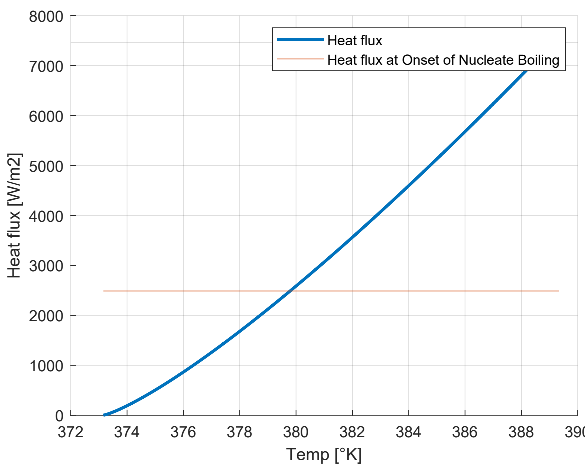
The time range in the *Natural Convection region* is set from 0 to 5s. It is a small range of time. This is because given the figure above it is possible to notice how quickly the *Onset of Nucleate Boiling* is reached. The data obtained at the Onset is the following:

$$\boxed{\begin{array}{l} t_{onb} = 1.7573[s] \\ T_{onb} = 106.6092[^{\circ}C] \end{array}}$$

These two parameters were calculated after verifying the value of the r_c with $r_{c,max}$. The critical radius of the *Davis Henderson Model* was greater, and so the temperature was to be discarded. For completeness purposes we show the values calculated by the model:

$$\boxed{\begin{array}{l} r_c = 4.9468\text{e-}04[m] \\ T_{DHonb} = 100.1544[^{\circ}C] \end{array}}$$

It is possible to notice how the temperature predicted by the model is actually lower than the official temperature needed to achieve the Onset, therefore it is a lower bound for nucleation.



In this figure we show the heat flux removed by the pool of liquid as the temperature of the surface increases. At the Onset the heat flux removed is:
q''_{onb}=2487.2 [W/m2]

B. 1 bar pressure Nucleate Boiling

We start by showing the value calculated at the *Critical Heat flux*:

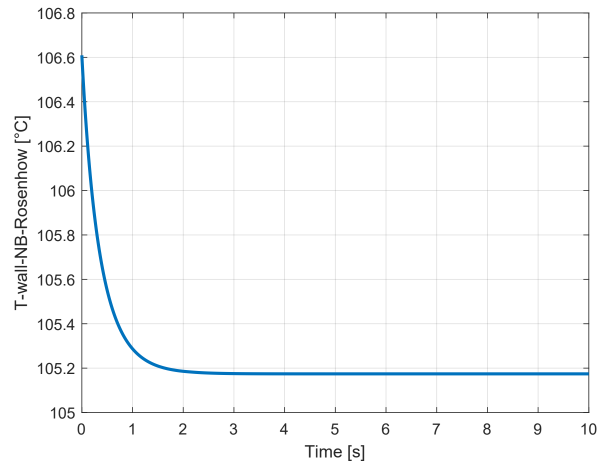
$$\boxed{q''_{chf}=1.3466\text{e+}06[W/m2]}$$

It is possible to notice that using this calculation that the value of the $q''_{rod} = 1.9615\text{e+}04$ [W/m2] is much smaller than the q''_{chf}, therefore the critical heat flux condition won't be reached, but that an equilibrium condition will be established in the *Nucleate Boiling region*. We start the analysis by showing the temperature at the Onset using the *Rosenhow Model* and the *Mostinski Model*:

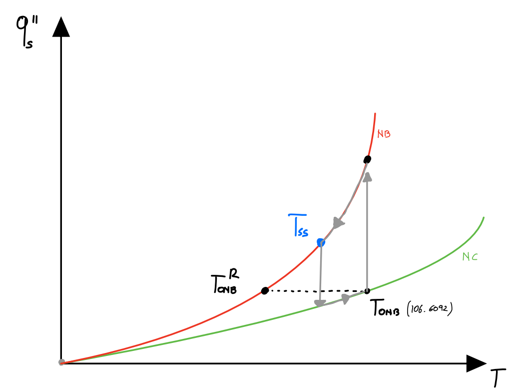
$$\boxed{\begin{array}{l} T_{onb-Rosenhow}=102.5997[m] \\ T_{onb-Mostinski}=103.4340[^{\circ}C] \end{array}}$$

The fact that both temperatures have a lower value respect to the T_{onb} means that the temperature needed to maintan boiling is lower than the temeperature neeeded to onset it. Furthemore the analysis show that a *Hysteresis cycle* is created in the conditions set by the problem at 1bar. We will use the *Rosenhow Model* to show this (also for the *Mostinski Model* this same condition is demonstrated.

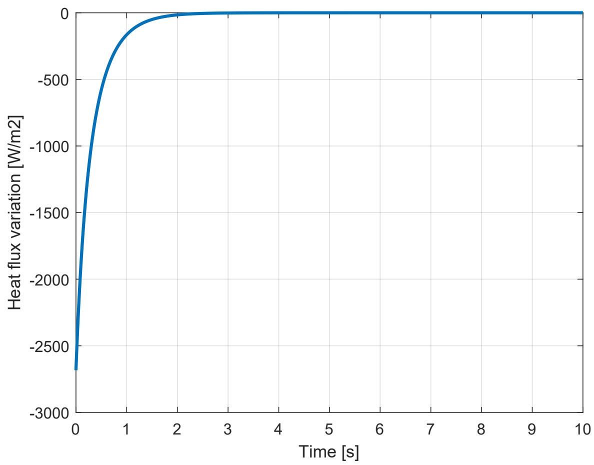
It possible to notice from the figure below that a steady state condition is reached starting from a $T_{wall_0} = T_{onb}$



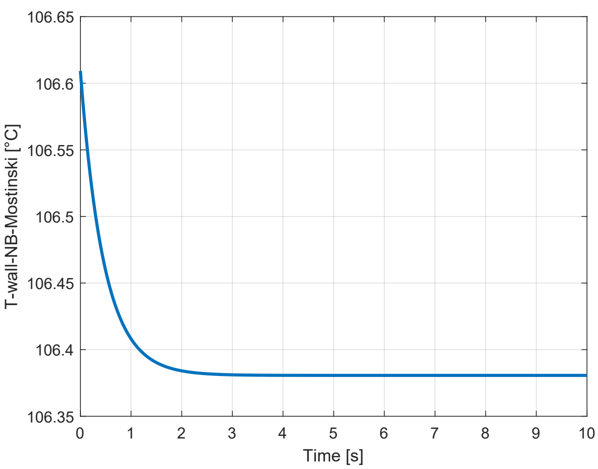
The temperature reached in *Steady state* is:
T_{ss}= 105.175 [$^{\circ}C$]. We have proven using the *Rosenhow Model* that an equilibrium condition is reached. But the fact that the temperature predicted at the *Steady state* is lower than the T_{onb} it means that the system returns spontaneously in the *Natural Convection region*. It then increases its temperature up to the T_{onb} since it is not an equilibrium state and a sudden jump in the *Nucleate Boiling region* is reached, but as soon this condition is matched, the system lowers its temperature up to the T_{ss} and the cycle repeats itself. We plot conceptually what is happening:



The figure at the top left of page 6 shows the difference between the heat flux removed and the heat flux of the rod (q''_s - q''_{rod}). This representation is an alternative way of viewing the steady state condition in terms of heat difference. As expected from equation (1) when the *Internal energy U* doesn't vary with time the heat input is equal to the heat removed.
The *Mostinski Model* was implemented to show a counterargument or an in favor argument regarding the *Hysteresis cycle* temperature behavior of the cladding wall. The advantage of using this model is that it was designed especially for water



conditions. The results show exactly the same behavior with a different value for the T_{ss}. The temperature calculated is T_{ss} = 106.381 [$°C$]. Also in this case the temperature calculated is below the T_{onb}, and so cycle similar to the one predicted by the *Rosenhow Model* is expected.

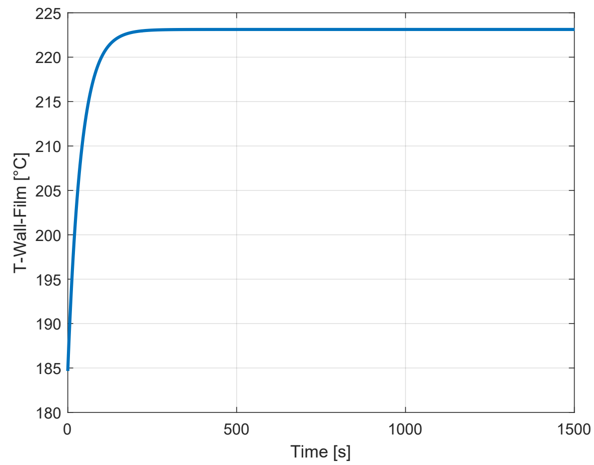


The objective of these models is not to have a precise value of the temperature at the *steady state* conditions nor to have a clear-cut value for *time* to achieve this condition, but to describe an overall behavior of the cladding wall, that in this case is best explained by a *Hysteresis cycle*.
It is important to mention that the *Biot Number* calculated in both models is less than 0.1.

C. 1 bar pressure Film Boiling

The *Film Boiling* simulation yields the following results, with a $T_{wall,Film,0}$=184.6631[$°C$] (Calculated using the *Berenson model*)

$q_{Leidenfrost}$=1.8791e+04[$W/m2$]
$T_{ss,Film}$=223.121[$°C$]



A steady state condition is reached in $t \simeq 450s$. It is important to notice the velocity with which the temperature rises. As an example, in just 15.65 s the temperature of the wall goes from 184.6631 [$°C$] up to 197.735 [$°C$]. This is due to the non-very-effective heat transfer removal in this region.

V. INPUT DATA 71.2 BAR

Here we will show the data input used for the analysis for **71.2 bar pressure**. (Dimensions and Heat data are taken from [Reference 3])

- L =3.588 [m]
- D = 0.0112 [m]
- Q_{core} = 3500 [MWth] (Used in Matlab code)
- N_{rods} = 74 (Used in Matlab code)
- $N_{assemblies}$ = 764 (Used in Matlab code)
- Q_{rod} = 2476.3 [Wth]
- q_{rod} = 7.0052e+6 [W/m3]
- ρ_{Zr} = 6511 [Kg/m3]
- c_{Zr} = 270 [J/Kg K]
- V_{clad} = 0.00035349 [m3]
- M_{rod} = 2.3016 [Kg]
- Cs = 621.4266 [J/K]
- A_w = 0.1262 [m3]
- T_{wall_0} = 560.3233 [K]
- P = 71.2 [bar]
- T_{sat} = 560.3232 [K]
- μ_l = 9.0600e-05 [Pa s]
- μ_g = 0.000009600 [Pa s]
- ρ_l = 737.2319 [Kg/m3]
- ρ_g = 37.3434 [Kg/m3]
- g = 9.81 [m/s2]
- k_l = 0.68 [W/(m K)]
- $c_{p,l}$ = 5.4289 [kJ/(Kg K)]

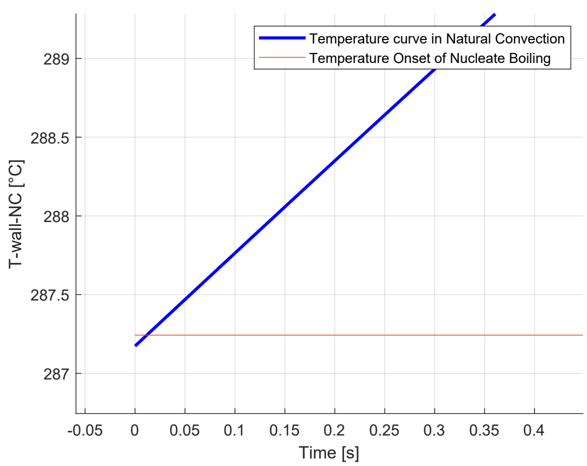
- $c_{p,g}$ = 5.4247 [kJ/(Kg K)]
- σ= 0.0173 [N/m]
- h_g = 2770.7 [kJ/Kg]
- h_l = 1274.6 [kJ/Kg]
- v_g = 0.0268 [m3/Kg]
- β = 7.51E-04 [1/K]

VI. RESULTS AND DISCUSSION 71.2 BAR

Every model described in the *THEORETICAL FRAMEWORK* section is used in the analysis of the paper with exactly the same order. - 71.2 Bar pressure results

A. 71.2 bar pressure Natural Convection-Onset of Nucleate Boiling

Implementing the *Natural Convection model* and solving the differential equation numerically using *Euler forward method* with a Twall0 = 287,1733 $[°C]$ (560,3233 $[K]$), we plot the *Temperature vs time*: (*Ra* is in the *Laminar flow*)



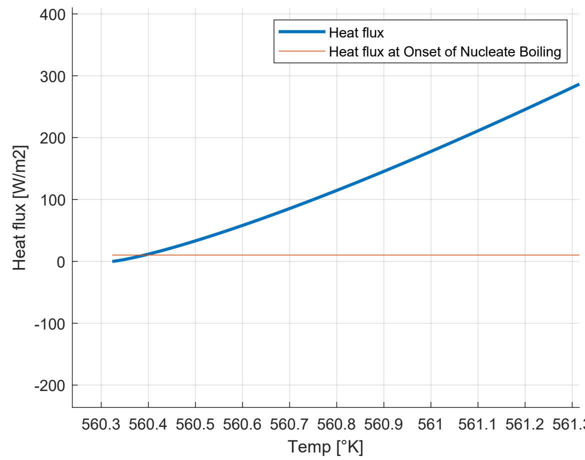
t_{onb} = 0.0117 $[s]$
T_{onb} =287.2427 $[°C]$

These two parameters were calculated after verifying the value of the r_c with $r_{c,max}$. The critical radius of the *Davis Henderson Model* was greater, and so the temperature was to be discarded. For completeness purposes we show the values calculated by the model:

r_c = 3.0204e-05 $[m]$
T_{DHonb} =287.2019 $[°C]$

It is possible to notice how the temperature predicted by the model is actually lower than the official temperature needed to achieve the Onset, therefore it is a lower bound for nucleation.

In the above figure we show the heat flux removed by the pool of liquid as the temperature of the surface increases. At



the Onset the heat flux removed is: q''_{onb}=10.1628 [W/m2]. The fact that the *Onset of Nucleate Boiling* is reached very quickly respect to the system at 1 bar pressure is due to the high pressure of the system. High values of pressure translate the Pool Boling curve towards lower values of ΔT_{onb} and so very low values of t_{onb}.

B. 71.2 bar pressure Nucleate Boiling

We start by showing the value calculated at the *Critical Heat flux*:

q''_{chf}=4.8307e+06$[W/m2]$

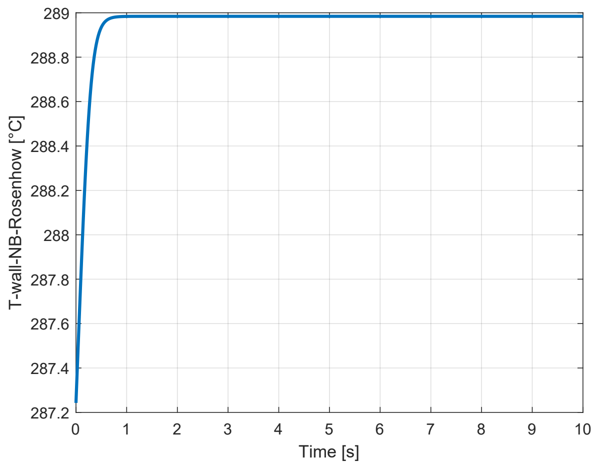
It is possible to notice that using this calculation that the value of the q''_{rod} = 1.9615e+04 [W/m2] is much smaller than the q''_{chf}, therefore the critical heat flux condition won't be reached, but that an equilibrium condition will be established in the *Nucleate Boiling region*. We start the Nucleate Boiling analysis by showing the temperature at the Onset using the *Rosenhow Model* and the *Mostinski Model*:

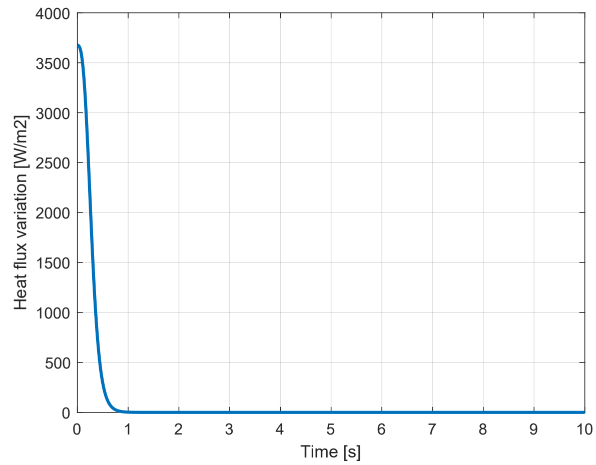
$T_{onb-Rosenhow}$=287.3007$[m]$
$T_{onb-Mostinski}$=287.3633$[°C]$

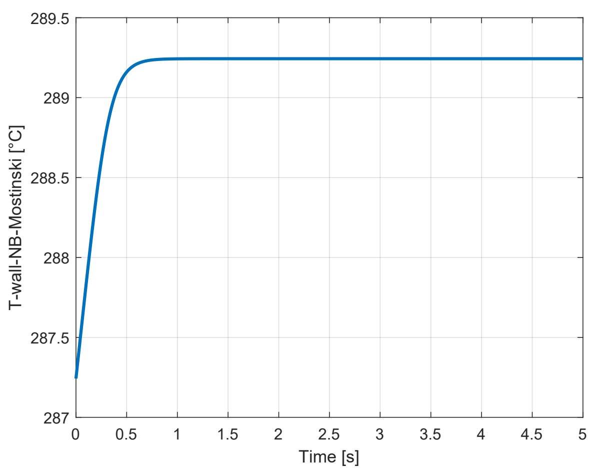
The fact that both temperatures have a higher value with respect to the T_{onb} means that the temperature needed to maintain boiling is higher than the temperature needed to onset it, therefore a *Hysteresis cycle* behaviour like in the 1 bar pressure condition won't be reached.
A steady state condition is reached starting from a T_{wall_0} = T_{onb} according to the *Rosenhow Model.*

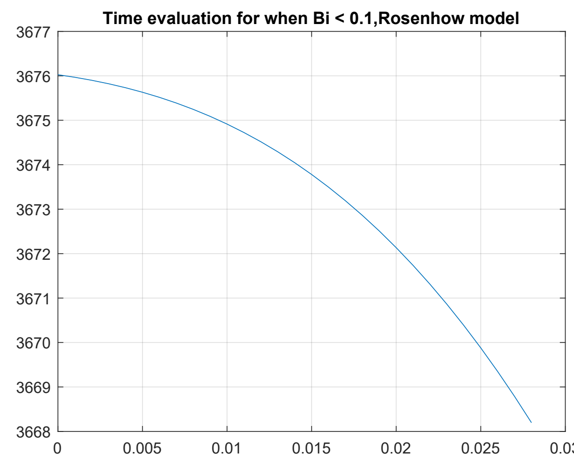
The temperature reached is:
T_{ss}= 288.984 $[°C]$. According to the *Rosenhow Model* an equilibrium condition is reached. According to the *Mostinski Model* the results show exactly the same behavior with different values for the T_{ss}. The temperature calculated is T_{ss} = 289.244 $[°C]$.

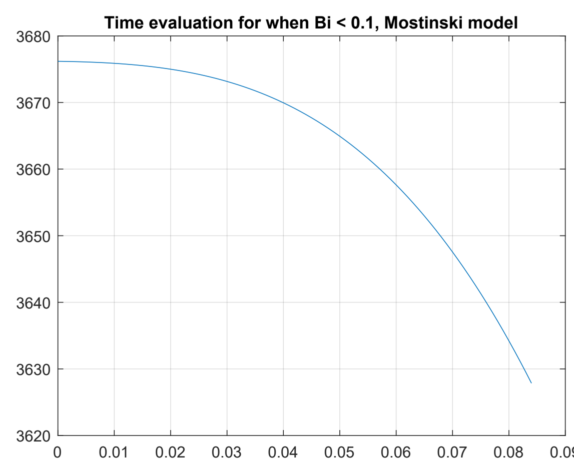
The figure at the top right of page 8 shows the difference between the heat flux removed and the heat flux of the rod (q''_s - q''_{rod}) for the *Rosenhow Model.* This representation











is an alternative way of viewing the steady state condition regarding heat difference. As expected from equation (1) when the *Internal energy* U doesn't vary with time, the heat input is equal to the heat removed.
It is clearly visible for both models that the *Steady state* condition is reached very quickly, this is because of the very high *Heat transfer* coefficient, in the Nucleate Boiling region. Both models show that within $t \simeq 1$ seconds, the system stabilizes.

It is important to mention that the description above for the cladding temperature in the *Nucleate Boiling region* is calculated assuming that the *Biot number* is less than 0.1 throughout the whole calculation. The points at which the approximation ceases to be valid is in the following points for both models:

$$t_{stopBi-Rosenhow} = 0.0280\ [s]$$
$$t_{stopBi-Mostinski} = 0.0840\ [s]$$
$$T_{stopBi-Rosenhow} = 560.558 [^\circ C]$$
$$T_{stopBi-Mostinski} = 560.885\ [^\circ C]$$

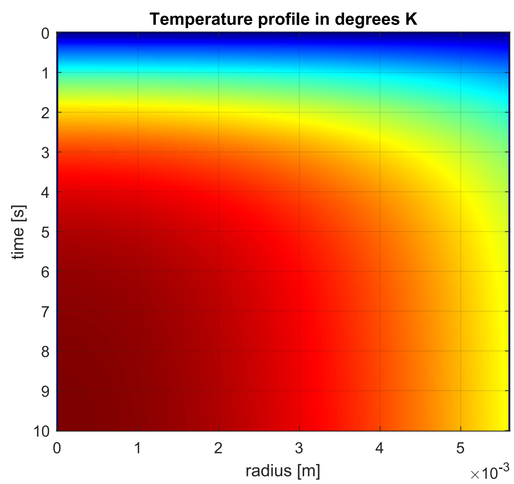
(On the *y axis* of Bi plots is represented the difference in ($Heat$), on the *x axis* is *time*)

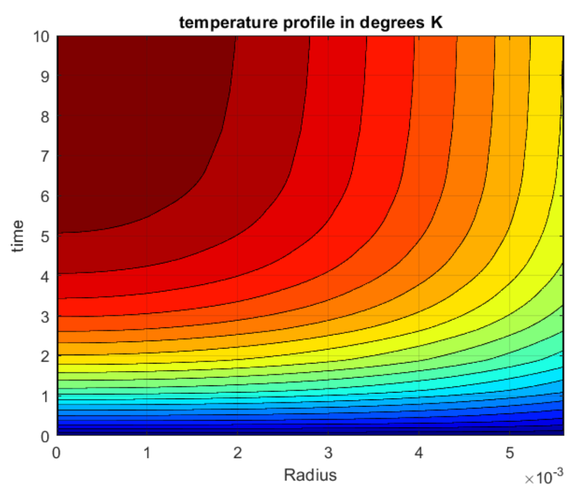
For both models is shown that there is still heat to be removed. An equilibrium condition will for sure be reached, but a more detailed model is needed to ensure that the heat is adequately removed in the absence of the approximation.

Finite difference method 71.2 bar

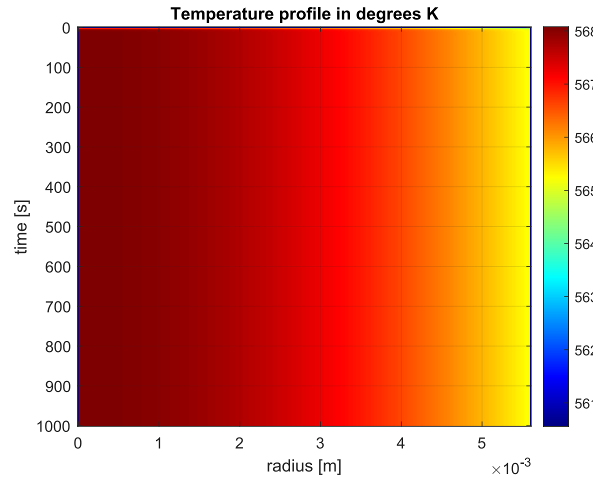
Because of the quick cease of the instantaneous *Uniform temperature approximation*, we implement the *Finite difference method*. The simulation was performed on the basis of the outputs of the *Rosenhow Model*. $\alpha_{conv} = 4.0172e+03$ $[W/m2\ K]$, is the value for which the *Biot number* becomes greater than 0.1. To simplify the simulation, this value is maintained constant. This simplification is not diminutive since the *Heat transfer coefficient* can only increase with an increase in the wall temperature, the heat removed can only increase. This simulation is therefore more conservative, if an equilibrium condition is reached we are sure that the system will stabilize also with higher *Heat transfer coeffficients* (A more detailed description of the values chosen are written in the Matlab code developed for this specific simulation). The simulation is divided into two-time frames, at *10 s* and *1000 s*:

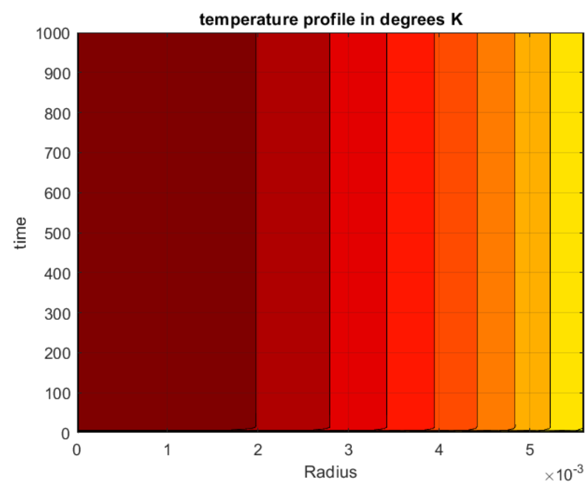
10 seconds graphs





1000 seconds graphs





The 10 seconds graphs show how the system still needs more time to reach a stable temperature distribution. A value of 1000 seconds was chosen to ensure that the system had enough time to reach a precise temperature distribution. A temperature of around 565 [K] on the border of the cladding is reached, and a temperature of around 568 [K] is reached in the center. Therefore the system stabilizes and is in safe conditions.

C. 71.2 bar pressure Film Boiling

The *Film Boiling region* simulation yields a different result, with respect to the analysis at 1 bar pressure. $T_{wall,Film,0}$=512.9466[°*C*] is the starting point for the *Leidenforst point* temperature according to the *Berenson Model*

$$\boxed{q_{Leidenfrost}=5.9663e+05[W/m2]}$$

The value of the $q_{Leidenfrost}$ is much larger than the heat flux input of the rod, therefore the Pool Boiling curve won't intersect the Heat flux input curve. The *Film Boiling region* is nonexistent with these specific Heat input conditions.

VII. CONCLUSION

The system stabilizes in both pressure conditions without a significant increase in the cladding wall temperature for a single vertical cylinder of Zirconium. Improvements can be made for more realistic future modeling. Possible improvements would inclde the modeling of a complete *Fuel rod bundle* with adequate pitches between the rods and new heat transfer correlations to describe the heat exchanged between the bundles with the confined channels of water. This more sophisticated model would improve simulation results since nuclear reactor meltdowns such as the one in Fukushima involve fuel bundles and not single rods of fuel.

VIII. TABLE INDEX

- L - Rod length
- D - Diameter of the rod
- Q_{core} - Thermal heat power of the core
- N_{rods} - Number of rods
- $N_{assemblies}$ - Number of assemblies
- Q_{rod} - Thermal heat power of the rod
- q_{rod} - Thermal heat power of the rod per unit volume
- k_{Zr} - Thermal conductivity of Zirconium
- ρ_{Zr} - Zirconium density
- c_{Zr} - Zirconium specific heat
- V_{clad} - Volume of the cladding
- M_{rod} - Mass of the rod
- Cs - Heat capacity of Zirconium
- A_w - Wetted area of the rod
- P - Pressure
- T_{sat} - Saturation temperature
- μ_l - Dynamic liquid water viscosity
- μ_g - Dynamic vapour water viscosity
- ρ_l - Saturated liquid density
- ρ_g - Saturated vapour density
- $\rho_{lg} = \rho_l - \rho_g$
- g - Gravity acceleration
- k_l - Thermal conductivity of saturated liquid water
- $c_{p,l}$ - Specif heat of saturated liquid water
- $c_{p,g}$ - Specific heat of saturated vapour water
- σ - Surface tension
- h_g - Specific Enthalpy of saturated vapor water
- h_l - Specific Enthalpy of saturated liquid water
- $h_{lg} = h_g$-h_l
- v_g - Specific volume of saturated vapour water
- β - Thermal expansion coefficient
- $\Delta T = T_{wall} - T_{sat}$

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