Principles of Robot Autonomy: Homework 2

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Other students worked with:
Time spent on homework: 6 hrs

Problem 1:

1. Input V is constant, omega is sin(t).

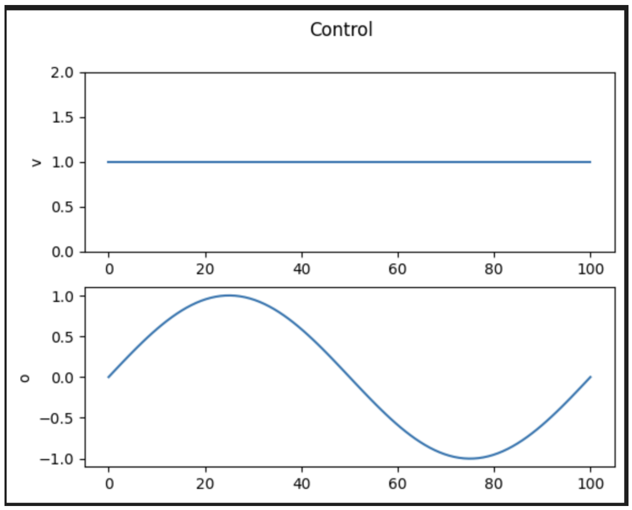

Figure 1: Input

State X and Y are approximately linear function of t, theta are trigonometry function of t.

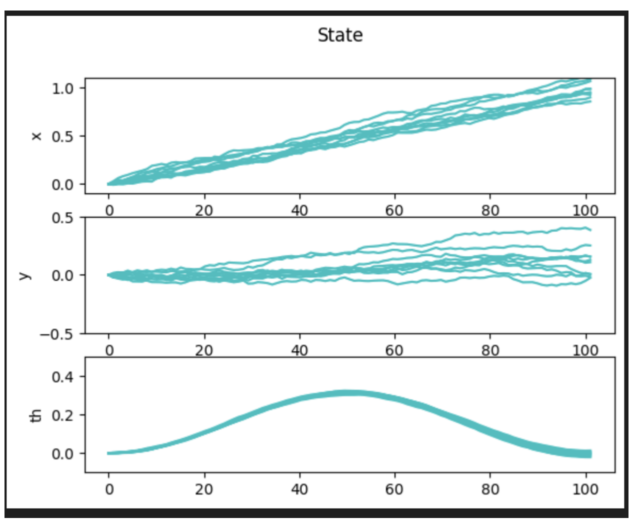

Figure 2: state output

2. $\mathbf{X_{t+1}} = \bar{\mathbf{A}}\mathbf{X_t} + \bar{\mathbf{B}}\mathbf{u_t}$

3. $\ddot{x}$ and $\ddot{y}$ are trigonometric function of t

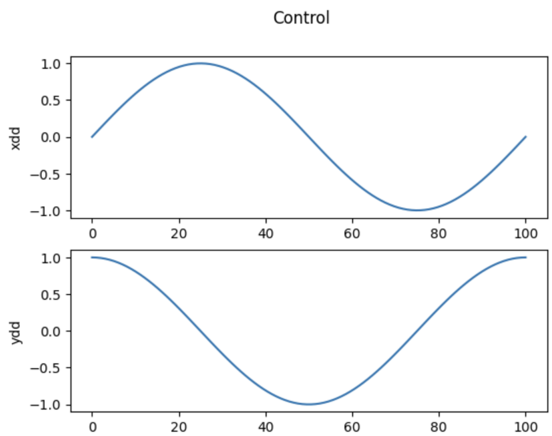

Figure 3: Input

In feed forward method, the state X = [x,y,θ], control input u = [v, ω]. As for double integrator dynmaics, the state X = [x,y,v_x,v_y], control input u = [a_x,a_y].
We can also compare figure 4 and figure 2 and find out that in feed forward fashion the state are more likely to be affected by disturbance, where x propagates to 1.0 and y to 0.5. However, when we vectorize dynamics, the state output are less deviated.

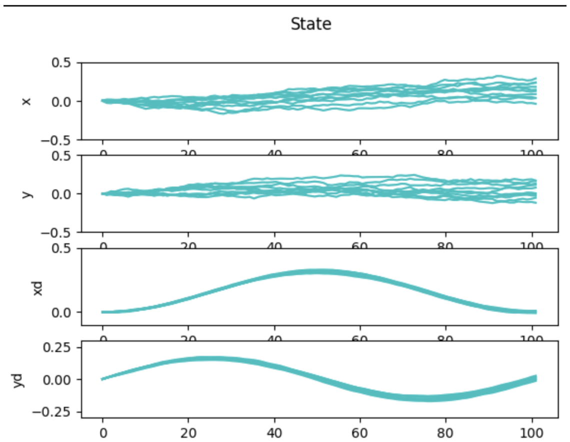

Figure 4: state output

Problem 2:

1. we have 8 conditions, 4 for x, 4 for y:
   $x(0) = 0 = x_1$
   $x(t_f) = 5 = x_1 + x_2 t_f + x_3 {t_f}^2 + x_4 {t_f}^3$
   $\dot{x}(0) = v(0)cos(\theta(0)) = x_2$
   $\dot{x}(t_f) = v(t_f)cos(\theta(t_f)) = x_2 + 2x_3 t_f + 3x_4 {t_f}^2$
   $y(0) = 0 = y_1$
   $y(t_f) = 5 = y_1 + y_2 t_f + y_3 {t_f}^2 + y_4 {t_f}^3$
   $\dot{y}(0) = v(0)sin(\theta(0)) = y_2$
   $\dot{y}(t_f) = v(t_f)sin(\theta(t_f)) = y_2 + 2y_3 t_f + 3y_4 {t_f}^2$
   These equation can be written as:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & t_f & {t_f}^2 & {t_f}^3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2t_f & 3{t_f}^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & t_f & {t_f}^2 & {t_f}^3 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2t_f & 3{t_f}^2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ -0.5 \\ -0.5 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{3}{125} \\ \frac{-2}{3125} \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \\ \frac{21}{250} \\ \frac{-7}{3125} \end{bmatrix} \tag{4}$$

2. Because the determinant of the J matrix equals to V, so when $v(t_f) = 0$ will cause invertible J matrix, which is also called singularity.

3. (a) $\theta = atan2(\frac{\dot{y}}{\dot{x}})$
   (b) Plotted trajectory match the initial and final conditions of $x(0) = 0, y(0) = 0, x(t_f) = 5, y(t_f) = 5, v(0) = 0.5, v(t_f) = 0.5, \theta(0) = -\pi/2, \theta(t_f) = -\pi/2$

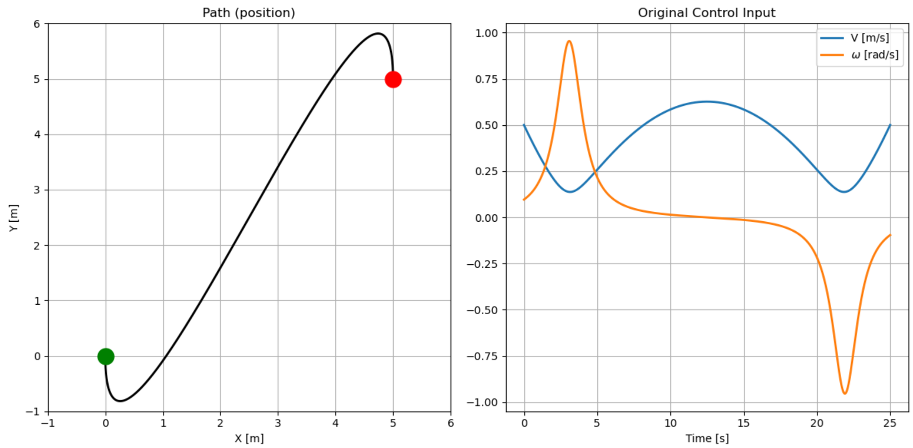

Figure 5: differential flatness

4. open-loop

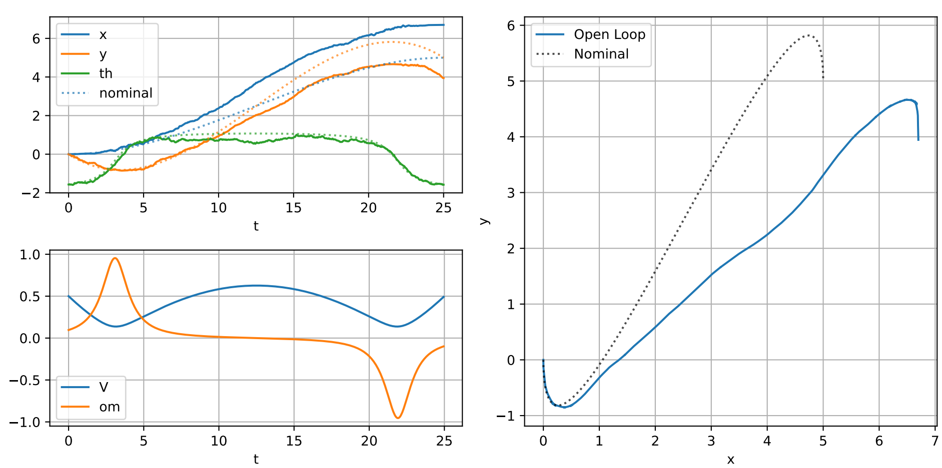

Figure 6: sim traj openloop

5. $a(t) = \dot{v}(t), \dot{\theta(t)} = \omega(t)$, if $v(t) \neq 0$ then

$$\begin{bmatrix} \dot{v}(t) \\ \omega \end{bmatrix} = \frac{1}{v(t)} \begin{bmatrix} v(t)cos(\theta(t)) & v(t)sin(\theta(t)) \\ -sin(\theta(t)) & cos(\theta(t)) \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \tag{5}$$

6. (a) Can simply use np.linalg,solve(J,u) to solve a and ω without computing inverse matrix by ourself.
(b) Closed-loop control control track the nominal trajectory quite well, we also can observe that close-loop control state track nominal state well. While open-loop state and trajectory deviate gradually with the existence of disturbance.

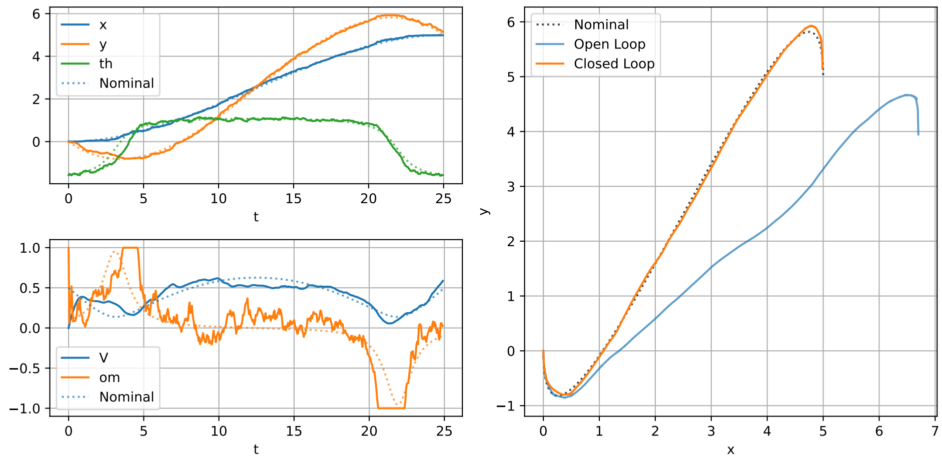

Figure 7: sim traj closedloop

Problem 3:

1. Dimension of state space of quadcopter is 6 , where x and y are position, v_x and v_y are translation velocity in x and y direction, pitch ϕ, pitch rate ω

$$X = \begin{bmatrix} x \\ v_x \\ y \\ v_y \\ \phi \\ \omega \end{bmatrix} \tag{6}$$

2. Dimension of control space is 2, T_1 and T_2 are thrust for the left and right propeller.

$$u = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \tag{7}$$

3. Use non-linear optimization of trajectory which follows the constraints.

4. (a) K_i is a 2x6 matrix
(b) We can modify the state weighting matrix Q and control input weighting matrix R to improve the gain correction.

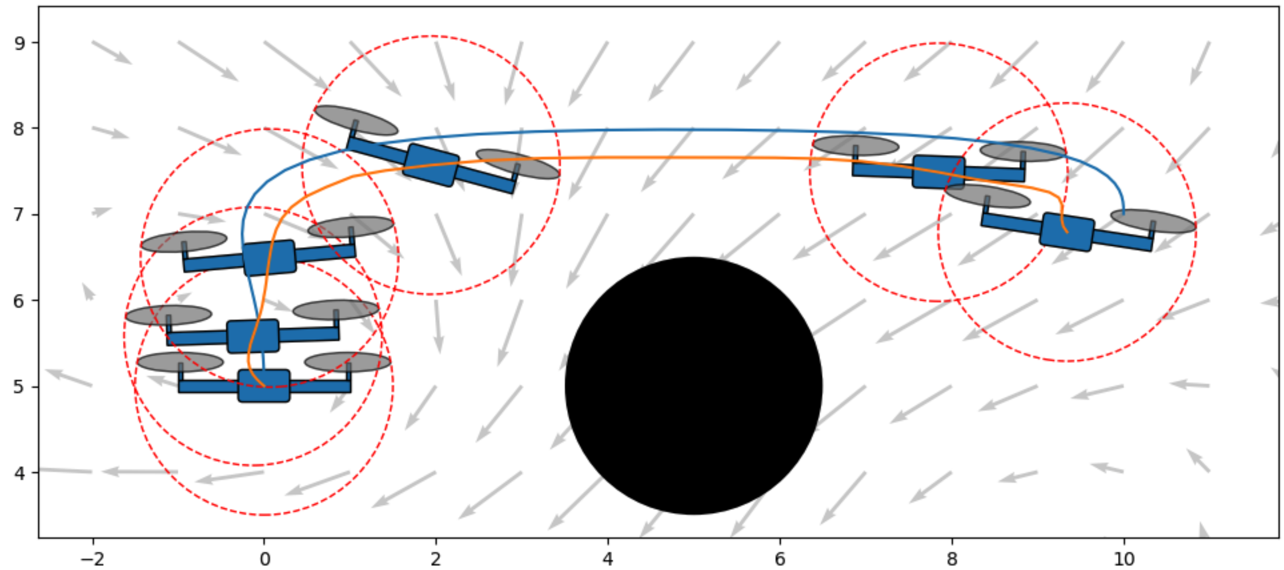

Figure 8: LQR