The effect of the recovery variable parameters on oscillating Izhikevich networks

Synchrony in a Strongly Connected Spiking Network

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The theory of neuronal group selection (TNGS) posits neuronal groups as functional units within the brain. Since the proposal of TNGS, studies have corroborated the existence of such units. With a myriad of different brain areas and rhythms, there are analogously various mechanisms for emergent synchrony that remain to be uncovered.

This work focuses on a subset of neuronal behaviours that occur within biology, by analysing a highly connected network of inhibitory and excitatory neurons, with a parallel to that of Central Pattern Generators (CPGs) in that the population bursts rhythmically. By extending the work of [2], and using Izhikevich neurons, we attain a computationally efficient implementation which captures the emergence of synchronised population bursting. We uncover and elaborate on why this synchronised bursting occurs in this specific model, and outline further interesting parallels.

Main Objectives

1. Test if population bursting depends on the single neuron sub-threshold oscillations
2. Test if single neuron firing can predict network bursting
3. Analyse the effect of the recovery variable parameter values on the neuron, and network

Materials and Methods

We extended the model of Oliveira et al. (2019) [2], by performing further exploration of the parameter-space, and a comparative analysis of the single neuron.

This led to observations which enabled us to explain the relationship between the model parameters and emergent firing rates. The Izhikevich neuron model employed is defined by,

$$\frac{dv}{dt} = 0.04v^2 + 5v + 140 - u + I_{\text{syn}} \quad (1) \qquad \frac{du}{dt} = a(bv - u) \quad (2)$$

$$\text{and if } v \geq 30\,\text{mV}, \text{ then } v \leftarrow c \text{ and } u \leftarrow u + d.$$

This work is primarily focused on the effect of the parameters a and b in regularly spiking networks, with two classes of neurons having $(c, d) = (-65, 8)$, and $(-65, 2)$, respectively.

The synaptic model is defined by a simplified differential equation,

$$\frac{dg}{dt} = -\frac{g}{\tau_g} \quad (3) \qquad I_{\text{syn}} = \sum_{i=1}^{N} g_i w \quad (4)$$

where g is reset to 1 upon spiking, and otherwise decays exponentially, as defined in Eq. 3. I_{syn} is modelled per neuron as the sum over each synaptic conductance times a synaptic weight constant $w \in [10, 20]$ (Eq. 4). At each time-step of the model simulation, a random neuron is stimulated by $I_{\text{external}} = 100\,\mu\text{A}$.

Results

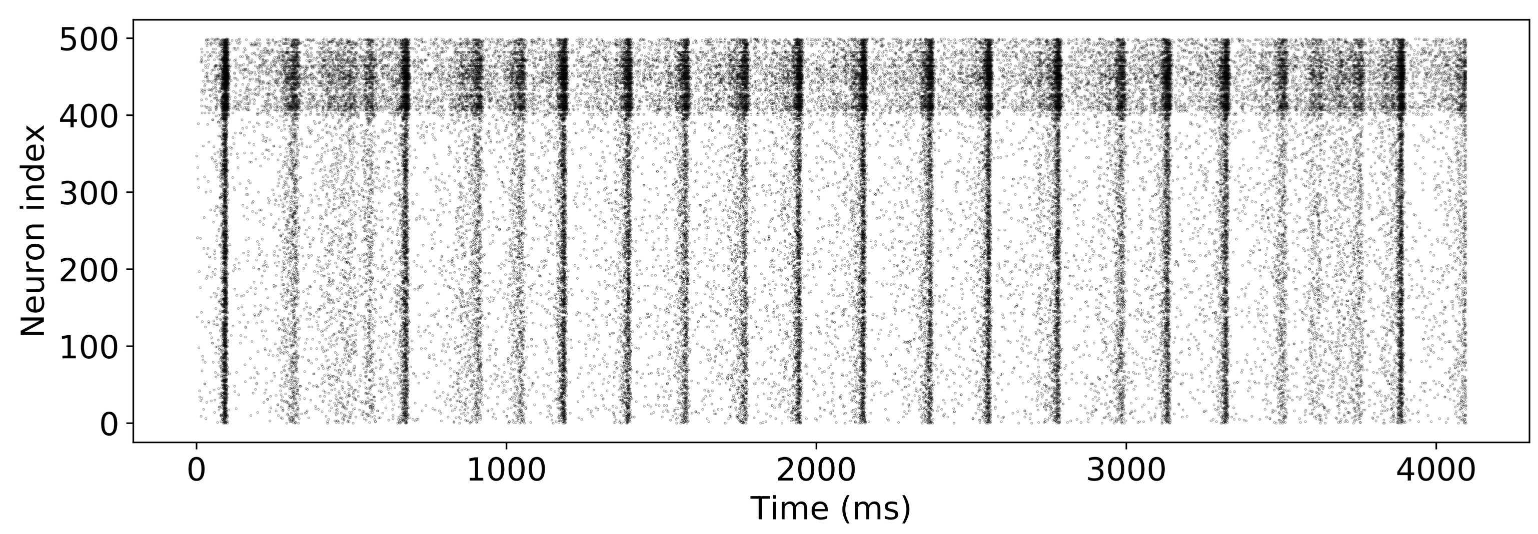

Figure 1: Raster plot for a network model consisting of $n = 500$ neurons, with parameter values $a = 0.005, b = 0.42$, a weight of $w = 11$, and $\tau_a = 5.0\,\text{ms}$.

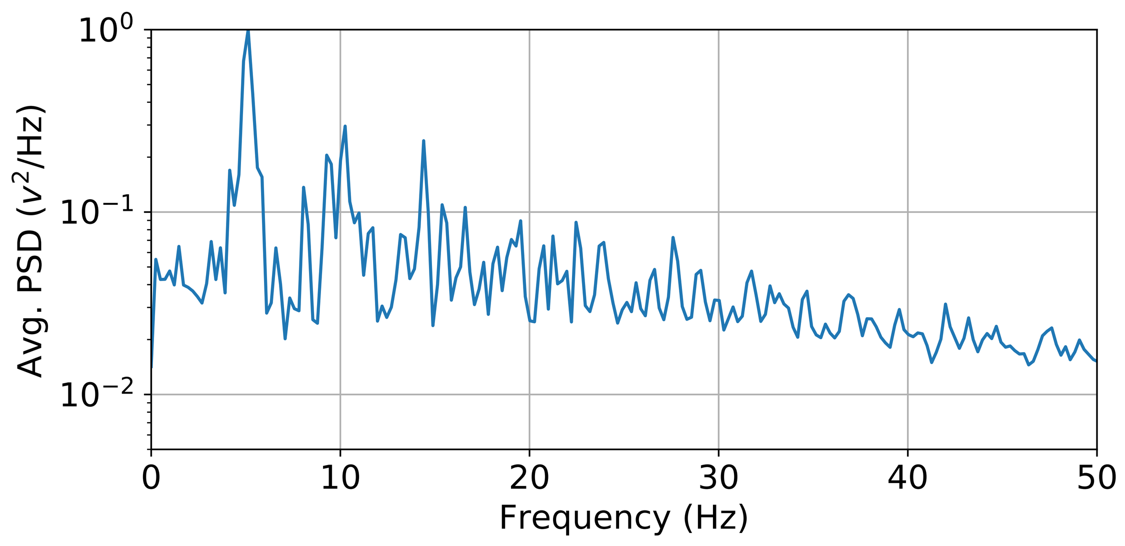

Figure 2: The power spectrum density reveals $\approx 4.15\,\text{Hz}$ as the most prominent burst rate.

- Inhibitory neurons are close to their bifurcation point; a slight stimulus leads to spiking
- Excitatory neurons drive synchronisation
- a and b determine the neuronal sub-threshold oscillation rate
- Network burst rate depends on the sub-threshold oscillation rate, and parameters a and b
- Population burst rate may be predicted by single excitatory neuron firing rate
- There is a minor effect of inhibitory neurons' negative stimulus on spike frequency
- Inhibitory neurons generally burst more due to: (1) a shorter refractory period (a higher a-value), and (2) a lower after-spike reset for the recovery variable (lower d)
- Bursting occurs due to the large input current, which is a result of the dense connectivity
- For lower weights w, I_{syn} is lower, and the network desynchronises

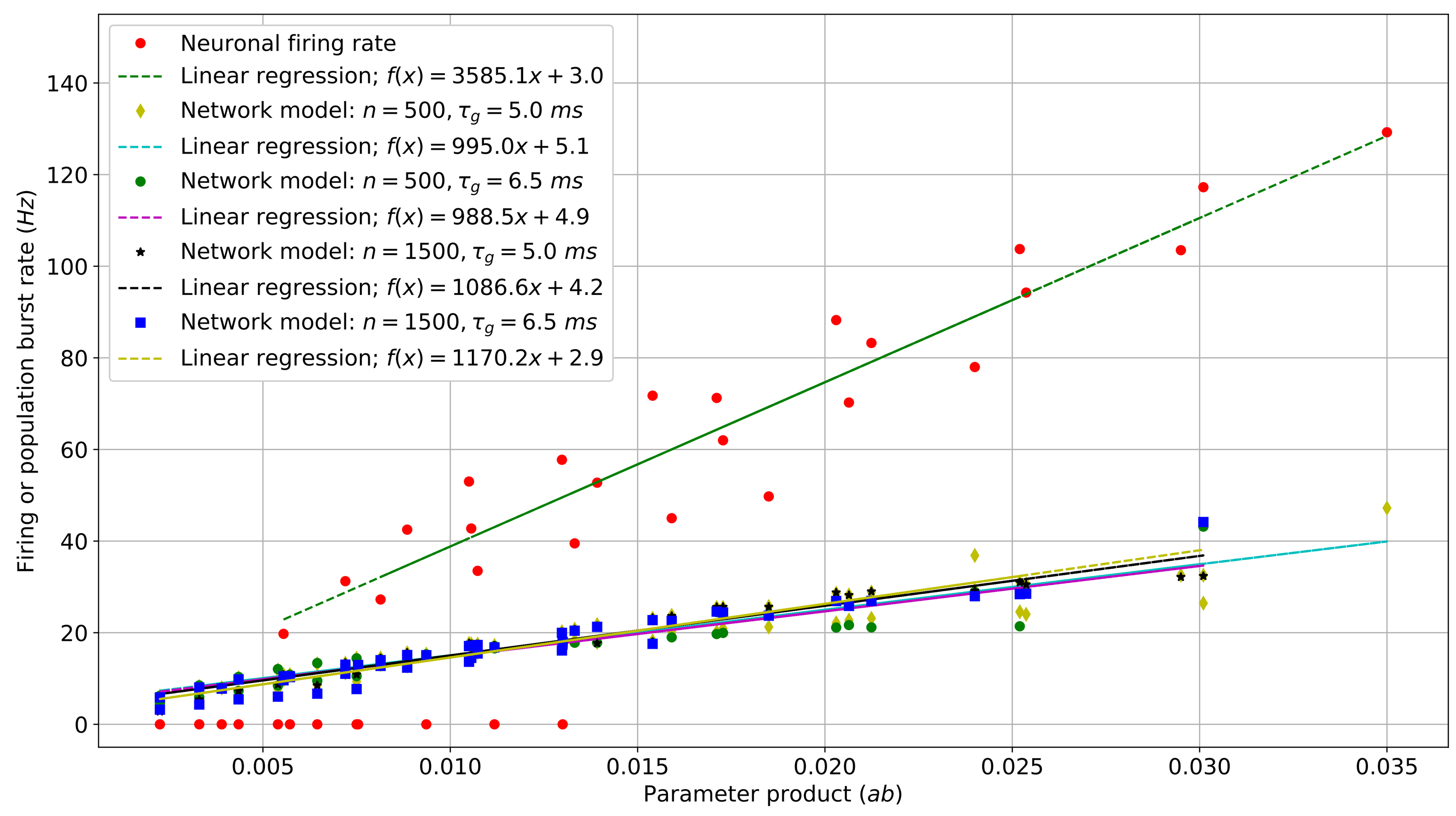

Figure 3: Neuronal sub-threshold oscillation rate and synchronised population burst rate reveal a linear relationship between ab and the burst rates for converged (synchronous) models. Data points for non-synchronous models are not included. Convergence to synchronous bursting is less likely for higher higher parameter-products and firing rates.

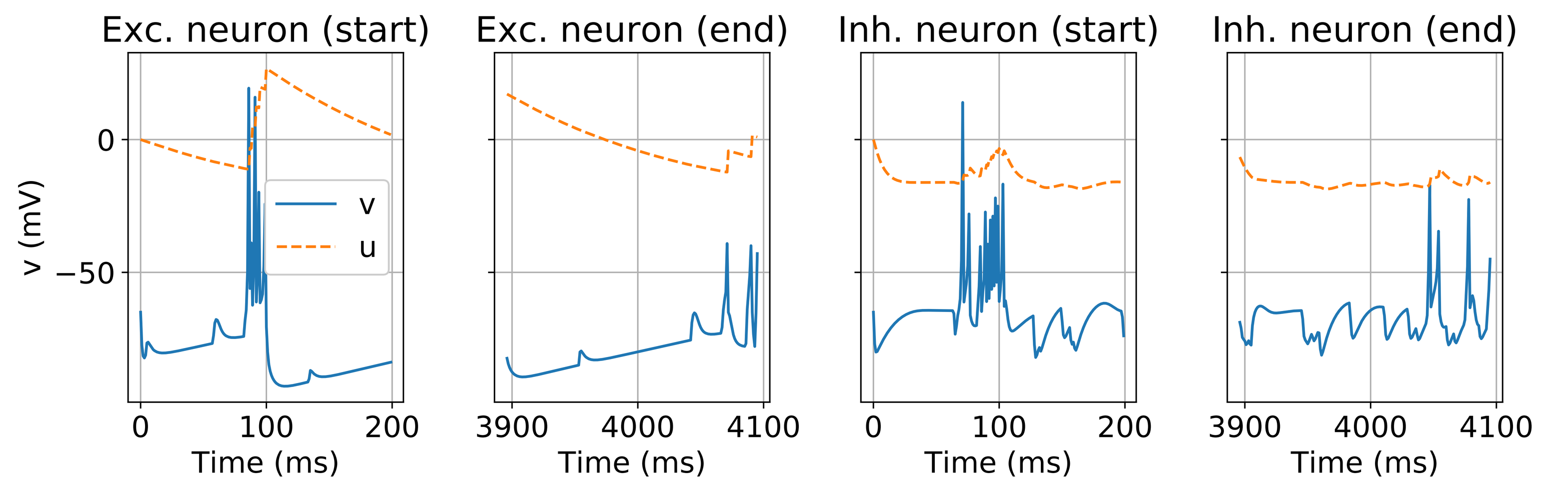

Figure 4: Voltage traces of two randomly chosen neurons for two time intervals. Inhibitory neurons fire longer bursts, due to their parametrisation of $(a, d) = (0.1, 2)$.

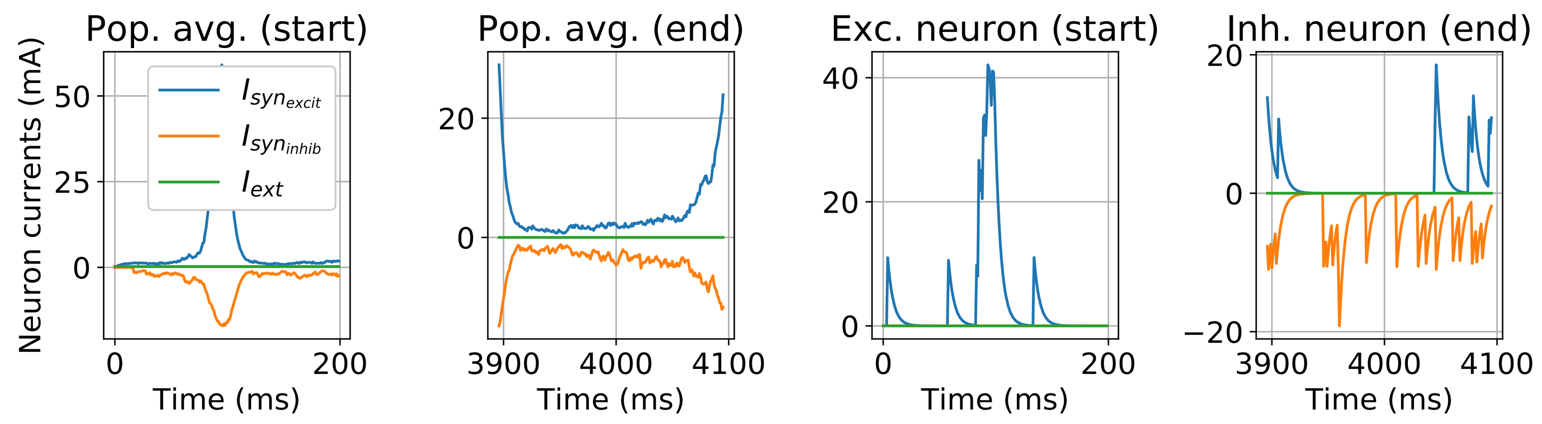

Figure 5: Currents at two time intervals for the population average, and two random neurons. Excitatory and inhibitory currents mirror one another on average due to synchronicity.

- With weights $w \in [10, 20]$, incrementing of u by $u \leftarrow (u + d)$ does not induce a refractory period for the neuron, as the signal I is still greater than the subtracted value u in $\frac{dv}{dt}$. This leads to a series of consecutive spikes and increments of u by d, or a burst, eventually inducing a refractory period.
- Because the inhibitory neurons have a slightly higher time scale a for the recovery variable u, they may impose a slightly higher negative current on the population, inducing a slightly longer build-up of the membrane potential v.
- Note that weights $w \geq 1$ is not necessarily biologically implausible, as they also incorporate a constant for the difference between the reverse and membrane potential (typically denoted $(E_{syn} - V_{post})$).

Forthcoming Research

(1) Using spiking Izhikevich networks to capture emergent neural assemblies as observed in vivo within the PPT/LDT (brainstem) of mice during wakefulness, REM and NREM sleep. **(2)** Analysing phase synchronisation in weakly coupled spiking networks with different rhythms. **(3)** Looking at the effect of brain topology on spiking Izhikevich network behaviour.

References

[1] Eugene M. Izhikevich. Simple model of spiking neurons. *IEEE Transactions on Neural Networks*, 14(6):1569–1572, nov 2003.

[2] Lucas D.R. Oliveira, Rogerio M. Gomes, Bruno A. Santos, and Henrique E. Borges. Effects of the parameters on the oscillation frequency of Izhikevich spiking neural networks. *Neurocomputing*, feb 2019.

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