PHYS 241: Signal Processing

William Homier[1]

[1] *McGill University Physics, 3600 Rue University, Montréal, QC H3A 2T8, Canada*

January 6^{th}, 2026

Abstract

Contents

1 Introduction **1**

2 Prerequisite knowledge **1**

3 Basics & Voltage, Current and Resistance **1**
- 3.1 Signal Types 1
  - 3.1.1 Digital Signal 1
  - 3.1.2 Analogue 1
- 3.2 Circuits 1
  - 3.2.1 DC 1
  - 3.2.2 AC 1
- 3.3 Waves 2
  - 3.3.1 Properties of waves 2
  - 3.3.2 Waveforms 2
- 3.4 Linear Systems 4
- 3.5 Electric Charge 4
- 3.6 Current flow 4
- 3.7 Electric Field & Potential 5
- 3.8 Potential & Ground 6
- 3.9 Electromotive Force 6

4 Circuit Theory Beyond Electronic **7**

5 Capacitors & Inductors **7**

6 RC and LR Circuits with AC Driving **7**

7 Impedance **7**

8 RLC Circuits **7**
- 8.1 Transient Response 7
- 8.2 Driven RLC Circuits 7
- 8.3 Power Input to RLC and Circuit Network 7

9 Circuit Networks **7**

10 Fourier Series **7**

11 Fourier Transforms **7**

12 Appendix **7**

13 Useful Links **7**

1 Introduction

2 Prerequisite knowledge

3 Basics & Voltage, Current and Resistance

3.1 Signal Types

3.1.1 Digital Signal

Definition 1. *A discretely sampled signal with a sequence of quantized values.*

3.1.2 Analogue

Definition 2. *A continuous signal (e.g., in time) representing (analogous to) some other quantity.*

Example 1. *Examples of analogue devices and computers are:*

- *thermometers*
- *sextants*
- *tide-predicting machine*

3.2 Circuits

3.2.1 DC

Definition 3. *Direct Current (DC) is a form of current where voltage and current are constant over time.*

DC Offset We often talk about adding a **DC offset** to an AC signal. This means adding a constant DC value to an AC signal. Doing this shifts the entire signal up or down relative to the 0 V level, without changing the shape of the AC signal.

Example 2. *Example of a source of DC current is a battery.*

3.2.2 AC

Definition 4. *Alternating Current (AC) is a form of current that changes over time, often in a sinusoidal manner.*

Example 3. *Example of a source of AC current is a transformer. Other examples of AC current are wall outlets.*

3.3 Waves

3.3.1 Properties of waves

To describe waves, considering a sinusoidal wave of the form $A_p sin(2\pi vt)$, we use the following terms

- Peak amplitude (A_p): maximum value of the wave from its equilibrium position .
- Peak-to-peak amplitude: total height of the wave from its maximum to its minimum value (i.e., $2A_p$).
- Frequency (v): number of cycles per second (Hz).
- Time (t): time variable.

To be able to describe AC signals effectively, we take the root mean square (RMS) amplitude $\frac{A_p}{\sqrt{2}}$ of values such as current and voltage.

$$I_{RMS} = \frac{I_p}{\sqrt{2}}$$

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$

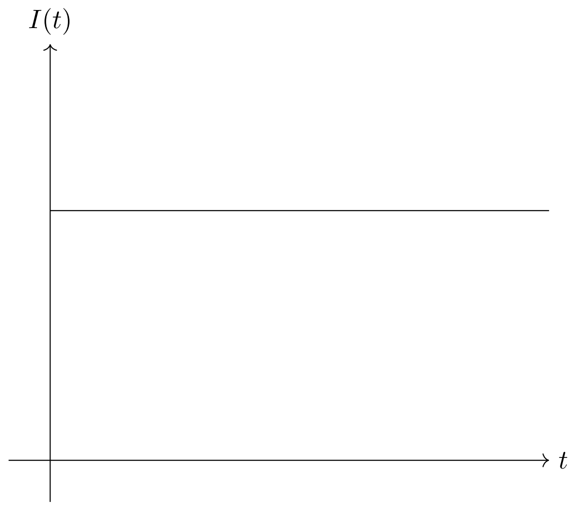
3.3.2 Waveforms

A waveform is the shape of a signal when plotted as a function of time. Common waveforms include:

- Sine wave
- Square wave
- Triangle wave
- Sawtooth wave

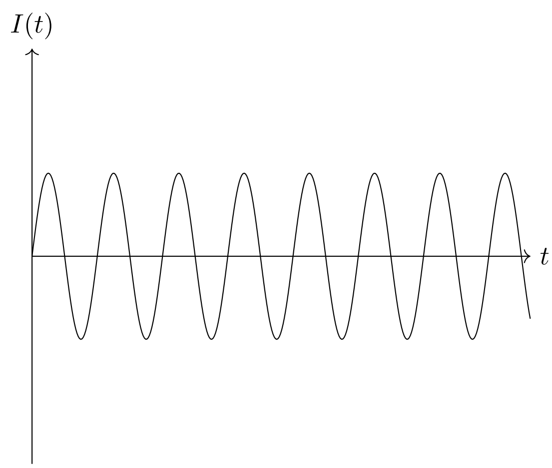
Direct current (DC). A direct current is constant in time, so its graph is a horizontal line.

Graph of a direct current $I(t)$:



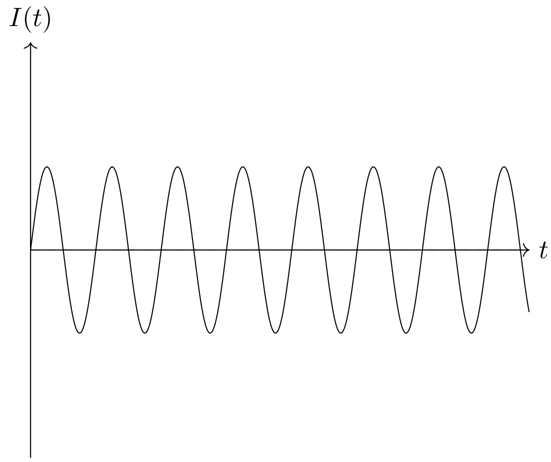
Alternating current (AC). An alternating current varies periodically in time and typically oscillates about zero.

Graph of an alternating current $I(t)$:



Pulsating current (DC + AC). A pulsating current is an alternating current superimposed on a non-zero DC level. This is just a matter of adding a constant DC level to an AC signal so that the signal is either shifted up or down with respect to the zero current level.

Graph of a pulsating current $I(t)$:



3.4 Linear Systems

Definition 5. *Linear systems obey the principles of superposition and scaling.*

Example 4. *Consider two inputs $x_1(t)$ and $x_2(t)$ to a linear system producing outputs $y_1(t) = H[x_1(t)]$ and $y_2(t) = H[x_2(t)]$, where H is some transformation function.*

A linear transformation must satisfy:

$$y_{total} = \alpha y_1(t) + \beta y_1(t) = H[\alpha x_1(t) + \beta x_2(t)],$$

where α and β are constants.

Example 5 (Superposition)**.**

$$H[x_1(t) + x_2(t)] = H[x_1(t)] + H[x_2(t)]$$

Example 6 (Scaling)**.**

$$H[\alpha x(t)] = \alpha H[x(t)]$$

3.5 Electric Charge

Definition 6. *Charge is a fundamental physical property that comes in two types: positive (+) and negative (-) (which cancel). Positive and negative charge are usually present in matter in exactly equal proportion (so matter is typically electrically neutral). Charge is conserved—can neither be created nor destroyed and is quantized in units of electronic charge $e = 1.6 \times 10^{-19}\,C$.*

3.6 Current flow

Definition 7 (Electric Current)**.** *Charge per unit time passing a given point in a circuit.*

Definition 8 (Current Flow). *The flow of electrons through a wire driven by an electric field potential energy difference. Defined as the rate of charge past a point in a circuit:*

$$I[ampere] = \frac{Q[coulomb]}{time[seconds]}$$

Definition 9 (Voltage). *The electric potential energy divided by the charge. It is the energy per unit charge.*

$$V[volt] = \frac{\Delta E_{Electric}[joule]}{Q[coulomb]}$$

Definition 10 (Electron-volt). *The energy gained or lost by an electron when it moves through an electric potential difference of one volt, which is a unit of energy equal to the work done on an electron in accelerating it through a potential difference of 1V, equal to* 1.6×10^{-19} *Joule.*

An electron between two charged plates has an electric potential energy given by

$$\Delta E_{Electric} = q\Delta V$$

which is converted into kinetic energy (KE) when it is ejected from between the plates.

When discussing current in a circuit, we follow the convention that it refers to the direction of the flow of positive charges.

Batteries store chemical energy potential. The cathode is surrounded by positively charged ions, and the anode by negatively charged ions. Electrons travel through a circuit from the anode to the cathode, releasing the stored chemical energy.

3.7 Electric Field & Potential

Charge emits an electric field that exerts a force on other charges. The electric field $\vec{E}$ at a point in space is defined as the force $\vec{F}$ experienced by a small positive test charge q placed at that point, divided by the magnitude of the test charge:

$$\vec{E} = \frac{\vec{F}}{q}$$

It takes work to move a charge against an electric field. The electric potential V at a point in space is defined as the work needed to move a positive test charge W per unit charge:

$$V = \frac{W}{q} = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E} \cdot d\vec{l}$$

If the electric field is uniform, this simplifies to

$$V = -\vec{E} \cdot \vec{d}$$

where $\vec{d}$ is the displacement vector from point $\vec{r_1}$ to point $\vec{r_2}$.

3.8 Potential & Ground

Definition 11 (Circuit ground)**.** *The reference point in a circuit at which the electric potential is defined to be zero. All voltages V in a circuit are measured relative to this point. The symbol for circuit ground is , and the symbol for chassis ground is .*

[1]

Definition 12 (Earth ground)**.** *The earth ground is a possible reference point, which is literally a metal stake or pipe tapped into the soil.*

3.9 Electromotive Force

Definition 13. *Electromotive Force (EMF) is the energy provided per unit charge by a source such as a battery or generator. It is the work done to move a charge through the source, measured in volts (V).*

[2]

To better understand the concept of EMF, consider the following analogy: a pump's action is similar to EMF in that it provides energy to move a fluid (like water) through a system. The gravitational potential at the top of a waterfall is analogous to the voltage drop ΔV, and gravity acting to drive the water down the hill is analogous to the electric field driving current in a circuit. Just as the pump provides energy to move the fluid through the system, the EMF provides energy to move charge through the circuit.

[1] Note the difference between the voltage drop ΔV across individual circuit elements (e.g. resistors) and the voltage at those elements measured relative to circuit ground.

[2] EMF is the total energy per unit charge from a source, while voltage is the potential difference measured across points in a circuit, typically less than the EMF due to internal resistance in real sources.

4 Circuit Theory Beyond Electronic

5 Capacitors & Inductors

6 RC and LR Circuits with AC Driving

7 Impedance

8 RLC Circuits

8.1 Transient Response

8.2 Driven RLC Circuits

8.3 Power Input to RLC and Circuit Network

9 Circuit Networks

10 Fourier Series

11 Fourier Transforms

12 Appendix

13 Useful Links