PHYS 241: Signal Processing

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Abstract

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1 Introduction

2 Prerequisite knowledge

3 Basics & Voltage, Current and Resistance

3.1 Signal Types

3.1.1 Digital Signal

Definition 1. *A discretely sampled signal with a sequence of quantized values.*

3.1.2 Analogue

Definition 2. *A continuous signal (e.g., in time) representing (analogous to) some other quantity.*

Example 1. *Examples of analogue devices and computers are:*

- *thermometers*
- *sextants*
- *tide-predicting machine*

3.2 Circuits

3.2.1 DC

Definition 3. *Direct Current (DC) is a form of current where voltage and current are constant over time.*

DC Offset We often talk about adding a **DC offset** to an AC signal. This means adding a constant DC value to an AC signal. Doing this shifts the entire signal up or down relative to the 0 V level, without changing the shape of the AC signal.

Example 2. *Example of a source of DC current is a battery.*

3.2.2 AC

Definition 4. *Alternating Current (AC) is a form of current that changes over time, often in a sinusoidal manner.*

Example 3. *Example of a source of AC current is a transformer. Other examples of AC current are wall outlets.*

3.3 Waves

3.3.1 Properties of waves

To describe waves, considering a sinusoidal wave of the form $A_p sin(2\pi vt)$, we use the following terms

- Peak amplitude (A_p): maximum value of the wave from its equilibrium position .
- Peak-to-peak amplitude: total height of the wave from its maximum to its minimum value (i.e., $2A_p$).
- Frequency (v): number of cycles per second (Hz).
- Time (t): time variable.

To be able to describe AC signals effectively, we take the root mean square (RMS) amplitude $\frac{A_p}{\sqrt{2}}$ of values such as current and voltage.

$$I_{RMS} = \frac{I_p}{\sqrt{2}}$$

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$

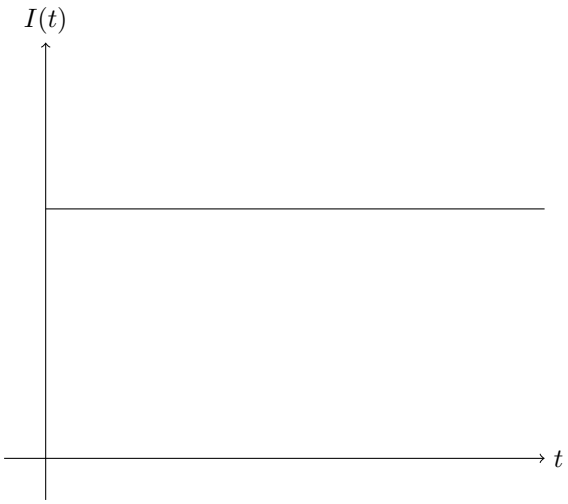
3.3.2 Waveforms

A waveform is the shape of a signal when plotted as a function of time. Common waveforms include:

- Sine wave
- Square wave
- Triangle wave
- Sawtooth wave

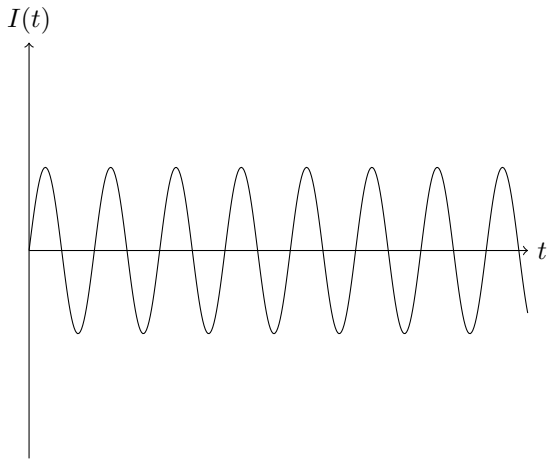
Direct current (DC). A direct current is constant in time, so its graph is a horizontal line.

Graph of a direct current $I(t)$:



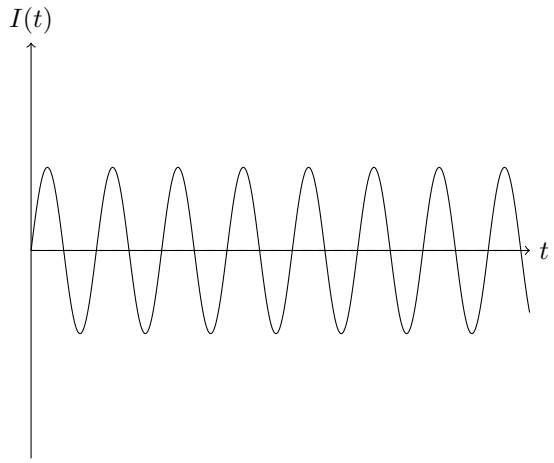
Alternating current (AC). An alternating current varies periodically in time and typically oscillates about zero.

Graph of an alternating current $I(t)$:



Pulsating current (DC + AC). A pulsating current is an alternating current superimposed on a non-zero DC level. This is just a matter of adding a constant DC level to an AC signal so that the signal is either shifted up or down with respect to the zero current level.

Graph of a pulsating current $I(t)$:



3.4 Linear Systems

Definition 5. *Linear systems obey the principles of superposition and scaling.*

Example 4. *Consider two inputs $x_1(t)$ and $x_2(t)$ to a linear system producing outputs $y_1(t) = H[x_1(t)]$ and $y_2(t) = H[x_2(t)]$, where H is some transformation function.*

A linear transformation must satisfy:

$$y_{total} = \alpha y_1(t) + \beta y_1(t) = H[\alpha x_1(t) + \beta x_2(t)],$$

where α and β are constants.

Example 5 (Superposition)**.**

$$H[x_1(t) + x_2(t)] = H[x_1(t)] + H[x_2(t)]$$

Example 6 (Scaling)**.**

$$H[\alpha x(t)] = \alpha H[x(t)]$$

3.5 Electric Charge

Definition 6. *Charge is a fundamental physical property that comes in two types: positive (+) and negative (-) (which cancel). Positive and negative charge are usually present in matter in exactly equal proportion (so matter is typically electrically neutral). Charge is conserved—can neither be created nor destroyed and is quantized in units of electronic charge $e = 1.6 \times 10^{-19} C$.*

3.6 Current flow

Definition 7 (Electric Current)**.** *Charge per unit time passing a given point in a circuit.*

Definition 8 (Current Flow)**.** *The flow of electrons through a wire driven by an electric field potential energy difference. Defined as the rate of charge past a point in a circuit:*

$$I[ampere] = \frac{Q[coulomb]}{time[seconds]}$$

Definition 9 (Voltage)**.** *The electric potential energy divided by the charge. It is the energy per unit charge.*

$$V[volt] = \frac{\Delta E_{Electric}[joule]}{Q[coulomb]}$$

Definition 10 (Electron-volt)**.** *The energy gained or lost by an electron when it moves through an electric potential difference of one volt, which is a unit of energy equal to the work done on an electron in accelerating it through a potential difference of 1V, equal to* 1.6×10^{-19} *Joule.*

3.7 Ohm's Law

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