PHYS 241: Signal Processing

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Abstract

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2 Foundations

2.1 Signal Types

2.1.1 Digital Signal

Definition 1 (Digital Signal)**.** *A discretely sampled signal with a sequence of quantized values.*

Example. *Examples of digital devices and computers include digital clocks, calculators, and modern computers that use binary code to process and store information.*

2.1.2 Analogue Signal

Definition 2 (Analogue Signal)**.** *A continuous signal (e.g., in time) representing (analogous to) some other quantity.*

Example. *Examples of analogue devices and computers are thermometers, sextants, and tide-predicting machines.*

2.2 Waves

2.2.1 Properties of waves

To describe waves, considering a sinusoidal wave of the form $A_p sin(2\pi vt)$, we use the following terms

- Peak amplitude (A_p): maximum value of the wave from its equilibrium position .
- Peak-to-peak amplitude: total height of the wave from its maximum to its minimum value (i.e., $2A_p$).
- Frequency (v): number of cycles per second (Hz).
- Time (t): time variable.

To be able to describe AC signals effectively, we take the root mean square (RMS) amplitude $\frac{A_p}{\sqrt{2}}$ of values such as current and voltage. This is because the mean value of an AC signal is zero, and the RMS is a more meaningful measure of the signal's amplitude as it gives a positive value that is proportional to the energy in the signal.

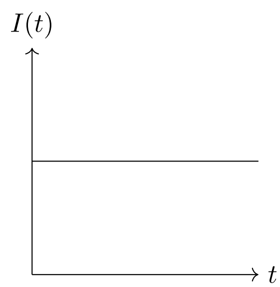
$$I_{RMS} = \frac{I_p}{\sqrt{2}}$$

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$

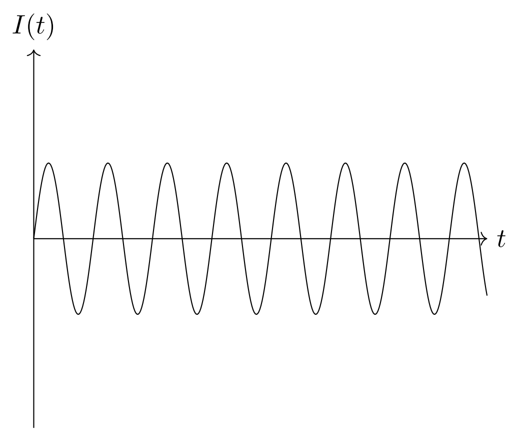
2.2.2 Waveforms

Definition 3 (Waveform)**.** *A waveform is a graphical representation of how a signal varies over time, and can take various forms such as a sine wave, square wave, triangle wave, or sawtooth wave.*

1. Direct current (DC). A direct current is constant in time, so its graph is a horizontal line.



2. Alternating current (AC). An alternating current varies periodically in time and typically oscillates about zero in a sinusoidal pattern.



3. Pulsating current (DC + AC). A pulsating current is an alternating current superimposed on a non-zero DC level. This is just a matter of adding a constant DC level to an AC signal so that the signal is either shifted up or down with respect to the zero current level.

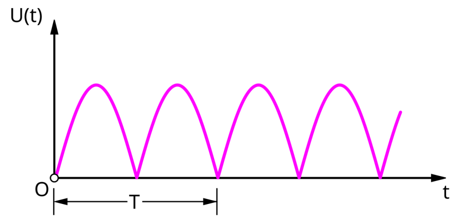


Figure 1: Pulsating direct current diagram

2.3 Linear Systems

Definition 4 (Linear System)**.** *A system H is linear if it satisfies superposition*

$$H[x_1(t) + x_2(t)] = H[x_1(t)] + H[x_2(t)],$$

and scaling

$$H[\alpha x(t)] = \alpha H[x(t)],$$

for all inputs $x_1(t), x_2(t)$ and constant α.

Example. *Consider two inputs $x_1(t)$ and $x_2(t)$ to a linear system producing outputs $y_1(t) = H[x_1(t)]$ and $y_2(t) = H[x_2(t)]$, where H is some transformation function.*

A linear transformation must satisfy:

$$y_{total} = \alpha y_1(t) + \beta y_1(t) = H[\alpha x_1(t) + \beta x_2(t)],$$

where α and β are constants.

3 Electricity and Magnetism

3.1 Electric Charge

Definition 5. *Charge is a fundamental physical property that comes in two types: positive (+) and negative (-) (which cancel). Opposite charges attract and equal amounts of positive and negative charge cancel, so most matter is electrically neutral. Electric charge is conserved, meaning it cannot be created or destroyed, only transferred. It is also quantized, existing in discrete multiples of the elementary charge*

$$e = 1.6 \times 10^{-19}\,[C],$$

where the coulomb (C) is the SI unit of charge.

3.2 Electric Field

Definition 6 (Electric Field). *The electric field $\vec{E}$ is a vector field produced by electric charges. It describes the force that a charge would experience at any point in space. Quantitatively, the electric field at a point is defined as the force $\vec{F}$ acting on a small positive test charge q placed at that point, divided by the magnitude of the charge:*

$$\vec{E} = \frac{\vec{F}}{q}.$$

The direction of $\vec{E}$ is the direction of the force on a positive test charge. Its SI units are newtons per coulomb (N/C), which are equivalent to volts per meter (V/m).

3.3 Electric Potential

Definition 7 (Electric Potential). *The electric potential $V(\vec{r})$ at a point is the electric potential energy per unit charge:*

$$V = \frac{U}{q}$$

It is a scalar quantity with units of volts (V) that represents the work done per unit charge to move a positive test charge from a reference point (often taken at infinity) to the point $\vec{r}$, without changing its kinetic energy. It is defined relative to a reference point $\vec{r}_0$ by

$$V(\vec{r}) - V(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l}$$

where $\vec{E}$ is the electric field. The electric field is related to the potential by

$$\vec{E} = -\nabla V.$$

Example (Uniform Electric Field). *If $\vec{E}$ is uniform, the potential difference between two points separated by $\vec{d}$ is*

$$\Delta V = -\vec{E} \cdot \vec{d}.$$

3.4 Electric Potential Energy

Definition 8 (Electric Potential Energy). *Electric potential energy U is the energy a charge possesses due to its position in an electric field. For a charge q at a point where the electric potential is V, the potential energy is*

$$U = qV,$$

and is measured in joules (J). Since electric potential is defined relative to a reference point, the potential energy is also defined relative to that same reference.

An electron placed between two charged plates experiences a potential difference ΔV. The change in its electric potential energy is

$$\Delta U = q\Delta V.$$

If the electron is allowed to move freely, this change in electric potential energy is converted into kinetic energy:

$$\Delta K = -\Delta U.$$

3.5 Magnetic Field

3.6 Magnetic Field

Definition 9 (Magnetic Field)**.** *A magnetic field $\vec{B}$ is a vector field produced by moving electric charges (currents) and intrinsic magnetic moments of particles. It exerts a force on moving charges given by the Lorentz force law:*

$$\vec{F} = q(\vec{v} \times \vec{B}),$$

where q is the charge and $\vec{v}$ is its velocity. The force is perpendicular to both the velocity and the magnetic field. Unlike electric fields, which act on charges whether stationary or moving, magnetic fields act only on moving charges. For a long straight current-carrying wire, the magnitude of the magnetic field at a distance r is

$$B = \frac{\mu_0 I}{2\pi r},$$

where μ_0 is the permeability of free space and I is the current. The SI unit of magnetic field is the tesla (T).

3.7 Magnetic Flux

Definition 10 (Magnetic Flux)**.** *Magnetic flux Φ_B through a surface is defined as the surface integral of the magnetic field $\vec{B}$ over that surface:*

$$\Phi_B = \int \vec{B} \cdot d\vec{A},$$

where $d\vec{A}$ is an infinitesimal area element of the surface, and the dot product accounts for the angle between the magnetic field and the normal to the surface. The SI unit of magnetic flux is the weber (Wb).

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3.8 Mutual Inductance

Definition 11 (Mutual Inductance)**.** *Mutual inductance is the property of two nearby circuits whereby a time-varying current in one coil induces an electromotive force (EMF) in another through magnetic flux linkage.*

If a current $I_1(t)$ flows in the primary coil, it produces a magnetic flux Φ_{21} through the secondary coil. According to Faraday's law,

$$\mathcal{E}_2 = -\frac{d\Phi_{21}}{dt}.$$

The mutual inductance M is defined by

$$\Phi_{21} = MI_1,$$

which gives

$$\mathcal{E}_2 = -M\frac{dI_1}{dt}.$$

The value of M depends on the number of turns in each coil, their geometry, their separation, and the magnetic permeability of the material between them.

3.8.1 Lenz's Law

Law 1 (Lenz's Law)**.** *The induced current in a circuit flows in a direction such that the magnetic field it produces opposes the change in magnetic flux that generated it.*

Consider any closed conducting loop. If the magnetic flux through the loop increases, the induced current produces a magnetic field that opposes the increase. If the flux decreases, the induced current produces a field that tends to restore it.

When the magnetic flux is increasing, the induced current is directed opposite to the orientation predicted by the right-hand rule for the increasing field. When the flux is decreasing, the induced current follows the orientation that attempts to reinforce the original field.

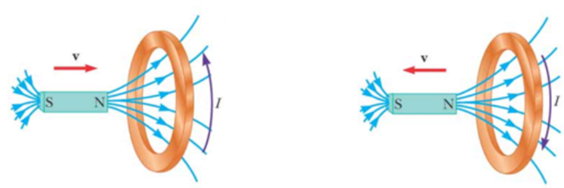

Figure 2: Direction of the induced current according to Lenz's Law.

4 Electrical Principles

4.1 Current flow

Definition 12 (Electric Current)**.** *Electric current is the flow of electrons through a wire driven by an electric field potential energy difference. Defined* *as the rate of charge past a point in a circuit*

$$I = \frac{dQ}{dt}, \tag{1}$$

and is measured in amperes (A), where

$$1\,A = 1\,C/s.$$

Definition 13 (Voltage). *The Voltage (or electric potential difference) is the difference in potential energy between two points in an electric field.*

Definition 14 (Electron-volt). *An electron-volt (eV) is a unit of energy equal to the energy gained or lost by an electron when it moves through an electric potential difference of one volt. Since energy is given by $U = qV$, for a charge equal to the elementary charge e, we have*

$$1\,eV = e \cdot 1\,V.$$

Numerically,

$$1\,eV = 1.602 \times 10^{-19}\,J.$$

When analyzing circuits, current is defined using the convention of positive charge flow. By convention, current flows from higher potential to lower potential, even though in metallic conductors the actual charge carriers are electrons moving in the opposite direction.

When the circuit is closed, electrons flow externally from the anode to the cathode, while conventional current flows from the cathode to the anode. As the charges move through the circuit, the battery's chemical energy is converted into electrical energy and then into other forms such as heat or light.

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4.2 Electromotive Force

Definition 15 (Electromotive Force (EMF)). *Electromotive Force (EMF)*[1] *is the energy supplied per unit charge by an energy source such as a battery or generator. It represents the work done by the source to move a charge around a complete circuit. EMF is measured in volts (V).*

To better understand the concept of EMF, consider the following analogy: EMF is like a voltage credit that is then all spent by dropping it over the elements of the circuit. Think of your checking account with two columns: credit (money in) and debit (money spent). EMF is like the "credit" that is then all "spent" by dropping it over the elements of the circuit.

[1] EMF is the total energy supplied per unit charge by a source. The terminal voltage across a real source is typically less than the EMF due to internal resistance.

4.2.1 Faraday's Law

Theorem 1 (Faraday's Law)**.** *Faraday's Law states that a changing magnetic flux through a closed loop induces an electromotive force (EMF) in the loop. Mathematically, it is expressed as:*

$$\mathcal{E}_{EMF} = -\frac{d\Phi_B}{dt}$$

where $\mathcal{E}_{EMF}$ is the induced EMF, and Φ_B is the magnetic flux through a wire loop. The negative sign indicates that the induced EMF creates a current whose magnetic field opposes the change in flux, as described by Lenz's Law.

4.3 Ohm's Law

Definition 16 (Ohm's Law)**.** *Ohm's Law states that the current I flowing through a conductor is directly proportional to the voltage V across it and inversely proportional to its resistance R. Mathematically, it is expressed as:*

$$I = \frac{V}{R}$$

Microscopically, Ohm's Law can be understood in terms of the motion of electrons in a conductor. When a voltage is applied across a conductor, it creates an electric field that exerts a force on the free electrons, causing them to move and create an electric current. The resistance of the conductor arises from collisions between the electrons and the atoms in the material, which impede their motion. The greater the resistance, the more collisions occur, resulting in a lower current for a given voltage. The current density J is given by Ohm's Law, which states that the current density is proportional to the electric field strength E:

$$J = \sigma E$$

Conversely, the electric field strength is given by the current density and the resistivity of the material:

$$E = J\rho_r$$

The current I is given by the current density and the cross-sectional area of the material:

$$I = AJ = A\sigma E$$

The electric field strength can also be expressed in terms of the potential difference ΔV across the material and its length l:

$$E = \Delta V/l$$

Substituting this into the expression for the current, we obtain:

$$I = \frac{A\sigma\Delta V}{l} = \frac{A\Delta V}{\rho_r l}$$

The potential difference across the material is also given by the resistance R and the current I as stated in Ohm's Law:

$$\Delta V = IR$$

Finally, the resistance can be expressed in terms of the resistivity of the material, the length of the material, and its cross-sectional area:

$$R = \frac{\rho_r l}{A}$$

Ohm's Law can be represented graphically using the Ohm's Law Wheel, which illustrates the relationships between current, voltage, resistance, and conductance. The wheel is a useful tool for quickly finding the value of one electrical property given the values of the other three.

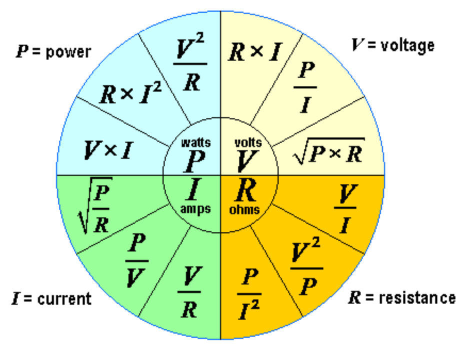


Figure 3: Ohm's Law Wheel and Current Density

4.4 Direct Current

Definition 17 (Direct Current (DC))**.** *Direct Current (DC) is a form of current where voltage and current are constant over time, could be found in batteries.*

Definition 18 (DC Offset)**.** *A DC offset is the addition of a constant DC value to an AC signal. This shifts the entire signal up or down relative to the* 0 *V level, without changing the shape of the AC signal.*

4.5 Alternating Current

Definition 19 (Alternating Current (AC))**.** *Alternating Current (AC) is a form of current that changes over time, often in a sinusoidal manner. AC currents are commonly used in power distribution systems, such as household electrical outlets.*

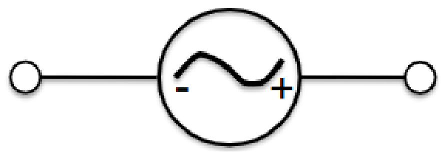


Figure 4: Alternating Current Symbol.

In general, an alternating current source is represented in circuit diagrams by a circular source symbol containing a sinusoidal waveform. This indicates that the source provides a time-varying current, typically sinusoidal in form.

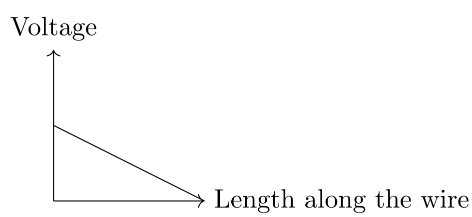
If the current is sinusoidal, it is commonly written as

$$I(t) = I_0 \sin(\omega t),$$

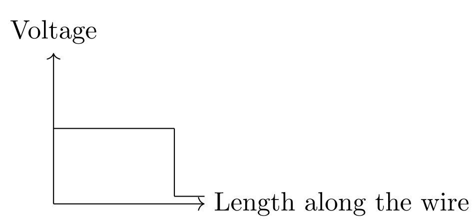
where I_0 is the amplitude and ω is the angular frequency.

4.6 Static Circuits and Electronics

Consider the variation of electric potential along a conducting wire.
If the wire has a small but non-negligible resistance that is uniformly distributed, the electric potential decreases approximately linearly with distance. This occurs because the voltage drop across a uniform conductor is proportional to its length.



In contrast, if a discrete resistor with resistance much larger than that of the wire is introduced, nearly all of the voltage drop occurs across the resistor. The potential along the wire remains approximately constant, with a sharp decrease only at the location of the resistor.



4.7 Circuits

Circuits may be classified according to whether they are linear or non-linear, and whether they contain active or passive elements. A linear circuit element has a linear (straight-line) relationship between voltage and current, meaning $V \propto I$. In contrast, a non-linear element does not exhibit a straight-line V-I relationship.

Definition 20 (Passive Linear Circuit)**.** *A passive linear circuit contains only passive linear elements, such as resistors, capacitors, inductors, and transformers.*

Definition 21 (Active Linear Circuit)**.** *An active linear circuit contains at least one active element, such as a transistor, operational amplifier, or independent source, while maintaining an overall linear V-I relationship.*

Definition 22 (Passive Non-linear Circuit)**.** *A passive non-linear circuit contains only passive components but includes at least one non-linear element, such as a diode.*

Definition 23 (Active Non-linear Circuit)**.** *An active non-linear circuit contains at least one active element and at least one non-linear element.*

5 Circuit Components

5.1 Resistors

Definition 24. *A resistor is a device that opposes the flow of current through it, resulting in a voltage drop across it. Resistors are characterized by their resistance value R, measured in ohms (Ω), which quantifies the amount of opposition they provide to the flow of current.*

When resistors are connected in series, their resistances add together to form a total resistance $R_{tot} = R_1 + R_2 + \cdots + R_n$. In contrast, when resistors are connected in parallel, their resistances add inversely to form a total resistance $\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$.

Example. *Look at the following examples of resistors, all of which are connected in parallel.*

5.1.1 Temperature Dependence

The temperature dependence of the resistance of many metals can be modelled by the following equation:

$$R_T = R_0[1 + \alpha(T - T_0)]$$

where R_0 is the resistance value at reference temperature T_0, and α is the linear temperature coefficient. This equation shows that the resistance of a metal increases linearly with temperature, which is a common phenomenon observed in many metals.

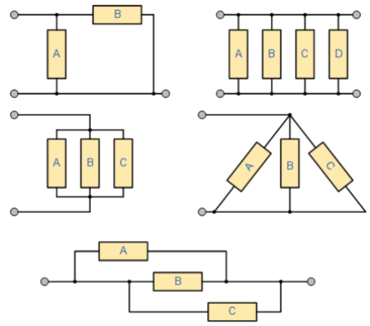


Figure 5: In all of these examples, the components are wired in parallel.

5.1.2 Power Dissipation

When current flows through a resistor, electrical energy is converted into heat energy due to the resistance of the material. The power P dissipated by a resistor can be calculated using the formula:

$$P = \Delta V \cdot I$$

Using Ohm's Law, we can express the power dissipated in terms of the resistance R and either the voltage drop ΔV or the current I:

$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

5.2 Batteries

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Definition 25 (Battery)**.** *A battery converts stored chemical energy into electrical energy by maintaining a potential difference between its terminals. Inside the battery, chemical reactions separate charge, creating an excess of negative charge at the anode and positive charge at the cathode.*

Definition 26 (Ideal Battery)**.** *An ideal battery is a voltage source with zero internal resistance that maintains a constant voltage regardless of the current drawn.*

Definition 27 (Real Battery)**.** *A real battery has internal resistance that causes the voltage output to decreases as the current increases, leading to a loss of energy in the form of heat due to the internal resistance.*

The circuit containing a real battery can be modeled as an ideal voltage source V_0 in series with an internal resistance r. The terminal voltage $V_{terminal}$ of the battery when a current I is drawn from it can be expressed as:

$$V_{terminal} = V_0 - Ir$$

However, if the circuit contained a resistor R connected to the battery, the current I flowing through the circuit can be calculated using Ohm's Law:

$$V_0 = Ir + IR.$$

5.3 Ground and Reference

Definition 28 (Circuit ground)**.** *The reference point in a circuit at which the electric potential is defined to be zero. All voltages V in the circuit are measured relative to this point. It does not have to be physically connected to the Earth. The symbol for circuit ground is* ⏚.

Definition 29 (Chassis Ground)**.** *Chassis ground is a conductive frame or enclosure of a device that is used as a common return path for current and often connected to circuit ground. It provides shielding and safety by preventing exposed metal parts from reaching dangerous voltages. In many systems, chassis ground may be connected to earth ground, but this is not required. The symbol for chassis ground is* ▽.

Definition 30 (Earth Ground)**.** *Earth ground is a physical connection to the Earth, typically made using a metal rod or pipe driven into the soil. It is used primarily for safety, allowing fault currents to safely dissipate into the ground.*

5.4 Capacitors

Definition 31 (Capacitor)**.** *A capacitor is a passive electrical component that stores energy in an electric field between two conductive plates separated by an insulating material (dielectric) or space. The capacitance C of a capacitor is defined as the ratio of the charge Q stored on one plate to the voltage V across the plates (or of the source of the circuit):*

$$C = \frac{Q}{V} \tag{2}$$

The SI unit of capacitance is the farad (F).

Capacitance can also be measured in terms of the physical characteristics of the capacitor. For a parallel plate capacitor, the capacitance is given by:

$$C = \frac{\varepsilon A}{d}$$

where ε is the dielectric constant, A is the area of the plates, and d is the distance between the plates.

Capacitors combine oppositely to resistors. In parallel,

$$C_{tot} = C_1 + C_2 + \cdots + C_n,$$

while in series,

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}.$$

From the definition of capacitance, the voltage-charge relation is

$$V = \frac{Q}{C}.$$

Because capacitors typically have very low internal resistance, they can charge and discharge much faster than batteries.

5.4.1 Energy Stored In a Capacitor

The energy stored equals the work required to move charge from one plate to the other. Since $V = \frac{Q}{C}$, the work done in charging the capacitor from 0 to Q is

$$W = \int_0^Q V\,dQ = \int_0^Q \frac{Q}{C}\,dQ = \frac{1}{2}CV^2.$$

Thus, the energy stored in a capacitor is

$$W = \frac{1}{2}CV^2.$$

5.4.2 Dielectric Insulator

Definition 32 (Dielectric Insulator)**.** *A dielectric is an insulating material that permits an electric field but does not conduct charge. When placed between capacitor plates, it increases capacitance by reducing the effective electric field, allowing more charge to be stored for a given voltage. Its permittivity is given by*

$$\varepsilon = \varepsilon_r \varepsilon_0,$$

where ε_r is the relative permittivity and ε_0 is the permittivity of free space.

5.4.3 Charging a Capacitor

For a capacitor, the current-voltage relationship, derived from (2) and (1), is

$$I = C\frac{dV}{dt},$$

where V is the voltage across the capacitor. Integrating,

$$V(t) = \frac{1}{C}\int I\,dt + V_0,$$

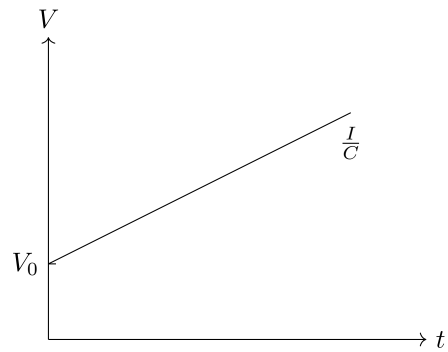
where V_0 is the initial voltage. If the current I is constant, then

$$\frac{dV}{dt} = \frac{I}{C},$$

so

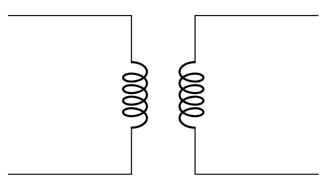
$$V(t) = V_0 + \frac{I}{C}t.$$

Thus, the voltage increases linearly with time, with slope $\frac{I}{C}$ and y-intercept V_0.



5.5 Transformers & Mutual Inductance

Definition 33 (Transformer)**.** *A transformer is a device consisting of two coils of wire, called the primary and secondary, magnetically coupled through a shared core. The main purpose of a transformer is to transfer energy between circuits to change voltage levels. A changing current in the primary coil induces a voltage in the secondary coil by mutual inductance. The symbol for a transformer is*



A transformer typically consists of two coils with N_1 and N_2 turns wound around a common core, which may be air or a ferromagnetic material to concentrate the magnetic field.

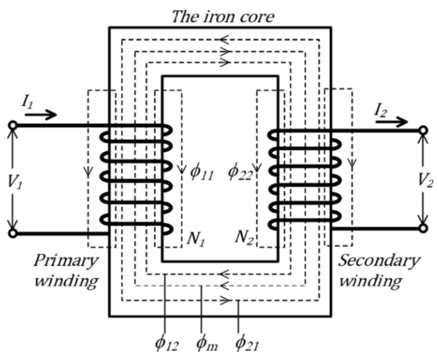


Figure 6: Visual representation of a transformer.

A time-varying current I_1 in the primary produces a magnetic field $\vec{B}$, creating a magnetic flux Φ_B through the secondary coil. By Faraday's law, a changing magnetic flux induces a voltage in the secondary:

$$\mathcal{E}_2 = -\frac{d\Phi_B}{dt}.$$

Since the magnetic flux is proportional to the primary current, the induced voltage is proportional to $\frac{dI_1}{dt}$. The proportionality constant is the mutual inductance M:

$$\mathcal{E}_2 = -M\frac{dI_1}{dt}.$$

The SI unit of mutual inductance is the henry (H), where

$$1\,\mathrm{H} = 1\,\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}.$$

The direction of the induced current depends on the relative winding orientation of the coils. Reversing the winding reverses the polarity of the induced voltage. If the primary current is sinusoidal, for example $I_1(t) = I_0\sin(\omega t)$, then

$$V_2(t) = M\frac{dI_1}{dt} = M\omega I_0\cos(\omega t),$$

so the secondary voltage is also sinusoidal but shifted in phase by $\frac{\pi}{2}$ relative to the current.
For an ideal transformer, energy losses are neglected and power is conserved:

$$V_1 I_1 = V_2 I_2.$$

This leads to the transformer relations

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1},$$

where $\frac{N_1}{N_2}$ is the turns ratio in the two coils.

Remark 1. *Long-distance power lines use high-voltage AC because it significantly reduces energy loss during transmission over long distances, as higher voltage means lower current, which in turn minimises heat loss within the power lines due to resistance; this is achieved efficiently with transformers that can easily step up and down AC voltage levels as needed.*

5.6 Inductors

Definition 34 (Inductor)**.** *An inductor is a passive electrical component that stores energy in a magnetic field created by the flow of current through a coil of wire. The inductance L of an inductor is defined as the ratio of the magnetic flux Φ linking the coil to the current I flowing through it:*

$$L = \frac{\Phi}{I}$$

The SI unit of inductance is the henry (H).

Common circuit symbols for inductors are shown below.

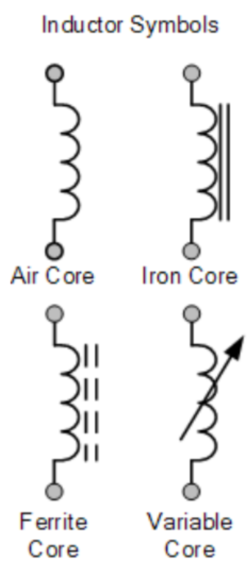
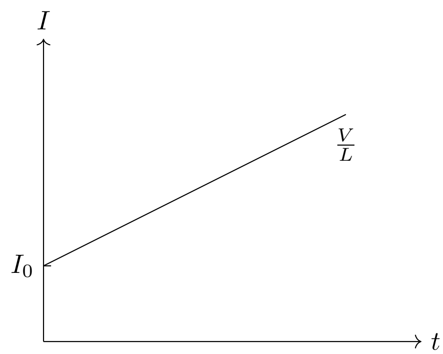


Figure 7: Circuit symbols for inductors with different core types.

5.6.1 Energy Storage

Energy is stored in the magnetic field within the inductor coil. If the voltage applied to the coil were constant, then the current in the coil would rise.



The energy stored is the amount of work done by the current, which is given by

$$W = \int_0^t V \cdot i dt = \int L \frac{di}{dt} i = \frac{1}{2} L I^2.$$

5.6.2 Networks of Inductors

Inductors in Series When inductors are connected in series, the same current flows through each element. The total voltage across the combination is the sum of the individual voltages:

$$V_{\text{total}} = \sum_{i=1}^{N} V_i.$$

Since $V_i = L_i \frac{dI}{dt}$ and the current I is the same in each inductor, we obtain

$$V_{\text{total}} = \left(\sum_{i=1}^{N} L_i \right) \frac{dI}{dt}.$$

Thus, the equivalent inductance is

$$L_{\text{total}} = \sum_{i=1}^{N} L_i.$$

Inductors in Parallel When inductors are connected in parallel, the voltage across each inductor is the same, while the total current is the sum of the individual currents:

$$I_{\text{total}} = \sum_{i=1}^{N} I_i.$$

Using $V = L_i \frac{dI_i}{dt}$ and the fact that the voltage is common to all branches, we obtain

$$\frac{1}{L_{\text{total}}} = \sum_{i=1}^{N} \frac{1}{L_i}.$$

These relations are formally similar to those of resistors, but they arise from the voltage-current relation of an inductor.

5.6.3 Solenoid

Definition 35 (Solenoid)**.** *A solenoid is a long cylindrical coil of wire designed to produce a nearly uniform magnetic field inside it when current flows.*

For a solenoid of length l, cross-sectional area $A = \pi r^2$, number of turns N, and core permeability μ, the inductance is

$$L = \frac{\mu N^2 A}{l}.$$

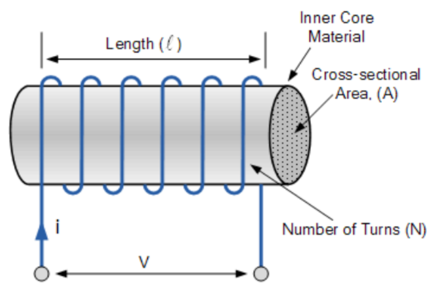


Figure 8: Schematic representation of a solenoid.

6 Circuits

6.1 RC Circuit

Definition 36 (RC Circuit)**.** *An RC circuit is a circuit consisting of a resistor R, a capacitor C, and a constant source V_0.*

Example. *An example of an RC circuit is shown in the figure below.*

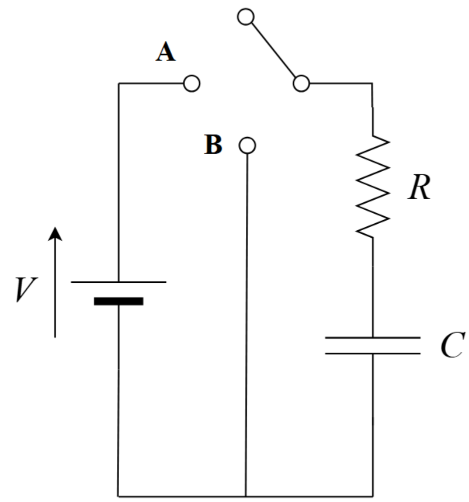


Figure 9: Visual representation of an RC circuit.

6.1.1 Charging

Consider an initially uncharged capacitor connected in series with a resistor and a constant source V_0. Applying Kirchhoff's Voltage Law,

$$V_0 = v_R(t) + v_C(t) = i(t)R + \frac{1}{C}\int_0^t i(\tau)\, d\tau.$$

Differentiating with respect to time gives

$$0 = R\frac{di(t)}{dt} + \frac{1}{C}i(t).$$

Multiplying by C,

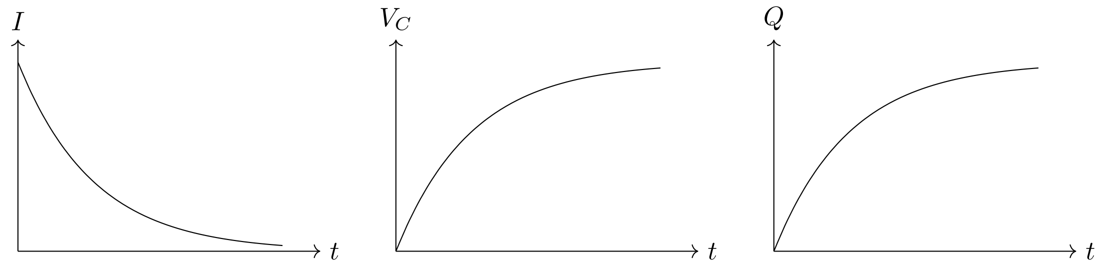
$$0 = RC\frac{di(t)}{dt} + i(t).$$

Solving,

$$I(t) = \frac{V_0}{R}e^{-t/\tau}, \quad V_C(t) = V_0\left(1 - e^{-t/\tau}\right),$$

where $\tau = RC$ is the time constant. As $t \to \infty$, the current and the resistor voltage decay exponentially to zero, while the capacitor voltage approaches V_0.

Charging Graphs



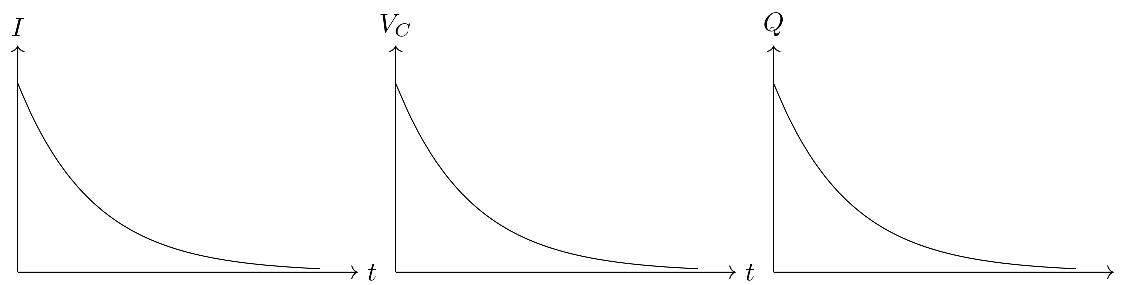
6.1.2 Discharging

If the source is removed and the capacitor initially has voltage V_0, the solutions become

$$I(t) = -\frac{V_0}{R}e^{-t/\tau}, \quad V_C(t) = V_0 e^{-t/\tau}.$$

Both current and voltage decay exponentially to zero.

Discharging Graphs



6.2 LR Circuit

Definition 37 (LR Circuit)**.** *An LR circuit is a circuit in which an inductor is connected in series with a resistor.*

Example. *Consider this circuit, at the moment it is turned on (switch connects to 'A'). If the inductor (coil) was replaced with a short, a current* $\frac{V}{R}$ *would flow immediately, with all of the voltage drop across the resistance* R*.*

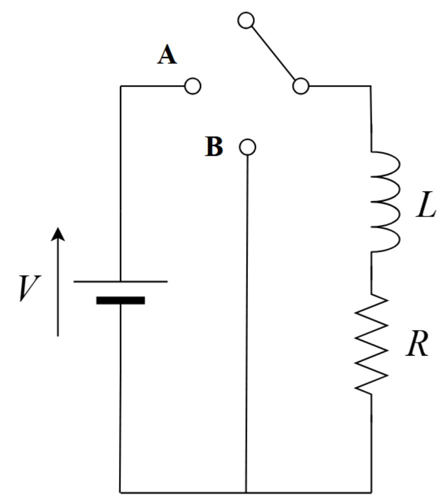


Figure 10: Visual representation of an LR circuit.

The current flow produces a magnetic flux through the coil. Lenz's law says that the coil will attempt to produce a field that opposes the change in B-field. To do this, it induces a voltage V (equal to the battery's voltage) across the coil, with the upper end positive and the lower negative. Therefore, just at the instant the switch is closed, the current flow will be zero.

For a single coil, the voltage-current relationship is

$$V = L\frac{di}{dt},$$

where L is called the self-inductance of the coil, or simply the inductance. L depends on the number of turns of wire, and the magnetic properties of the material within the coil. Any current-carrying conductor even a straight wire is surrounded by a magnetic field, and hence has some self-inductance.

When the switch is connected to terminal A in Figure (6.2), the current in the inductor increases according to

$$I(t) = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right),$$

while the voltage across the inductor decreases as

$$V_L(t) = Ve^{-\frac{Rt}{L}}.$$

The characteristic time constant of the circuit is

$$\tau = \frac{L}{R}.$$

7 RC and LR Circuits with AC Driving

January 29, 2026

7.1 Driving Frequency and Time Constant

For a periodic input signal with period T and angular frequency ω,

$$\omega = \frac{2\pi}{T} = 2\pi\nu.$$

The characteristic time constants are:

$$\tau_{RC} = RC$$

$$\tau_{LR} = \frac{L}{R}$$

The circuit behaviour depends on the comparison between T and τ:

- Low frequency: $T \gg \tau$
- High frequency: $T \ll \tau$

The key idea is that the response depends on the ratio between the driving period and the circuit time constant.

7.2 RC Circuit with Square-Wave Input

Low Frequency ($T \gg \tau$)

- Capacitor fully charges and discharges.
- Current similar to pure resistor case.
- Very little voltage remains across the capacitor.
- Capacitor behaves approximately like a short circuit.

High Frequency ($T \ll \tau$)

- Capacitor does not have time to charge.
- Voltage dropped mostly across the capacitor.
- Current appears as sharp spikes at switching.
- Capacitor behaves approximately like an open circuit.

7.3 Integrator and Differentiator Behaviour (RC)

For slowly varying signals (low frequency),

$$V_C(t) \approx \frac{1}{RC} \int V_{in}(t)\, dt$$

The capacitor acts as an integrator. A square wave input produces a triangle-like voltage output. For rapidly varying signals (high frequency),

$$V_R(t) \approx RC \frac{dV_{in}}{dt}$$

The resistor behaves as a **differentiator**. Sharp transitions in the input produce current spikes.

7.4 LR Circuit with AC Input

The LR circuit is mathematically analogous to the RC circuit.

$$\tau_{LR} = \frac{L}{R}$$

- Inductor resists changes in current.
- At low frequency, the inductor behaves approximately like a short circuit.
- At high frequency, the inductor behaves approximately like an open circuit.

In an LR circuit, current plays a role analogous to voltage in an RC circuit.

7.5 Frequency Behaviour Summary

Circuit	Low Frequency ($T \gg \tau$)	High Frequency ($T \ll \tau$)
RC	Capacitor $\approx$ short	Capacitor $\approx$ open
LR	Inductor $\approx$ short	Inductor $\approx$ open

Important physical facts:

- A capacitor blocks DC.
- An inductor passes DC (after long time).
- A capacitor resists changes in voltage.
- An inductor resists changes in current.

8 Appendix

9 Useful Links

To understand visually inductors: https://www.youtube.com/shorts/lXk4CQDbr4Q
to visually understand capacitors: https://www.youtube.com/shorts/PYrTLQctdwg
to understand resistors: https://www.youtube.com/watch?v=5HI33hSZoH4