# Coding Exercise 1: Solving the Continuous-Time Consumption Euler Equation

ECON 202A

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In this problem set, you will solve the continuous-time consumption Euler equation both analytically and numerically, and then compare the two solutions.

The consumption Euler equation is given by:

$$\frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\theta}$$

where:

- $C(t)$ is the consumption at time $t$,
- $r(t)$ is the interest rate at time $t$,
- $\theta$ is the inverse of the intertemporal elasticity of substitution (IES),
- $\rho$ is the time discount rate.

The terminal condition is:

$$C(T) = C_T$$

where $T$ is the terminal time and $C_T$ is the consumption at time $T$.

Your task is to solve for the consumption path $C(t)$ over time, from $t = 0$ to $t = T$, using both the integrating factor method (analytical) and the finite difference method (numerical).

The model is parameterized as follows:

- $C_T = 2.0$ (terminal consumption at time $T$),
- $r(t) = r_0 + \alpha t$, where $r_0 = 0.05$ and $\alpha = 0.01$,
- $\theta = 2.0$ (inverse of the intertemporal elasticity of substitution),
- $\rho = 0.03$ (time discount rate),
- $T = 10$ (terminal time).

Please complete the following tasks and submit both your written answers and the code you used. Ensure that your submission includes plots and a brief explanation of your results. Refer to the section syllabus for detailed code submission guidelines.

1. Solve the equation analytically using the integrating factor method. Then, plot the analytical solution as a function of time $t$ and consumption $C(t)$.

2. Solve the equation numerically using the finite difference method with 100 time steps. Plot the numerical and analytical solutions together in a single figure for comparison.

3. Repeat the numerical solution using 10 time steps. Plot the numerical and analytical solutions together in a single figure.

4. Compare the analytical and numerical solutions. Briefly discuss the differences observed with different time step sizes.