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Topology of the South African stock market network across the 2008 financial crisis

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HIGHLIGHTS

- There is substantial clustering and homogeneity on the JSE.
- The most connected nodes are in the financial and resources sectors.
- The MST shrank before and during the crisis, and slowly expanded afterwards.
- Network clusters are significantly affected by diversification and credit dynamics.

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ABSTRACT

This study uses the cross-correlations in the daily closing prices of the South African Top 100 companies listed on the JSE All share index (ALSI) from June 2003 to June 2013 to compute minimum spanning tree maps. In addition to the full sample, the analysis also uses three sub-periods to investigate the topological evolution before, during, and after the 2008 financial crisis. The findings show that although there is substantial clustering and homogeneity on the JSE, the most connected nodes are in the financial and resources sectors. The sub-sample results further reveal that the JSE network tree shrank in the run-up to, and during the financial crisis, and slowly expanded afterwards. In addition, the different clusters in the network are connected by various nodes that are significantly affected by diversification and credit market dynamics.

1. Introduction

With the advent of behavioural finance and econophysics, new empirical models that apply a combination of physics and statistical theories have been employed in an attempt to explain the complex nature of stock markets [1]. Network theory is one such approach that offers a unique way to study stock market network patterns and identify evolving trends in a graphical tree topology [2]. A common network approach that has been used to examine the interdependency and dynamic evolution of stock markets is the minimum spanning tree (MST) methodology.

This study applies the minimum spanning tree approach to construct a network map for the Top 100 companies listed on the Johannesburg Stock exchange (JSE) between June 2003 and June 2013. An analysis of the JSE is of interest for three reasons. First, the JSE is the largest equity market in Africa and among the twenty largest stock markets in the world [3]. Second, following the country's financial liberalization in 1995, South Africa has increasingly integrated into the global economy and thus the JSE forms part of global equity market structures. Third, although the JSE includes diverse sectors,

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it is also characterized by a high degree of sectorial and liquidity concentration in resources and financials [4–6], and consequently, the JSE was significantly impacted by financial and trade contagion arising from the 2008 financial crisis [7]. Thus, in addition to examining the interdependence and correlation among the different sectors over the long-run (11 years), the analysis also investigates the possible topological evolution over the periods before the financial crisis (June 2003 to 15th September 2008), during the crisis (15th September 2008 to 1st July 2009),[1] and after the crisis (from 1st July 2009 to the end of the sample).

The remainder of the paper is structured as follows. Section 2 discusses the literature on financial market networks. Section 3 introduces the minimum spanning tree methodology. Section 4 briefly discusses the data. Section 5 presents the empirical results, and Section 5.1 concludes the paper with a summary of the key findings.

2. Literature review

Early work on the minimum spanning tree (MST) methodology was conducted during the 1950s by Kruskal [10] and Prim [11], who formulated the various sub-concepts that were used in deriving trees and the sub-concepts of a graph and nodes. Since then, MSTs have been used to study asset correlations in a number of developed market networks. For example, Bonanno et al. [12] examine the top 100 companies listed in the New York stock exchange (NYSE) between January 1995 and December 1998 and find clusters of stocks in similar sectors have a significant tree topology that filters information in a manner useful for portfolio optimization. Coelho et al. [13] apply the MST to study the top 67 companies listed in the UK FTSE100 equity index between August 1996 and June 2005. In addition to identifying the topological structure, their results also show that the movements and positioning of certain nodes change as the structure is impacted by external market shocks such as currency, banking, or equity market crises. Onnela et al. [14] apply MST to assess dynamic asset trees using 116 companies listed in the S&P 500 between 1982 and 2000, and their results similarly find evolutionary restructuring with shrinkage of tree length at different periods in response to stock market shocks.

With regard to the use of MSTs to examine emerging market networks. Sinha and Pan [15] examine stock market co-movements over the period of 1996 to 2006 in order to assess correlational strength and dominance influence in the National Stock Market of India. The results find that a few companies significantly shape the Indian market network. Gałązka [16] studies the Polish stock market network in order to identify the companies that influence the market topology between January and December 2007. His results find that a few companies significantly influence the entire market, and thus conclude that price fluctuations can significantly influence divergent stocks in the same market.

On a sectorial basis, Zhuang et al. [17] examine 260 companies listed on the Chinese stock market as represented by the Shanghai 300 index between 2004 and 2007. Despite using a relatively short period, they find that topological behaviour based on the MST estimates show strong connections between stock price movements in terms of clusters formed per sector behaviour. Tabak et al. [18] study the Brazilian stock market between 2000 and 2008 in order to identify the relative importance of the sector clusters in the network topology. Their results find significant evidence of long term topological evolution on a sectorial basis. Similarly, Situngkir and Surya [19] find that the financial and basic material sectors tend to dominate the Indonesian stock market network between October and December 2004. Hence, the studies for both the developed and emerging stock markets show that there are nodal companies and sectors that have the dynamic power to shape and influence the topological structures. However, the literature further shows that these nodes and structures change over time as they are impacted by crises.

Among the first to study the effects of crises on MST topologies is Onnela et al. [14] who study the responses to the Black Monday crisis in October 1987. Using asset prices from 116 companies listed on the S&P 500 index over the period of 1982 to 2000, they find that there is tree length shrinkage in the aftermath of the crisis and thus conclude that a tree reconfiguration as a result of such a shrinkage indicates dynamic evolution of the asset tree. Jung et al. [20], study the evolution and internal properties of the Korean equity market over the period of 2001 to 2004 and finds that there is a reconfiguration of company-specific and sector-specific nodes in the stock market network prior to, and after the 1997 Asian financial crisis.

Using MST analysis to examine the co-movement between government bond markets in Europe during and after the sovereign debt crisis in 2009, Gilmore et al. [21] find high cross-correlations, indicating a reorganized evolution. Roy and Sarkar [22] use a MST to study the impact of Lehman Brothers collapse on the network of global indices between 2006 and 2010. Their findings show reconfiguration of the network tree away from the US indices towards Japanese and United Kingdom indices, thus highlighting the US role in spreading the crisis through the global market network. Khashanah and Miao [23] study the structural change in the US market network in response to the 2007 recession. Their results show increased integration on the onset of the recession and considerable change in the significant leading factors, from the US risk index (VIX) prior to recession to the 3-month treasury bill rate during the recession. Dias [24] investigates the internal network restructuring associated with the 2010 sovereign debt crisis in the European Union using daily government bond yield rates between April 2007 and October 2010, and finds divergence after the crisis between strong economies such as Germany, and weak economies such as Portugal. Thus these studies indicate that during financial crises, heightened equity market volatility within or across markets or asset classes can significantly reorganize market topologies.

[1] The crisis period start date of 15th September 2008 has been selected on the basis that this marks the time where crisis contagion significantly impacted African stock markets [8], while the crisis period end date of 1st July 2009 is when the country officially exited the crisis induced recession [9].

However, recent studies have further shown that during crises periods, the interactions between developed and emerging markets also affects the topological dynamics. Sandoval [25] studies the correlations of global financial markets from the Black Monday crash in 1987 through the 2008 banking crisis. The results show that there are higher correlations between emerging and developed markets, but also between emerging markets during crisis periods. Trancoso [26] examines emerging market correlations in the global economic network between 1996 and 2010 and finds convergence between emerging market economies (especially India, Indonesia, and South Korea) and developed economies over time. Although emerging markets are commonly associated with greater volatility during crises, they also tend to react differently during crises in terms of severity of impact and recovery duration. Sensoy and Tabak [27] evaluate the Asian pacific markets between June 1992 and June 2013 and find significant shrinking of the dynamic spanning tree across the financial crisis period. In addition, the results show that stability of the spanning tree decreases when the Hong Kong market is excluded. This is because, being the key financial market in the region, the exclusion of Hong Kong removes key market data thus leading to an unstable network. Hence, these studies show that in addition to market topologies being shaped by the changing influence of specific nodal companies and sectors, the network is also severely affected by crises and contagion.

3. Methodology

This study uses the cross-correlations in the daily closing prices of the South African Top 100 companies by market capitalization (in ZAR) listed on JSE All share index (ALSI) from June 2003 to June 2013 to compute a minimum spanning tree network. In addition, the sample is divided into three sub-periods to investigate the topological evolution before the financial crisis (June 2003 to 15th September 2008), during the crisis (15th September 2008 to 1st July 2009), and after the crisis (from 1st July 2009 to the end of the sample).

The MST methodology applies the logarithmic return of the shares and converts their respective correlation matrix into diagrammatic ultrametric distances. In the resultant MST, each node symbolizes a listed company in the JSE, and their distances can then be analysed to assess their respective strength and length. Although the MST method tends to simplify the data contained in the correlation and distance matrices, which can then be observed as a random map of weakly connected nodes, the methodology is widely used due to its robustness in mapping complex stock market data in a simple graphical tree topology and its ability to assess the topological evolution of network maps [2].

The mechanics of the minimum spanning tree analysis are as follows. If $X_i(t)$ is the share-price of company i ($i = 1, \ldots, N$) at time t; then the log-returns for the share-price after a time interval (Δt) can be calculated as:

$$Y_I\left(t\right) = \ln X_i\left(t\right) - \ln X_i\left(t-1\right). \tag{1}$$

The Pearson correlation coefficient of the return of shares ith and jth is then given as:

$$\rho_{i,j} = \frac{\left\langle Y_i \times Y_j\right\rangle - \left\langle Y_i\right\rangle \left\langle Y_j\right\rangle}{\sqrt{\left(\left\langle Y_i^2\right\rangle - \left\langle Y_i\right\rangle^2\right)\left(\left\langle Y_j^2\right\rangle - \left\langle Y_j\right\rangle^2\right)}} \tag{2}$$

where $\langle Y_i\rangle$ represents the statistical mean of $Y_{i,j}$ for duration of the study (Mantegna and Stanley, 2000). The correlation coefficient $\rho_{i,j}$ denotes the weight allocated to the edge joining from nodes i to j. The correlation band is between -1 and 1 i.e. $-1 \le \rho_{i,j} \le 1$, which then provides the following scenarios:

$\rho_{i,j} = 1$: indicates perfect correlation between share prices i and j
$\rho_{i,j} = 0$: indicates no-correlation between share prices i and j
$\rho_{i,j} = -1$: indicates perfect opposite correlation between share prices i and j.

The correlation matrix for a set of companies ($N \times N$) is then converted to a matrix that captures distance in the tree network, denoted by [28]:

$$d_{ij} = \sqrt{2(1-\rho_{i,j})} \tag{3}$$

where d_{ij} is the metric distance between nodes i and j; and $\rho_{i,j}$ is the cross-correlation in variation between shares i and j. The minimum spanning tree, symbolized as T, is then computed using this metric distance, essentially indicating the joined graph linking nodes N using $N-1$ edges in a way that minimizes the total weight of the edges [14]:

$$T = \sum_{(i,j)\in T} d_{ij}. \tag{4}$$

When computing T, the data metric is changed from $N \times (N-1)/2$ cross-correlations coefficients to $V-1$ isolated edges. After the correlation coefficient of shares ith and jth is identified ($\rho_{i,j}$). Eq. (3) is applied to estimate the metric distance (d_{ij}). The companies studied represent the number of nodes joined with their respective edges and weighing W. The weighted, linked and directionless graph (G) is captured in the form of $G = (N, E, W)$, which is then adapted into the minimum spanning tree T using a Kruskal algorithm [10].

Thereafter, four sub-metrics are used to observe the sectorial properties of the JSE market network: Normalized tree lengths, degree centrality, betweenness centrality, and domination strength. Normalized tree lengths are used to assess the length of the MST across the period of study and how it changes towards and after the financial crisis. The normalized tree length formula is denoted as $L(t)$, such that:

$$L(t) = (1/(N-1)) \times \sum_{(i,j)\in T^t} d_{ij}. \tag{5}$$

Closeness centrality is used to assess the level of power associated with a particular node in a network where higher values indicate more closeness of the node to other nodes. This measure is used to determine the nodes (i.e. companies) that are closer to other companies or sectors in the network and is denoted as:

$$C(k) = 1 \Big/ \sum_{h\in G} d_G(k, h) \tag{6}$$

where $d_G(k, h)$ symbolizes the smallest distance from node k to node h. Betweenness centrality is used to estimate a number of times a node acts as shortest link between two nodes, or clusters. It is used to assess the intermediary role played by certain nodes in transferring information in the network [29]. It is mathematically defined as follows:

$$C_B(k) = \sum_{s\neq k\neq t\in V} \frac{\sigma_{st}(k)}{\sigma_{st}} \tag{7}$$

where σ_{st} is the aggregate number of shortest links. The final metric that is used is domination strength, which investigates the supremacy of specific vertices with respect to significance so as to determine which companies or sectors most significantly influence the network. Domination strength is calculated as follows:

$$\beta(i) = \sum_{j=1}^{n} (\omega(i,j)/\lambda(j)) \tag{8}$$

where $\omega(i, j)$ indicates weight of joints linking node i, while $\lambda(j)$ is the power density of node j defined as:

$$\lambda(j) = \sum_{j=1}^{n} [\omega(i,j)]. \tag{9}$$

Essentially, a weighted metric is used to estimate both the number of closed triplets in the region of a vertex, as well as their aggregate comparative masses with regard to power of the vertices [30]. It is measured in a band between 0 and 1 whereby 1 is the value assigned to the node with maximum strength in a given cluster and 0 for nodes without neighbours around. It is defined as follows:

$$c_i^{\omega} = \left(\frac{1}{s_i(k_i-1)}\right) \times \sum_{j.h} \frac{(\omega_{i.j} + \omega_{i.h})}{2} a_{ij}a_{ih}a_{jh} \tag{10}$$

where k_i denotes degree of node i, s_i is node strength, while a_{ij} is equal to 1 should there be a link joining i and j. The crucial estimate for strength of the particular node is based on [31]:

$$s_i = \sum_{j\in\gamma(i)} \omega_{ij}. \tag{11}$$

Vertex strength is taken as ordinary simplification of the connectivity since it incorporates such connectivity info with the significance of the weights of its edges. This measure is used to assess nodes (i.e. companies and sectors) that play a crucial role as a hub in the network.

4. Data

The analysis makes use of the closing prices of the Top 100 companies listed in the JSE All-Share Index (ALSI) from June 2003 to June 2013. The time series price data has been gathered from *I-Net Bridge*. Prior to running the analysis, the selected companies were arranged in their respective sectors as classified by the JSE. These include Financials, Industrials, Consumer Services, Consumer Goods, Basic Materials, Health Care, Technology, Oils and Gas Producers and Telecommunication. The analysis of the data makes use of two different software tools. The raw time series data was downloaded from the database and stored in *Microsoft Excel* files. These files were then exported to *R-studio* software for application of the Kruskal algorithm [10] and the generation of the minimum spanning tree network diagrams.

5. Findings

This section presents the findings of the minimum spanning tree analysis conducted on the Top 100 companies by market capitalization on the JSE ALSI's daily closing prices between June 2003 and June 2013 measured by logarithmic returns. The analysis and discussion in this section therefore focuses on the resultant correlation and distance matrices followed by discussion around the minimum spanning tree structure, as well as the robustness network measures.

Table 1
Table of summary observations across pre-crisis, crisis, post-crisis and overall datasets.

	Correlation coefficient			Distance		
	Average	Minimum	Maximum	Average	Minimum	Maximum
Pre-crisis	0.134	−0.061	0.794	1.307	0.642	1.456
Crisis	0.193	−0.237	0.857	1.258	0.534	1.573
Post-crisis	0.140	−0.093	0.814	1.301	0.610	1.478
Overall	0.145	−0.062	0.818	1.298	0.604	1.457

Note that the maximum correlation coefficient and minimum distance values include the matrix loadings of the nodal pairings of the companies with themselves (e.g. AGL with AGL, etc.).

5.1. Correlation coefficients and distance matrices

The full sample investigation reveals a matrix of Pearson correlation coefficients between −0.126 and 1, and distance (d_{ij}) values between 0 and 1.456, which are marginally higher than the 1.38 average for the Brazilian market observed by Tabak et al. [18], and thus indicate greater clustering and homogeneity on the JSE.[2]

To assess the changes before, during and after the financial crisis, the dataset was then divided into three sub-sets: A pre-crisis sub-set running from the beginning of June 2003 to 15th September 2008; a crisis period sub-set from 15th September 2008 to 1st July 2009; and a post-crisis sub-set, running from 1st July 2009 to June 3rd 2013. The changes in the correlation coefficients and distance variables across the JSE companies in the three datasets are summarized in Table 1.[3] The results show that the pre-crisis period has lower average correlations, thus indicating relatively weaker clusters. In contrast, the crisis period correlations are higher by 44% compared to pre-crisis period, and then decrease to an average of 0.14 after the crisis, slightly higher than the average pre-crisis correlation of 0.134. The highest correlation is observed during the crisis period, between Anglo American PLC (AGL) and BHP Billiton PLC (BIL) with a correlation coefficient of 0.857 and a resultant shortest distance metric of 0.534. This pair also has the highest correlation of 0.814 in the post crisis period.

The minimum correlation pair is similar before and after the crisis, consisting of Resilient Property Fund (RES) and Bell Equipment Limited (BEL) at −0.06 and consequent maximum distance of 1.457. The highest correlated sector in the overall period is the basic material sector, with the mining companies of BHP Billion (BIL) and Anglo American PLC (AGL) once again having a 0.818 correlation coefficient with a resultant low distance metric of 0.604. This result could be explained by both companies having primary listing outside South Africa (BHP in Australia and AGL in London) and being engaged in mining activities with core selling prices being determined on global markets. However, given South Africa's economic dependence on mining, it is not unexpected that these two multinational mining companies have a high correlation coefficient across the samples.[4]

5.2. Dynamic evolution of correlations and distances

In order to study the consistency and dynamic evolution of the correlations and distances among the JSE Top 100, the datasets were divided up into equal windows (W) of 250 trading days across the period. The correlations and distance metrics are then examined in each window[5] based on the approach of Situngkir and Surya [19]. Table 2 shows the 11 windows with their starting and end period, and their respective mean correlation and distances across the timeline of the study. The results indicate that the average correlation peaks at W4, two window periods before the crisis; and remains high throughout the crisis period before gradually decreasing towards former levels at around W7, which was also the onset of recovery in South Africa. By the end of the sample period (W11), the average correlation is twice as much compared to the beginning period. This indicates that even though the average tree distance returns to pre-crisis levels, the reconfiguration of the tree after the crisis results in an increased correlation, which is similar to the findings for the UK [13] and for the S&P 500 [14].

5.3. Ultrametric space and minimum spanning tree results

Using Eq. (4), the Kruskal algorithm is used to transform the distance matrix into ultrametric space with total minimum weights represented as a minimum spanning tree. Each node represents a company, with a total of 100 companies used in

2 The data was divided in half, and assess any significant changes in the variables. Only minor differences were observed in terms of correlation coefficients and distances signifying presence of similar attributes between full dataset as well as divided dataset.

3 These are the cross-correlations between each pair of nodes in the study. It is only used to create a distance matrix, hence not ideal to be extracted and aggregated per sector. Therefore, the explanation that follows is based on the underlying companies and not sectors.

4 Similar justification was given by Sandoval [32] when describing the position of Petrobras Company in the Brazilian stock market. Petrobras exhibited fluctuations and correlations dependent on the internationally sold commodity (Oil) as opposed to internal market factors.

5 The final window had only 113 days data points.

Table 2
Summary of the dynamic evolution of mean correlation and distances.

	Start period	End period	Average distance	Average correlation
W1	2003-05-30	2004-05-13	1.362	0.061
W2	2004-05-14	2005-04-28	1.358	0.066
W3	2005-04-29	2006-04-13	1.316	0.121
W4	2006-04-14	2007-03-29	1.243	0.214
W5	2007-03-30	2008-03-13	1.271	0.180
W6	2008-03-14	2009-02-26	1.260	0.190
W7	2009-02-27	2010-02-11	1.310	0.129
W8	2010-02-12	2011-01-27	1.285	0.158
W9	2011-01-28	2012-01-12	1.266	0.180
W10	2012-01-13	2012-12-27	1.326	0.108
W11	2012-12-28	2013-06-03	1.310	0.127

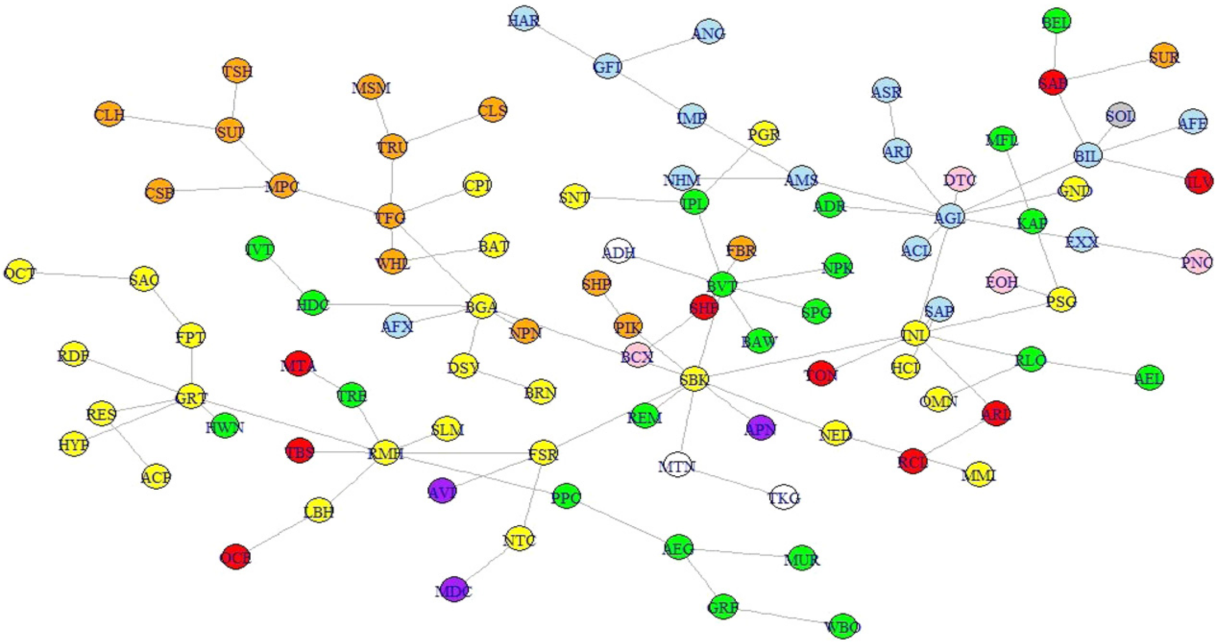

Fig. 1. Minimum spanning tree map of the JSE (June 2003–June 2013). Colour coding: Financials in yellow, basic materials in light blue, oil and gas in grey, industrials in green, consumer goods in red, health care in purple, consumer services in orange, telecommunication in white, and technology in pink. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the study. Once again, the data was separated into three sub-periods (pre-crisis, crisis period and post-crisis) to study the evolving connectivity, positions and resultant influence of the companies (nodes).

The overall JSE network across the entire period under investigation is presented in Fig. 1. The results show that there are three key clusters: the consumer services sector (orange) in the top left, the basic materials sector (light blue) spread across the top centre and to the right, and the financial sector (yellow) spread across the bottom left, with insurers and banks in the middle. Industrial (green) and consumer goods (red) are also spread throughout the map indicating homogeneity in the sectors. In terms of connectivity,[6] there are seven key nodes, which are Anglo American (AGL, 9), Standard Bank (SBK, 9), Investec Plc. (INL, 8), Rand Merchant Bank (RMH, 7), Bidvest Group (BVT, 7), Barclays Group Africa (BGA, 6), and Growth Point (GRT, 6). Hence, the finding that the most connected nodes in the MST are in the financial and resources sectors reflects the JSE's high concentration in these sectors. However, the existence of a significant but limited number of key nodes in the MST is not unusual for an emerging market as similar dynamics have been observed in Brazil [18], China [17], India [15], and Indonesia [19].

Fig. 2 shows the pre-crisis minimum spanning tree map of the JSE network. As can be seen, there are seven principle nodes: BHP Billiton (BIL, 11), Standard Bank Group (SBK, 9), The Foschini Group (TFG, 8), Growth Point (GRT, 7), FirstRand (FSR, 6), Rand Merchant Bank (RMH, 6), and Investec PLC (INL, 6). Thus the MST highlights the strong reliance on banking, mining, property and consumer services in the pre-crisis South African economy.

6 The number beside each stock identifier indicates the number of connections per node.

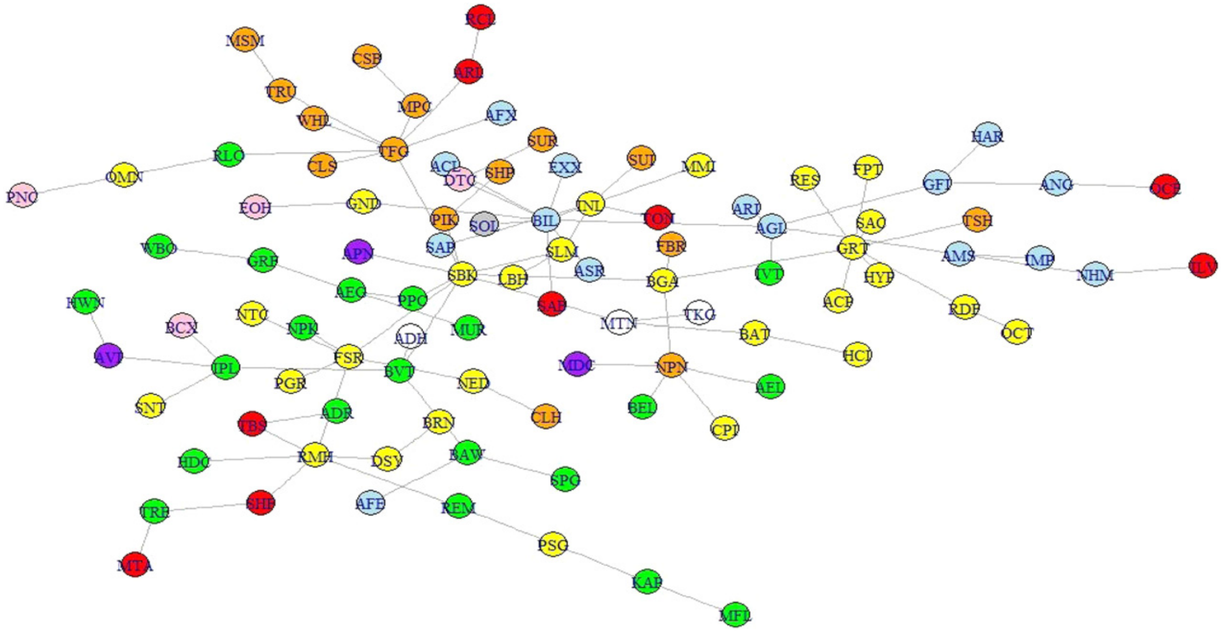

Fig. 2. Pre-crisis minimum spanning tree map of the JSE network (June 2003–September 2008). Colour coding: Financials in yellow, basic materials in light blue, oil and gas in grey, industrials in green, consumer goods in red, health care in purple, consumer services in orange, telecommunication in white, and technology in pink. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The central role played by the financial sector can be seen by the scattering of financial companies throughout the map, thus suggesting that the financial crisis spread to the other sectors through the financial sector. The orange cluster in the top left corner shows that most of the consumer service sector, including retailers such as Massmart (MSM), Truworths (TRU), Woolworths (WHL), and Mr. Price (MPC) connect to other parts of the network through The Foschini Group (TFG). A second sectorial cluster is the property funds (yellow) on the top right, while the consumer goods companies (red) are scattered on the periphery. Hence, the pre-crisis map shows that the consumer goods sector was less connected and thus less exposed to shocks in the network.

The crisis period minimum spanning tree map of the JSE network is presented in Fig. 3 and shows that the two most connected nodes are Anglo American (AGL, 7) and Barclays Group Africa (BGA, 7), while the nodes of Fountainhead Property Trust (FPT), The Foschini Group (TFG), Truworths (TRU), Standard Bank (SBK), and Exxaro Resources Limited (EXX) have five connections each. Thus the crisis MST shows that there has been a weakening in the number of connections compared to the pre-crisis period. The results further reveal that the financial sector nodes (yellow) split into two groups, with big banks and insurance companies on the right side while the residential companies are clustered on the left. This split possibly reflects the different reactions to the risks posed by the crisis as the residential cluster separates from the main banking cluster. In addition, the construction companies comprising Murray and Roberts (MUR), Aveng (AEG), Group 5 (GRP), and Wilson-Bailey (WBO) connect to the centre of the crisis period map through FirstRand (FSR) instead of to Standard Bank Group (SBK) as in the pre-crisis map (Fig. 2). Similarly, the consumer services cluster (orange) connects to the centre of the map through Barclays Group Africa (BGA) during the crisis compared to Standard Bank Group (SBK) in the pre-crisis map. Hence, the crisis map indicates that companies with similar risk profiles tended to be affected in a similar manner.

The results also show that there is no longer a direct link between the retail sector and the Standard Bank Group (SBK) node. In the pre-crisis map there is a direct link from The Foschini Group (TFG) to the Standard Bank Group (SBK), but during the crisis, the retail sector connects to the Standard Bank Group (SBK) node through Barclays Group Africa (BGA) instead. This change could reflect the increased foreign ownership of ABSA from Barclays Plc, essentially making Barclays Group Africa (BGA, ABSA at the time) the bank with the highest foreign ownership after 2009 [33]. The implication of this change translates into ABSA's (BGA) increased ability to issue consumer credit, which spurred credit retail sales, and explains the role of Barclays Group Africa (BGA) in linking the consumer retail sector with the banking cluster dominated by Standard Bank Group (SBK).

The post-crisis minimum spanning tree map of the JSE network is presented in Fig. 4. As can be seen the two most significant nodes are Rand Merchant Bank (RMH, 9) and Anglo American (AGL, 7), followed by Investec Plc (INL, 6), Sanlam (SLM, 6), Standard Bank (SBK, 6), and BHP Billiton Plc (BIL, 5). Thus, although three of the pre-crisis nodes (TFG, GRT, and FSR) are no longer as connected after the crisis, the connectivity of the remaining nodes (RMH, AGL, INL, and SBK) has generally returned to their pre-crisis levels. The one exception however, is BHP Billiton Plc (BIL), which dropped from 11 connections before the crisis to just 5 after the crisis with Anglo American Plc. (AGL) replacing BHP Billiton (BIL) as the central node, possibly reflecting the changing degree of diversification among these companies. Prior to the crisis, BHP Billiton was

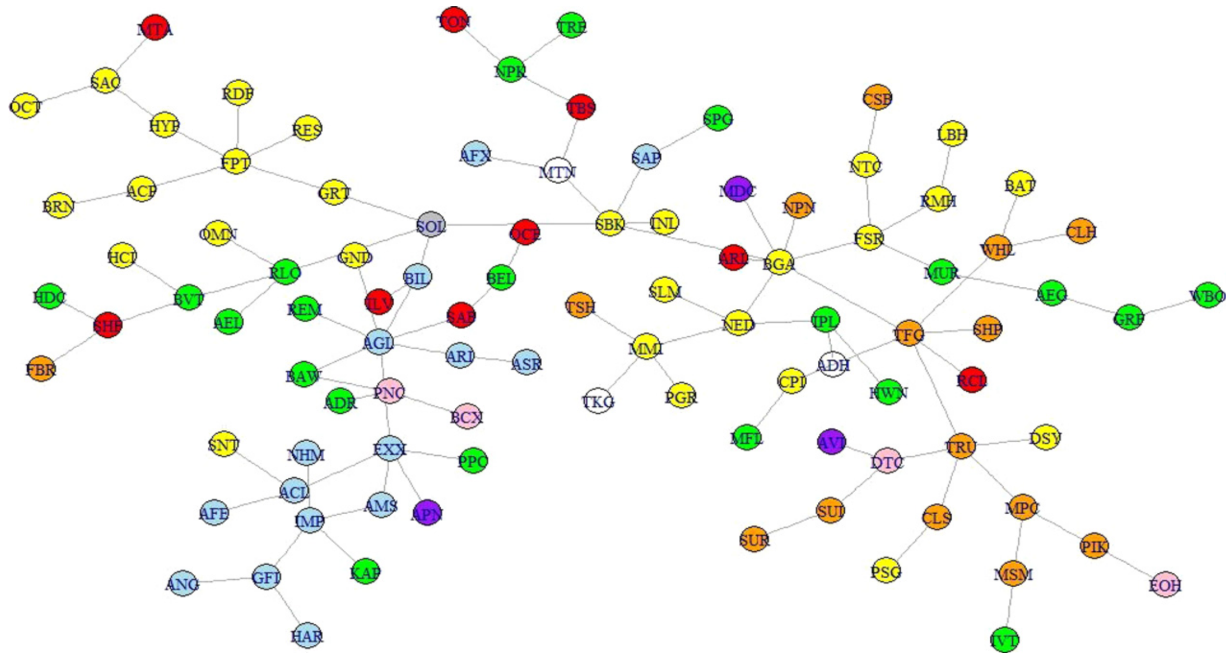

Fig. 3. Crisis period minimum spanning tree map of the JSE network (15th September 2008 to 1st July 2009). Colour coding: Financials in yellow, basic materials in light blue, oil and gas in grey, industrials in green, consumer goods in red, health care in purple, consumer services in orange, telecommunication in white, and technology in pink. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

more diversified than Anglo American Plc., but after the crisis, Anglo American Plc. became increasingly diversified with investment in companies such as Kumba iron ore and Anglo-Platinum (AMS), Mondi and De Beers. Hence, Anglo American Plc. (AGL) has steadily increased its influence in the post-crisis network compared to BHP Billiton (BIL) as it has become increasingly diversified.

The results further show that as is to be expected, following the financial crisis, the contagion effect is observed by a re-arrangement and re-configuration of many nodes in the map. The financial sector companies (yellow), the resource sector (blue), as well as the consumer services sector (orange) show further consolidation in their respective clusters, possibly to accommodate and withstand the impact of the crisis. This signifies a reduced connectivity of these sectors' nodes from the rest of the network, an isolation that suggests movement away from post-shock information flows. Moreover, during periods of extreme volatility or market anomalies such as a financial crisis, defencive stocks are expected to perform better than their peers due to their non-cyclicality [34], which is verified by the increased influence and number of the defencive counters (red nodes) being among the most centrally spread but linking with highly connected nodes in the post-crisis network compared to their positioning on the periphery on the pre-crisis map. Generally, the peripheral positioning of these nodes indicates reduced connectivity in the map, which could be translated as reduced links with the riskier sectors.

A further change can be found in the readjustment of the retail cluster and its connection to the centre of the map. Prior to the crisis, the retail cluster connected to the rest of the network through The Foschini Group (TFG) and Standard Bank Group (SBK). This link changed from Standard Bank Group (SBK) to Barclays Group Africa (BGA) during the crisis period. However, in the post-crisis map (Fig. 4), the Truworths (TRU) node becomes the central node instead of The Foschini Group (TFG), restoring the link with the financial cluster through Standard Bank Group (SBK). This change possibly reflects Truworth's over-reliance on credit sales compared to the Foschini Group (70% and 30% respectively according to Thomas [35]). Hence, the change in nodes can be related to an increase in credit impairment suffered by Barclays Group Africa as opposed to Standard Bank Group's stability.

5.4. MST sub-metrics

To study the dynamic evolution of the minimum spanning tree lengths, the data used to produce the normalized tree lengths is once again divided into 11 windows covering the period from June 2003 to June 2013. The results in Table 3 suggest that the tree lengths generally shrink in the run-up to the financial crisis period in W6, although the minimum tree length once again occurs in W4.[7] The increased correlation during the crisis tends to pull the nodes together hence lowering their distances and thus shrinking the MST (as observed from 136.185 in first window to 126.106 in the sixth window). The

[7] 21-day windows were used in this case for a less smoothed curve.

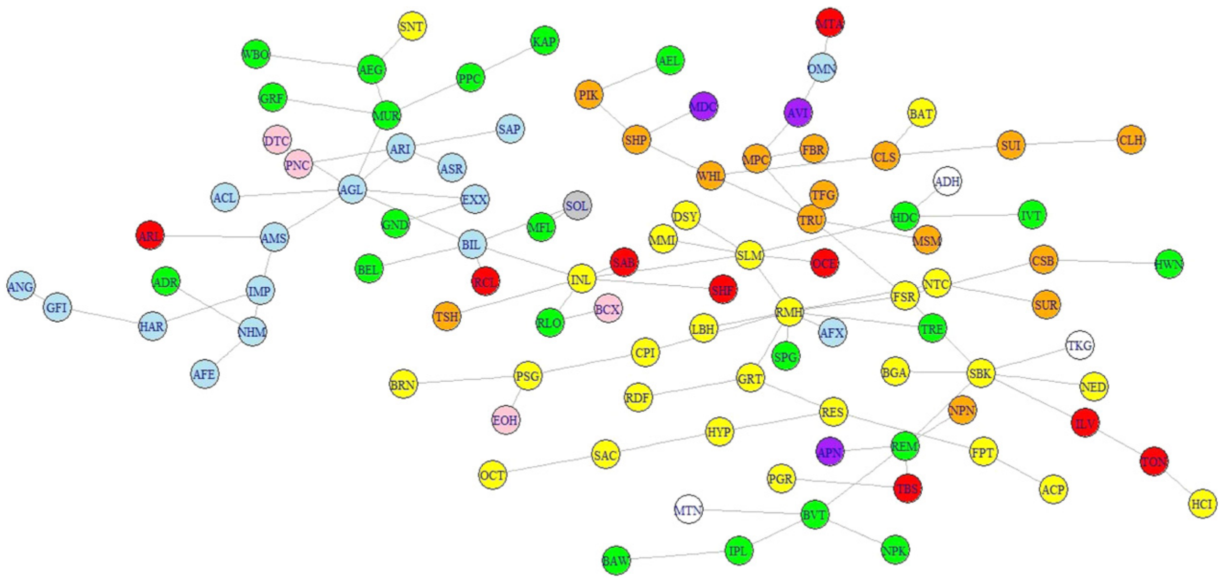

Fig. 4. Post-crisis minimum spanning tree map of the JSE network (1st July 2009 to June 3rd 2013). Colour coding: Financials in yellow, basic materials in light blue, oil and gas in grey, industrials in green, consumer goods in red, health care in purple, consumer services in orange, telecommunication in white, and technology in pink. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3
Summary of the dynamic evolution of tree length.

Window	Start period	End period	Total distance	Normalized tree length
W1	2003-05-30	2004-05-13	13 346.28	136.185
W2	2004-05-14	2005-04-28	13 309.74	135.812
W3	2005-04-29	2006-04-13	12 897.68	131.607
W4	2006-04-14	2007-03-29	12 182.71	124.312
W5	2007-03-30	2008-03-13	12 460.32	127.145
W6	2008-03-14	2009-02-26	12 358.53	126.106
W7	2009-02-27	2010-02-11	12 835.44	130.973
W8	2010-02-12	2011-01-27	12 595.34	128.523
W9	2011-01-28	2012-01-12	12 406.08	126.592
W10	2012-01-13	2012-12-27	12 998.31	132.635
W11	2012-12-28	2013-06-03	12 839.12	131.010

results further show that the tree lengths then increase after the crisis (from 126.106 in W6 to 130.973 in W7), indicating a reduction in the correlation coefficients as the economy recovers.

The analysis next assesses the sectorial degree centrality of the MST. As highlighted by Barthelemy et al. [31], the strength of a node can be studied to identify key hubs in the network by observing the number of connections through each node. Scale-free networks often exhibit a distribution that follows a power law identified by having many nodes with low degrees and a few nodes with degrees above average. These few nodes with a high degree are often regarded as hubs in the networks. As can be seen from Figs. 5(a)–(d), the JSE network appears to exhibit this distribution across the three periods.[8]

The figures show that a majority of the Top 100 companies have a small degree of centrality of less than two average node degrees with only a few having very high degrees above five across all the periods. Also, the maximum number of only eight is low during the crisis period compared to the maximum of ten before and after the crisis. This further indicates a reconfiguration of the JSE network map to accommodate a period of extreme volatility. Among the companies that consistently exhibit average high node degrees (thus indicating a key role in a hub) are Standard Bank Group (SBK) with nine degrees, Anglo American Plc (AGL) with eight degrees, and Investec Plc (INL) with seven degrees, which is not unexpected given the JSE's concentration in mining and financial services.

The degree centrality depicted in Fig. 6 further shows that prior to the financial crisis, the financial sector nodes had an average of nearly three incident edges, followed closely by the basic material sector with an average of two. However, during the crisis, the average number of incident edges per financial node decreases while that of basic material sector nodes increases. This indicates that there has been a change in hub nodes away from the financial sector to the basic material sector during the crisis as the JSE network reduced connectivity to the crisis-prone financial sector. The other sectors that display a

8 It should be noted however that there is insufficient data in the plots to definitively conclude that there is a scale-free distribution.

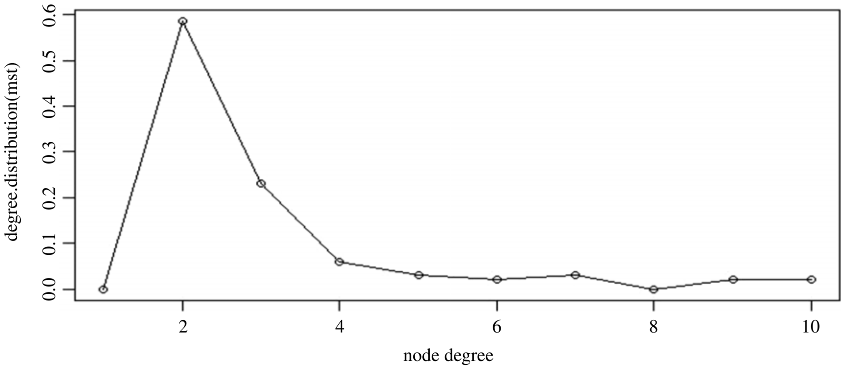

Fig. 5(a). Pre-crisis degree distribution in the JSE network.

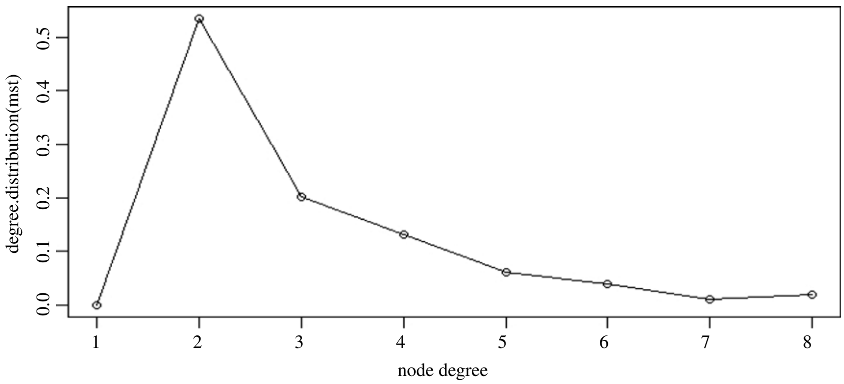

Fig. 5(b). Crisis period degree distribution in the JSE network.

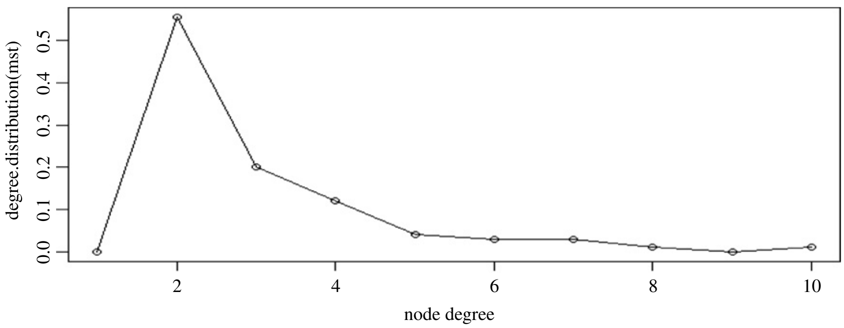

Fig. 5(c). Post-crisis degree distribution in the JSE network.

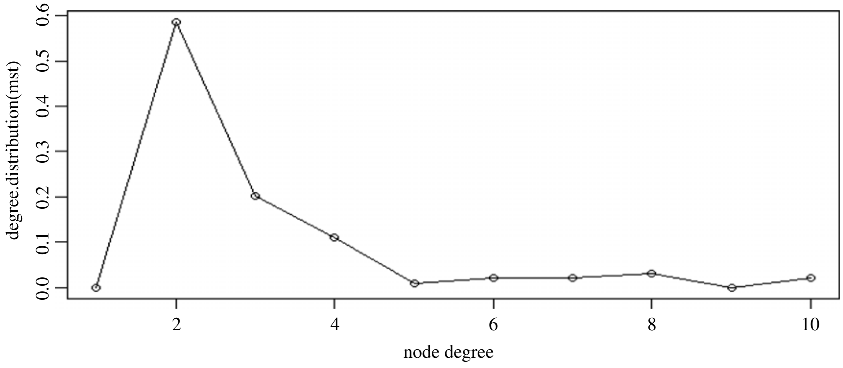

Fig. 5(d). Overall degree distribution in the JSE network: June 2003–June 2013.

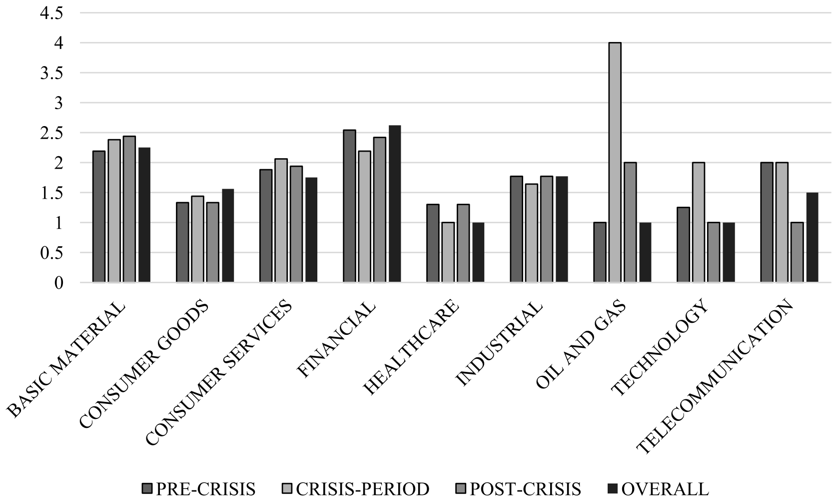

Fig. 6. Degree centrality for the JSE sector network.

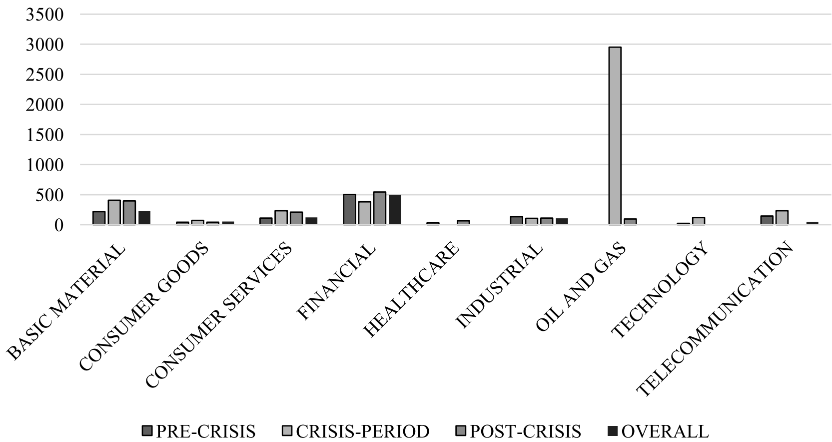

Fig. 7. Betweenness centrality for JSE sector network.

temporary increase in connectivity during the crisis are consumer goods and services, and technology; while in contrast, the health care and industrial sectors show reduced connectivity. The crisis-period spike in number of edge incidents though the oil and gas sector can be explained by the increased primary economic activities during and after the crisis. A comparison between the crisis and post-crisis centrality metrics further shows that there is a recovery in connectivity in the basic material, financial, healthcare and industrial sectors but ongoing weakness in the consumer goods and services, oil and gas, technology, and telecommunications sectors.

With regard to sectorial betweenness centrality, Fig. 7 shows that although the financial and basic materials sectors are most significant, the financial sector has the highest number of short routes, indicating a strong intermediary role. Similarly, the significance of the betweenness centrality of the basic materials sector highlights the crucial role of mining in the South African economy. This is indicated by the increase in average intermediary routes from pre-crisis to crisis period followed by a slight decrease after the crisis. The relatively high betweenness centrality of consumer services is possibly a reflection of the credit market cycle and thus shows the post-crisis decline due to heightened credit impairment. The spike in the intermediary role of the oil and gas sector is similar to the role of multinational oil companies in the Brazilian market network as described by Sandoval [32]. Sasol in South Africa, like Petrobras in Brazil, produces commodities that are priced in the international market and thus shows fewer links and a lower degree of influence in the domestic market as prices and economic activity are driven by global GDP [36]. Hence, the post-crisis spike in the oil and gas centrality metrics reflects the increased mediational influence of the sector during the global recovery phase.

Domination strength of the graph or node in the network describes the interconnectedness between the nodes/clusters and thus identifies the sectors that exhibit significant strength in the network. High values of graph strengths are associated with high exposure of the respective sectors to market shocks [37]. Fig. 8 depicts the average sector strength in the three periods studied as well as overall period. The results show a particularly high pre-crisis strength for the financial sector, which once again explains the sector's susceptibility to external shocks prior to the crisis, and its role in transmitting shocks

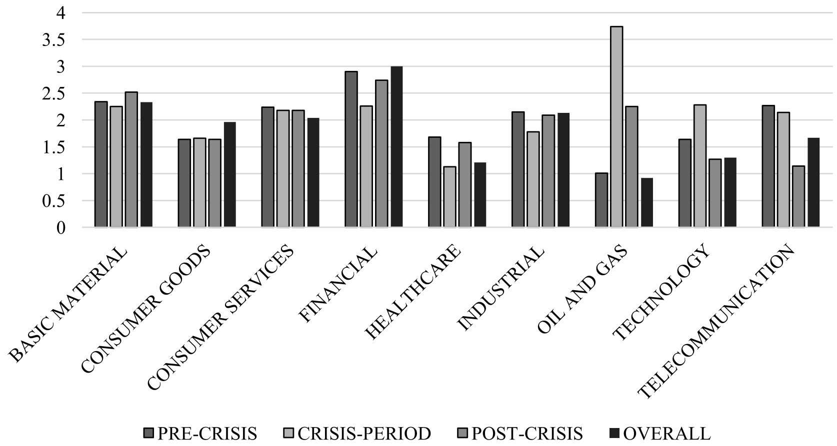

Fig. 8. Domination strength in the JSE network.

to the rest of the network. During the crisis period, the basic materials, financial, healthcare, and industrial sectors show a reduced strength, while the oil and gas, and technology sectors show a significantly increased strength. In the post-crisis period, there is a recovery in the basic materials, financial, healthcare, and industrial sectors but a decline in the strength of the oil and gas, technology, and telecommunications sectors. Hence, only the consumer goods and services sector remains relatively unaffected, while only the basic material sector shows an increase in strength after the crisis.

6. Conclusion

This study used graph theory concepts to map the network of JSE Top 100 companies. Cross correlations were transformed into ultrametric spaces with respective distances used to construct the minimum spanning tree of the network across three samples: a pre-crisis sub-sample covering the period of 2nd June 2003 to 15th September 2008; a crisis period sub-sample covering the period of 15th September 2008 to 1st July 2009; a post-crisis sub-sample covering the period of 1st to 3rd June 2013; and a full sample covering the period of 2nd June 2003 to 3rd June 2013 period.

Overall, the full sample investigation reveals that there is substantial clustering and homogeneity on the JSE. However, the sub-sample results show that there was relatively weak clustering in the pre-crisis period, followed by a substantial increase in the correlation coefficients during the crisis, and then a post-crisis decrease back to correlation coefficients only slightly higher than the pre-crisis levels. On a more detailed basis, the results of the full sample period find that there are three significant clusters in the consumer services, basic materials, and financial sectors connected, which connect to the rest of the network through the seven key nodes of Anglo American (AGL), Standard Bank (SBK), Investec Plc. (INL), Rand Merchant Bank (RMH), Bidvest Group (BVT), Barclays Group Africa (BGA), and Growth Point (GRT, 6).

With regard to the sub-sample results, the pre-crisis MST shows that the financial companies are scattered throughout the map, implying that the financial crisis spread to the other sectors through the financial sector. In the case of the crisis topology, the results show that the two most connected nodes are Anglo American (AGL) and Barclays Group Africa (BGA) while the remaining nodes have five or less connections, indicating a topological weakening compared to the pre-crisis period. The results further reveal that the financial sector splits into two groups comprising banks and insurance companies in one and property companies in the other, possibly reflecting the different reactions to the risks posed by the crisis. The post-crisis MST reveals that the two most significant nodes are Rand Merchant Bank (RMH) and Anglo American (AGL), followed by Investec Plc (INL), Sanlam (SLM), Standard Bank (SBK), and BHP Billiton Plc (BIL); and although three of the pre-crisis nodes (TFG, GRT, and FSR) are no longer as connected after the crisis, the connectivity of the remaining nodes (RMH, AGL, INL, and SBK) generally returned to their pre-crisis levels.

In addition to producing the MST maps, the results were further assessed using degree centrality, sectorial betweenness centrality, and domination strength sub-metrics. Overall, the sub-metrics show that the financial and basic material sectors are most significant but there is a crisis-period spike in the oil and gas sector. The degree centrality metrics find that the majority of the companies on the JSE have less than 2 average node degrees with only a few having degrees above five across all the periods. Before and after the financial crisis, the maximum number of node degrees is ten, while during the crisis period the maximum decreases to eight, indicating a reconfiguration of the network to accommodate a period of extreme volatility. The sectorial betweenness centrality metrics further show that there is a relatively high centrality in the consumer services sector, possibly reflecting the post-crisis credit impairment effects. Finally, the domination strength metrics indicate that only the consumer goods and services sector remains relatively unaffected across the crisis sub-periods, while only the basic material sector shows an increase in strength after the crisis.

Thus in summary, the findings show that although there is substantial clustering and homogeneity on the JSE, the most connected nodes are in the financial and resources sectors. However, the findings further show that the JSE network tree shrank in the run-up to, and during the financial crisis, and slowly expanded afterwards. In addition, the different clusters in the network before, during and after the financial crisis are connected by various nodes that are significantly affected by diversification and credit market dynamics. Hence, these results indicate that the JSE network remains structurally vulnerable to global business and commodity cycle fluctuations, as well as to domestic credit and consumer cycles.

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