Econometrics II

Notes - Midterm

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September 19, 2020

Chapter 1

Introduction

1.1 Motivation

This course will be dedicated to *time series analysis.* Informally, a *time series* is any type of data collected over time – or, more formally, it is the realization of a stochastic process indexed in time. We usually denote the time series as follows:

$$y_1, ..., y_T; \quad \{y_t\}_{i=1}^T; \quad \{y_t\}_t$$

Time series analysis is useful for a number of different applications:

- **Forecasting.**
  - Uni and multivariate models
  - **ARIMA** models: mean and confidence interval forecasting
  - **ARCH** models: variance forecasting – especially useful in finance for volatility and risk
- **Dynamics.** Evaluate the impact of one variable in another over time.
  - Multivariate models including VAR, ECM
  - Contemporaneous lagged structural relations

It is important to address a first and simple question. **Why time series are different from other data?** The answer is also simple but incredibly relevant: *time series observations are not serially independent!*

$$Y_t \not\perp Y_{t-j}$$

In fact, they don't even have to be identically distributed:

$$F_{Y_t} \neq F_{Y_{t-j}}$$

This means that the essential *iid* hypothesis for traditional Econometrics *does not hold.* This means that we'll have to make some adjustments to our methods. That is the task of time series analysis.

1.2 Statistics with dependence

Let's begin with a proper definition of a time series.

1.2.1 Definition of a time series

Suppose that we have a probability space $(\Omega, S, \mathbb{P})$. Ω is the sample space; S is the set of all events; $\mathbb{P}$ is a measure of probability $\mathbb{P} : S \to [0, 1]$. From this, we define a random variable $Y : \Omega \to \mathbb{R}$. A realization of this r.v. is denoted by $y = Y(\omega)$ with fixed ω.

From this, we can define multiple random variables in the same sample space, indexed by integers:

$$Y = \{..., Y_{t-2}, Y_{t-1}, Y_t, ...\}$$

This is equivalent to writing:

$$Y : \Omega x \mathbb{Z} \to \mathbb{R}$$

We now arrive at our formal definition of a time series: $\{Y_t, t \in \mathbb{Z}\}$ is a time-indexed stochastic process.

- $Y(\cdot, t) : \Omega \to \mathbb{R}$ is a r.v. for fixed t.
- $Y(\omega, \cdot) : \mathbb{Z} \to \mathbb{R}$ is a *sequence of real numbers* for a fixed ω. In other words, this represents the *observed time series.*
- For fixed t, ω, $Y(\omega, t) \in \mathbb{R}$.

1.2.2 Unconditional expectation

An important concept to make clear here is *unconditional expectation.* With fixed t,

$$\mathbb{E}(Y_t) = \int_{-\infty}^{\infty} x f_{Y_t}(x) dx$$

Note the Y_t subscript on the probability density function f_{Y_t}. This means that $\mathbb{E}(Y_t)$ is not calculated with the values assumed by Y_{t-1}, Y_{t+1}. This raises an important problem: *how would you be able to estimate* $\mathbb{E}(Y_t)$*?* Note that we only observe $Y_t = y_t$, i.e., one realization of the r.v.

1.2.3 Statistical dependence

For any random variables X, Y, we can define multiple measures of dependency:

- **Linear:** $Cov(X, Y) \equiv \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y)$
- **Quadratic:** $Cov(X^2, Y^2)$
- **General:** $Cov(f(X), g(Y))$. This is a measure of covariance between two general functional forms of X and Y.

With this general definition, we arrive at an equivalent definition for independent random variables:

- $F_{X,Y}(x, y) = F_X(x) * F_Y(y)$, i.e., joint pdf is equal to the product of the marginal pdfs.
- $Cov(f(X), g(Y)) = 0$ for every pair of bounded functions f, g.

From this, we now define the *autocovariance and autocorrelation functions.*

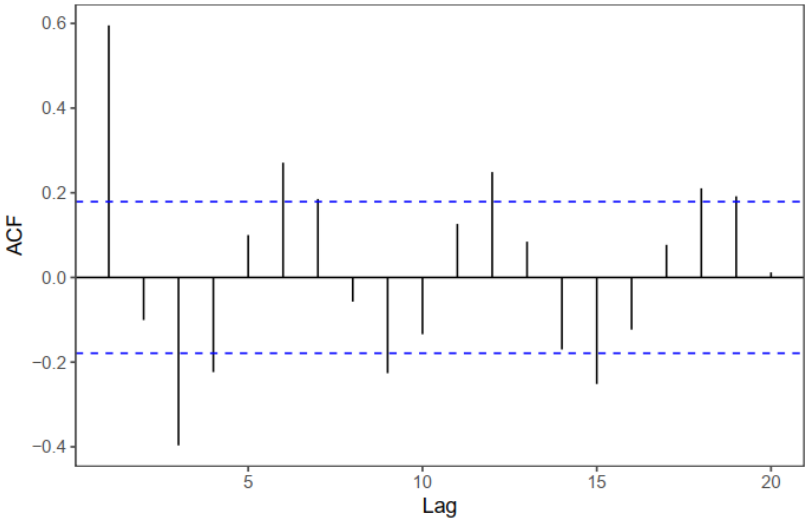
Definition 1.2.1. $\gamma_{j,t} := Cov(Y_t, Y_{t-j})$ *is the autocovariance function for a given time series* $\{Y_t, t \in \mathbb{Z}\}$.

Definition 1.2.2. $\rho_{j,t} := \frac{\gamma_{j,t}}{\sqrt{\gamma_{0,t}\gamma_{0,t-j}}}$ *is the autocorrelation function for a given time series* $\{Y_t, t \in \mathbb{Z}\}$.

Note that, if *iid* holds:

$$\gamma_{j,t} = \begin{cases} 0 & j \neq 0, \forall t \\ Var(Y) & otherwise \end{cases}$$

This is an example of an autocorrelation function.



1.2.4 Asymptotic theory with dependence

Some form of asymptotic theory is needed to enable *any kind of statistical analysis.*