Econometrics II - Problem 9

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Ref: IPEA Discussion Paper 230, Lecture 8 - Notes, Enders 5.1-5.3.

In this problem, we'll apply ARDL modelling to the Phillips Curve. Namely, we'll focus on two hypothesis for the curve:

- **Inertial inflation.** There's a backward-looking component to any Phillips curve.
- **Rational expectations.** The curve is forward-looking.

From these hypothesis, it is also possible to test a hybrid approach that incorporates both a backward and a forward component.

ARDL models

Consider the following model:

$$Y_t = \alpha + \sum_{j=0}^{q} \theta_j X_{t-j} + \varepsilon_t$$

This is the *distributed lag* model for q lags of X. Note that Y_t has serial correlation because of its relation with X_{t-j}.

An *Autoregressive distributed lag* model is a combination of an $AR(p)$ with a $DL(q)$:

$$Y_t = \alpha + \sum_{i=1}^{p} \phi_i Y_{t-i} + \sum_{j=0}^{q} \theta_j X_{t-j} + \varepsilon_t$$

Or, with lag operator notation:

$$\Phi(L)Y_t = \alpha + \Theta(L)X_t + \varepsilon_t$$

We can further define $\Psi := \Phi^{-1}(L)\Theta(L) = \psi_0 + \psi_1 L + ...$ This implies that we can rewrite the ARDL(p,q) model as:

$$Y_t = \Phi^{-1}(1)\alpha + \Psi(L)X_t + \Phi^{-1}(L)\varepsilon_t$$

Also note that we can expand this definition by letting $X_{t,s}$ be a matrix of s processes.

Phillips Curve and its developments

Phillips famously postulated a negative relation – i.e., a trade-off – between inflation rate and unemployment. We can define this curve as follows:

$$\pi_t = \alpha - \gamma u_t,$$

where π_t is inflation, u_t is unemployment and $\gamma > 0$. Note that this is a *static model.* Furthermore, note that this formulation proposes a *long term trade-off* between these variables.

Friedman and Phelps disputed the claims of these long run effects by including *expectations* in the model. The monetarist Phillips curve asserted that inflation expectations would be formed as a function of *past inflation rates* – i.e., under the assumption of adaptative expectations. This model has the following representation:

$$\pi_t = \mathbb{E}_{t-1}(\pi_t) - \gamma(u_t - \bar{u}),$$

where $\bar{u}$ represents the natural rate of unemployment (NAIRU). This is called the Accelerationist Phillips Curve (APC), given its *backward-looking* – hence, inertial – nature. Now, the model is *dynamic*, and can be translated to the ARDL framework with $Y_t := \pi_t, X_t := \mathbb{E}_{t-1}(\pi_t)$.

The rational expectations revolution of Lucas and Sargent, which implied fundamental critiques to the Keynesian system, prompted a revaluation of its framework, including the Phillips Curve. The New Keynesian Phillips Curve (NKPC) incorporates *rational expectations* in the model:

$$\pi_t = \beta\mathbb{E}_t(\pi_{t+1}) + \gamma x_t,$$

where x_t is a measure of output gap. Note that this model is essentially *forward-looking*.

> This model implies that there is no need for gradualist policies to reduce inflation. According to the NKPC, low inflation can be achieved immediately by the central bank announcing (and the public believing) that it is committing itself to eliminating positive output gaps in the future. (IPEA, p. 11)

We can interpret this model in the ARDL framework as a DL model with $X_t := \mathbb{E}_t(\pi_{t+1})$.

Some authors have also proposed a NKPC with a backward-looking element. This is called the Hybrid Philips Curve:

$$\pi_t = \gamma_b\pi_{t-1} + \gamma_f\mathbb{E}_t(\pi_{t+1}) + \kappa x_t.$$

This can be interpreted as an ARDL(1,1) model, with $X_t := \mathbb{E}_{t-1}(\pi_t)$.

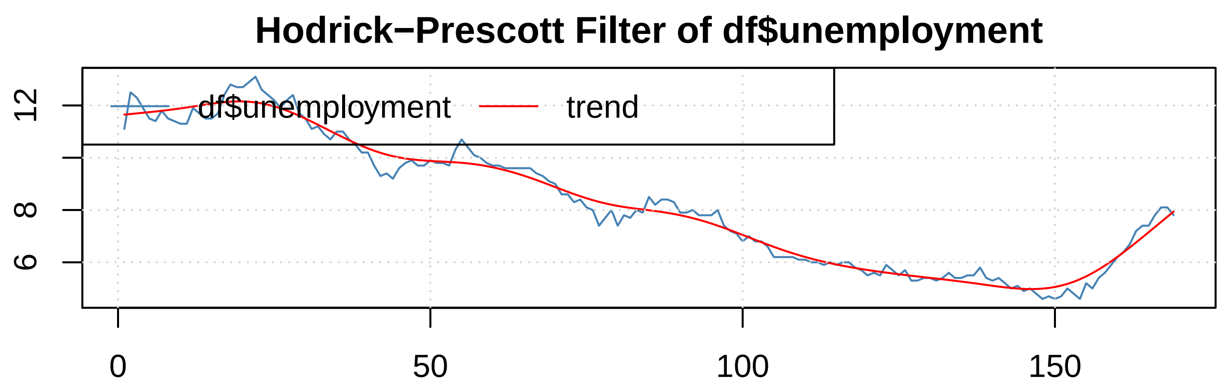
Estimation

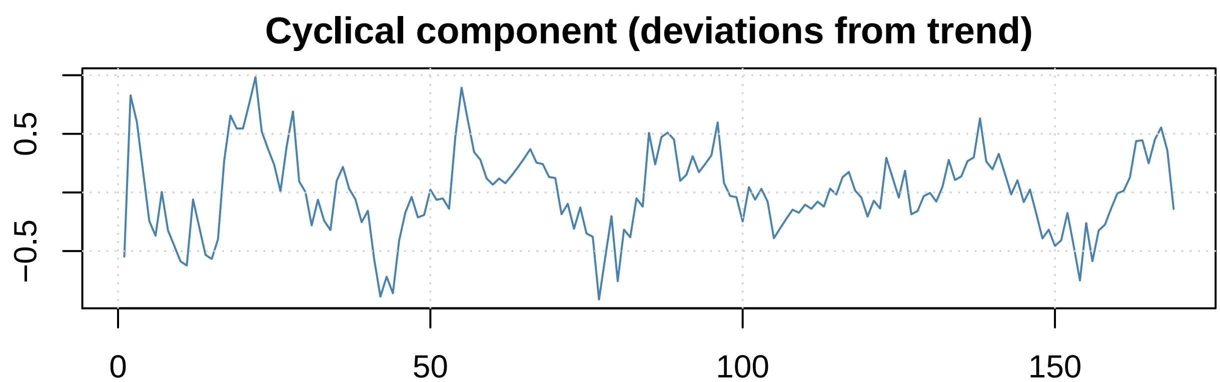
These hypothesis will now be tested with Brazilian time series data for inflation, unemployment and inflation expectation.

```r
# APC

nairu <- hpfilter(df$unemployment, 1600, type = 'lambda')

plot(nairu)
```






```r
nairu_trend <- nairu$trend

u_dev <- df$unemployment - nairu_trend

df$u_dev = u_dev

exp_lag = dplyr::lag(df$exp_IPCA, k=1)

df$exp_lag = exp_lag

df2 = df[2:length(df$IPCA),]

auto_apc <- auto_ardl(IPCA ~ exp_lag + u_dev, data = df2, max_order = 12)
```

```
## Warning: The `x` argument of `as_tibble.matrix()` must have unique column names if `.name_repair` is
## Using compatibility `.name_repair`.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_warnings()` to see where this warning was generated.
```

```r
summary(auto_apc$best_model)
```

```
##
## Time series regression with "ts" data:
## Start = 13, End = 168
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
```

```
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.50165 -0.13225 -0.00937  0.15074  0.53030
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -0.20593    0.15248  -1.351 0.179360
## L(IPCA, 1)      1.36220    0.09409  14.477  < 2e-16 ***
## L(IPCA, 2)     -0.25922    0.15632  -1.658 0.099869 .
## L(IPCA, 3)     -0.15454    0.14216  -1.087 0.279168
## L(IPCA, 4)      0.03670    0.14287   0.257 0.797728
## L(IPCA, 5)     -0.00382    0.13884  -0.028 0.978097
## L(IPCA, 6)      0.01040    0.13806   0.075 0.940101
## L(IPCA, 7)     -0.28244    0.13509  -2.091 0.038651 *
## L(IPCA, 8)      0.23050    0.13592   1.696 0.092482 .
## L(IPCA, 9)      0.16376    0.13237   1.237 0.218452
## L(IPCA, 10)    -0.19093    0.07024  -2.718 0.007531 **
## exp_lag         0.19976    0.14986   1.333 0.185038
## L(exp_lag, 1)  -0.21667    0.27641  -0.784 0.434648
## L(exp_lag, 2)   0.11636    0.29766   0.391 0.696551
## L(exp_lag, 3)   0.05002    0.27272   0.183 0.854787
## L(exp_lag, 4)  -0.11920    0.26643  -0.447 0.655394
## L(exp_lag, 5)   0.13768    0.26194   0.526 0.600115
## L(exp_lag, 6)  -0.08363    0.25155  -0.332 0.740129
## L(exp_lag, 7)  -0.13985    0.24522  -0.570 0.569529
## L(exp_lag, 8)   0.19974    0.24368   0.820 0.414009
## L(exp_lag, 9)   0.43565    0.23764   1.833 0.069222 .
## L(exp_lag, 10) -0.88295    0.23142  -3.815 0.000216 ***
## L(exp_lag, 11)  0.21339    0.21510   0.992 0.323144
## L(exp_lag, 12)  0.23022    0.10933   2.106 0.037301 *
## u_dev           0.17855    0.10235   1.744 0.083618 .
## L(u_dev, 1)    -0.17677    0.12056  -1.466 0.145184
## L(u_dev, 2)     0.17243    0.12087   1.427 0.156279
## L(u_dev, 3)    -0.02066    0.11855  -0.174 0.861926
## L(u_dev, 4)    -0.15435    0.11717  -1.317 0.190220
## L(u_dev, 5)    -0.10897    0.11954  -0.912 0.363813
## L(u_dev, 6)     0.09109    0.11789   0.773 0.441180
## L(u_dev, 7)     0.09352    0.11742   0.796 0.427340
## L(u_dev, 8)    -0.07746    0.11673  -0.664 0.508205
## L(u_dev, 9)    -0.07030    0.11578  -0.607 0.544857
## L(u_dev, 10)    0.19848    0.10036   1.978 0.050240 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2358 on 121 degrees of freedom
## Multiple R-squared:  0.9945, Adjusted R-squared:  0.993
## F-statistic: 643.1 on 34 and 121 DF,  p-value: < 2.2e-16
```

```
Box.test(auto_apc$best_model$residuals)
```

```
##
##  Box-Pierce test
##
```

```
## data:  auto_apc$best_model$residuals
## X-squared = 0.020209, df = 1, p-value = 0.887
```

```r
AIC(auto_apc$best_model)
```

```
## [1] 24.24382
```

```r
BIC(auto_apc$best_model)
```

```
## [1] 134.0386
```

```r
# APC as DL of order 1

apc1 <- dynlm(IPCA ~ exp_lag + u_dev, data = df2)

summary(apc1)
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 168
##
## Call:
## dynlm(formula = IPCA ~ exp_lag + u_dev, data = df2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0369 -1.0285 -0.3057  0.7819  5.8672
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.83045    0.48542  -5.831 2.83e-08 ***
## exp_lag      1.73784    0.08576  20.265  < 2e-16 ***
## u_dev        1.43364    0.33701   4.254 3.51e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.53 on 165 degrees of freedom
## Multiple R-squared:  0.7139, Adjusted R-squared:  0.7105
## F-statistic: 205.9 on 2 and 165 DF,  p-value: < 2.2e-16
```

```r
Box.test(apc1$residuals) # Modelo claramente inconsistente
```

```
##
##  Box-Pierce test
##
## data:  apc1$residuals
## X-squared = 130.33, df = 1, p-value < 2.2e-16
```

```r
# NKPC

auto_nkpc <- auto_ardl(IPCA ~ exp_IPCA + u_dev, data = df, max_order = 18)

summary(auto_nkpc$best_model) # Unit root?
```

```
##
## Time series regression with "ts" data:
## Start = 15, End = 169
##
```

```
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.37515 -0.10114 -0.00144  0.10024  0.42758
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -0.240062   0.126758  -1.894 0.060824 .
## L(IPCA, 1)        1.350684   0.085512  15.795  < 2e-16 ***
## L(IPCA, 2)       -0.184944   0.138564  -1.335 0.184677
## L(IPCA, 3)       -0.244035   0.120283  -2.029 0.044847 *
## L(IPCA, 4)        0.042681   0.119997   0.356 0.722745
## L(IPCA, 5)        0.024477   0.118250   0.207 0.836391
## L(IPCA, 6)        0.082208   0.118457   0.694 0.489123
## L(IPCA, 7)       -0.340537   0.117336  -2.902 0.004462 **
## L(IPCA, 8)        0.249078   0.122336   2.036 0.044107 *
## L(IPCA, 9)        0.020930   0.122176   0.171 0.864289
## L(IPCA, 10)      -0.060628   0.117853  -0.514 0.607961
## L(IPCA, 11)       0.155070   0.115509   1.342 0.182150
## L(IPCA, 12)      -0.458104   0.113047  -4.052 9.39e-05 ***
## L(IPCA, 13)       0.400469   0.111369   3.596 0.000482 ***
## L(IPCA, 14)      -0.104736   0.058148  -1.801 0.074366 .
## exp_IPCA          0.601175   0.116501   5.160 1.08e-06 ***
## L(exp_IPCA, 1)  -0.685972   0.229083  -2.994 0.003385 **
## L(exp_IPCA, 2)   0.073614   0.259025   0.284 0.776784
## L(exp_IPCA, 3)  -0.029816   0.258443  -0.115 0.908360
## L(exp_IPCA, 4)   0.340617   0.256786   1.326 0.187385
## L(exp_IPCA, 5)  -0.324089   0.230055  -1.409 0.161680
## L(exp_IPCA, 6)   0.290928   0.218800   1.330 0.186334
## L(exp_IPCA, 7)  -0.250743   0.212432  -1.180 0.240363
## L(exp_IPCA, 8)  -0.078165   0.215093  -0.363 0.716990
## L(exp_IPCA, 9)   0.321380   0.218641   1.470 0.144393
## L(exp_IPCA, 10)  0.176315   0.216452   0.815 0.417047
## L(exp_IPCA, 11) -0.711509   0.216809  -3.282 0.001376 **
## L(exp_IPCA, 12)  0.229051   0.199822   1.146 0.254125
## L(exp_IPCA, 13)  0.170695   0.101511   1.682 0.095444 .
## u_dev            -0.006651   0.093741  -0.071 0.943567
## L(u_dev, 1)      -0.069862   0.108958  -0.641 0.522716
## L(u_dev, 2)       0.120614   0.102093   1.181 0.239941
## L(u_dev, 3)      -0.002564   0.101930  -0.025 0.979980
## L(u_dev, 4)      -0.133487   0.100473  -1.329 0.186687
## L(u_dev, 5)      -0.128222   0.101271  -1.266 0.208093
## L(u_dev, 6)       0.180646   0.100089   1.805 0.073784 .
## L(u_dev, 7)      -0.031687   0.099571  -0.318 0.750901
## L(u_dev, 8)      -0.013898   0.098087  -0.142 0.887583
## L(u_dev, 9)      -0.119316   0.098170  -1.215 0.226768
## L(u_dev, 10)      0.089325   0.097712   0.914 0.362590
## L(u_dev, 11)      0.126947   0.098010   1.295 0.197902
## L(u_dev, 12)      0.196699   0.096418   2.040 0.043696 *
## L(u_dev, 13)     -0.343949   0.084819  -4.055 9.29e-05 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.194 on 112 degrees of freedom
## Multiple R-squared:  0.9963, Adjusted R-squared:  0.9949
## F-statistic: 715.4 on 42 and 112 DF,  p-value: < 2.2e-16
```

```r
adf.test(df2$IPCA) # UNIT ROOT!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  df2$IPCA
## Dickey-Fuller = -2.6908, Lag order = 5, p-value = 0.288
## alternative hypothesis: stationary
```

```r
######## Correcting for unit root ############

df3 = data.frame(diff(df$IPCA), diff(df$exp_IPCA), df2$u_dev)

colnames(df3) = c("IPCA", "exp_IPCA", "u_dev")

diff_lag = dplyr::lag(df3$exp_IPCA, 1)

df3$diff_lag = diff_lag

df4 = df3[2:length(df3$IPCA),]

# APC

auto_apc_diff <-  auto_ardl(IPCA ~ diff_lag + u_dev, data = df4, max_order = 24)

summary(auto_apc_diff$best_model)
```

```
##
## Time series regression with "ts" data:
## Start = 18, End = 167
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.41343 -0.10932  0.00661  0.09394  0.58493
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)      0.010941   0.018254   0.599 0.550277
## L(IPCA, 1)       0.409004   0.105330   3.883 0.000185 ***
## L(IPCA, 2)       0.088896   0.112047   0.793 0.429432
## L(IPCA, 3)       0.119181   0.114087   1.045 0.298704
## L(IPCA, 4)       0.110875   0.109923   1.009 0.315575
## L(IPCA, 5)      -0.006091   0.107789  -0.057 0.955050
## L(IPCA, 6)       0.060649   0.090617   0.669 0.504854
## L(IPCA, 7)      -0.258467   0.086586  -2.985 0.003564 **
## L(IPCA, 8)       0.127535   0.089633   1.423 0.157887
```

```
## L(IPCA, 9)       0.058985   0.091137   0.647 0.518977
## L(IPCA, 10)     -0.017815   0.089833  -0.198 0.843201
## L(IPCA, 11)      0.137300   0.083879   1.637 0.104796
## L(IPCA, 12)     -0.266226   0.089314  -2.981 0.003610 **
## L(IPCA, 13)      0.037546   0.090901   0.413 0.680463
## L(IPCA, 14)     -0.099533   0.086609  -1.149 0.253207
## L(IPCA, 15)      0.191537   0.087305   2.194 0.030560 *
## L(IPCA, 16)      0.001053   0.086328   0.012 0.990289
## L(IPCA, 17)     -0.009501   0.074321  -0.128 0.898535
## diff_lag         0.249735   0.156337   1.597 0.113329
## L(diff_lag, 1)  -0.157402   0.170257  -0.924 0.357453
## L(diff_lag, 2)  -0.146646   0.175684  -0.835 0.405868
## L(diff_lag, 3)   0.080162   0.171792   0.467 0.641784
## L(diff_lag, 4)   0.053050   0.174345   0.304 0.761547
## L(diff_lag, 5)   0.119037   0.177688   0.670 0.504453
## L(diff_lag, 6)   0.258261   0.176193   1.466 0.145844
## L(diff_lag, 7)  -0.261561   0.169356  -1.544 0.125640
## L(diff_lag, 8)   0.189110   0.149533   1.265 0.208929
## L(diff_lag, 9)   0.284296   0.144779   1.964 0.052347 .
## L(diff_lag, 10) -0.283618   0.151988  -1.866 0.064964 .
## L(diff_lag, 11) -0.294228   0.154530  -1.904 0.059782 .
## L(diff_lag, 12)  0.191864   0.155691   1.232 0.220712
## L(diff_lag, 13) -0.198643   0.149325  -1.330 0.186453
## L(diff_lag, 14)  0.151608   0.119171   1.272 0.206256
## u_dev            0.007234   0.114981   0.063 0.949963
## L(u_dev, 1)     -0.221264   0.129559  -1.708 0.090773 .
## L(u_dev, 2)      0.111406   0.127527   0.874 0.384436
## L(u_dev, 3)      0.102516   0.126804   0.808 0.420745
## L(u_dev, 4)     -0.021851   0.125640  -0.174 0.862280
## L(u_dev, 5)     -0.204757   0.126623  -1.617 0.109014
## L(u_dev, 6)      0.216744   0.117440   1.846 0.067913 .
## L(u_dev, 7)     -0.062466   0.119477  -0.523 0.602249
## L(u_dev, 8)      0.014275   0.118463   0.121 0.904325
## L(u_dev, 9)     -0.112848   0.114714  -0.984 0.327621
## L(u_dev, 10)     0.140497   0.114542   1.227 0.222853
## L(u_dev, 11)     0.074526   0.114624   0.650 0.517065
## L(u_dev, 12)     0.270649   0.113429   2.386 0.018912 *
## L(u_dev, 13)    -0.296732   0.112686  -2.633 0.009800 **
## L(u_dev, 14)    -0.019484   0.117377  -0.166 0.868494
## L(u_dev, 15)    -0.241873   0.117920  -2.051 0.042867 *
## L(u_dev, 16)     0.169188   0.106435   1.590 0.115084
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2132 on 100 degrees of freedom
## Multiple R-squared:  0.8526, Adjusted R-squared:  0.7804
## F-statistic: 11.81 on 49 and 100 DF,  p-value: < 2.2e-16
```

```r
n = 12
x = matrix(NA, nrow = n, ncol = 1)

for (i in (1:n)) {

  auto_apc_diff <- auto_ardl(IPCA ~ diff_lag + u_dev, data = df4, max_order = i)
```

```r
  x[i,] = Box.test(auto_apc_diff$best_model$residuals)$p.value

}


# NKPC


auto_nkpc_diff <- auto_ardl(IPCA ~ u_dev | exp_IPCA, data = df3, max_order = 18)

summary(auto_nkpc_diff$best_model)
```

```
##
## Time series regression with "ts" data:
## Start = 19, End = 168
##
## Call:
## dynlm::dynlm(formula = full_formula, data = data, start = start,
##     end = end)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.84193 -0.13569 -0.01028  0.14147  0.56659
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.005060   0.020142  -0.251 0.802125
## L(IPCA, 1)    0.509132   0.089594   5.683 1.08e-07 ***
## L(IPCA, 2)   -0.012886   0.104356  -0.123 0.901953
## L(IPCA, 3)    0.116841   0.101546   1.151 0.252360
## L(IPCA, 4)    0.105208   0.098757   1.065 0.289042
## L(IPCA, 5)   -0.026791   0.098619  -0.272 0.786383
## L(IPCA, 6)    0.027351   0.097234   0.281 0.779015
## L(IPCA, 7)   -0.112602   0.088553  -1.272 0.206182
## L(IPCA, 8)    0.063236   0.089340   0.708 0.480546
## L(IPCA, 9)   -0.039058   0.080590  -0.485 0.628883
## L(IPCA, 10)   0.043902   0.076831   0.571 0.568881
## L(IPCA, 11)  -0.045146   0.076777  -0.588 0.557718
## L(IPCA, 12)  -0.404506   0.082443  -4.906 3.20e-06 ***
## L(IPCA, 13)   0.194478   0.087748   2.216 0.028709 *
## L(IPCA, 14)  -0.101528   0.091232  -1.113 0.268175
## L(IPCA, 15)   0.208424   0.089401   2.331 0.021540 *
## L(IPCA, 16)   0.079960   0.090906   0.880 0.380980
## L(IPCA, 17)  -0.009173   0.090564  -0.101 0.919501
## L(IPCA, 18)  -0.128494   0.075047  -1.712 0.089657 .
## u_dev        -0.010612   0.123057  -0.086 0.931435
## L(u_dev, 1)  -0.227780   0.142390  -1.600 0.112510
## L(u_dev, 2)   0.169796   0.140662   1.207 0.229952
## L(u_dev, 3)   0.152551   0.141213   1.080 0.282356
## L(u_dev, 4)  -0.040788   0.137601  -0.296 0.767464
## L(u_dev, 5)  -0.303975   0.134514  -2.260 0.025786 *
## L(u_dev, 6)   0.056251   0.137073   0.410 0.682324
## L(u_dev, 7)  -0.101813   0.126234  -0.807 0.421657
```

```
## L(u_dev, 8)   0.179746   0.124871   1.439 0.152835
## L(u_dev, 9)  -0.062434   0.123264  -0.507 0.613503
## L(u_dev, 10)  0.107862   0.124115   0.869 0.386695
## L(u_dev, 11)  0.048279   0.122524   0.394 0.694310
## L(u_dev, 12)  0.199816   0.122730   1.628 0.106339
## L(u_dev, 13) -0.263087   0.120391  -2.185 0.030970 *
## L(u_dev, 14)  0.103658   0.121815   0.851 0.396634
## L(u_dev, 15) -0.333328   0.126655  -2.632 0.009703 **
## L(u_dev, 16) -0.013391   0.134179  -0.100 0.920681
## L(u_dev, 17)  0.096555   0.132828   0.727 0.468808
## L(u_dev, 18)  0.098262   0.116650   0.842 0.401393
## exp_IPCA      0.521329   0.136527   3.818 0.000222 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2401 on 111 degrees of freedom
## Multiple R-squared:  0.7925, Adjusted R-squared:  0.7215
## F-statistic: 11.16 on 38 and 111 DF,  p-value: < 2.2e-16
```

```r
Box.test(auto_nkpc_diff$best_model$residuals)
```

```
##
##  Box-Pierce test
##
## data:  auto_nkpc_diff$best_model$residuals
## X-squared = 0.40611, df = 1, p-value = 0.524
```

```r
nkpc_dyn <- dynlm(IPCA ~ exp_IPCA + u_dev, data = df3)

summary(nkpc_dyn)
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 168
##
## Call:
## dynlm(formula = IPCA ~ exp_IPCA + u_dev, data = df3)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.61527 -0.17231  0.01181  0.24129  2.06516
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.01222    0.03659   0.334 0.738822
## exp_IPCA     0.51091    0.08887   5.749 4.24e-08 ***
## u_dev       -0.35898    0.10400  -3.452 0.000707 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.4739 on 165 degrees of freedom
## Multiple R-squared:  0.2366, Adjusted R-squared:  0.2273
## F-statistic: 25.57 on 2 and 165 DF,  p-value: 2.124e-10
```

```r
Box.test(nkpc_dyn$residuals)
```

```
##
##  Box-Pierce test
##
## data:  nkpc_dyn$residuals
## X-squared = 71.621, df = 1, p-value < 2.2e-16
```

```r
# HPC

hpc_dyn <- dynlm(IPCA ~ exp_IPCA + L(IPCA, 1) + u_dev, data = df3)

summary(hpc_dyn)
```

```
## Warning in summary.lm(hpc_dyn): essentially perfect fit: summary may be
## unreliable
```

```
##
## Time series regression with "numeric" data:
## Start = 1, End = 168
##
## Call:
## dynlm(formula = IPCA ~ exp_IPCA + L(IPCA, 1) + u_dev, data = df3)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -1.136e-15 -3.240e-18  4.370e-18  1.270e-17  1.012e-16
##
## Coefficients:
##               Estimate Std. Error    t value Pr(>|t|)
## (Intercept)  4.051e-18  7.073e-18  5.730e-01    0.568
## exp_IPCA     1.651e-16  1.882e-17  8.773e+00 2.15e-15 ***
## L(IPCA, 1)   1.000e+00  1.504e-17  6.647e+16  < 2e-16 ***
## u_dev       -1.121e-17  2.081e-17 -5.390e-01    0.591
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.158e-17 on 164 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 1.929e+33 on 3 and 164 DF,  p-value: < 2.2e-16
```

```r
Box.test(hpc_dyn$residuals)
```

```
##
##  Box-Pierce test
##
## data:  hpc_dyn$residuals
## X-squared = 0.34931, df = 1, p-value = 0.5545
```

```r
# The results suggest that inflation is both forward and backward looking.
```