

Econometrics II - Problem 6

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Part 2: Unit roots

As has been discussed during the lecture, when the population model is a *random walk*:

$$Y_t = 1 * Y_{t-1} + \varepsilon_t, \quad \varepsilon \sim wn(0, \sigma^2),$$

it happens that the series *is not ergodic* (nor is it stationary). Therefore, the usual asymptotic properties *do not hold*, as all innovations have *permanent effects*.

$$Y_t = c + \delta t + \sum_{i=1}^t \varepsilon_i$$

The random walk, defined above, is an example of the more general class of *unit root processes*:

$$Y_t = c + \delta t + u_t,$$

u_t has an ARMA(p,q) representation:

$$\Phi_p(L)u_t = \Theta_q(L)\varepsilon_t, \quad \varepsilon \sim wn(0, \sigma^2)$$

Suppose that one of the roots of $\Theta_p(L)$ is equal to 1:

$$\Theta_p(L) = (1 - [1]L)(1 - \lambda_2 L) \dots (1 - \lambda_p L)$$

$$(1 - L)u_t = (1 - \lambda_2 L)^{-1} \dots (1 - \lambda_p L)^{-1} \Theta_q(L)\varepsilon_t =: \Psi(L)\varepsilon_t$$

We can now rewrite this as:

$$(1 - L)Y_t = (1 - L)c + (1 - L)\delta t + (1 - L)u_t$$

$$(1 - L)Y_t = \delta + \Psi(L)\varepsilon_t$$

Defining ΔY_t , we have:

$$\Delta Y_t := (1 - L)Y_t = Y_t - Y_{t-1} = \delta + \Psi(L)\varepsilon_t$$

With this concept, we can define ARIMA(p,d,q) processes:

$$\Phi_p(L)(1 - L)^d Y_t = c + \Theta_q(L)\varepsilon_t, \quad \varepsilon_t \sim wn(0, \sigma^2)$$

Hypothesis testing

It turns out that testing for unit root presence presents some challenges. Under the null ($a_1 = 1$), its distribution *is not standard* and does not present the usual asymptotic properties. We can circumvent this issue with the use of numeric methods, such as the *Monte Carlo simulation*. It will now be employed, following Dickey and Fuller (1979).

First, some notation:

We begin with a simple model:

$$Y_t = a_1 y_{t-1} + \varepsilon_t$$
$$\Delta Y_t = \gamma y_{t-1} + \varepsilon_t, \quad \gamma := a_1 - 1$$

Dickey and Fuller constructed the following regression equations:

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t \tag{1}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + \varepsilon_t \tag{2}$$

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \varepsilon_t \tag{3}$$

(1) has no intercept and represents a simple random walk. (2) includes an intercept. (3) also adds a deterministic time trend.

Note that the critical values of the t-statistics are *different* between these regressions. This will now be shown with our Monte Carlo simulation.

For equation (1):

```
set.seed(76345)

# length of ts
T <- 100

# Loops
S <- 10000

e <- rnorm(T, 0, 1)

# As  $y_{t+1} = y_t + \varepsilon_t$ , we can write this as an MA( $\infty$ ) model.  $y_t = \sum \varepsilon_i$ .
y <- cumsum(e)

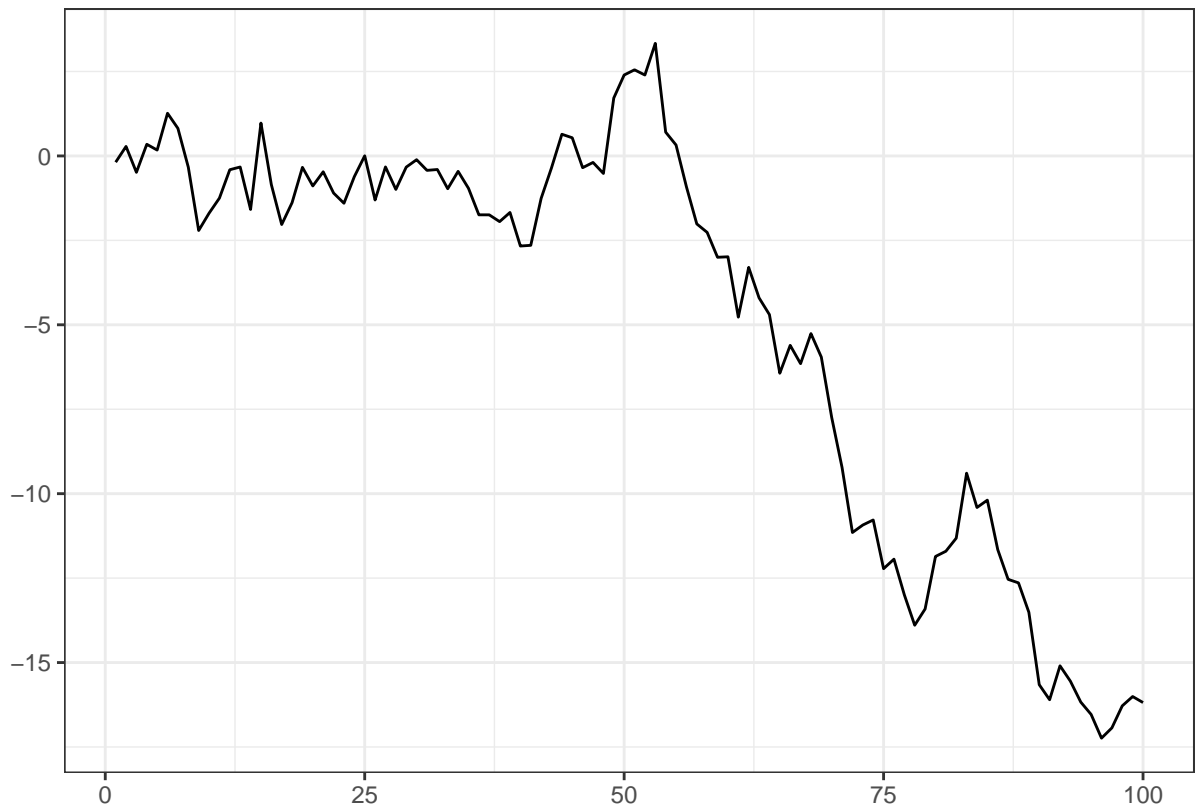
y <- vector()

y[1] = e[1]

for (i in (2:T)) {
  y[i] <- y[(i-1)] + e[i]
}

y <- as.ts(y)

autoplot(y) + theme_bw()
```



```
# Taking the first difference

y_diff <- diff(y)

# Regression of I(1) model

reg <- dynlm(y_diff ~ 0 + L(y,1)) # no lag (x1 = 0)

reg <-summary(reg)

reg$coefficients[1,3] # t value
```

```
## [1] 1.110447
```

```
# Loop

tau <- vector()

k <- 1

delta <- 0

for (i in 1:S) {

  e <- rnorm(T,0,1)
  y = k + delta*seq(1:T) + cumsum(e)
  y <- as.ts(y)
```

```

y_diff <- diff(y)

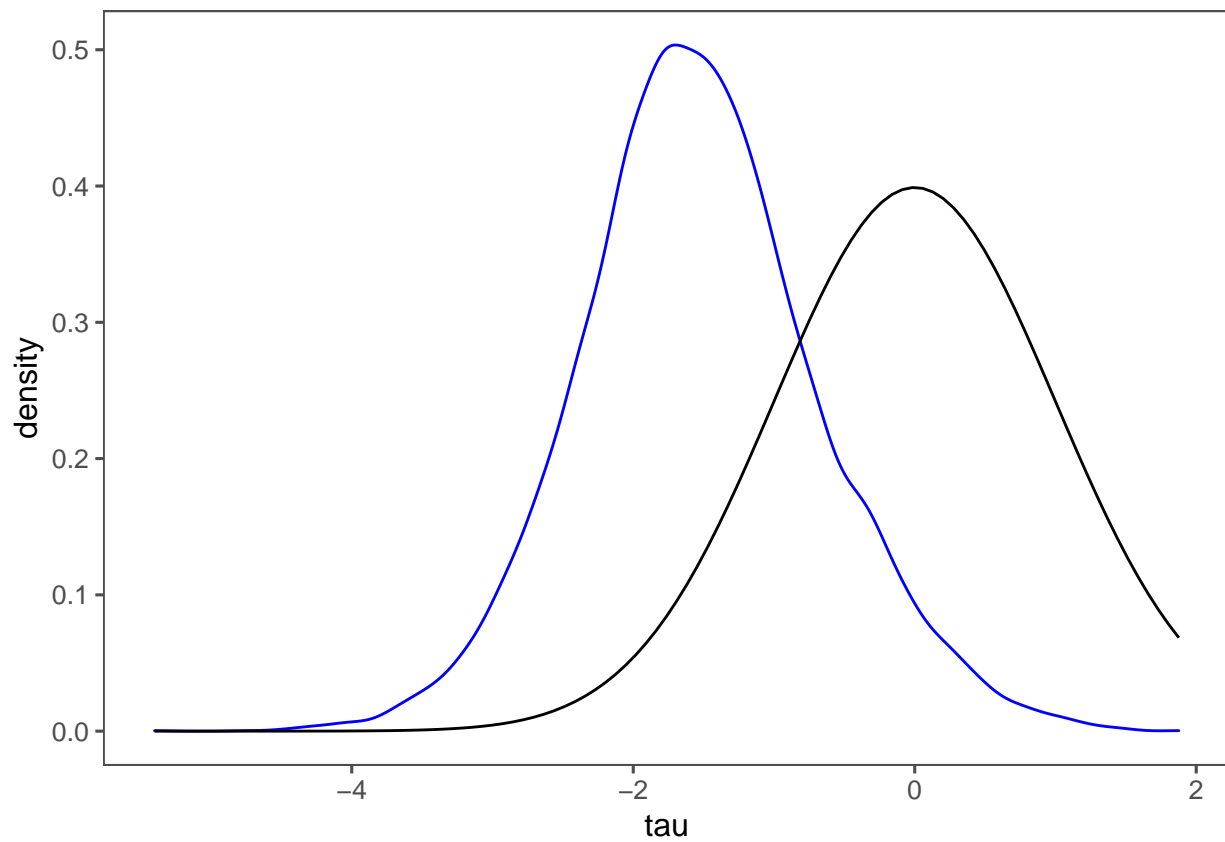
reg <- summary(dynlm(y_diff ~ 1 + L(y, 1)))

tau[i] <- reg$coefficients[2,3]
}

tau.df <- data.frame(tau)

ggplot(data = tau.df, aes(x = tau)) + geom_density(color = "blue") + stat_function(fun = dnorm, n = 101

```



```

jarque.bera.test(tau) # We reject H0 at the 1% significance level.

```

```

##
## Jarque Bera Test
##
## data: tau
## X-squared = 86.138, df = 2, p-value < 2.2e-16

```

```

tau.ci <- quantile(tau, c(0.01, 0.05, 0.1))

```

```

tau.ci

```

```

##          1%          5%          10%
## -3.494124 -2.880240 -2.567557

```

For equation (2) – with an intercept:

```
# Loop

tau_mu <- vector()

for (i in 1:S) {

  e <- rnorm(T, 0, 1)
  y <- cumsum(e)
  y <- as.ts(y)

  y_diff <- diff(y)

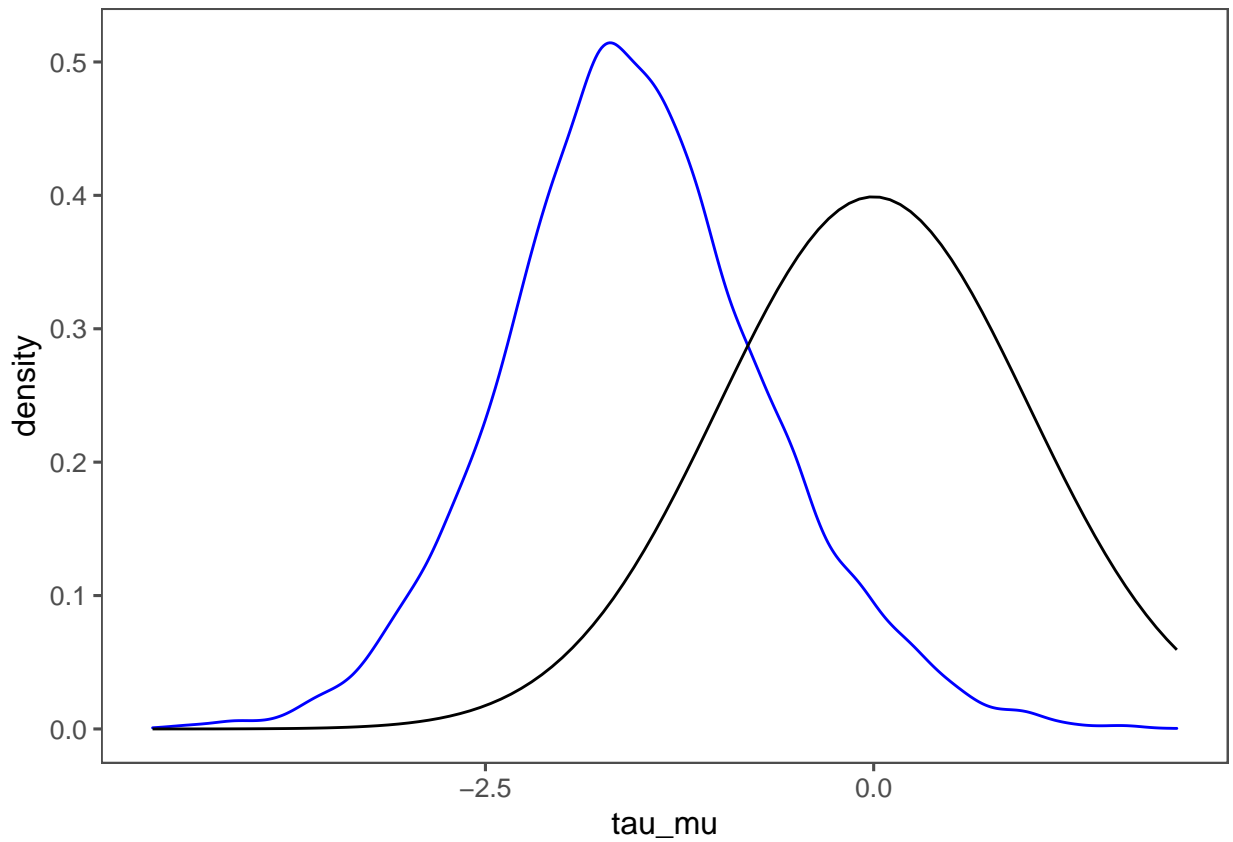
  reg <- summary(dynlm(y_diff ~ 1 + L(y, 1)))

  tau_mu[i] <- reg$coefficients[2, 3]

}

tau_mu.df <- data.frame(tau_mu)

ggplot(data = tau_mu.df, aes(x = tau_mu)) + geom_density(color = "blue") +
  stat_function(fun = dnorm, n = 101, args = c(mean = 0, sd = 1)) +
  theme_few()
```



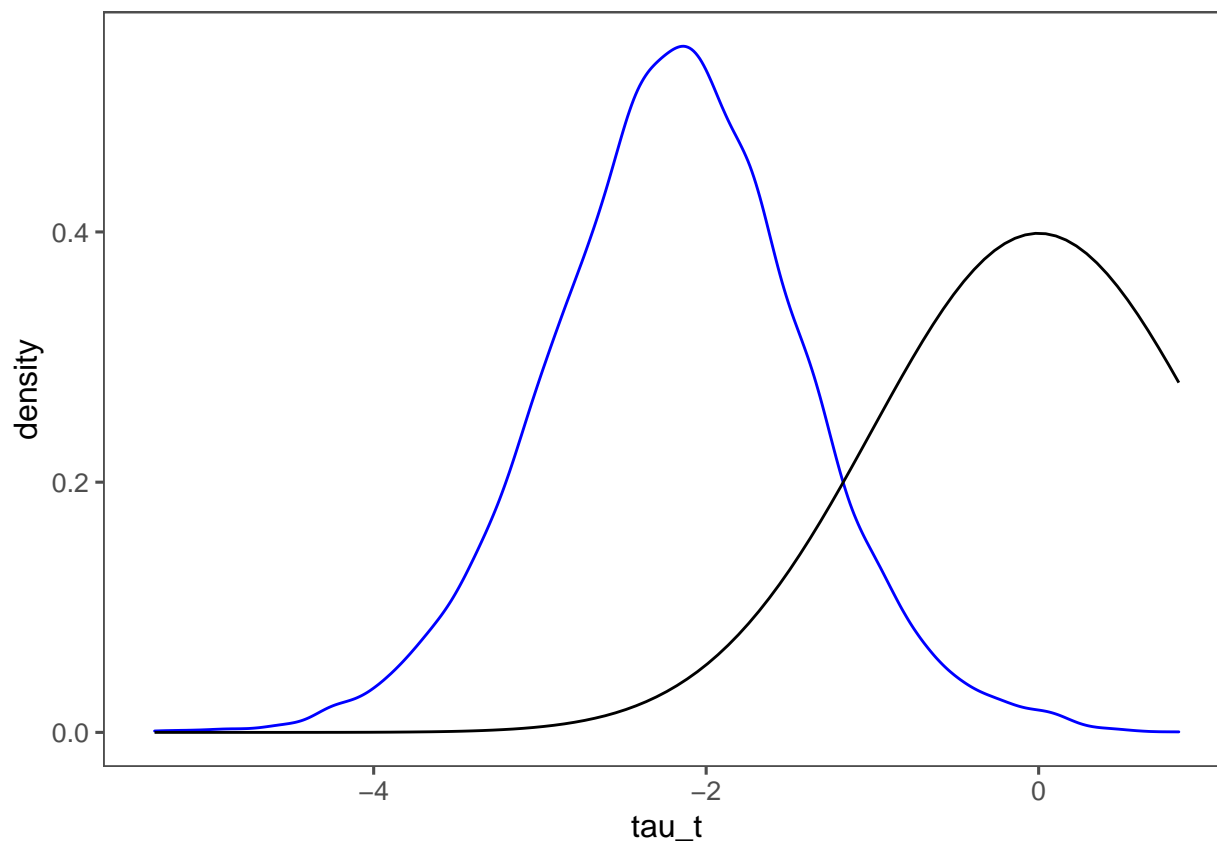
```
jarque.bera.test(tau_mu) # We reject H0 at the 1% significance level.
```

```
##  
## Jarque Bera Test  
##  
## data: tau_mu  
## X-squared = 96.894, df = 2, p-value < 2.2e-16  
tau_mu.ci <- quantile(tau_mu, c(0.01, 0.05, 0.1))  
  
tau_mu.ci
```

```
##          1%          5%          10%  
## -3.494281 -2.895757 -2.577903
```

And finally, for equation (3) – with an intercept and a deterministic time trend:

```
# Loop  
  
tau_t <- vector()  
time <- c(1:T)  
  
for (i in 1:S) {  
  
  e <- rnorm(T, 0, 1)  
  y <- cumsum(e)  
  y <- as.ts(y)  
  
  y_diff <- diff(y)  
  
  reg <- summary(dynlm(y_diff ~ 1 + L(y, 1) + time[-1])) # removed 1 dimension for no. of obs.  
  
  tau_t[i] <- reg$coefficients[2, 3]  
  
}  
  
tau_t.df <- data.frame(tau_t)  
  
ggplot(data = tau_t.df, aes(x = tau_t)) + geom_density(color = "blue") +  
  stat_function(fun = dnorm, n = 101, args = c(mean = 0, sd = 1)) +  
  theme_few()
```



```
jarque.bera.test(tau_t) # We reject H0 at the 1% significance level.
```

```
##
##  Jarque Bera Test
##
## data:  tau_t
## X-squared = 58.223, df = 2, p-value = 2.275e-13
tau_t.ci <- quantile(tau_t, c(0.01, 0.05, 0.1))

tau_t.ci
```

```
##          1%          5%          10%
## -4.065067 -3.462447 -3.158227
```

Let's now change the distributions of the errors for equation (3):

```
# Loop

tau_t_pois <- vector()
time <- c(1:T)

for (i in 1:S) {

  e <- rpois(T, 1)
  y <- cumsum(e)
  y <- as.ts(y)
```

```

y_diff <- diff(y)

reg <- summary(dynlm(y_diff ~ 1 + L(y, 1) + time[-1])) # removed 1 dimension for no. of obs.

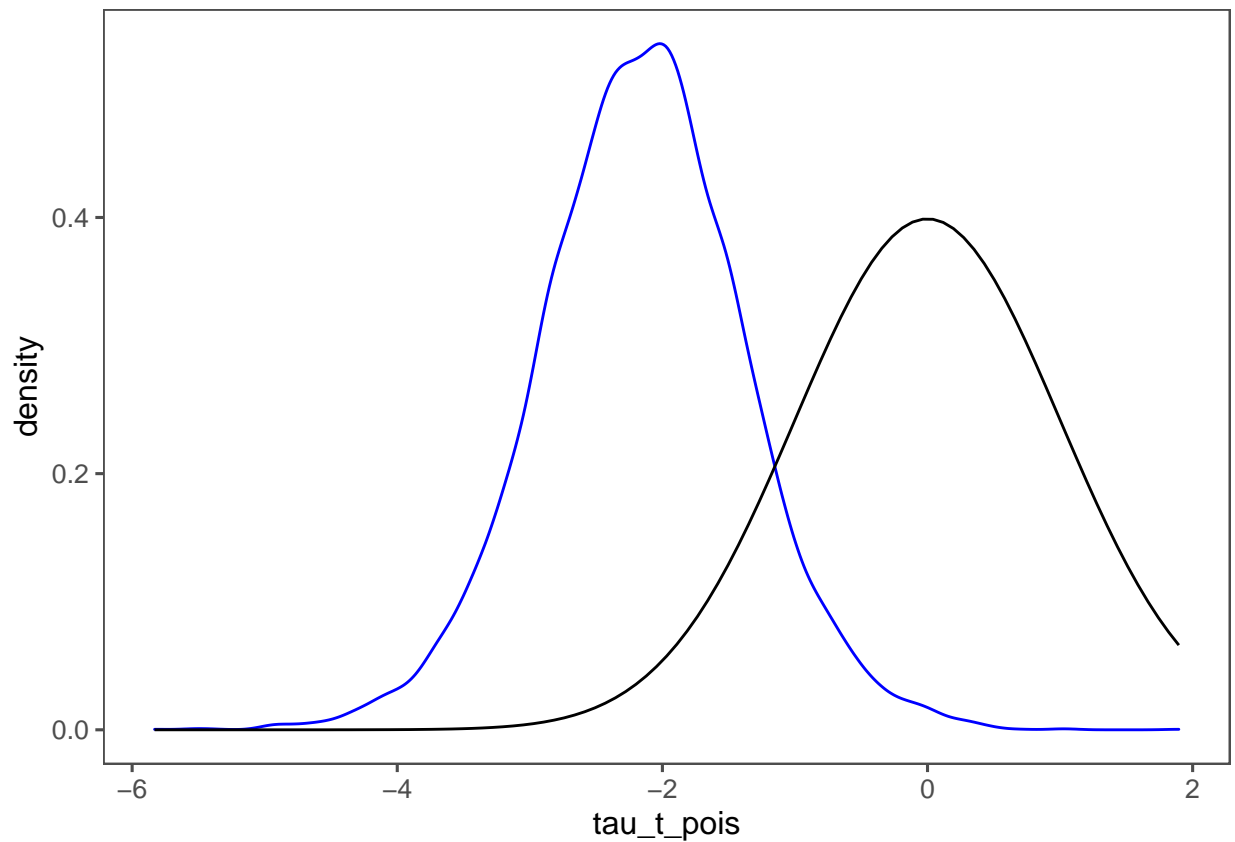
tau_t_pois[i] <- reg$coefficients[2, 3]

}

tau_t_pois.df <- data.frame(tau_t_pois)

ggplot(data = tau_t_pois.df, aes(x = tau_t_pois)) + geom_density(color = "blue") +
  stat_function(fun = dnorm, n = 101, args = c(mean = 0, sd = 1)) +
  theme_few()

```



```

jarque.bera.test(tau_t_pois) # We reject H0 at the 1% significance level.

```

```

##
## Jarque Bera Test
##
## data: tau_t_pois
## X-squared = 120.61, df = 2, p-value < 2.2e-16
tau_t_pois.ci <- quantile(tau_t_pois, c(0.01, 0.05, 0.1))

tau_t_pois.ci

```

```

##          1%          5%          10%

```



```
## -4.085753 -3.436365 -3.127525
```

Part 3: Applying the Dickey-Fuller test for GDP

Loading the data from the previous problem:

```
pib <- get_sidra(6612, variable = 9318, category = 90707, period = "all")
```

```
## Considering all categories once 'classific' was set to 'all' (default)
```

```
pib_limpo <- pib[(pib$`Setores e subsetores (Código)` == 90707),  
  ]
```

```
pib <- pib_limpo
```

```
pib_limpo2 <- pib[, c(5, 13)]
```

```
pib <- pib_limpo2
```

```
names(pib)[1] <- "t"
```

```
names(pib)[2] <- "v"
```

```
pib$t <- as.numeric(pib$t)
```

```
names(pib)
```

```
## [1] "t" "v"
```

```
head(pib)
```

```
##           t           v  
## 18  199601 170920.0  
## 40  199602 176708.8  
## 62  199603 189844.3  
## 84  199604 184112.9  
## 106 199701 176732.2  
## 128 199702 185109.5
```

```
tail(pib)
```

```
##           t           v  
## 2042 201901 292647.6  
## 2064 201902 297748.9  
## 2086 201903 305150.9  
## 2108 201904 302108.7  
## 2130 202001 291912.5  
## 2152 202002 263699.7
```

```
pib <- ts(pib$v)
```

```
# Choosing the correct model with auto.arima
```

```
aa_pib <- auto.arima(pib, stepwise = F)
```

```
summary(aa_pib)
```

```
## Series: pib
```

```
## ARIMA(2,1,2) with drift
##
## Coefficients:
##          ar1          ar2          ma1          ma2          drift
##        -0.0092   -0.9764    0.2721    0.956    864.9855
## s.e.    0.0233    0.0171    0.0455    0.117    537.6284
##
## sigma^2 estimated as 23455415:  log likelihood=-961.03
## AIC=1934.06   AICc=1934.99   BIC=1949.51
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE      ACF1
## Training set 64.56963 4692.48 3219.806 0.05416274 1.322955 0.5203698 0.01746157
```

auto.arima yields an ARIMA(2,1,2) model with a drift parameter:

$$\Delta y_t = a_0 + a_1 \Delta y_{t-1} + a_2 \Delta y_{t-2} + a_3 \Delta \varepsilon_{t-1} + a_4 \Delta \varepsilon_{t-2} + \varepsilon_t$$

Let's decompose this process. First, we'll perform the Dickey-Fuller test on the GDP ts.

```
adf.test(pib)
```

```
## Warning in adf.test(pib): p-value greater than printed p-value
##
## Augmented Dickey-Fuller Test
##
## data:  pib
## Dickey-Fuller = 0.12028, Lag order = 4, p-value = 0.99
## alternative hypothesis: stationary
```

As we have not been able to reject H_0 , it follows that the ts includes (at least one) unit root. Now, let's find the optimal model for the regression.

```
max_p <- 5
max_q <- 5
max_d <- 2

models1 <- vector("list", (max_p + 1) * (max_q + 1))
models2 <- vector("list", (max_p + 1) * (max_q + 1))

# Updating the model

fit1 <- vector("list", (max_p + 1) * (max_q + 1))
fit2 <- vector("list", (max_p + 1) * (max_q + 1))

model_info1 <- data.frame(matrix(NA, nrow = ((max_p + 1) * (max_q + 1)), ncol = 3))

# Updating the model
```

```

for (u in 0:max_q) {
  for (j in 0:max_p) {

    fit1[(((max_p + 1) * j) + u + 1)] <- Arima(pib, order = c(j,
      1, u))

    model_info1[(((max_p + 1) * j) + u + 1), 1:2] <- c(fit1[(((max_p +
      1) * j) + u + 1)]$aic, fit1[(((max_p + 1) * j) +
      u + 1)]$bic)

  }
}

names(model_info1) <- c("AIC", "BIC", "void")

which.min(model_info1$AIC)

## [1] 23

which.min(model_info1$BIC)

## [1] 15

fit1[[which.min(model_info1$AIC)]]

## Series: pib
## ARIMA(3,1,4)
##
## Coefficients:
##      ar1      ar2      ar3      ma1      ma2      ma3      ma4
##    -0.9688 -0.9834 -0.9685  1.4659  1.3755  1.2705  0.5008
## s.e.   0.0305   0.0210   0.0270  0.1344  0.1682  0.1821  0.1484
##
## sigma^2 estimated as 21987482:  log likelihood=-956.26
## AIC=1928.53  AICc=1930.17  BIC=1949.13

fit1[[which.min(model_info1$BIC)]]

## Series: pib
## ARIMA(2,1,2)
##
## Coefficients:
##      ar1      ar2      ma1      ma2
##    -0.0084 -0.9757  0.2769  0.9673
## s.e.   0.0236   0.0175  0.0408  0.1130
##
## sigma^2 estimated as 23692560:  log likelihood=-962.3
## AIC=1934.6  AICc=1935.26  BIC=1947.47

```

```

# For I(2)

model_info2 <- data.frame(matrix(NA, nrow = max_d * ((max_p +
  1) * (max_q + 1)), ncol = 3))

for (u in 0:max_q) {
  for (j in 0:max_p) {

    fit2[(((max_p + 1) * j) + u + 1)] <- Arima(pib, order = c(j,
      2, u))

    model_info2[(((max_p + 1) * j) + u + 1), 1:2] <- c(fit2[(((max_p +
      1) * j) + u + 1)]$aic, fit2[(((max_p + 1) * j) +
      u + 1)]$bic)

  }
}

names(model_info2) <- c("AIC", "BIC", "void")

which.min(model_info2$AIC)

## [1] 24

which.min(model_info2$BIC)

## [1] 16

fit2[[which.min(model_info2$AIC)]]

## Series: pib
## ARIMA(3,2,5)
##
## Coefficients:
##          ar1      ar2      ar3      ma1      ma2      ma3      ma4      ma5
##      -0.9700 -0.9835 -0.9674  0.4748  0.0162 -0.0121 -0.6534 -0.3766
## s.e.   0.0315   0.0218   0.0277  0.1511  0.1211   0.1508   0.1090   0.1793
##
## sigma^2 estimated as 22148502: log likelihood=-946.73
## AIC=1911.47  AICc=1913.56  BIC=1934.55

fit2[[which.min(model_info2$BIC)]]

## Series: pib
## ARIMA(2,2,3)
##
## Coefficients:
##          ar1      ar2      ma1      ma2      ma3
##      -0.0054 -0.9795 -0.6754  0.6720 -0.7903
## s.e.   0.0248   0.0162   0.1121  0.0786   0.1482
##
## sigma^2 estimated as 23858414: log likelihood=-951.82
## AIC=1915.63  AICc=1916.58  BIC=1931.02

```