FERROMAGNETIC

Chain

$$\pm 2 \text{ JS } (1 - \cos(x)) \tag{1}$$

Square

$$\pm 4 \text{ JS } (2 - \cos(x) + \cos(y)) \tag{2}$$

Simple Cube

$$\pm 4 \text{ JS } (3 - \cos(x) - \cos(y) - \cos(z)) \tag{3}$$

Face-Centered Cube

$$\pm 8 \text{ JS } (3 - \cos(x)\cos(y) - \cos(x)\cos(z) - \cos(y)\cos(z)) \tag{4}$$

Body-Centered Cube

$$\pm 16 \text{ JS } (1 - \cos(x)\cos(y)\cos(z)) \tag{5}$$

ANTIFERROMAGNETIC

Chain

$$\pm 2 \text{ JS } \sin(x) \tag{6}$$

Square

$$\pm 4 \text{ JS } \sqrt{(2 - \cos(x) - \cos(y))(2 + \cos(x) + \cos(y))} \tag{7}$$

Simple Cube

$$\pm 4 \text{ JS } \sqrt{(3 - \cos(x) - \cos(y) - \cos(z))(3 + \cos(x) + \cos(y) + \cos(z))} \tag{8}$$

Face-Centered Cube

$$\pm 8 \text{ JS } \sqrt{(3 - \cos(y)\cos(z) - \cos(x)(\cos(y) + \cos(z)))(3 + \cos(y)\cos(z) + \cos(x)(\cos(y) + \cos(z)))} \tag{9}$$

Body-Centered Cube

$$\pm 16 \text{ JS } \sqrt{1 - \cos^2(x)\cos^2(y)\cos^2(z)} \tag{10}$$

Name	Formula
Ferromagnetic	
Chain	± 2 JS $(1 - \cos(x))$
Square	± 4 JS $(2 - \cos(x) + \cos(y))$
Simplie Cube	± 4 JS $(3 - \cos(x) - \cos(y) - \cos(z))$
Face-Centered Cube	± 8 JS $(3 - \cos(x)\cos(y) - \cos(x)\cos(z) - \cos(y)\cos(z))$
Body-Centered Cube	± 16 JS $(1 - \cos(x)\cos(y)\cos(z))$
Antiferromagnetic	
Chain	± 2 JS $\sin(x)$
Square	± 4 JS $\sqrt{(2 - \cos(x) - \cos(y))(2 + \cos(x) + \cos(y))}$
Simplie Cube	± 4 JS $\sqrt{(3 - \cos(x) - \cos(y) - \cos(z))(3 + \cos(x) + \cos(y) + co}$
Face-Centered Cube	± 8 JS $\sqrt{(3 - \cos(y)\cos(z) - \cos(x)(\cos(y) + \cos(z)))(3 + \cos(y)\cos(z) + \cos(x}$
Body-Centered Cube	± 16 JS $\sqrt{1 - \cos^2(x)\cos^2(y)\cos^2(z)}$