Maxwell’s equations along with constitutive relations in the time-harmonic case is given by



 represents source electric current density while  is an extra quantity which will be zero for any physical system (can be interpreted as a source magnetic current density).

Two fields  and  taken from two separate systems with possibly different material parameters have the following relation



Substituting in the time-harmonic Maxwell’s equations yields



The general form of the sensitivity equation is derived by considering three systems. The first system A is the system of interest with specific source electric current density. The second system Aδ is the same as A but with material properties perturbed by a small amount. The third system B is an auxiliary system that must be solved to compute the sensitivity. The fields and material properties for the three systems are listed in the following table.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| A | **E**A | **H**A | *σ* | *ε* | *µ* | **J***s*A | **0** |
| Aδ | **E**A**+**δ**E**A | **H**A**+**δ**H**A | *σ+*δ*σ* | *ε+*δ*ε* | *µ+*δ*µ* | **J***s*A | **0** |
| B | **E**B | **H**B | *σ* | *ε* | *µ* | **J***s*B | **L***s*B |

The first substitution sets system 1 quantities in equal to those of system Aδ and system 2 as system B. The result is



Subtracting from the same equation with perturbations set to zero and neglecting higher-order terms yields



The second substitution sets system 1 as system B and system 2 as system Aδ. Again, subtracting zero perturbation and neglecting higher-order terms yields



Finally, subtracting from and rearranging yields



The surface integral in vanishes by setting appropriate boundary conditions for system B.

Consider the measured quantity to be of the integral form



A perturbation of the measured quantity is linearly approximated as



Comparing and reveals that by setting  and , the sensitivities are



In our case, the measured quantity is magnetic flux density.



where  is the Dirac-delta function centered at the position of the sensor and  is the sensing axis of the sensor. Thus, sources in system B should be set to



In actual numerical computations, the Dirac-delta function can be approximated by an adequately small volume.