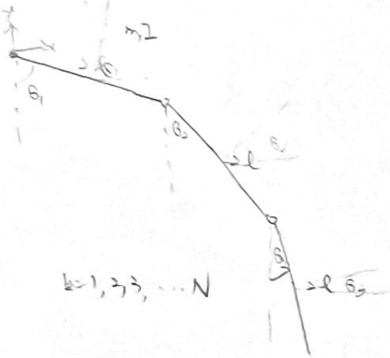

$k = 1, 2, 3, \ldots N$

$$\vec{V}_1 = l\dot{\theta}_1 \cos\theta_1\, \hat{i} + l\dot{\theta}_1 \sin\theta_1\, \hat{j}$$

$$\vec{V}_2 = 2l\dot{\theta}_1 \cos\theta_1\, \hat{i} + 2l\dot{\theta}_1 \sin\theta_1\, \hat{j} + l\dot{\theta}_2 \cos\theta_2 + l\dot{\theta}_2 \sin\theta_2$$

$$= (2l\dot{\theta}_1 \cos\theta_1 + l\dot{\theta}_2 \cos\theta_2)\, \hat{i} + (2l\dot{\theta}_1 \sin\theta_1 + l\dot{\theta}_2 \sin\theta_2)\, \hat{j}$$

$$\vec{V}_3 = (2l\dot{\theta}_1 \cos\theta_1 + 2l\dot{\theta}_2 \cos\theta_2 + l\dot{\theta}_3 \cos\theta_3)\, \hat{i} + (2l\dot{\theta}_1 \sin\theta_1 + 2l\dot{\theta}_2 \sin\theta_2 + l\dot{\theta}_3 \sin\theta_3)\, \hat{j}$$

$$\vec{V}_k = \left(\sum_{i=1}^{k-1} 2l\dot{\theta}_i \cos\theta_i + l\dot{\theta}_k \cos\theta_k\right)\hat{i} + \left(\sum_{i=1}^{k-1} 2l\dot{\theta}_i \sin\theta_i + l\dot{\theta}_k \sin\theta_k\right)\hat{j}$$

$$T = \sum_{k=1}^{N} \frac{1}{2} I \dot{\theta}_k^2 + \sum_{k=1}^{N} \frac{1}{2} m \left[\left(\sum_{i=1}^{k-1} 2l\dot{\theta}_i \cos\theta_i + l\dot{\theta}_k \cos\theta_k\right)^2 + \left(\sum_{i=1}^{k-1} 2l\dot{\theta}_i \sin\theta_i + l\dot{\theta}_k \sin\theta_k\right)^2 \right]$$

$$y_1 = -l\cos\theta_1$$

$$y_2 = -2l\cos\theta_1 - l\cos\theta_2$$

$$y_3 = -2l\cos\theta_1 - 2l\cos\theta_2 - l\cos\theta_3$$

$$y_k = \sum_{i=1}^{k-1} -2l\cos\theta_i - l\cos\theta_k$$

$$V = \sum_{k=1}^{N} mg\left(\sum_{i=1}^{k-1} -2l\cos\theta_i - l\cos\theta_k\right)$$

$$L = T - V$$

$$\frac{\partial T}{\partial \dot\theta_j} = \sum_{k=1}^{N} I \dot\theta_k \delta_{jk} + \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \cos\theta_i + l \dot\theta_k \cos\theta_k \right) \left(\sum_{i=1}^{k-1} 2 l \cos\theta_i \delta_{ij} + l \cos\theta_k \delta_{jk} \right)$$

$$+ \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \sin\theta_i + l \dot\theta_k \sin\theta_k \right) \left(\sum_{i=1}^{k-1} 2 l \sin\theta_i \delta_{ij} + l \sin\theta_k \delta_{jk} \right)$$

$$\cancel{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_j}\right) = \sum_{k=1}^{N} I \ddot\theta_k \delta_{jk} + \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} (2 l \ddot\theta_i \cos\theta_i - 2 l \dot\theta_i^2 \sin\theta_i) + (l \ddot\theta_k \cos\theta_k - l \dot\theta_k^2 \sin\theta_k) \right) \left(\sum_{i=1}^{k-1} -2 l \sin\theta_i \dot\theta_i \delta_{ij} - l \sin\theta_k \dot\theta_k \delta_{jk} \right)}$$

$$\frac{\partial T}{\partial \theta_j} = \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \cos\theta_i + l \dot\theta_k \cos\theta_k \right) \left(\sum_{i=1}^{k-1} -2 l \dot\theta_i \sin\theta_i \delta_{ij} - l \dot\theta_k \sin\theta_k \delta_{kj} \right)$$

$$+ \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \sin\theta_i + l \dot\theta_k \sin\theta_k \right) \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \cos\theta_i \delta_{ij} + l \dot\theta_k \cos\theta_k \delta_{kj} \right)$$

$$+ \sum_{k=1}^{N} m g \left(\sum_{i=1}^{k-1} -2 l \sin\theta_i \delta_{ij} - l \sin\theta_k \delta_{kj} \right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_j}\right) - \frac{\partial T}{\partial \theta_j} = 0 \quad \forall j = 1, 2, \dots, N$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_j}\right) = \sum_{k=1}^{N} I \ddot\theta_k \delta_{jk} + \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} (2 l \ddot\theta_i \cos\theta_i - 2 l \dot\theta_i^2 \sin\theta_i) + (l \ddot\theta_k \cos\theta_k - l \dot\theta_k^2 \sin\theta_k) \right) \left(\sum_{i=1}^{k-1} 2 l \cos\theta_i \delta_{ij} + l \cos\theta_k \delta_{jk} \right)$$

$$+ \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \cos\theta_i + l \dot\theta_k \cos\theta_k \right) \left(\sum_{i=1}^{k-1} -2 l \sin\theta_i \dot\theta_i \delta_{ij} - l \sin\theta_k \dot\theta_k \delta_{jk} \right)$$

$$+ \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} (2 l \ddot\theta_i \sin\theta_i + 2 l \dot\theta_i^2 \cos\theta_i) + (l \ddot\theta_k \sin\theta_k + l \dot\theta_k^2 \cos\theta_k) \right) \left(\sum_{i=1}^{k-1} 2 l \sin\theta_i \delta_{ij} + l \sin\theta_k \delta_{jk} \right)$$

$$+ \sum_{k=1}^{N} m \left(\sum_{i=1}^{k-1} 2 l \dot\theta_i \sin\theta_i + l \dot\theta_k \sin\theta_k \right) \left(\sum_{i=1}^{k-1} 2 l \cos\theta_i \dot\theta_i \delta_{ij} + l \cos\theta_k \dot\theta_k \delta_{kj} \right)$$

$j=j$

$j \leq N-2$

$$I\ddot{\theta}_j + \sum_{k=j+1}^{N} \Big\{ m\Big(\sum_{i=1}^{k-1}(2l\ddot{\theta}_i\cos\theta_i - 2l\dot{\theta}_i^2\sin\theta_i) + (l\ddot{\theta}_k\cos\theta_k - l\dot{\theta}_k^2\sin\theta_k)\Big)(2l\cos\theta_j) + m\Big(\sum_{i=1}^{k-1}2l\dot{\theta}_i\cos\theta_i + l\dot{\theta}_k\cos\theta_k\Big)(-2l\sin\theta_j\,\dot{\theta}_j)$$

$$+ m\Big(\sum_{i=1}^{k-1}(2l\ddot{\theta}_i\sin\theta_i + 2l\dot{\theta}_i^2\cos\theta_i) + (l\ddot{\theta}_k\sin\theta_k + l\dot{\theta}_k^2\cos\theta_k)\Big)(2l\sin\theta_j) + m\Big(\sum_{i=1}^{k-1}2l\dot{\theta}_i\sin\theta_i + l\dot{\theta}_k\sin\theta_k\Big)(2l\cos\theta_j\,\dot{\theta}_j)$$

$$- m\Big(\sum_{i=1}^{k-1}2l\dot{\theta}_i\cos\theta_i + l\dot{\theta}_k\cos\theta_k\Big)(-2l\dot{\theta}_j\sin\theta_j) - m\Big(\sum_{i=1}^{k-1}2l\dot{\theta}_i\sin\theta_i + l\dot{\theta}_k\sin\theta_k\Big)(2l\dot{\theta}_j\cos\theta_j) - mg(-2l\sin\theta_j)\Big\}$$

$$+ m\Big(\sum_{i=1}^{j-1}(2l\ddot{\theta}_i\cos\theta_i - 2l\dot{\theta}_i^2\sin\theta_i) + (l\ddot{\theta}_j\cos\theta_j - l\dot{\theta}_j^2\sin\theta_j)\Big)(l\cos\theta_j) + m\Big(\sum_{i=1}^{j-1}2l\dot{\theta}_i\cos\theta_i + l\dot{\theta}_j\cos\theta_j\Big)(-l\sin\theta_j\,\dot{\theta}_j)$$

$$+ m\Big(\sum_{i=1}^{j-1}(2l\ddot{\theta}_i\sin\theta_i + 2l\dot{\theta}_i^2\cos\theta_i) + (l\ddot{\theta}_j\sin\theta_j + l\dot{\theta}_j^2\cos\theta_j)\Big)(l\sin\theta_j) + m\Big(\sum_{i=1}^{j-1}2l\dot{\theta}_i\sin\theta_i + l\dot{\theta}_j\sin\theta_j\Big)(l\cos\theta_j\,\dot{\theta}_j)$$

$$- m\Big(\sum_{i=1}^{j-1}2l\dot{\theta}_i\cos\theta_i + l\dot{\theta}_j\cos\theta_j\Big)(-l\dot{\theta}_j\sin\theta_j) - m\Big(\sum_{i=1}^{j-1}2l\dot{\theta}_i\sin\theta_i + l\dot{\theta}_j\sin\theta_j\Big)(l\dot{\theta}_j\cos\theta_j) - mg(-l\sin\theta_j) = 0$$

$$\Rightarrow I\ddot{\theta}_j + \sum_{k=j+1}^{N}\Big\{ m\sum_{i=1}^{k-1}4l^2\ddot{\theta}_i\cos\theta_i\cos\theta_j - m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i^2\sin\theta_i\cos\theta_j + m2l^2\ddot{\theta}_k\cos\theta_k\cos\theta_j - m2l\dot{\theta}_k^2\sin\theta_k\cos\theta_j \cancel{- m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i\dot{\theta}_j\cos\theta_i\sin\theta_j} \cancel{- m2l^2\dot{\theta}_k\dot{\theta}_j\cos\theta_k\sin\theta_j}$$

$$+ m\sum_{i=1}^{k-1}4l^2\ddot{\theta}_i\sin\theta_i\sin\theta_j + m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i^2\cos\theta_i\sin\theta_j + m2l^2\ddot{\theta}_k\sin\theta_k\sin\theta_j + m2l\dot{\theta}_k^2\cos\theta_k\sin\theta_j \cancel{+ m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i\dot{\theta}_j\sin\theta_i\cos\theta_j} \cancel{+ m2l^2\dot{\theta}_k\dot{\theta}_j\sin\theta_k\cos\theta_j}$$

$$\cancel{+ m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i\dot{\theta}_j\cos\theta_i\sin\theta_j} \cancel{+ m2l^2\dot{\theta}_k\dot{\theta}_j\cos\theta_k\sin\theta_j} \cancel{- m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i\dot{\theta}_j\sin\theta_i\cos\theta_j} \cancel{- m2l^2\dot{\theta}_k\dot{\theta}_j\sin\theta_k\cos\theta_j} + 2mgl\sin\theta_j\Big\}$$

$$+ m\sum_{i=1}^{j-1}2l^2\ddot{\theta}_i\cos\theta_i\cos\theta_j - m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i^2\sin\theta_i\cos\theta_j + ml^2\ddot{\theta}_j\cos\theta_j\cos\theta_j \cancel{- l\dot{\theta}_j^2\sin\theta_j\cos\theta_j} \cancel{- m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i\dot{\theta}_j\cos\theta_i\sin\theta_j} \cancel{- ml^2\dot{\theta}_j^2\cos\theta_j\sin\theta_j}$$

$$+ m\sum_{i=1}^{j-1}2l^2\ddot{\theta}_i\sin\theta_i\sin\theta_j + m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i^2\cos\theta_i\sin\theta_j + ml^2\ddot{\theta}_j\sin\theta_j\sin\theta_j \cancel{+ l^2\dot{\theta}_j^2\cos\theta_j\sin\theta_j} + m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i\dot{\theta}_j\sin\theta_i\cos\theta_j \cancel{+ ml^2\dot{\theta}_j^2\sin\theta_j\cos\theta_j}$$

$$\cancel{+ m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i\dot{\theta}_j\cos\theta_i\sin\theta_j} \cancel{+ ml^2\dot{\theta}_j^2\cos\theta_j\sin\theta_j} \cancel{- m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i\dot{\theta}_j\sin\theta_i\cos\theta_j} \cancel{- ml^2\dot{\theta}_j^2\sin\theta_j\cos\theta_j} + mgl\sin\theta_j = 0$$

$$\Rightarrow I\ddot{\theta}_j + \sum_{k=j+1}^{N}\Big\{ m\sum_{i=1}^{k-1}4l^2\ddot{\theta}_i\cos(\theta_i-\theta_j) + m2l^2\ddot{\theta}_k\cos(\theta_k-\theta_j) - m\sum_{i=1}^{k-1}4l^2\dot{\theta}_i^2\sin(\theta_i-\theta_j) - m2l^2\dot{\theta}_k^2\sin(\theta_k-\theta_j) + 2mgl\sin\theta_j\Big\}$$

$$+ m\sum_{i=1}^{j-1}2l^2\ddot{\theta}_i\cos(\theta_i-\theta_j) + ml^2\ddot{\theta}_j - m\sum_{i=1}^{j-1}2l^2\dot{\theta}_i^2\sin(\theta_i-\theta_j) + mgl\sin\theta_j$$

$$A\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \vdots\\ \vdots\\ \ddot{\theta}_N\end{bmatrix} + \underset{sum}{\overset{B}{\Longrightarrow}} = 0$$

$j=N$

a_{N1}	a_{N2}	$a_{N(N-3)}$	$a_{N(N-2)}$	$a_{N(N-1)}$	a_{NN}	
$2m\ell^2\cos(\theta_1-\theta_N)$	$2m\ell^2\cos(\theta_2-\theta_N)$	$2m\ell^2\cos(\theta_{N-3}-\theta_N)$	$2m\ell^2\cos(\theta_{N-2}-\theta_N)$	$2m\ell^2\cos(\theta_{N-1}-\theta_N)$	$I+m\ell^2$	
b_{N1}	b_{N2}	$b_{N(N-3)}$	$b_{N(N-2)}$	$b_{N(N-1)}$	b_{NN}	
$-2m\ell^2\dot{\theta}_1^2\sin(\theta_1-\theta_N)$	$-2m\ell^2\dot{\theta}_2^2\sin(\theta_2-\theta_N)$	$-2m\ell^2\dot{\theta}_{N-3}^2\sin(\theta_{N-3}-\theta_N)$	$-2m\ell^2\dot{\theta}_{N-2}^2\sin(\theta_{N-2}-\theta_N)$	$-2m\ell^2\dot{\theta}_{N-1}^2\sin(\theta_{N-1}-\theta_N)$	0	$+mg\ell\sin\theta_N$

$j=N-1$

$a_{(N-1)1}$	$a_{(N-1)2}$	$a_{(N-1)(N-3)}$	$a_{(N-1)(N-2)}$	$a_{(N-1)(N-1)}$	$a_{(N-1)N}$	
$2m\ell^2\cos(\theta_1-\theta_{N-1})$	$2m\ell^2\cos(\theta_2-\theta_{N-1})$	$2m\ell^2\cos(\theta_{N-3}-\theta_{N-1})$	$2m\ell^2\cos(\theta_{N-2}-\theta_{N-1})$	$I+m\ell^2$	0	
$4m\ell^2\cos(\theta_1-\theta_{N-1})$	$4m\ell^2\cos(\theta_2-\theta_{N-1})$	$4m\ell^2\cos(\theta_{N-3}-\theta_{N-1})$	$4m\ell^2\cos(\theta_{N-2}-\theta_{N-1})$	$4m\ell^2$	$2m\ell^2\cos(\theta_N-\theta_{N-1})$	
$b_{(N-1)1}$	$b_{(N-1)2}$	$b_{(N-1)(N-3)}$	$b_{(N-1)(N-2)}$	$b_{(N-1)(N-1)}$	$b_{(N-1)N}$	
$-2m\ell^2\dot{\theta}_1^2\sin(\theta_1-\theta_{N-1})$	$-2m\ell^2\dot{\theta}_2^2\sin(\theta_2-\theta_{N-1})$	$-2m\ell^2\dot{\theta}_{N-3}^2\sin(\theta_{N-3}-\theta_{N-1})$	$-2m\ell^2\dot{\theta}_{N-2}^2\sin(\theta_{N-2}-\theta_{N-1})$	0	0	$+mg\ell\sin\theta_{N-1}$
$-4m\ell^2\dot{\theta}_1^2\sin(\theta_1-\theta_{N-1})$	$-4m\ell^2\dot{\theta}_2^2\sin(\theta_2-\theta_{N-1})$	$-4m\ell^2\dot{\theta}_{N-3}^2\sin(\theta_{N-3}-\theta_{N-1})$	$-4m\ell^2\dot{\theta}_{N-2}^2\sin(\theta_{N-2}-\theta_{N-1})$	$-4m\ell^2\dot{\theta}_{N-1}^2\sin(\theta_{N-1}-\theta_N)$	$-2m\ell^2\dot{\theta}_N^2\sin(\theta_N-\theta_{N-1})$	$+2mg\ell\sin\theta_{N-1}$

$j=N-2$

$a_{(N-2)1}$	$a_{(N-2)2}$	$a_{(N-2)(N-3)}$	$a_{(N-2)(N-2)}$	$a_{(N-2)(N-1)}$	$a_{(N-2)N}$	
$2m\ell^2\cos(\theta_1-\theta_{N-2})$	$2m\ell^2\cos(\theta_2-\theta_{N-2})$	$2m\ell^2\cos(\theta_{N-3}-\theta_{N-2})$	$I+m\ell^2$	0	0	
$4m\ell^2\cos(\theta_1-\theta_{N-2})$	$4m\ell^2\cos(\theta_2-\theta_{N-2})$	$4m\ell^2\cos(\theta_{N-3}-\theta_{N-2})$	$4m\ell^2$	$2m\ell^2\cos(\theta_{N-1}-\theta_{N-2})$	0	
$4m\ell^2\cos(\theta_1-\theta_{N-2})$	$4m\ell^2\cos(\theta_2-\theta_{N-2})$	$4m\ell^2\cos(\theta_{N-3}-\theta_{N-2})$	$4m\ell^2$	$4m\ell^2\cos(\theta_{N-1}-\theta_{N-2})$	$2m\ell^2\cos(\theta_N-\theta_{N-2})$	
$b_{(N-2)1}$	$b_{(N-2)2}$	$b_{(N-2)(N-3)}$	$b_{(N-2)(N-2)}$	$b_{(N-2)(N-1)}$	$b_{(N-2)N}$	
$-2m\ell^2\dot{\theta}_1^2\sin(\theta_1-\theta_{N-2})$	$-2m\ell^2\dot{\theta}_2^2\sin(\theta_2-\theta_{N-2})$	$-2m\ell^2\dot{\theta}_{N-3}^2\sin(\theta_{N-3}-\theta_{N-2})$	0	0	0	$+mg\ell\sin\theta_{N-2}$
$-4m\ell^2\dot{\theta}_1^2\sin(\theta_1-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_2^2\sin(\theta_2-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_{N-3}^2\sin(\theta_{N-3}-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_{N-2}^2\sin(\theta_{N-2}-\theta_{N-2})$	$-2\ell^2\dot{\theta}_{N-1}^2\sin(\theta_{N-1}-\theta_{N-2})$	0	$+2mg\ell\sin\theta_{N-2}$
$-4m\ell^2\dot{\theta}_1^2\sin(\theta_1-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_2^2\sin(\theta_2-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_{N-3}^2\sin(\theta_{N-3}-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_{N-2}^2\sin(\theta_{N-2}-\theta_{N-2})$	$-4m\ell^2\dot{\theta}_{N-1}^2\sin(\theta_{N-1}-\theta_{N-2})$	$-2m\ell^2\dot{\theta}_N^2\sin(\theta_N-\theta_{N-2})$	$+2mg\ell\sin\theta_{N-2}$

$$(\Delta\theta)_{ij} = \theta_j - \theta_i$$

$$\underline{\underline{\Delta\theta}} = \begin{bmatrix} \theta_1-\theta_1 & \theta_2-\theta_1 & \cdots & \theta_{N-1}-\theta_1 & \theta_N-\theta_1 \\ \theta_1-\theta_2 & \theta_2-\theta_2 & & & \\ \vdots & & & & \\ & & & \theta_{N-1}-\theta_{N-1} & \theta_N-\theta_{N-1} \\ \theta_1-\theta_N & & & \theta_{N-1}-\theta_N & \theta_N-\theta_N \end{bmatrix} \qquad \underline{\underline{W}} = \begin{bmatrix} \dot{\theta}_1^2 & & & \\ & \dot{\theta}_2^2 & & \\ & & \ddots & \\ & & & \dot{\theta}_N^2 \end{bmatrix}$$

$$\underline{\underline{C}} = \begin{bmatrix} 18 & 18 & & 18 & 14 & 14 & 10 & 6 & 2 \\ 14 & 14 & & 14 & 14 & 10 & 10 & 6 & 2 \\ 10 & 10 & & 10 & 10 & 10 & 8 & 6 & 2 \\ 6 & 6 & & 6 & 6 & 6 & 6 & 4 & 2 \\ 2 & 2 & & 2 & 2 & 2 & 2 & 2 & 0 \end{bmatrix}$$

$$\left[ml^2\, \underline{\underline{C}} \odot \cos(\underline{\underline{\Delta\theta}}) + (I + ml^2)\, \underline{\underline{I}}\right]\left(\;\right) - ml^2\left(\underline{\underline{C}} \odot \sin(\underline{\underline{\Delta\theta}})\right)\begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \\ \vdots \\ \dot{\theta}_N^2 \end{bmatrix} + mgl \begin{bmatrix} 2N-1 \\ 2N-3 \\ \vdots \\ 5 \\ 3 \\ 1 \end{bmatrix} \sin\left(\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \end{bmatrix}\right) = 0$$

$$\left(ml^2 \begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & \cos(\theta_1-\theta_2) \\ \cos(\theta_1-\theta_2) & 1 \end{bmatrix} + \begin{bmatrix} I+ml^2 & 0 \\ 0 & I+ml^2 \end{bmatrix}\right)\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} - ml^2\left(\begin{bmatrix} 4 & 2 \\ 2 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & -\sin(\theta_1-\theta_2) \\ \sin(\theta_1-\theta_2) & 0 \end{bmatrix}\right)\begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} + mgl \begin{bmatrix} 3\sin\theta_1 \\ \sin\theta_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} I+5ml^2 & 2ml^2\cos(\theta_1-\theta_2) \\ 2ml^2\cos(\theta_1-\theta_2) & I+ml^2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} - \begin{bmatrix} -2ml^2\dot{\theta}_2^2\sin(\theta_1-\theta_2) \\ 2ml^2\dot{\theta}_1^2\sin(\theta_1-\theta_2) \end{bmatrix} + \begin{bmatrix} 3mgl\sin\theta_1 \\ mgl\sin\theta_2 \end{bmatrix} = 0$$