

natural number category
 $n \in \mathbb{N} \rightsquigarrow n$ objects: $\{0, 1, 2, \dots, n\}$

Morphism $i \rightarrow i + 1$

functor
simplicial set $X : \Delta^{\text{op}} \rightarrow \text{Sets}$, i.e. X_n is a set.

$$m \rightarrow n \rightsquigarrow X_n \rightarrow X_m$$

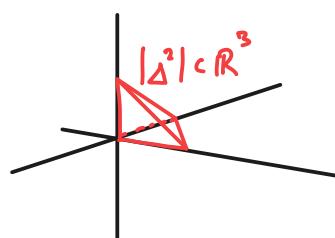
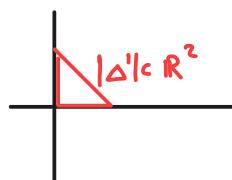
Ex. Consider $[n] \mapsto |\Delta^n| = \{(t_1, \dots, t_n) \in \mathbb{R}^{n+1} : \sum_{i=1}^n t_i = 1, t_i \geq 0\}$

functional: $f: [m] \rightarrow [n]$

$$\rightsquigarrow g_{\alpha}: |\beta^n| \rightarrow |\delta^n|$$

$$\Theta_{\pi}(t_0 \dots t_n) = (s_0 \dots s_n)$$

$$s_i = \sum_{t_j \in \theta^{-1}(i)} t_j.$$



T top. Space.

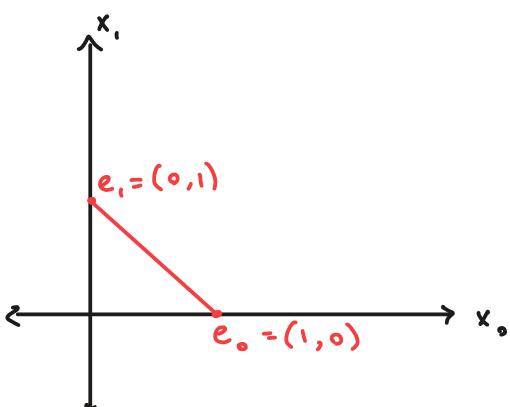
$$\text{hom}(\Delta^{\text{op}}, \text{Top}^{\text{op}}) \rightarrow \text{Set}$$

$$\text{So } h \circ k(x) \circ \theta : \Delta^{\circ 1} \rightarrow \text{Sets}.$$

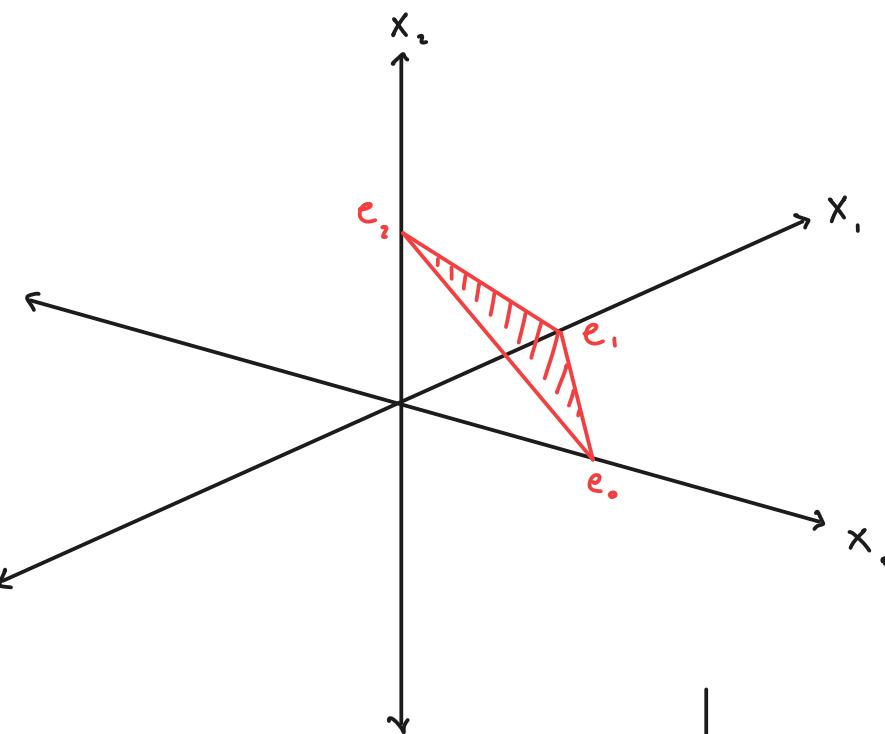
Δ_0

$$\longleftrightarrow \quad c_0 = 1$$

$$I^{\text{th}} \text{ face of } \Delta_0 = \begin{cases} \emptyset & I = \emptyset \\ \Delta_0 & I = \{0\} \end{cases}$$

 Δ_1 

$$I^{\text{th}} \text{ face of } \Delta_1 = \begin{cases} \emptyset & I = \emptyset \\ \{e_0\} \cong \Delta_0 & I = \{0\} \\ \{e_1\} \cong \Delta_0 & I = \{1\} \\ \Delta_1 & I = \{0,1\} \end{cases}$$

 Δ_2 

$$I^{\text{th}} \text{ face of } \Delta_2 \cong \begin{cases} \emptyset & I = \emptyset \\ \Delta_0 & |I| = 1 \\ \Delta_1 & |I| = 2 \\ \Delta_2 & |I| = 3 \end{cases}$$

$$\begin{aligned} I &= \text{im}(f) \\ f: [m] &\longrightarrow [n] \text{ order-pres.} \\ &\Downarrow \\ \Delta_f: \Delta_m &\longrightarrow \Delta_n \\ (x_0, \dots, x_m) &\longmapsto (y_0, \dots, y_n) \\ \text{where } y_i &= \sum_{j \in f^{-1}(i)} x_j. \end{aligned}$$

- gluing data:
- $X_{(n)}$ n-simplices for each i .
 - for $f: [m] \longrightarrow [n]$ functor,
 - $XF: X_{(n)} \longrightarrow X_{(m)}$ functorial.