To the University of Toronto Department of Mathematics,

I am writing to apply for a USRA, UTEA, or MDRA in summer 2026. I am completing my third year as a Mathematics Specialist and Computer Science Minor at the University of Toronto, with a GPA of ███ and an expected graduation date of June 2027. I was a Dean's List Scholar in 2023-24 and 2024-25, and have been awarded the ██████████████████████████████████ ████████████████████████████. I have notably completed MAT1300 Differential Topology and MAT1190 Algebraic Geometry I, and optimistically, I will have taken MAT1301 Algebraic Topology and Algebraic Geometry II by this summer.

My mathematical activities beyond coursework have included learning about topological K-theory as part of the department's Directed Reading Program in winter 2025, participating in a casual reading course in commutative algebra and homological algebra with Professor Jacob Tsimerman during summer 2025, investigating the representation theory of quantum groups at the 2025 Fields Undergraduate Summer Research Program under the supervision of Professor Hadi Salmasian at the University of Ottawa, leading a reading group on basic knot theory with the undergraduate mathematics student union at the level of Murasugi's book in fall 2025, and presenting a colloquium on September 4th, 2025 illustrating Smale's $h$-cobordism theorem and subsequent solution to the Poincaré conjecture in dimensions greater than or equal to $5$. It is worth mentioning that my research project at the 2025 FUSRP involved a nontrivial amount of coding in Mathematica to compute relations describing a quantization of the Weyl algebra and a quantum group acting on it. For a sample of my mathematical writing, please see my exposition of the Cohen–Seidenberg "going up and going down" theorems in commutative algebra, written at a level accessible to students of MAT347.

My mathematical interests range broadly across topology and geometry, concerning objects such as manifolds, schemes, Lie algebras and Lie groups, with an appetite for algebraic techniques and categorical language. I find knot theory to be an ideal sandbox for the application of algebraic tools to the study of low-dimensional manifolds, routinely supplying fascinating ideas such as the homology theories of Heegard–Floer and Khovanov; quantum groups and the Yang–Baxter equation; atoms and braids. I would love the opportunity to work directly with a frontier tool such as Professors Bar-Natan and van der Veen's $\theta$ invariant for knots, and I have discussed with Professor Dror Bar-Natan his project on *Knot Families and their $\theta$ Invariant*.

As an aspiring mathematician interested in the forces of algebra in low-dimensional topology, I would be humbled to contribute to mathematical research at the University of Toronto this summer. I believe my mathematical fluency, computer programming skills, and commitment to effective mathematical communication would prove invaluable to the projects in which I have indicated interest. Thank you for your consideration.

Sincerely,


William Gao