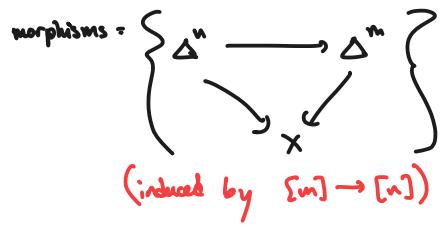


realization $| \cdot | : S \longrightarrow \text{Top}$

↓ cat of simplicial sets + natural transforms

cat. $\Delta^1 X$ of simplices of X : objs = $\{\sigma: \Delta^n \rightarrow X\}$



Also $X \cong \lim_{\substack{\longrightarrow \\ \Delta^n \rightarrow X \text{ in } \Delta^1 X}} \Delta^n$.

Why? \in small cat \Rightarrow functor $\mathcal{C} \rightarrow \text{Set}$ is colim of representable functors. \square

So we define $|X| := \lim_{\substack{\longrightarrow \\ \Delta^n \rightarrow X \text{ in } \Delta^1 X}} |\Delta^n|$

Prop. $| \cdot |$ is left adj to singular functor.

Pf. $\text{hom}_{\text{Top}}(|X|, Y) \cong \lim_{\substack{\longrightarrow \\ \Delta^n \rightarrow X}} \text{hom}_{\text{Top}}(|\Delta^n|, Y)$
 $\cong \lim_{\substack{\longrightarrow \\ \Delta^n \rightarrow X}} \text{hom}_S(\Delta^n, S(Y))$
 $\cong \text{hom}_S(X, S(Y))$.

Thus $| \cdot |$ preserves colimits (which always exist in S)

In fact $|X|$ is CW, hence CGHaus.