realization $|\cdot| : S \longrightarrow Top$

$\hookrightarrow$ cat of simplicial sets + natural transforms

cat. $\Delta\downarrow X$ of simplices of X : objs = $\{\sigma : \Delta^n \to X\}$

morphisms = $\left\{ \begin{tikzcd} \Delta^n \arrow[rr] \arrow[dr] & & \Delta^m \arrow[dl] \\ & X & \end{tikzcd} \right\}$

(induced by $[m] \to [n]$)

$\exists$ iso $X \cong \varinjlim_{\Delta^n \to X \text{ in } \Delta\downarrow X} \Delta^n$.

why? $\mathcal{C}$ small cat $\Rightarrow$ functor $\mathcal{C} \to Set$ is colim of representable functors. $\square$

So we define $|X| := \varinjlim_{\Delta^n \to X \text{ in } \Delta\downarrow X} |\Delta^n|$

Prop. $|\cdot|$ is left adj to singular functor.

Pf.
$$\hom_{Top}(|X|, Y) \cong \varinjlim_{\Delta^n \to X} \hom_{Top}(|\Delta^n|, Y)$$
$$\cong \varinjlim_{\Delta^n \to X} \hom_S(\Delta^n, S(Y))$$
$$\cong \hom_S(X, SY).$$

Thus $|\cdot|$ preserves colimits (which always exist in S)

In fact $|X|$ is CW, hence CGHaus.