

natural number $n \in \mathbb{N} \rightsquigarrow$ category \mathcal{M} objects: $\{0, 1, 2, \dots, n\}$

morphism $i \rightarrow i+1$

category Δ	objects	morphisms
	0	$n \rightarrow m$
	\vdots	$i \rightarrow j \Rightarrow \theta(i) \rightarrow \theta(j)$
	n	

functor $X: \Delta^p \rightarrow \text{Sets}$. i.e. X_n is a set.

$$m \rightarrow n \rightsquigarrow X_m \rightarrow X_n$$

ex. consider $[n] \mapsto |\Delta^n| = \{(t_0, \dots, t_n) \in \mathbb{R}^{n+1} : \sum t_i = 1, t_i \geq 0\}$

functorial: $\theta: [m] \rightarrow [n]$

$$\rightsquigarrow \theta_*: |\Delta^m| \rightarrow |\Delta^n|$$

$$\theta_*(t_0, \dots, t_m) = (s_0, \dots, s_n)$$

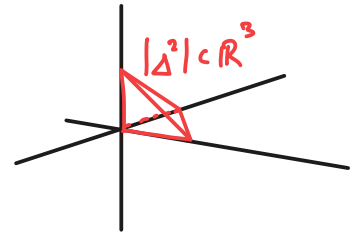
$$s_i = \sum_{t_j \in \theta^{-1}(i)} t_j$$

\mathcal{T} top. space.

$$\theta: \Delta^p \rightarrow \text{Top}^{\text{op}}$$

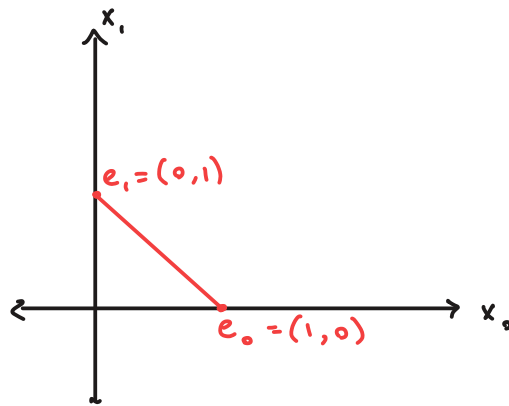
$$\text{hom}(_, \mathcal{T}): \text{Top}^{\text{op}} \rightarrow \text{Set}$$

$$\text{so } \text{hom}(\mathcal{T}) \circ \theta: \Delta^p \rightarrow \text{Sets}.$$

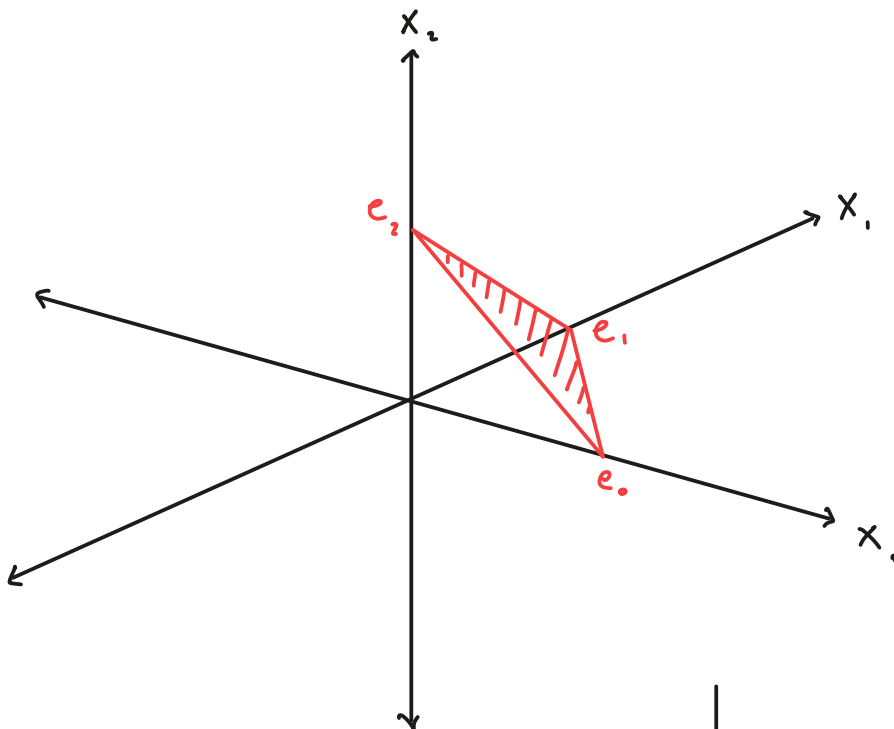


Δ_0 

$$I^{\text{th}} \text{ face of } \Delta_0 = \begin{cases} \emptyset & I = \emptyset \\ \Delta_0 & I = \{0\} \end{cases}$$

 Δ_1 

$$I^{\text{th}} \text{ face of } \Delta_1 = \begin{cases} \emptyset & I = \emptyset \\ \{e_0\} \cong \Delta_0 & I = \{0\} \\ \{e_1\} \cong \Delta_0 & I = \{1\} \\ \Delta_1 & I = \{0, 1\} \end{cases}$$

 Δ_2 

$$I^{\text{th}} \text{ face of } \Delta_2 \cong \begin{cases} \emptyset & I = \emptyset \\ \Delta_0 & |I| = 1 \\ \Delta_1 & |I| = 2 \\ \Delta_2 & |I| = 3 \end{cases}$$

$$\begin{aligned} I &= \text{im}(f) \\ f: [m] &\longrightarrow \hat{[n]} \text{ order-pres.} \\ \downarrow \\ \Delta_f: \Delta_m &\longrightarrow \Delta_n \\ (x_0 \dots x_m) &\longmapsto (y_0 \dots y_n) \\ \text{where } y_i &= \sum_{j \in f^{-1}(i)} x_j. \end{aligned}$$

glueing data: • $X_{(n)}$ n -simplices for each i .
 • for $f: [m] \longrightarrow [n]$ functor,
 $Xf: X_{(m)} \longrightarrow X_{(n)}$ functorial.