Quantum algorithm for solving differential equations

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We investigate a quantum algorithm due to Xin et al, 2020 for solving systems of linear differential equations. We formulate 3 elementary examples, and implement them in qiskit. We introduce the notion of success probability in the context of this algorithm. We point out that the problem of determining certain important operators in this algorithm (not addressed by Xin et al, 2020) can be regarded as a quantum state preparation problem. We expect that this algorithm does not have an advantage over classical algorithms.

Circuit 1

This circuit solves a simple homogeneous linear differential equation,

$$\frac{d\mathbf{y}(t)}{dt} = X\mathbf{y}(t), \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{1}$$

with the initial condition

$$\mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

Circuit 2

This circuit approximately solves the homogeneous linear differential equation,

$$\frac{d\mathbf{y}(t)}{dt} = R\mathbf{y}(t), \qquad R = Ry(\frac{2\pi}{5}) = \begin{pmatrix} \cos(\frac{\pi}{5}) & -\sin(\frac{\pi}{5}) \\ \sin(\frac{\pi}{5}) & \cos(\frac{\pi}{5}) \end{pmatrix} \tag{3}$$

and the same initial condition (2).

Circuit 3

This circuit solves the non homogeneous linear differential equation,

$$\frac{d\mathbf{y}(t)}{dt} = X\mathbf{y}(t) + \mathbf{b} \tag{4}$$

with

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

and the same initial condition (2)