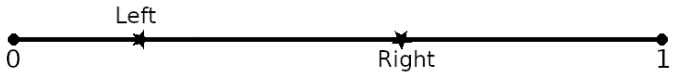
A Broken Stick Problem

A famous question goes like this:

> **Given a stick that is broken at two random locations, what is the probability that you can form a triangle with the three pieces?**

Let's explore this problem with spreadsheets. You can simulate breaking a unit-length stick with these steps:

1. Generate two random numbers, Break1 and Break2.
2. Determine the Left break and Right break:
   - Left = min(Break1, Break2)
   - Right = max(Break1, Break2)
3. The three lengths are:
   - L1 = Left
   - L2 = Right - Left
   - L3 = 1 - Right

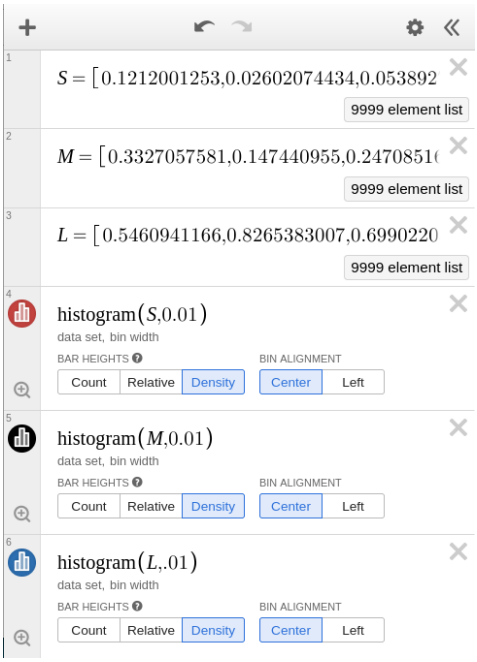


4. The shortest piece is min(L1,L2,L3), the longest pieces is max(L1,L2,L3), and the third piece has length of one minus those two lengths.

This can all be done in a single row, so you have something like:

	A	B	C	D	E	F	G	H	I	J	K
1	Break 1	Break 2	Left	Right	L1	L2	L3		Shortest Piece	Middle Piece	Longest Piece
2	0.9439487746	0.4860745245	0.4860745245	0.9439487746	0.4860745245	0.4578742502	0.05605122536		0.05605122536	0.4578742502	0.4860745245

Once you have this written with formulas, you can drag down for several thousands of rows. Select the values in the shortest piece column and copy-paste them into Desmos where you can create a density histogram. Do the same for the medium and long values and your formula panel should look something like this:



Work to be done

1. Record your observations of the histograms. What is the longest the shortest piece can be? What is the shortest the longest piece can be? Did you already know this would be the case? Include questions if you have them.

2. Find a probability density function (PDF) for each of the lengths: shortest, middle, and longest. These might be piecewise functions. Be sure to state the domain.

3. In order for a triangle to exist, the longest piece needs to be at least or at most some length (I can't remember). What is the constraint on the length of the longest piece so that a triangle is possible?

4. Using the PDF for the longest piece, determine the probability that the pieces can form a triangle.

5. You can generate a Monte Carlo estimation for the triangle probability in your spreadsheet. In a new column, use a formula with IF to check if the longest piece satisfies the constraint for a triangle. Place a 1 in the cell if that row has a triangle, and a 0 if it does not. Then find the sum of that column and determine the experimental probability of getting a triangle.

6. By now, you should have a very good idea about the probability that a broken stick can form a triangle. Can you prove this probability in another way?