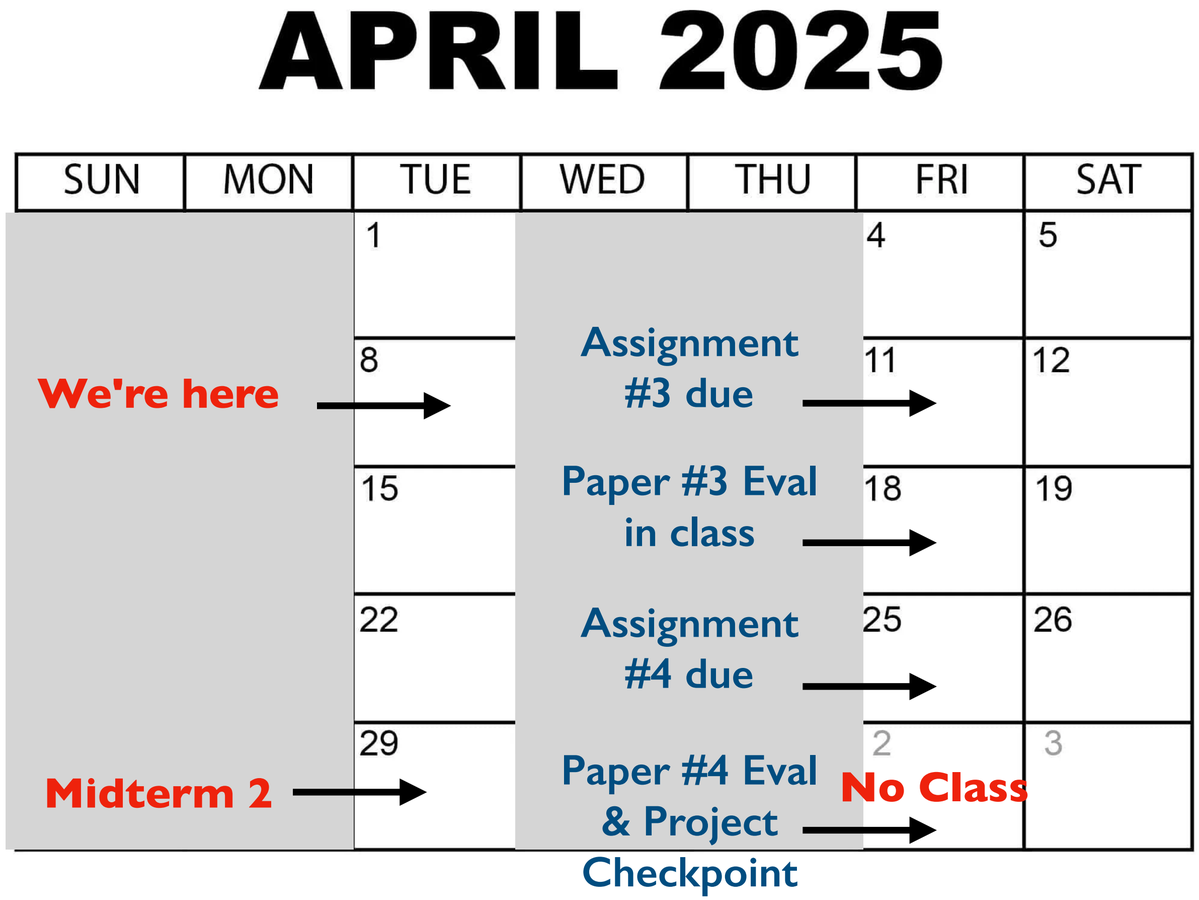
CSCI 357: Algorithmic Game Theory

Lecture 13: Voting & Social Choice II

Shikha Singh

Announcements and Logistics

- Pick up **HW 6**, due Tues April 16
- **Paper Eval #3** (partner assignment):
  - Meet with me to discuss topic of interest
  - Look over posted papers before the meeting
  - Read selected paper and write a paper report
  - Just this opportunity to build background for project



Questions?

ChatGPT Use Guidelines

- This is for Paper Eval #3/4 and Project:
  - You are **allowed to use ChatGPT or similar tools** to do "research" for finding papers and project topics
  - In particular, you can find it to find resources for topics, find related papers, etc
  - You can use it as a search tool to guide your creative thinking forward
  - You are **not allowed to use it to generate text** for your project report
    - The report should be in your words that describes your own findings
    - Use it like a helpful librarian/more powerful search tool
- ChatGPT use is not allowed for HWs and Assignments (which are meant for practice)

Recap: Plurality & Ranked-Choice

- Discussed plurality and ranked choice voting

November 2024

Arizona, Colorado, Idaho, Montana, Nevada, Oregon and South Dakota had ballot measures that would have replaced party primaries with nonpartisan contests and/or created a ranked choice voting system in their elections.

ELECTIONS

Ballot measures to upend state election systems failed across the country

NOVEMBER 8, 2024 · 4:30 PM ET

Ashley Lopez

The New York Times

Feb 2024

Some on the Right Flirt With a Voting Method the Left Loves

Ranked-choice voting could be on the November ballot in four states, a sign of the system's rising popularity. Most conservatives have opposed it. But some say that could be changing.

Share full article 297

Ranked-Choice Voting

- Not Condorcet consistent
- Consider an example with alternatives $A = \{a, b, c\}$ and 5 voters with votes
- Condorcet winner: b but get eliminated in the first round in ranked-choice

2 voters	1 voters	2 voters
a	b	c
b	c	b
c	a	a

Borda Count

- Well known voting rule: often used in sports, also used in Eurovision song contest
- Voters submit their full ranked lists: an alternate gets $|A|$ for each first-choice vote, $|A|-1$ points for each second-choice vote, and so on and 1 point for each last-choice vote
- Example:
  - a gets 15 points, b gets 12 points
  - c gets 10 points, d gets 13 points
- Borda count would elect a
  - In contrast to ranked-choice b

	Voters #1,2	Voters #3,4	Voter #5
1st Choice	a	b	c
2nd choice	d	a	d
3rd choice	c	d	b
4th choice	b	c	a

Borda Count

- **Question.** Is Borda count strategyproof?

Winner
b

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

Borda Count

- Is Borda count strategyproof?
  - **Idea**: incentive to rank closest competitor to preferred candidate last
- In example, what is the Borda score of a and b?
  - a's score: $2 \cdot 3 + 4 = 10$
  - b's score: $2 * 4 + 3 = 11$
- If voter 3 moves b to the last place
  - b's score: $8 + 1 = 9$

Winner
b

1	2	3
b	b	a
a	a	b
c	c	c
d	d	d

1	2	3
b	b	a
a	a	c
c	c	d
d	d	b

Winner
a

Borda Count

- **Question.** Does Borda count satisfy Condorcet criterion?
    - Question in next homework

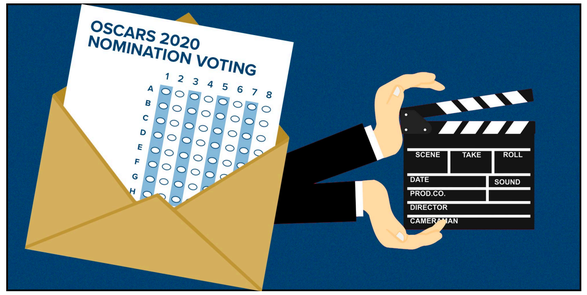
Positional Scoring Rules

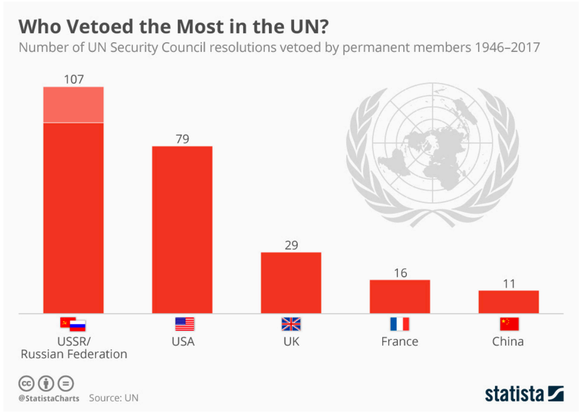
- In general, you can have different ways to score each position
- For each vote, a **positional-scoring rule** on $m = |A|$ alternatives assigns a score of α_j to the alternative ranked in jth place. The alternative with maximum total score (across all votes) is selected.
  - Assume $\alpha_1 \geq \alpha_2 \geq \ldots \alpha_m$ and $\alpha_1 > \alpha_m$
  - E.g., plurality gives 1 point for first-choice, zero for others
- Many positional scoring rules have been studied
  - You might see some on the homework/ papers you read

Comparison of preferential electoral systems [hide]

System	Mono-tonic	Condorcet winner	Majo-rity	Condorcet loser	Majority loser	Mutual majority	Smith	ISDA	LIIA	Independence of clones	Reversal symmetry	Participation, consistency	Later-no-harm	Later-no-help	Polynomial time	Resol-vability
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	No	Yes	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes
Tideman's Alternative	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No	Yes	No
Nanson	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	No	Yes	Yes
Black	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	Yes	No	No	No	Yes	Yes
Instant-runoff voting	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	Yes	Yes	Yes	Yes
Smith/IRV	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes	Yes
Borda	Yes	No	No	Yes	Yes	No	No	No	No	No	Yes	Yes	No	Yes	Yes	Yes
Baldwin	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes
Bucklin	Yes	No	Yes	No	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes
Plurality	Yes	No	Yes	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes
Contingent voting	No	No	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Coombs[4]	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No	Yes	Yes
MiniMax	Yes	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	Yes	Yes
Anti-plurality[4]	Yes	No	No	No	Yes	No	No	No	No	No	No	Yes	No	No	Yes	Yes
Sri Lankan contingent voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Supplementary voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Dodgson[4]	No	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	No	Yes

Many Rules, Many Applications





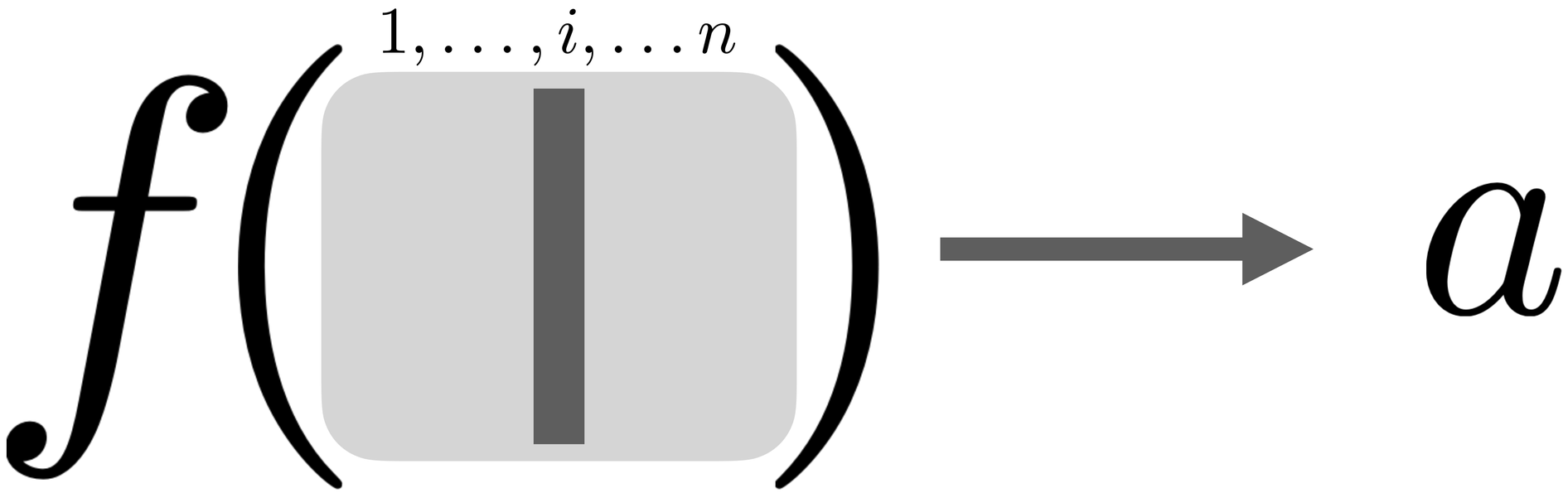
https://rohitvaish.in/Teaching/2022-Spring/Slides/Lec%202.pdf

One to Rule them All?

- For the same input profile, plurality, Borda and ranked-choice can all output a different winner!
    - Can you construct such an example?
- Changing the voting rule changes the outcome of the mechanism
- Leads to contention on which voting rule is the “best”
- Voting theorists have an "axiomatic" approach to study voting rules
- Identify "desirable" properties that one would like
- Compare rules based on that
- **Question**: Is there any voting rule that is strategyproof and reasonable?

Properties of Voting Rules

- **Onto**: For any candidate a, there exists an input profile where a wins



- Are Borda, plurality, ranked-choice etc onto?
    - Yes, can always construct a profile to make any candidate win

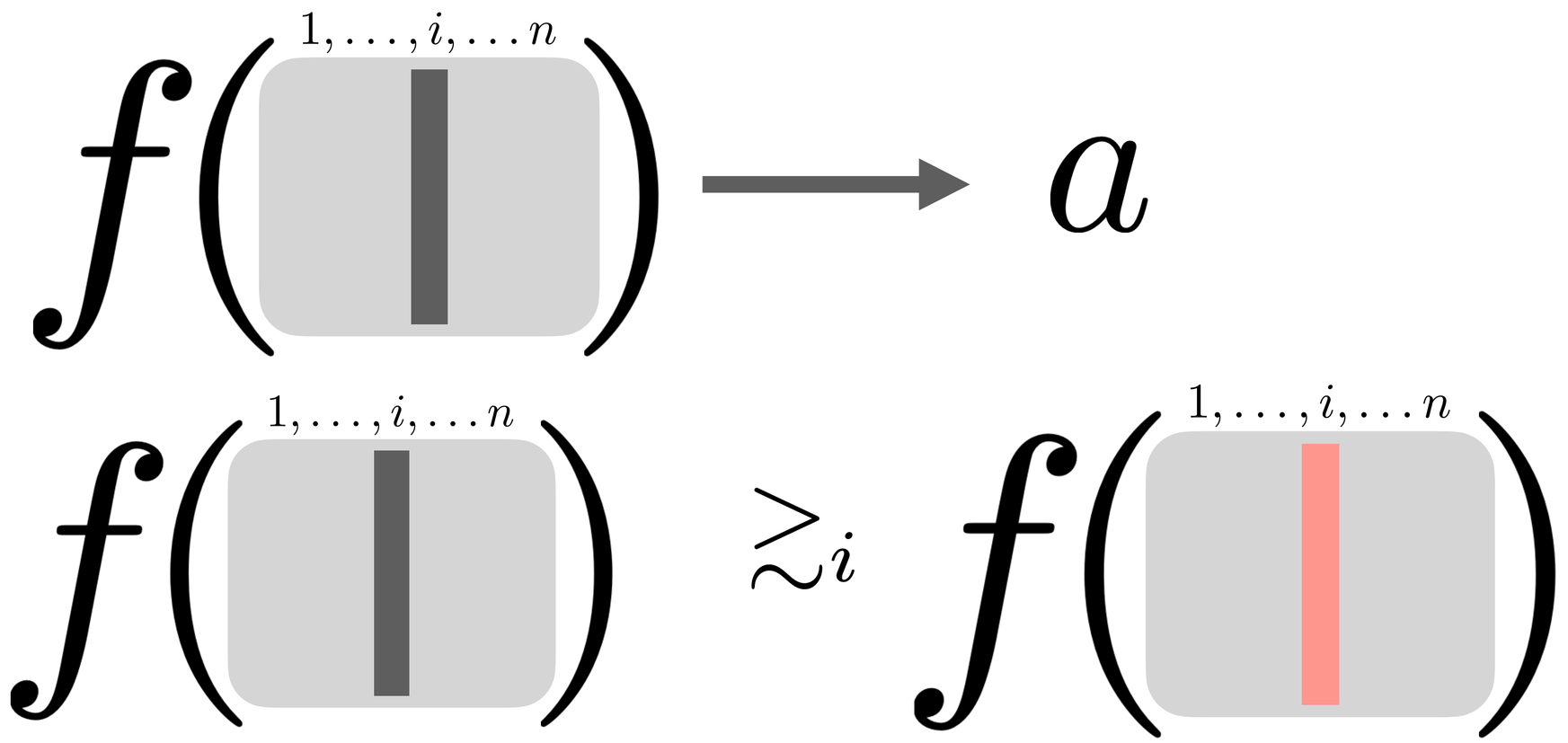
Properties of Voting Rules

- **Strategyproof**: No voter can improve by misreporting preferences

- Are Borda, plurality, ranked-choice etc strategyproof?
  - No

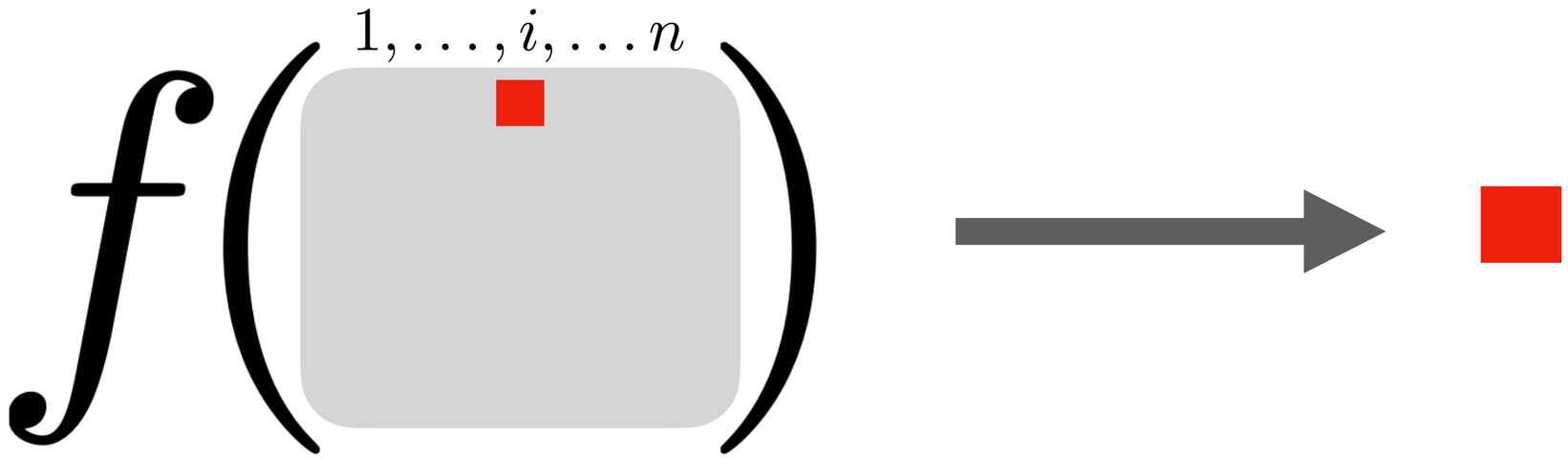
Onto and Strategyproof

- (3 or more alternatives) onto but not strategyproof? Borda, Plurality, Ranked-choice
- (3 or more alternatives) strategyproof **AND** onto?



A Bad Voting Rule

- **Dictatorship** : A voting rule is **dictatorial** if there is a voter i such that the rule always elects i's first choice (regardless of others' preferences)



- Is a dictatorship straregyproof?
- Is a dictatorship onto?

[Gibbard '73, Satterthwaite '75]

When there are 3 or more alternatives, a voting rule is strategyproof and onto if and only if it is dictatorial.

[GS Theorem] With three or more candidates, a voting rule is **strategyproof** and **onto** if and only if it is a **dictatorship**.

Goal. Strategyproof + Onto $\Longrightarrow$ Dictatorship

[Proof Outline]

Part 1. Strategyproof $\Longleftrightarrow$ Monotonicity

Part 2. Monotone + Onto $\Longrightarrow$ Unanimous

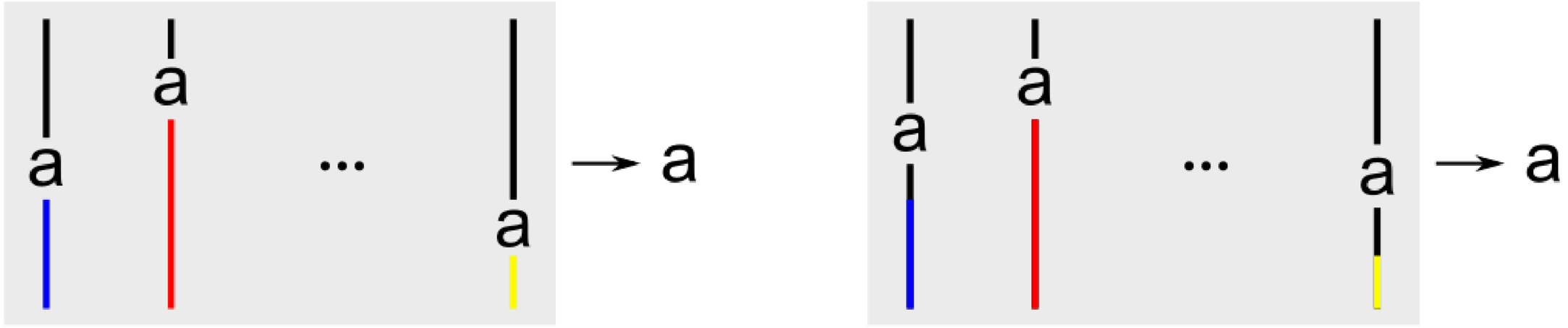
Part 3. Monotone + Unanimous $\Longrightarrow$ Dictatorship

Exposition

- https://rohitvaish.in/Teaching/2022-Spring/Slides/Lec%202.pdf

Monotonicity

- **Definition**. Suppose a is the current winner (on profile L). For all input profiles L', in which for all voters, any candidate who was ranked below a in L is still ranked below a in L', then a should continue to win in L'.
  - Support of a either increases or stays the same: a's outcome cannot get worse
- **Theorem**. Strategyproof $\iff$ monotone



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Strategyproof $\Longrightarrow$ Monotone

- Suppose a rule is strategyproof but not monotone
- Strategyproof means:
  - No voter can change their individual ranking to make a more preferred candidate win
- **Not monotone** means:
  - Suppose a is the current winner (on profile L). For all input profiles L', in which for all voters, any candidate who was ranked below a in L is still ranked below a in L', then it is still possible for another candidate b to win in L'.

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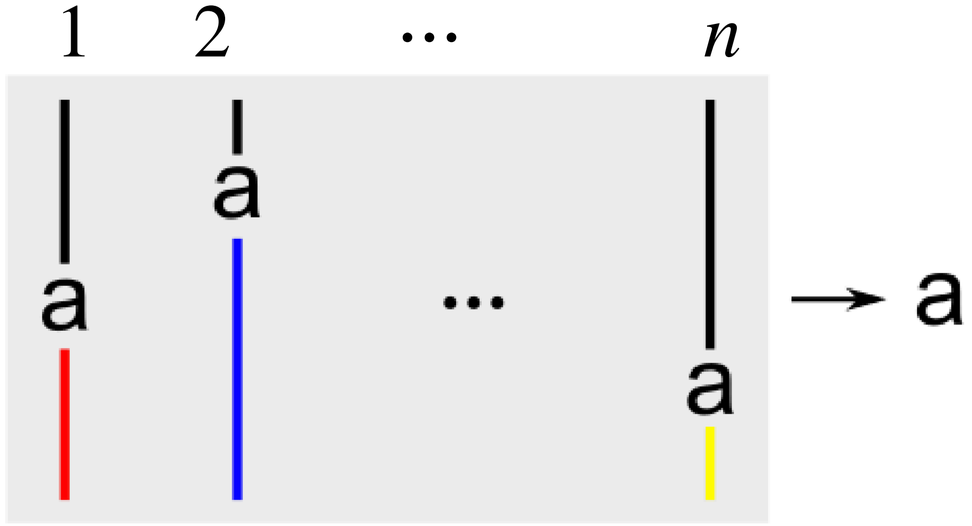
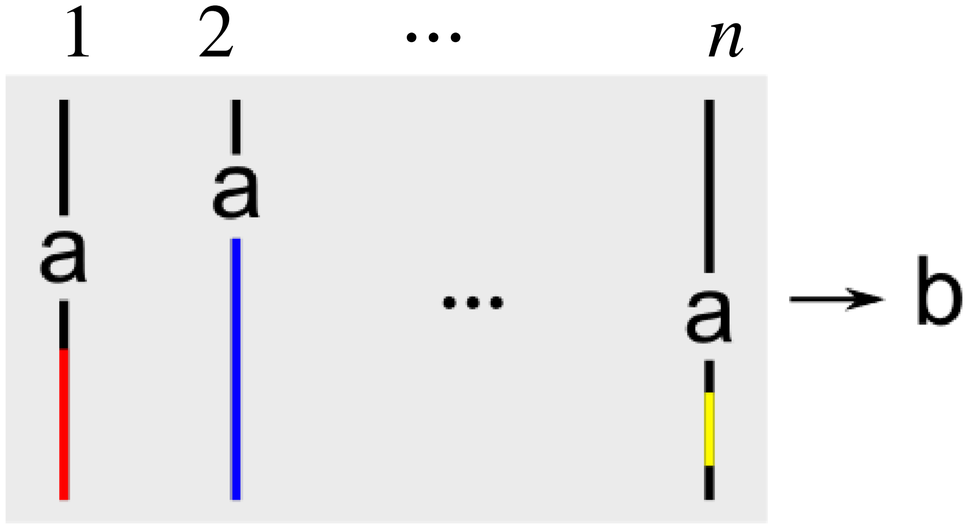
Strategyproof $\Longrightarrow$ Monotone

- Suppose a rule is strategyproof but not monotone





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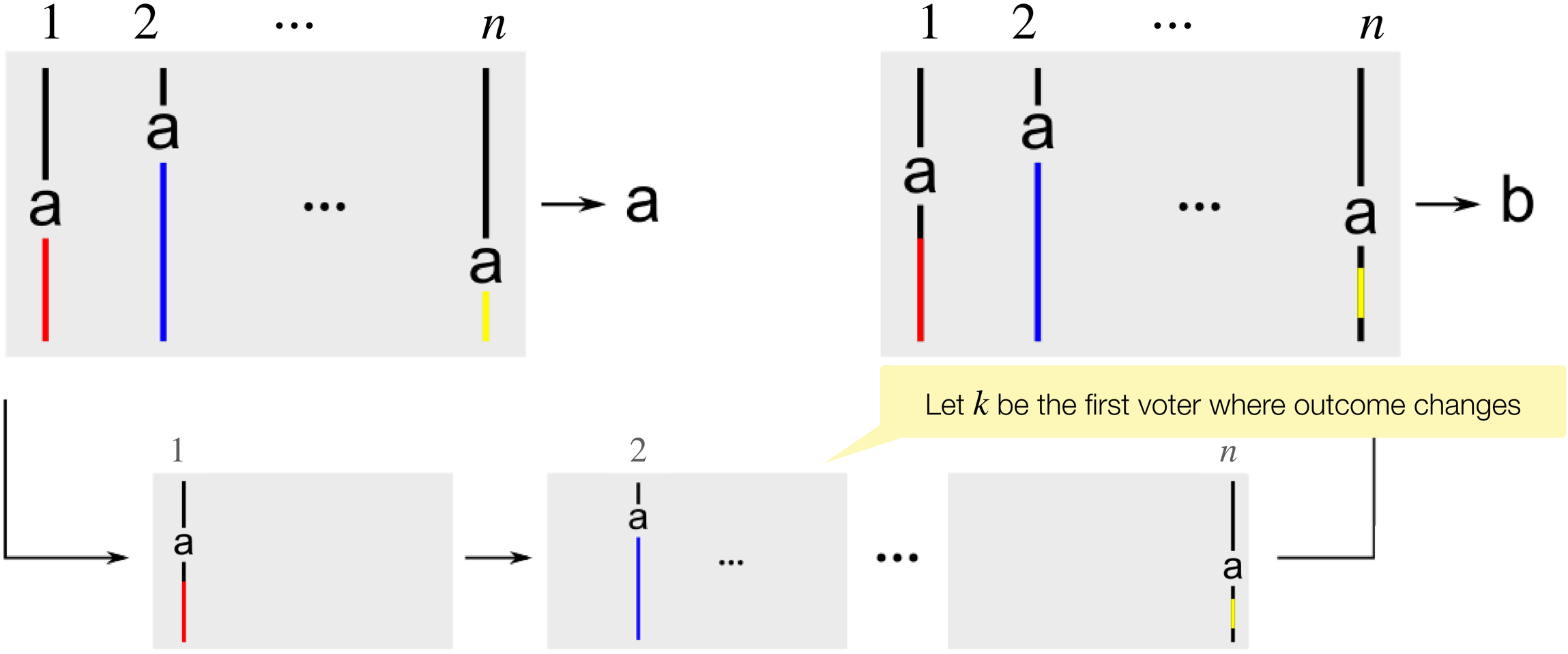
Strategyproof $\Longrightarrow$ Monotone

- Suppose a rule is strategyproof but not monotone



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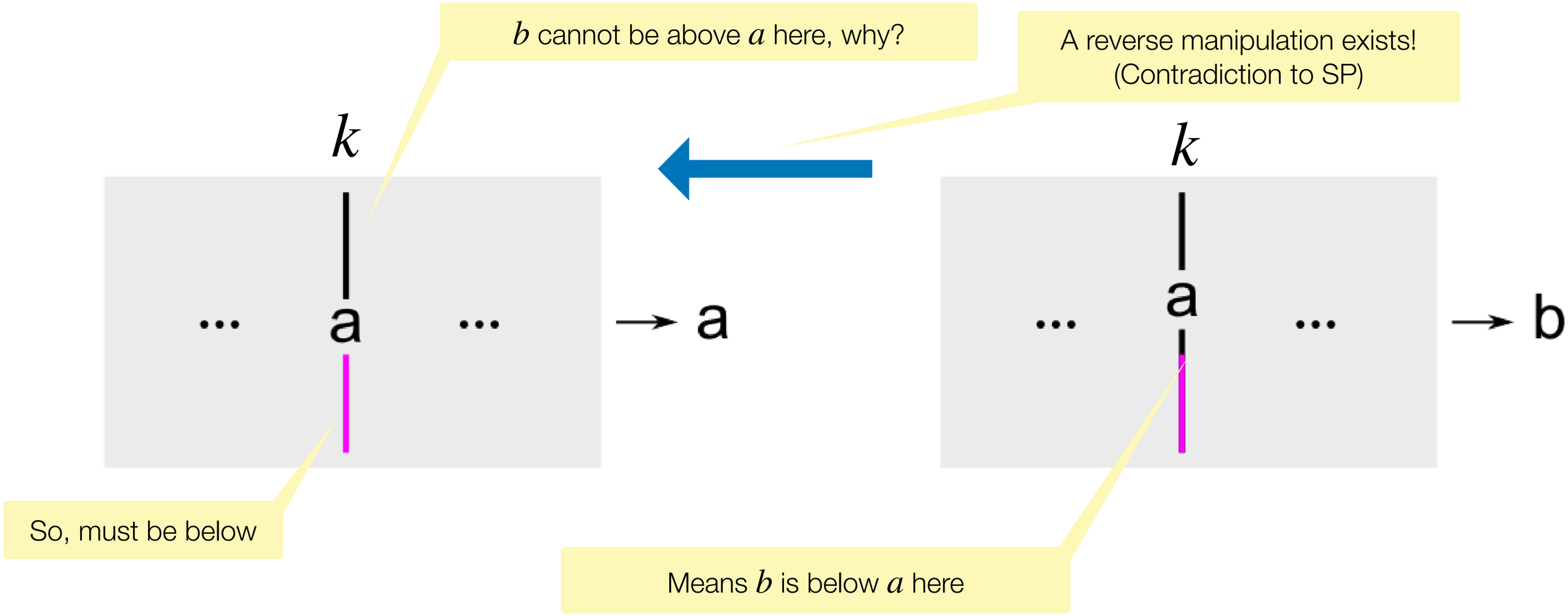
Strategyproof $\Longrightarrow$ Monotone



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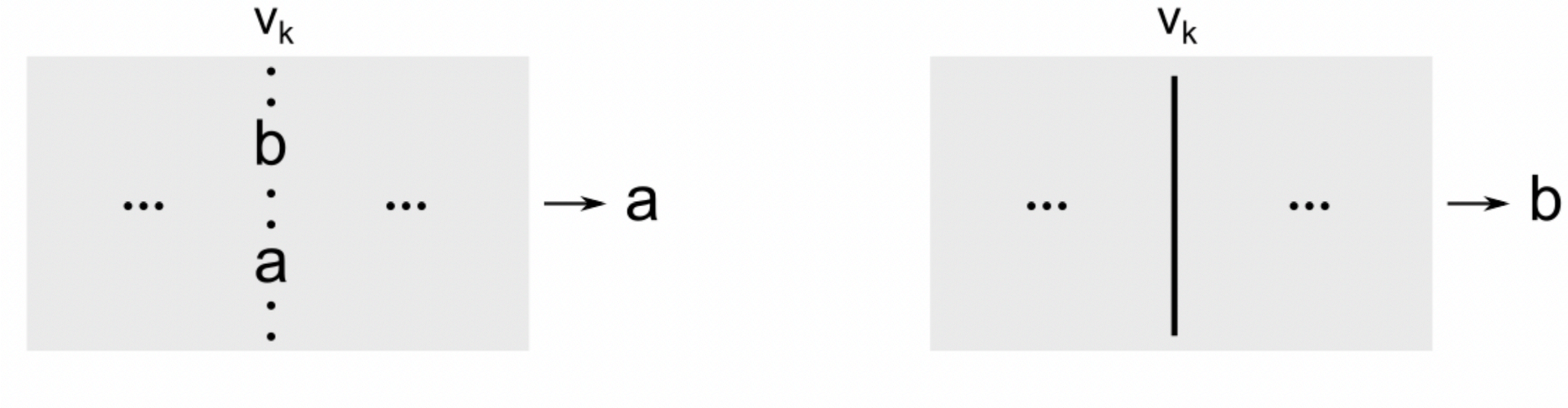
Monotone $\Longrightarrow$ Strategyproof

- Suppose there is a voter v_k that prefers b to a
- Consider truthful instance on left where a wins



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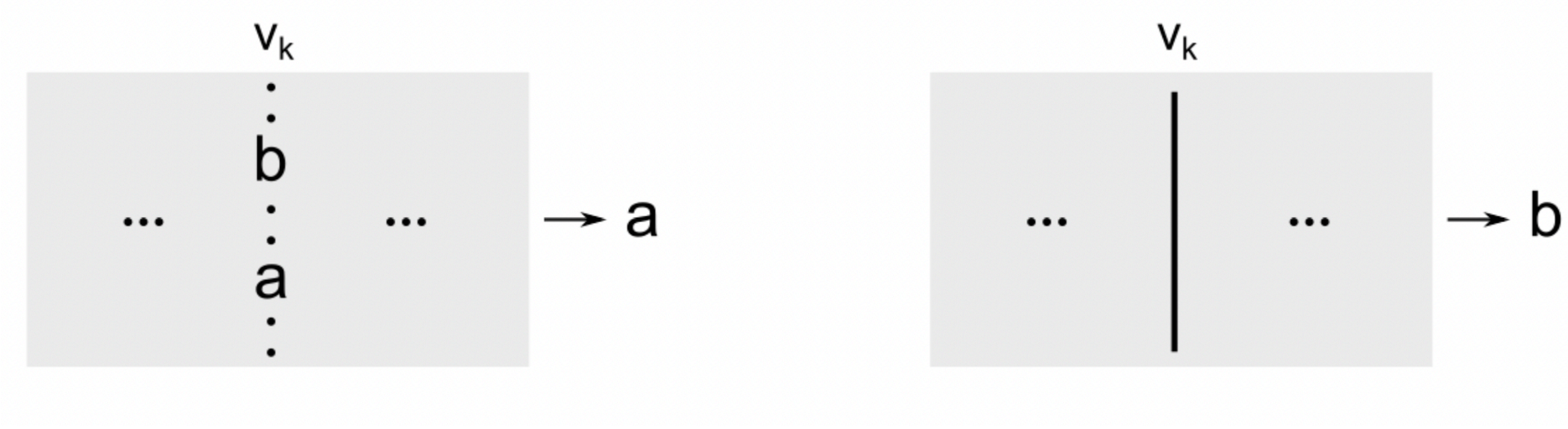
Monotone $\Longrightarrow$ Strategyproof

- Suppose there is a voter v_k that prefers b to a
- Consider truthful instance on left where a wins
- Suppose v_k can misreport and make candidate b win (keeping other preferences fixed)



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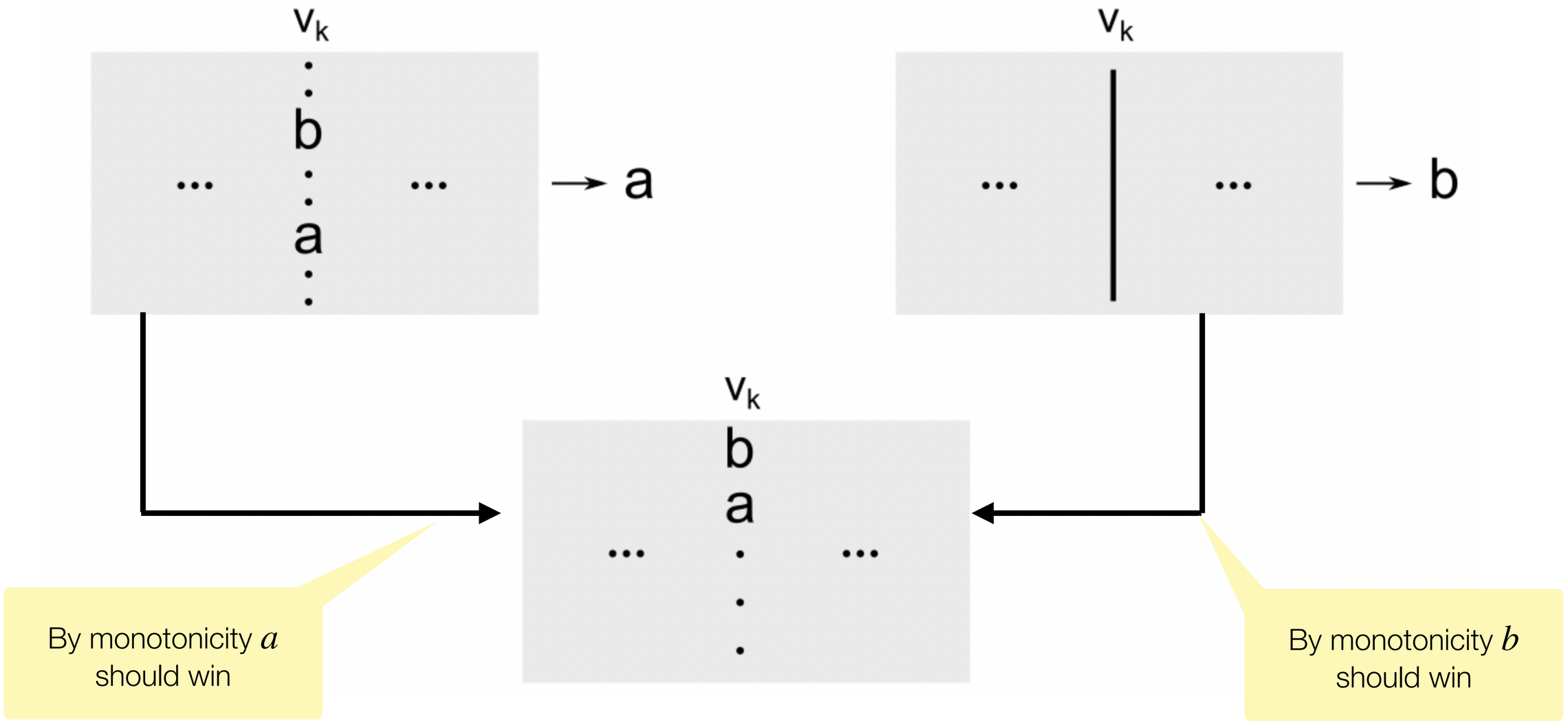
Monotone $\Longrightarrow$ Strategyproof



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Goal. Strategyproof + Onto $\Longrightarrow$ Dictatorship

[Proof Outline]

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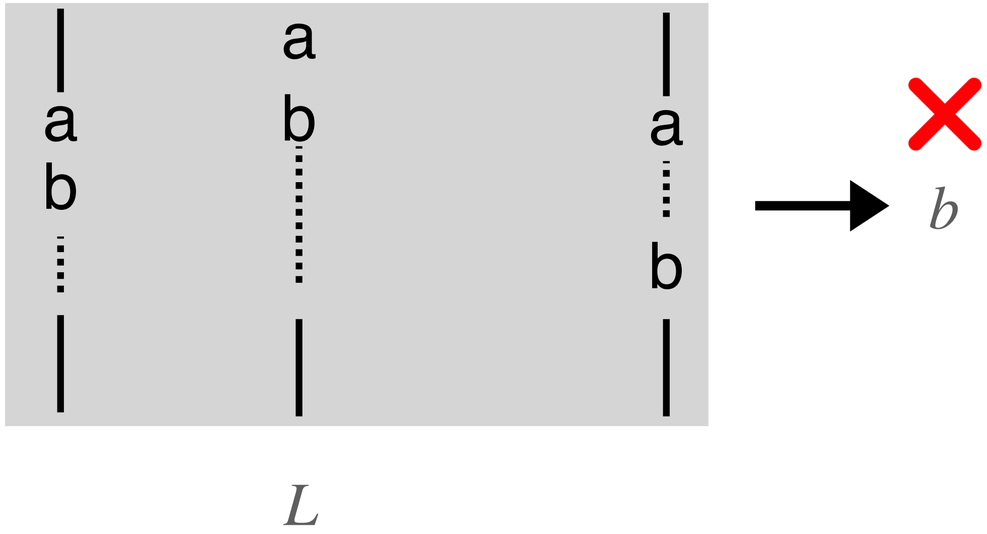
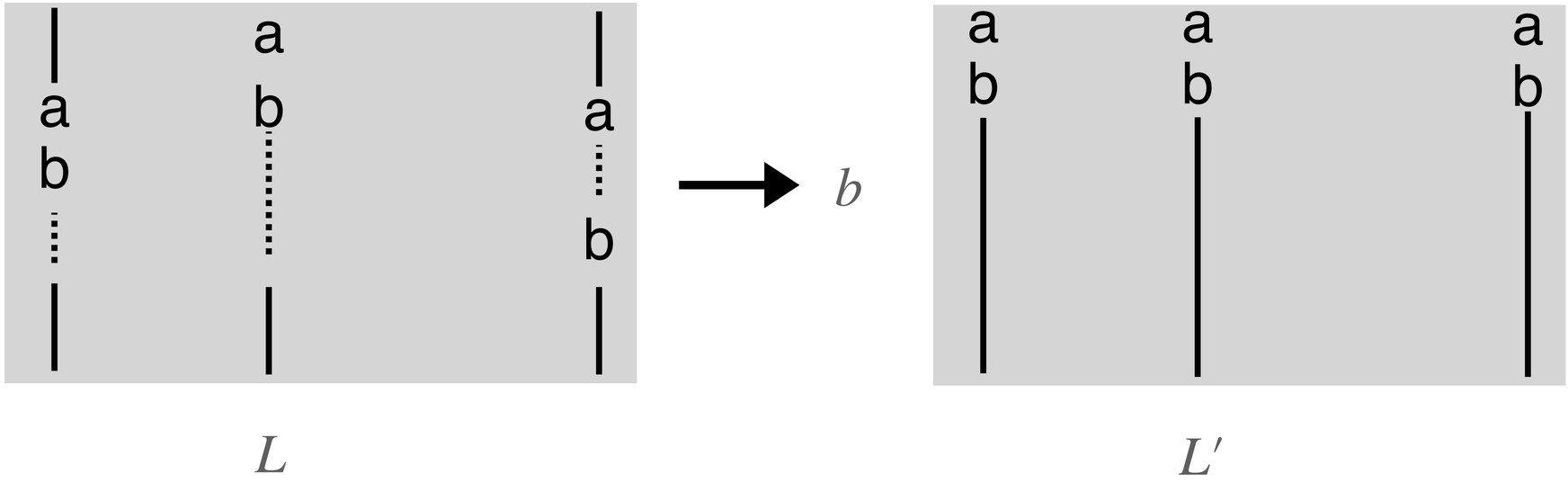
SP + Onto $\Longrightarrow$ Unanimous

- **Definition (Unanimity)**. Given preference profile L, if there is an alternative a that every voter prefers to b, then $f(L) \neq b$.
- **Lemma**. SP + Onto $\Longrightarrow$ Unanimous



SP + Onto $\Longrightarrow$ Unanimous

- **Definition (Unanimity)**. Given preference profile L, if there is an alternative a that every voter prefers to b, then $f(L) \neq b$.
- **Lemma**. SP + Onto $\Longrightarrow$ Unanimous
- **Proof**. Suppose $f(L) = b$. Consider L' below. $f(L') = ?$



L L'

SP + Onto $\Longrightarrow$ Unanimous

- **Definition (Unanimity)**. Given preference profile L, if there is an alternative a that every voter prefers to b, then $f(L) \neq b$.
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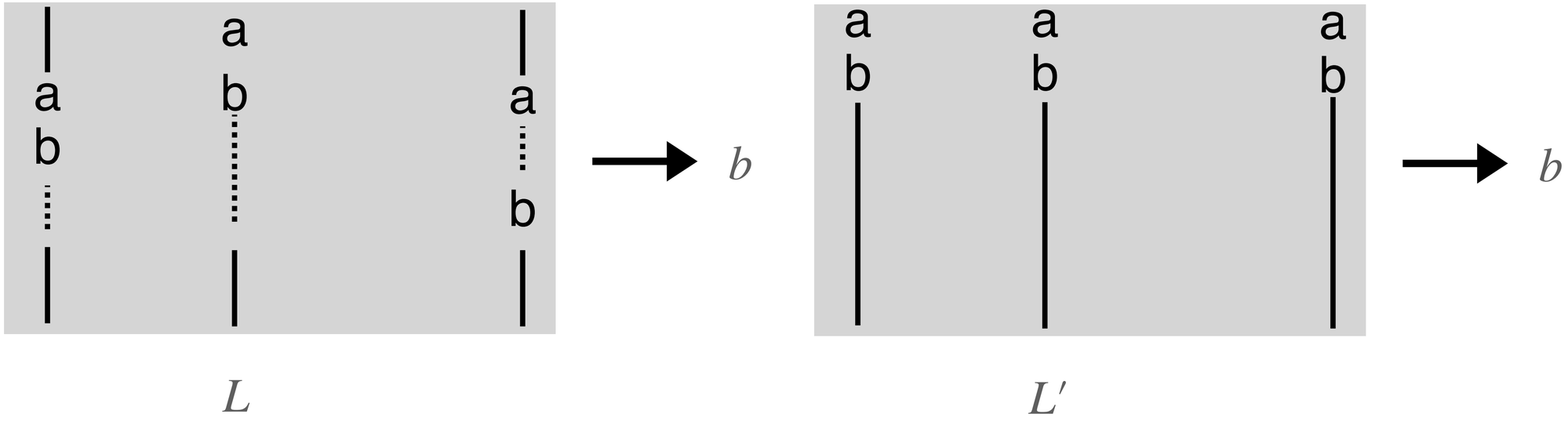
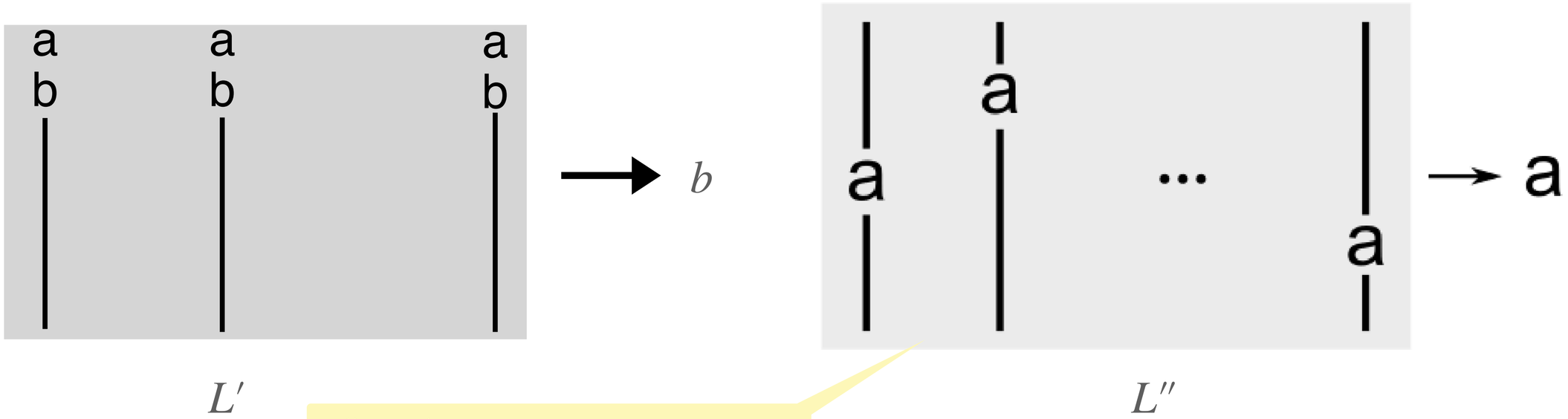


SP + Onto $\Longrightarrow$ Unanimous

- **Definition (Unanimity)**. Given preference profile L, if there is an alternative a that every voter prefers to b, then $f(L) \neq b$.
- **Lemma**. SP + Onto $\Longrightarrow$ Unanimous
- **Proof**. We know $f(L') = b$ by monotonicity. By onto, there exists a profile L'' where a wins.



[GS Theorem] With three or more candidates, a voting rule is **strategyproof** and **onto** if and only if it is a **dictatorship**.

Goal. Strategyproof + Onto $\Longrightarrow$ Dictatorship

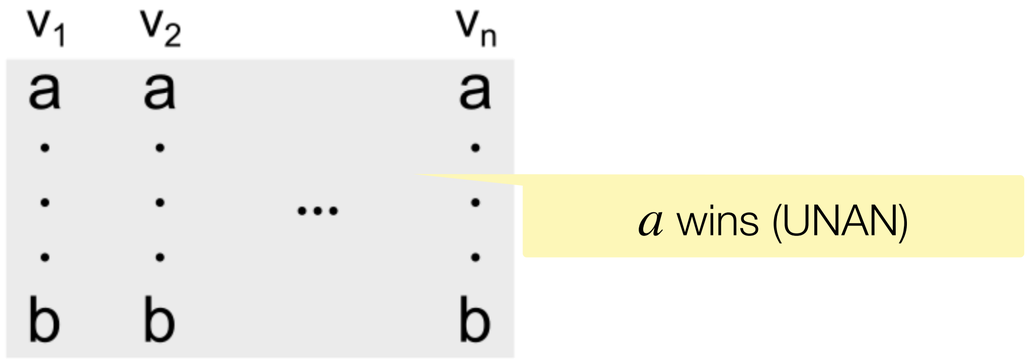
[Proof Outline]

Part 1. Strategyproof $\Longleftrightarrow$ Monotonicity

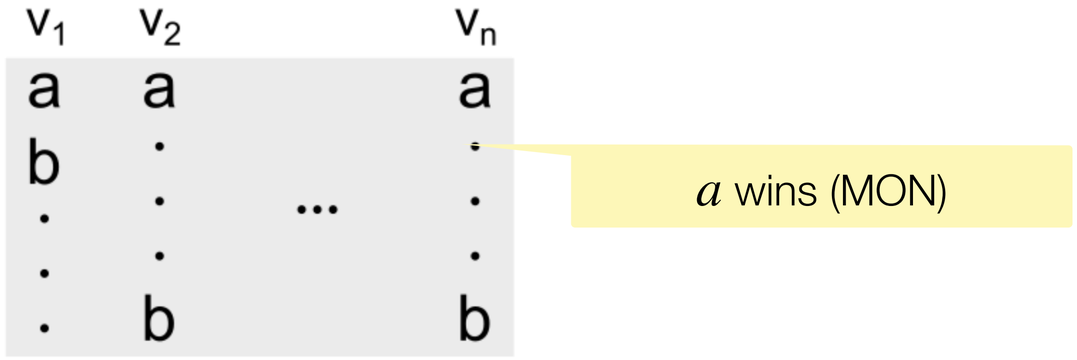
Part 2. Monotone + Onto $\Longrightarrow$ Unanimous

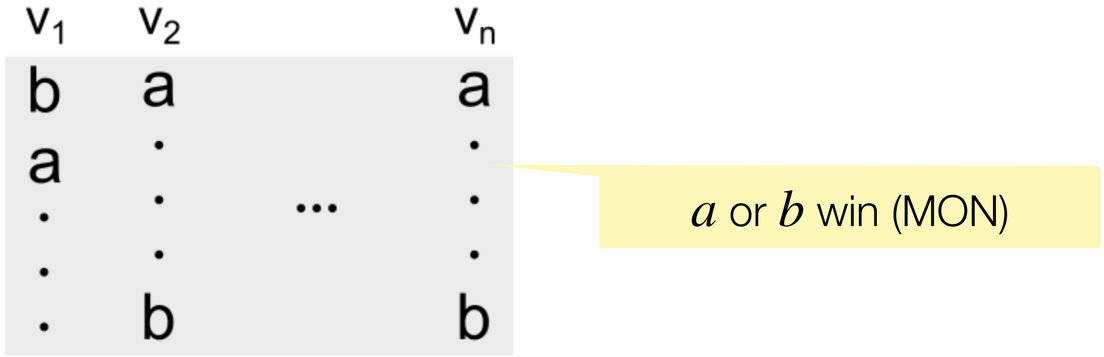
Part 3. Monotone + Unanimous $\Longrightarrow$ Dictatorship

MON + Unanimous $\Longrightarrow$ Dictatorship

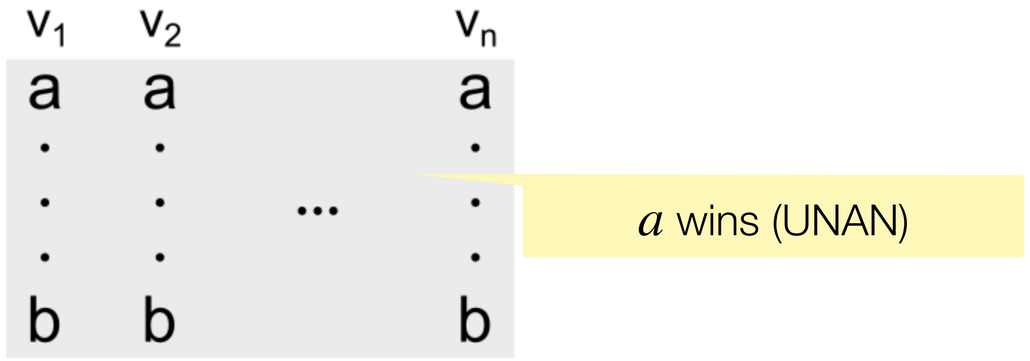


- Gradually promote b in each voters preference list

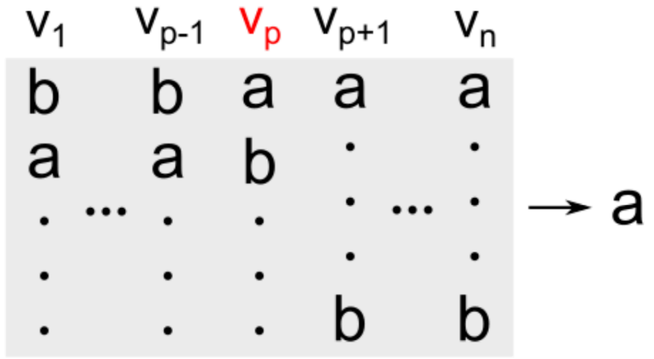


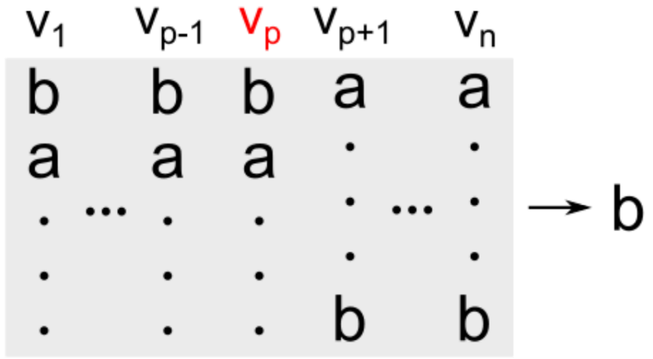


MON + Unanimous $\Longrightarrow$ Dictatorship

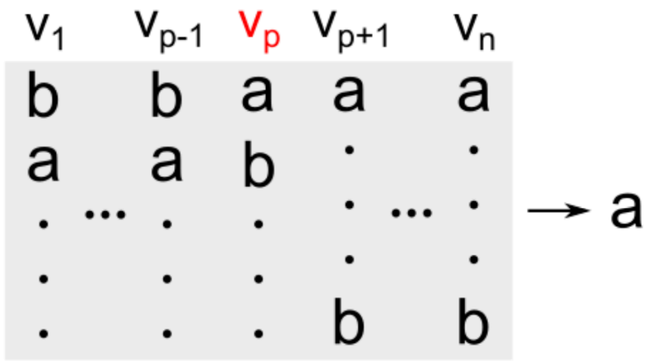


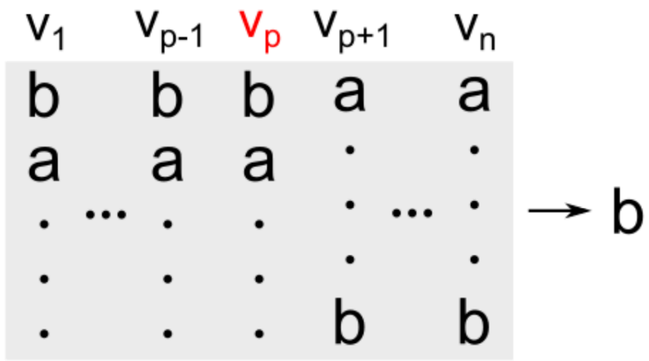
- At some point as we promote b, there must be a step where the winner switches from a to b
- Let v_p be such a **pivotal voter**



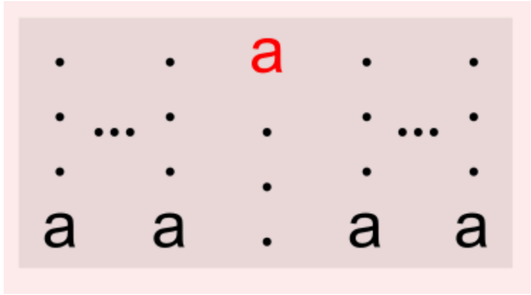


MON + Unanimous $\Longrightarrow$ Dictatorship





- **Lemma.** Show that v_p can make a win even if everyone else ranks a last
- That is, v_p is the dictator for candidate a
- Since a was arbitrary, every candidate has a dictatorship for it
- Cannot have distinct dictators for different candidates
- So v_p must be the dictator for all candidates

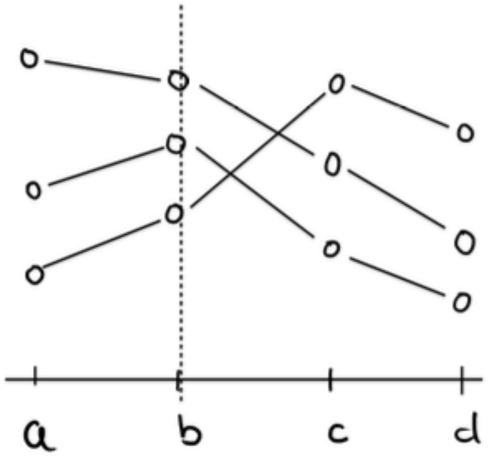


Circumventing GS

Approximation

Money

Incomplete information



Restricted preference profiles

Computational complexity