Paper Evaluation 2: Part B

LaTeX Source: `https://www.overleaf.com/read/gxhcjqycgbjh#f50ecb`

School Choice: A Mechanism Design Approach. These questions are based on the Abdulkadiroğlu and Sönmezon paper [1].

What you need for the write up. Each group must submit answers to both short-answer questions below and at least one proof from the analysis section.

1 Short Answer Questions

1. Consider the Boston mechanism that the paper discusses, which was used to assign students to high schools until 2005.[1]

   - Each students submits a complete ranked list of their preferences.
   - The students are ordered in some way (e.g., by lottery numbers)
   - *Phase 1.* The students are considered in this order. When student i is considered, if her top-ranked school is still available, then she is assigned to that school. Otherwise, she is not assigned in this phase.
   - *Phase 2.* The unassigned students are considered in the same order as before. When student i is considered, if her second-ranked school is still available, then she is assigned to that school. Otherwise, she is not assigned in this phase.
   - And we continue similarly with Phase 3 considered third-choices of unassigned students, ... Phase i considering ith choices, etc. until all students are assigned.

   Given an explicit counter example that shows that this mechanism is not dominant strategyproof.

2. Abdulkadiroglu and Sonmez say school choice can be formulated as a two-sided matching problem. Students have preferences over schools and schools have preferences *priorities* over students. School priorities are often determined by the school zone policies, and factor things like where students live, where their siblings attend school, etc. Because priorities are not preferences, emphasis if often given to preferences of students when comparing the outcomes of these mechanisms.

   The paper discusses two mechanisms that are strategyproof on the student side and allow schools to set a priority structure: the deferred acceptance (DA) mechanism and the top-trading cycle (TTC) mechanism. We will explore the trade-offs between these mechanisms when it comes to the welfare of the students discussed in the paper using specific examples.

   Consider the following preferences orders for students s_1, s_2, s_3, and priority orders for schools t_1, t_2, t_3. Assume that each school has capacity one.

[1] They switched soon after this paper was published.

Table 1: Preferences of students

s_1	t_2	t_1	t_3
s_2	t_1	t_2	t_3
s_3	t_1	t_2	t_3

Table 2: Priorities of schools

t_1	s_1	s_3	s_2
t_2	s_2	s_1	s_3
t_3	s_2	s_1	s_3

(a) Interpreting priority orders as preference orders, use student-proposing DA to find the student-optimal stable matching. If we ignore school priorities, is there a matching that Pareto dominates this matching for students?

(b) They present the TTC mechanism for this problem which works as follows:

In each step, each school with remaining capacity points to the unmatched student with most priority, and each unmatched student points to the most preferred school with remaining capacity. Paths alternate between students and schools, and "trading on a cycle" corresponds to each student on the cycle being matched with its requested school.

Run the generalized TTC mechanism on the above example. Is the outcome Pareto optimal for students? Is the outcome stable? Justify your answers.

2 Analysis of TTC Algorithm

The paper establishes the following two guarantees of the top-trading cycle algorithm. Prove at least on these properties in your write up. You are expected to prove this from scratch in your own words and notation, rather than restating the paper's arguments. You can refer to the paper's proof for guidance.

1. **Property 1.** The top-trading cycle algorithm outputs a matching that is Pareto optimal (not Pareto dominated by another matching).

2. **Property 2.** The top-trading cycle algorithm is strategyproof.

References

[1] Atila Abdulkadiroğlu and Tayfun Sönmez. School choice: A mechanism design approach. *American economic review*, 93(3):729–747, 2003.