CS 357: Algorithmic Game Theory **Spring 2025**

Assignment 1 (due 02/21/2025)

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LaTeX Template (Source): https://www.overleaf.com/read/zmfrcczpwjgz#620752

Instructions. This is a **partner assignment**. You must use the LaTeX solution template provided to write your answers and submit a joint PDF your their partner on Gradescope. Points will be awarded for *clarity, correctness and completeness* of the answers. Assignments are different from homeworks and formal proofs as the justification is expected for each question. Assignments are typically going to be slightly longer and due Wednesday night. But because we lose a lecture to winter carnival, this assignment is shorter and due Friday (02/21) at noon on Gradescope.

Pair-proof-writing tips. Best practices for writing proofs together with a partner are similar to best practices of pair programming.

- Try to sit down with a piece of paper or near a blackboard and first discuss the problem—unpack it and discuss definitions.
- One person should be the *driver* and the other the *navigator*. Switch roles frequently, even for the same question.
- The driver proposes the idea, the navigator reviews the logic. The navigator can then either find a bug in the idea or try to extend it to a formal proof.
- Once you both agree on a solution on the blackboard or a piece of paper, you can divide the task of cleaning it up and typing it in LaTeX. However, the quick and dirty "divide-and-conquer approach (splitting the questions up) is not recommended. It may get the job done in the short term but is not optimal for learning.

Problem 1. Prove that if only a single action profile a survives iterated elimination of *strongly* dominated strategies (refer to the algorithm from HW 1), then a is the unique pure-strategy Nash equilibrium of the game. *Show that the surviving strategy is a Nash equilibrium and then argue that it must be unique.*

Solution.

□

Reading for the next two questions. Read about second-price auction basics: Roughgarden Ch 2. For a more informal overview, see EK Section 9.4.

Problem 2. An auction is **dominant strategyproof** (called *dominant-strategy incentive compatible* in the literature DSIC in the readings) if truthful bidding is a dominant strategy for every bidder. (Truthful bidding is reporting your true value as your bid.)

Consider a sealed-bid single-item auction with at least three bidders, where the item is awarded to the highest bidder, at a price equal to the third-highest bid. Prove that this auction is not dominant strategyproof .

Solution.

□

Problem 3. (Collusion) In class, we showed that in the second-price auction, no bidder has an incentive to deviate unilaterally from bidding truthfully, regardless of the other bidders' actions. But the auction is susceptible to *collusion*, that is, a situation where two or more bidders can coordinate and decide how each of them should submit their individual bids to benefit the group as a whole.

Consider a single-item second-price auction with $N = \{1, 2, \ldots, n\}$ bidders in which all but a subset $S \subseteq N$ of the bidders bid truthfully. The members of S attempt to collude to increase their utility as a group $u(S)$, defined as—the sum of their valuations for the item $\sum_{i \in S} v_j$ minus the price p they pay for the item, if someone from S receives it. The group utility is zero if no one from S receives the item.[1]

A collusion by S is considered *useful* if the group utility of S when colluding is more than the group utility if each member bid truthfully.

State and prove necessary and sufficient conditions on the valuations of the bidders in S (relative to the others) such that it is possible for them to have a useful collusion.

You must state the conditions as well as prove both directions (necessary and sufficient).

Solution.

□

[1] Even though one person in S has to pay the auctioneer in case they win, collusion implies a prior agreement among the group on how to share the payment. This model makes sense for *transferable goods/services*—a setting where agents can pool their resources to win an item and the item is something each person in S can enjoy as if they own it as long as someone in the group has it, e.g., a subscription to a streaming service with no restrictions on number of viewers.

Citation Sources

Using question-specific prompts on the Internet is a violation of the honor code. If you referred to lecture notes or other resources on the topic and you find that information helpful towards these questions, you must cite them here.