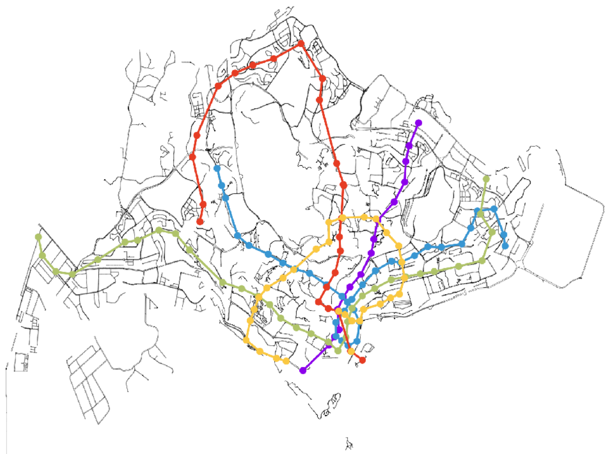
CS 357: Algorithmic Game Theory

Lecture 8: Auctions Wrap Up

Shikha Singh



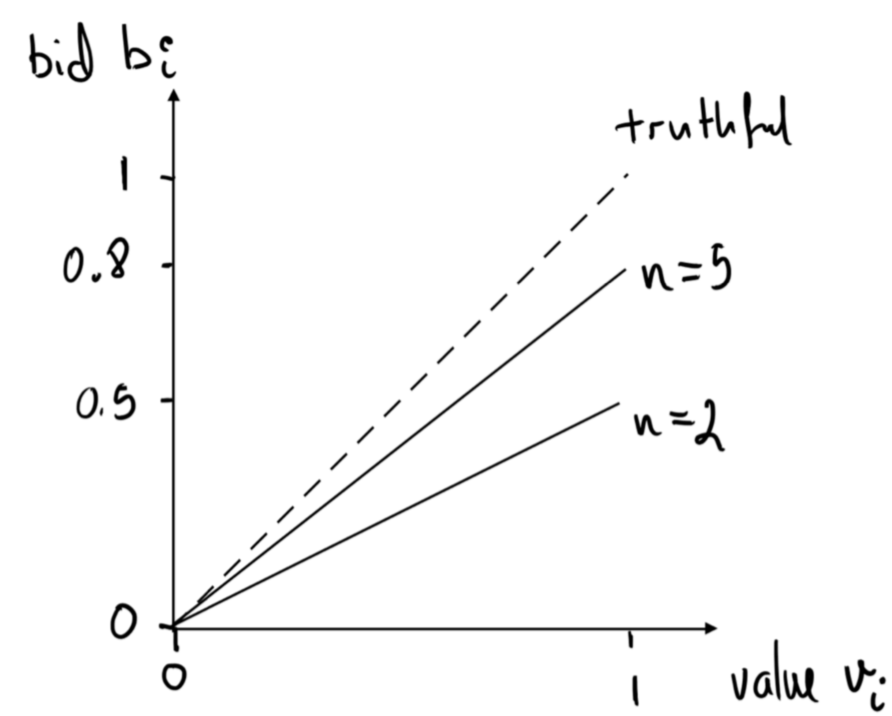
Announcements

- **HW 3** solutions on GLOW
- **HW 4** due next Tuesday in class
- **Assignment 2** was due at noon
  - Solutions will be posted on GLOW tonight
- **Assignment 1** graded: setting expectations on notation/ formality
- **Exam 1** will be held in class on **Friday March 14**
  - Short-"ish" questions, mostly HW style questions with one or two open/ended answers or proofs
  - Cover everything until today: review HWs, assignments, readings and lectures to prepare
  - Closed book but can bring prepared notes (no more than 5 pages)

First Price: n Bidders

- **Theorem.** Assume each of the n bidders have values drawn i.i.d. from uniform distribution on $[0,1]$. Then, the strategy $s(v_i) = \dfrac{n-1}{n} \cdot v_i$ is a symmetric Bayes Nash equilibrium of the sealed-bid first price auction.

- **Takeaway**: the more the competition, the more the bidders need to bid closer to their value if they want to win.



First Price: n Bidders Derivation

$$\mathbb{E}(u_1) = (v_1 - b_1) \cdot \text{Pr}(1 \text{ wins with bid } b_1) + 0 \cdot \text{Pr}(1 \text{ loses with bid } b_1)$$

$$= (v_1 - b_1) \cdot \text{Pr}[b_1 \geq \max_{i=2}^{n} b_i]$$

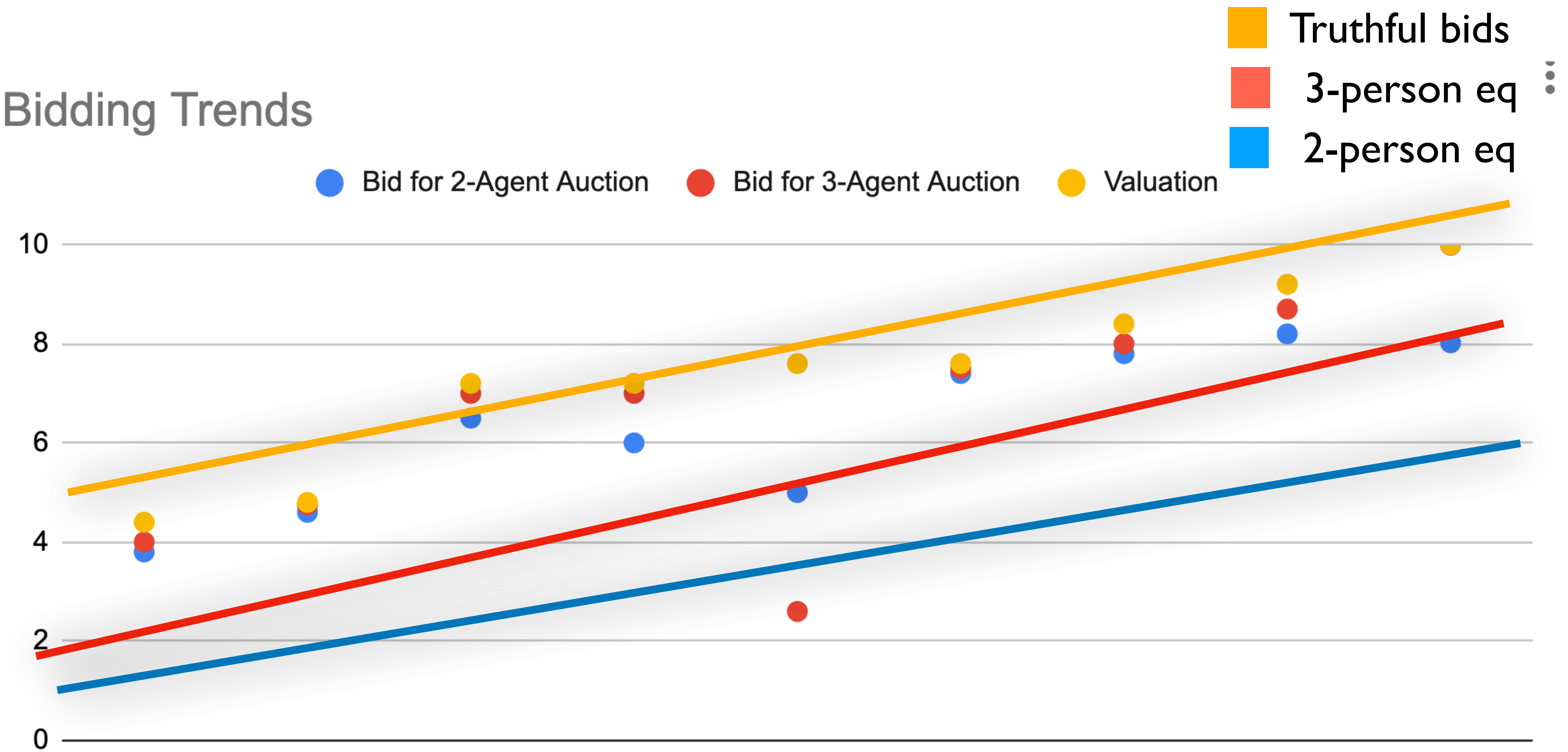
$$= (v_1 - b_1) \cdot \text{Pr}(b_1 \geq b_2 \cap b_2 \geq b_3 \cdots \cap b_1 \geq b_n)$$

Set $b_i = \alpha \cdot v_i$ for each $i = 2, \ldots, n$. As values are independent, we get:

$$\mathbb{E}(u_1) \ = (v_1 - b_1) \cdot \text{Pr}(v_2 \leq \frac{b_1}{\alpha}) \cdots \text{Pr}(v_n \leq \frac{b_1}{\alpha}) = (v_1 - b_1) \cdot \left(\frac{b_1}{\alpha}\right)^{n-1}$$

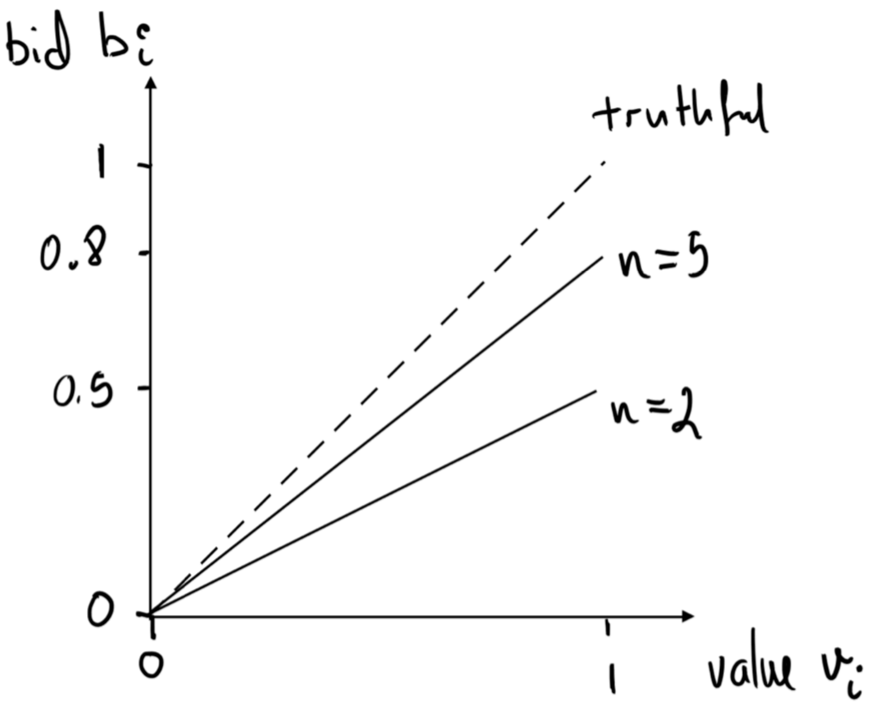
To find the bid b_1 that maximizes this utility, can differentiate wrt b_1 and set to zero, which gives us $b_1 = \dfrac{n-1}{n} \cdot v_1$

Empirical Bids vs Equilibrium



First Price Auction Guarantees

- Generalizes to arbitrary i.i.d. distributions
- Bayes Nash equilibrium is **unique**
  - Maximizes welfare at this equilibrium
  - Generates the same expected revenue as second-price auction



Revenue of First Price Auction

Revenue Equivalence

- **Theorem.** If bidder's values are uniform i.i.d., then the expected revenue of the first-price auction is equal to that of the second-price auction at equilibrium.

- **Proof.** Let $E[R_1]$ and $E[R_2]$ be the expected revenues of the first and second-price auction.

- $E[R_2] = E[\text{second-highest bid}] = E[\text{second-highest value}]$

- $E[R_1] = E[b_{\max}] = E\left[\dfrac{n-1}{n} \cdot v_{\max}\right] = \dfrac{n-1}{n} \cdot E[v_{\max}]$
    - This step uses linearity of expectation $E(a \cdot X + b \cdot Y) = a \cdot E(X) + b \cdot E(Y)$ where a and b are constants

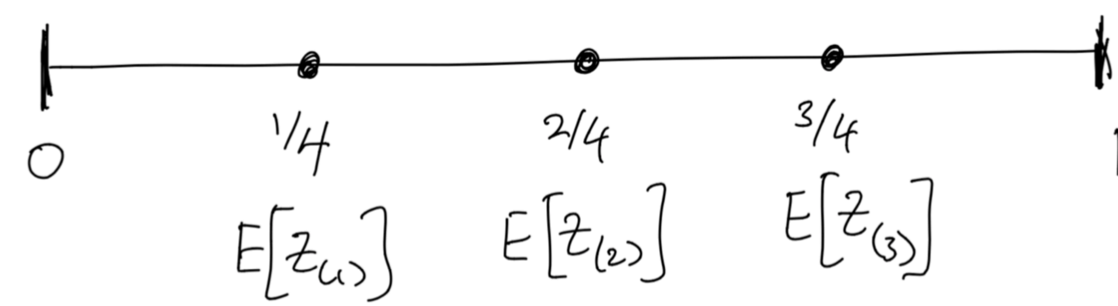
- These expected values are called **order statistics**

Review: Order Statistics

- Let $X_1, X_2, \ldots X_n$ be n independent samples drawn identically from the uniform distribution on $[0,1]$
- The first-order statistic $X_{(1)}$ is the max value of the samples, the second-order statistic $X_{(2)}$ is the second-max value of the samples, etc
- Expected value $X_{(k)}$ for n i.i.d samples from $U(a,\ b)$ is

$$E[X_{(k)}] = \ a + \frac{n-(k-1)}{n+1} \cdot (b-a)$$

- Remember: a uniform random variable evenly divides the interval it is over



Expected kth order statistic for 3 samples, uniform $[0,1]$

Revenue Equivalence

- **Theorem.** If bidder's values are uniform i.i.d., then the expected revenue of the first-price auction is equal to that of the second-price auction at equilibrium.

- **Proof.** Let $E[R_1]$ and $E[R_2]$ be the expected revenues of the first and second-price auction.

- $E[R_2] = E[\text{second-highest bid}] = E[\text{second-highest value}] = \dfrac{n-1}{n+1}$

- $E[R_1] = E[b_{\max}] = E\left[\dfrac{n-1}{n} \cdot v_{\max}\right] = \dfrac{n-1}{n} \cdot E[v_{\max}]$

$$= \frac{n-1}{n} \cdot \frac{n}{n+1} = \frac{n-1}{n+1} \quad \blacksquare$$

Myerson Lemma for BNE

- Myerson's lemma also characterizes BNE in single-parameter mechanisms
- **Statement:** A strategy profile s is a Bayes' Nash equilibrium in $(\mathbf{x}, \mathbf{p})$ if and only if for all i

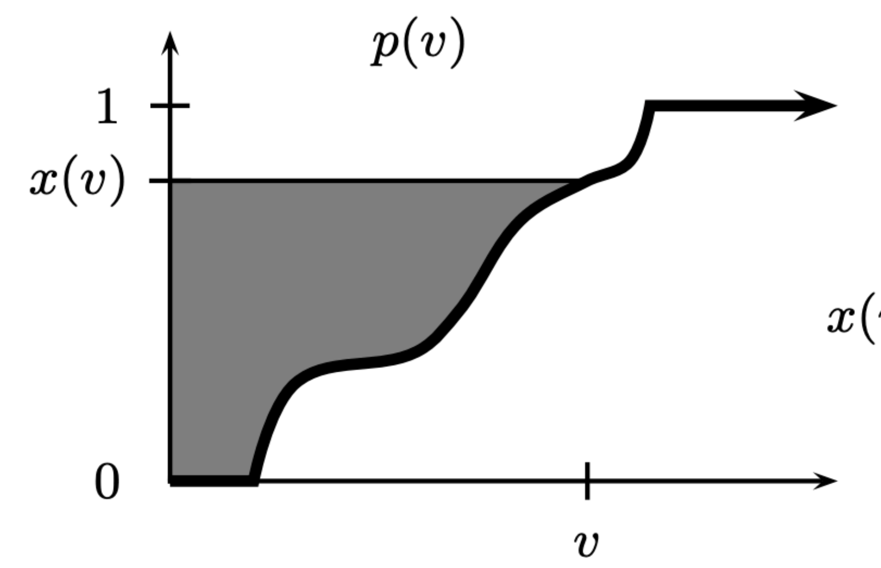
(a) (monotonicity) the allocation probability $x_i(v_i)$ is monotone non decreasing

(b) (payment identity) **agent i's expected payment** is given by:

$$p_i(v_i) = v_i \cdot x_i(v_i) - \int_0^{v_i} x_i(z)\, dz$$

Assuming that $p_i(0) = 0$.

Proof is analogous to the DSE case.



Revenue Equivalence

- Myerson's lemma tells us something surprising these mechanisms
  - If two mechanisms have the same distribution of agent values and same allocation (at BNE), then they generate the same revenue

If we want to increase the (expected) revenue, **changing payments or charging more won't do it!** You need to change how you allocate!

Wrapping Up This Unit

- Two things we have not yet discussed

1. Revenue maximization as primary optimization objective

2. Ascending/descending auction formats for multiple goods

Wrapping Up This Unit

- Two things we have not yet discussed

1. Revenue maximization as primary optimization objective:
   eBay/ Yahoo's example as a case study

2. Ascending/descending auction formats for multiple goods
   FCC spectrum auctions overview

Revenue Maximization

Let's Talk About Revenue

- So, far revenue is incidental: payments were necessary to maximize social welfare
- Start with one bidder with private value v and one item
- Unique dominant-strategyproof welfare-maximizing auction?
    - Allocate the item to the the bidder
    - Charge critical bid: zero
- Any ideas on how we can improve the revenue?
    - Post a minimum price; ignore bids below it: don’t sell
    - If bid is above it, charge posted price

Single Item and Single Buyer

- Suppose we knew the bidders value v (maximum willingness to pay)
    - What should the posted price r be?
    - $r = v$ (also called reservation/reserve price or the monopoly price)
- Unfortunately we don't know v
- What are the tradeoffs of setting r too high or too low?
    - Set it too high, might not sell the item
    - Set it too high, might get less revenue than is possible
- What if the seller knew the distribution F from which v is drawn?

Posted Price for One Bidder

- Suppose reserve price is r and the value v is drawn from a distribution F
    - If $v < r$: no sale
    - Otherwise we
- What is the expected re

$E[R] = r \cdot \text{Pr(sale)} + 0 \cdot \text{Pr}$

$= r \cdot (1 - F(r))$

Notice that we **sometimes don't sell the item, i.e. (this is not surplus maximizing):** revenue equivalence says we must allocate item differently to generate more revenue

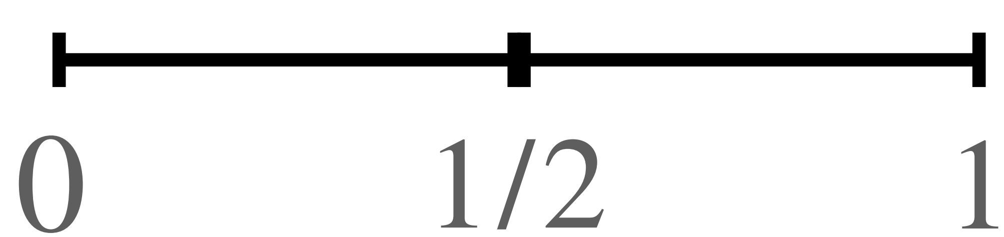
- If $v \sim$ i.i.d. $U(0,1)$, then $F(r) = r$

$E[R] = r(1 - r)$, maximized at $r = 1/2$

- Achieving an expected revenue of $1/4$

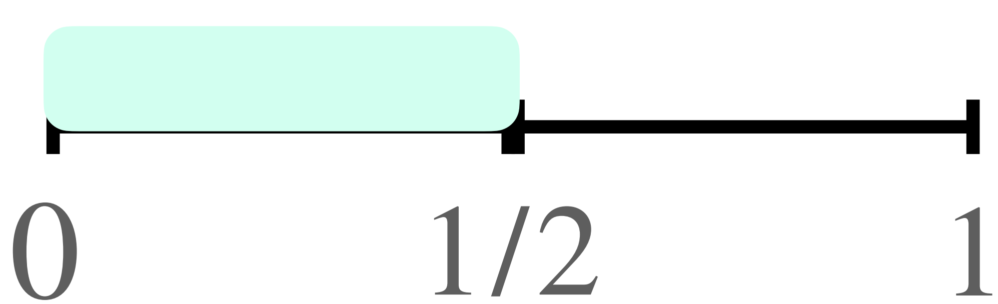
Second Price With Reserve

- Suppose now we have two bidders: suppose both their values is drawn uniform i.i.d. from $U(0,1)$ and no reserve price
- Revenue of second price auction without reserve
  - $E[R_2] = \dfrac{n-1}{n+1} = \dfrac{1}{3}$
- Can we improve this revenue if we have a reserve price?
- Suppose $r = 1/2$



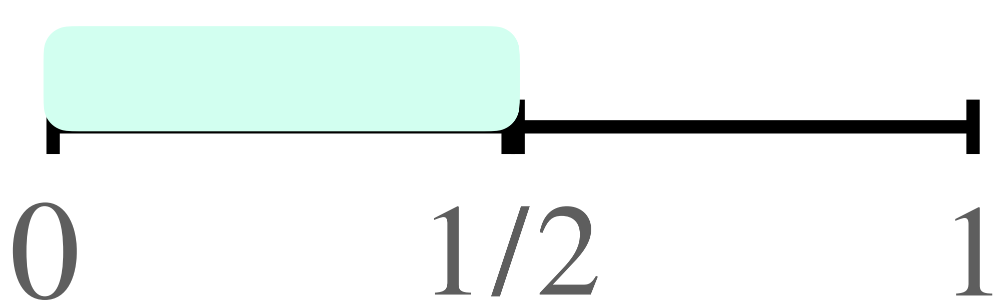
Second Price With Reserve

- Suppose now we have two bidders: suppose both their values is drawn uniform i.i.d. from $U(0,1)$ and $r = 1/2$
- Probability that both bidder values are below $1/2$
    - Probability that two uniformly randomly thrown balls fall into the first half (when both are thrown independently)
        - $1/2 \cdot 1/2 = 1/4$
    - Expected revenue in this case?
        - 0



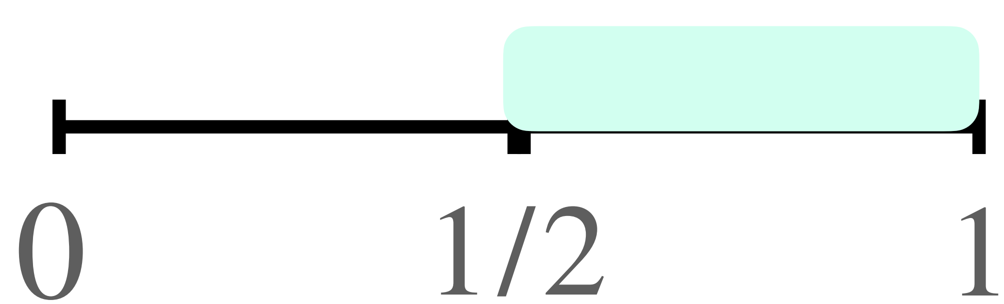
Second Price With Reserve

- Suppose now we have two bidders: suppose both their values is drawn uniform i.i.d. from $U(0,1)$ and $r = 1/2$
- Probability that one bidder value is above $1/2$, other below
    - Probability that at exactly one ball (thrown uniformly randomly and independently) lands in the first half
        - $1/2 \cdot 1/2 + 1/2 \cdot 1/2 = 1/2$
- Expected revenue in this case?
    - Reserve price $r = 1/2$



Second Price With Reserve

- Suppose now we have two bidders: suppose both their values is drawn uniform i.i.d. from $U(0,1)$ and $r = 1/2$
- Probability that both bidder values are above $1/2$
  - Probability that two uniformly randomly thrown balls fall into the second half (when both are thrown independently)
    - $1/2 \cdot 1/2 = 1/4$
- Expected revenue in this case?
  - Expected value of second-highest sample when two samples are drawn iid from $U(0.5,\ 1)$



Second Price With Reserve

- We use the fact that a uniform random variable evenly divides the interval its over
- In this case the interval is $[a, b]$ where $a = 0.5,\ b = 1$
- Expected value of kth order statistic for n samples drawn iid from the range $[a, b]$ is $\frac{1}{2} + \frac{2-(2-1)}{2+1} \cdot \frac{1}{2} = \frac{2}{3}$

$$a + \frac{n-(k-1)}{n+1} \cdot (b-a)$$

Second Price With Reserve

- Suppose now we have two bidders: suppose both their values is drawn uniform i.i.d. from $U(0,1)$ and $r = 1/2$
- Putting it all together: $\dfrac{1}{4} \cdot 0 + \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{1}{4} \cdot \dfrac{2}{3} \;\; = \dfrac{5}{12}$
- Expected revenue increased!
  - Without reserve $1/3$, with a reserve of $1/2$ it is $5/12$
- **Question.** Can we do better? By using a different reserve price or using a totally different auction format?
- **(Side question.)** Is this auction still dominant-strategyproof?
  - Pretend there is an another bidder with bid r

Revenue Optimal Auctions: Main Takeaways

- **Theorem [Myerson].** If we know the distribution from which bidder's values are drawn IID (and it is **"regular"**), then the Vickrey (second-price) auction with a suitable reserve price is dominant-strategyproof and **maximizes expected revenue.**
  - The above generalizes to all single-parameter settings!
- **Theorem [Bulow Klemperer].** Vickrey auction (with no reserve) with $n+1$ bidders generates just as much expected revenue as the revenue-optimal auction with n bidders!

Takeaways

- Going from welfare maximization to revenue is not a big jump
  - We just need to add a suitable reserve price!
- Revenue optimal auctions are simple
- Myerson received the 2007 Nobel Prize in part for this work
- **Main Takeaway.** Better to spend resources in increasing competition rather than figuring out buyer

Theory vs Practice

- eBay Auctions are essentially second price with a suitable "opening bid" (a reserve price)
- Thus the theory we developed argues that eBay auctions are the best possible for revenue and are strategyproof!

Reserve Price in Yahoo

- Does our optimal auction theory apply well in practice?
- Ostrovsky and Schwarz did a field experiment in 2008 exploring the affect of reserve price in Yahoo! Keyword auctions
- Before 2008, Yahoo had been using relatively small reserve prices: around 1 or 5 cents and the same reserve price for all keywords

Reserve Prices in Internet Advertising Auctions:
A Field Experiment*

Michael Ostrovsky[†] Michael Schwarz[‡]

YAHOO!

Reserve Price in Yahoo

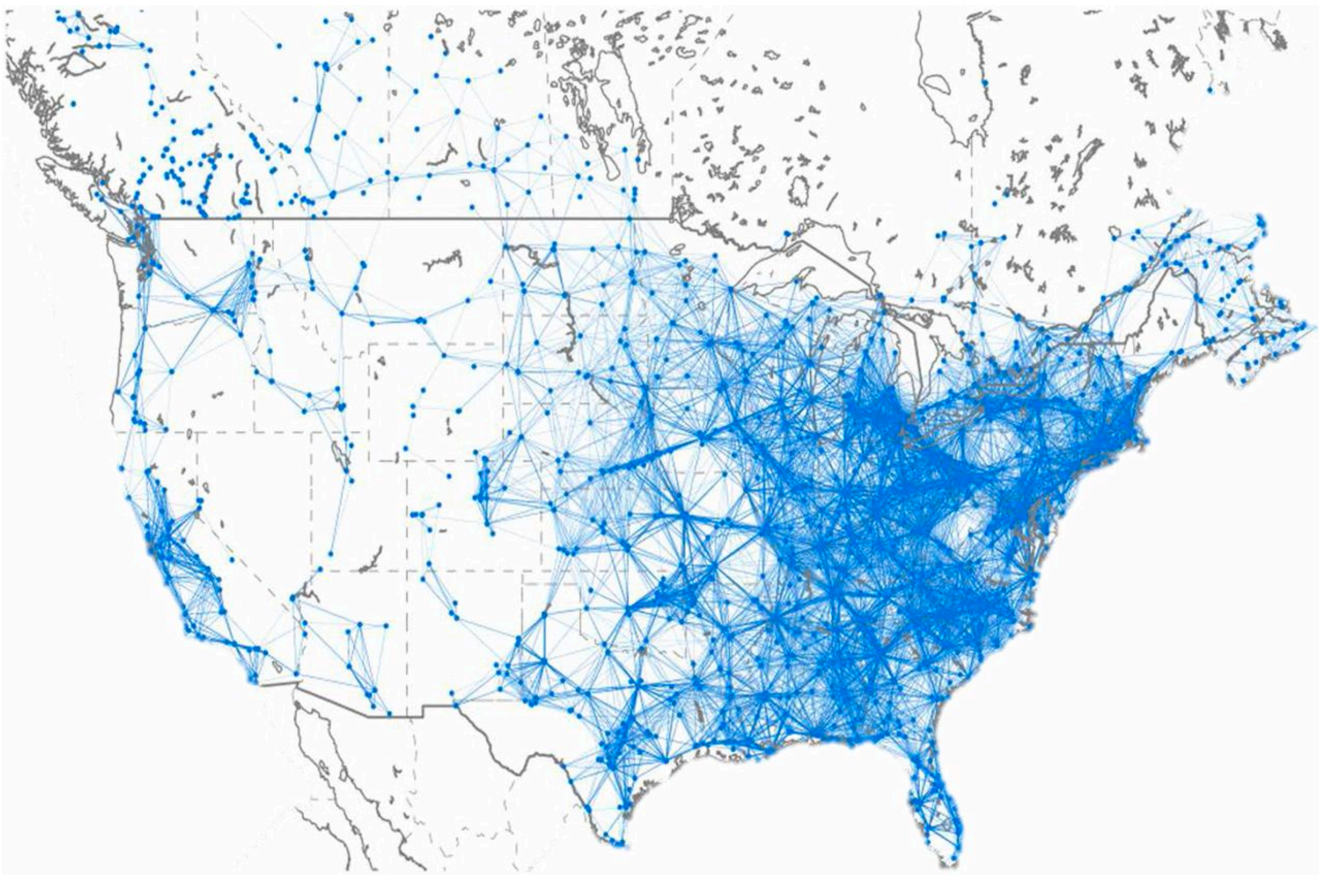
- And it worked!
- Yahoo's revenue went up several percent (of a huge number!)
- The change was especially effective in "thin" markets: not as competitive (less than 6 bidders)

On the [revenue per search] front I mentioned we grew 11% year-over-year in the quarter [. . .], so thats north of a 20% gap search growth rate in the US and that is a factor of, attributed to rolling out a number of the product upgrades we've been doing. [Market Reserve Pricing] was probably the most significant in terms of its impact in the quarter. We had a full quarter impact of that in Q3, but we still have the benefit of rolling that around the world.

Sue Decker, President, Yahoo! Inc. Q3 2008 Earnings Call.[18,19]

Application: Spectrum Auctions

Spectrum Auctions



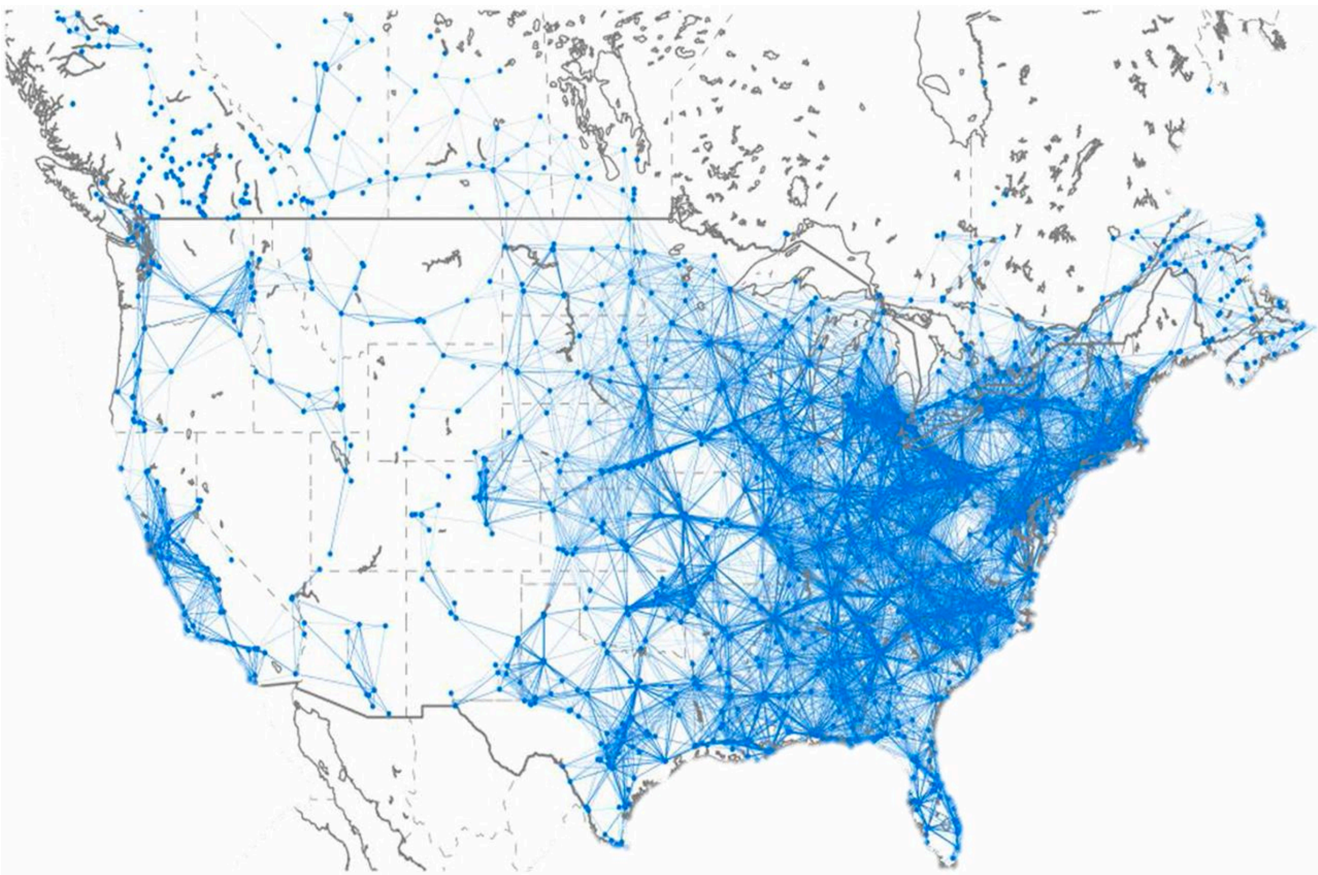
What is Spectrum?

Spectrum refers to the invisible radio frequencies that wireless signals travel over. Those signals are what enable us to make calls from our mobile devices, tag our friends on Instagram, call an Uber, pull up directions to a destination, and do everything on our mobile devices.

The frequencies we use for wireless are only a portion of what is called the electromagnetic spectrum.

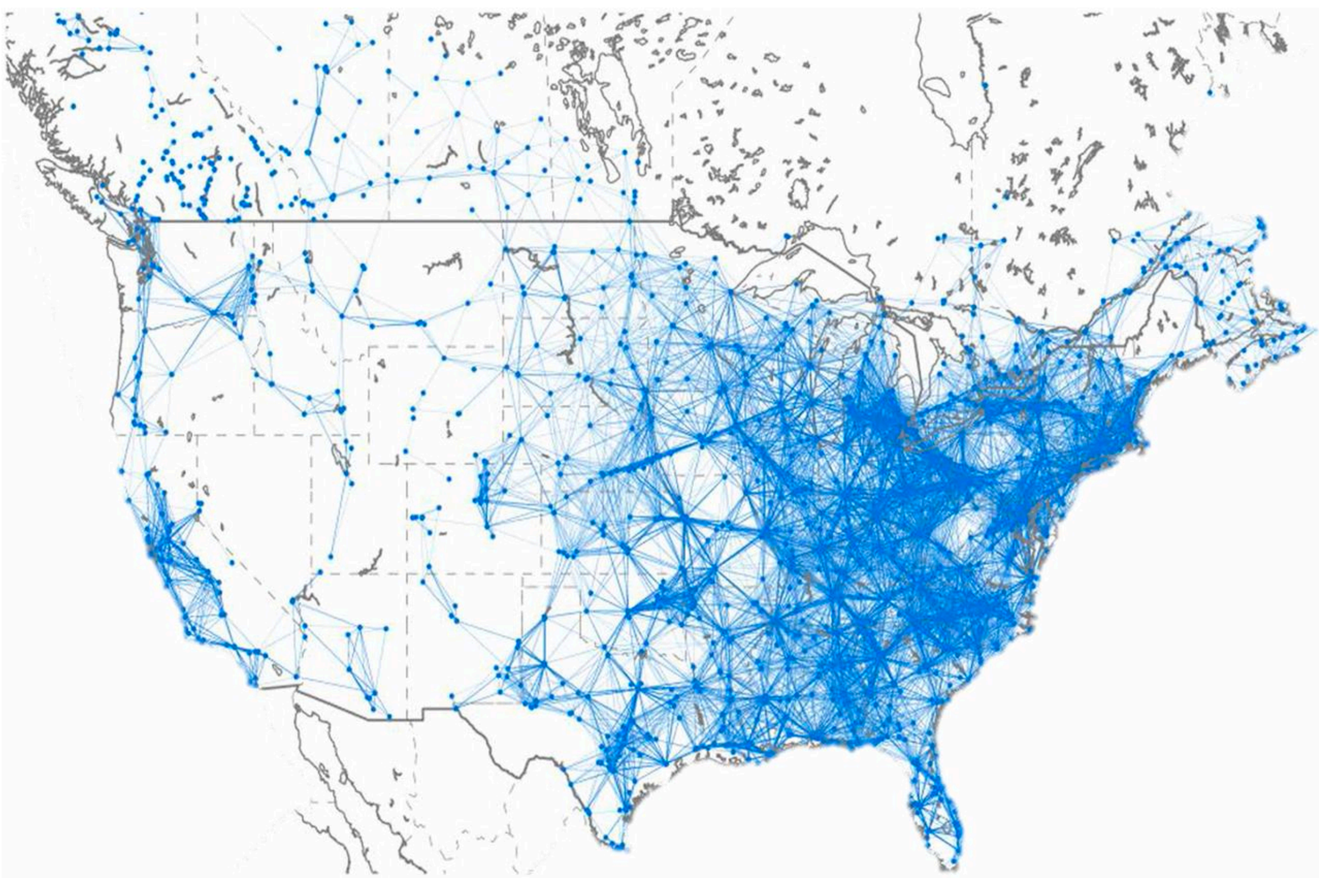
https://www.ctia.org/news/what-is-spectrum-a-brief-explainer

Wireless Spectrum



"The easiest way to understand what spectrum really is and how it provides services is to look at your radio. When you tune your radio to 93.9 FM, you are tuning into a station that is broadcasting at 93.9 megahertz. If you want to a listen to a different station, like one that only plays country music or jazz, you turn the dial to another frequency like 104.7 FM. And a different radio station will be transmitting over that particular frequency on a different setting on your radio dial. No two stations transmit over the same spectrum at the same time in the same area, because if they did, they'd cause interference with one another." -- https://www.cnet.com/news/wireless-spectrum-what-it-is-and-why-you-should-care/

Wireless Spectrum



The New York Times

F.C.C. Backs Proposal to Realign Airwaves

By Edward Wyatt

Sept. 28, 2012

WASHINGTON — The government took a big step on Friday to aid the creation of new high-speed wireless Internet networks that could fuel the development of the next generation of smartphones and tablets, and devices that haven't even been thought of yet.

The five-member Federal Communications Commission unanimously approved a sweeping, though preliminary, proposal to reclaim public airwaves now used for broadcast television and auction them off for use in wireless broadband networks, with a portion of the proceeds paid to the broadcasters.

The initiative, which the F.C.C. said would be the first in which any government would pay to reclaim public airwaves with the intention of selling them, would help satisfy what many industry experts say is booming demand for wireless Internet capacity.

Mobile broadband traffic will increase more than thirtyfold by 2015, the commission estimates. Without additional airwaves to handle the traffic, officials say, consumers will face more dropped calls, connection delays and slower downloads of data.

The F.C.C. will issue proposed rules for what it calls incentive auctions — the sale of airwaves that are voluntarily given up by broadcasters in exchange for a portion of the auction proceeds.

FCC Incentive Auction

- For over 20 years, US and other countries have used spectrum auctions to sell licenses for wireless spectrum
- What's new and different that's happened this decade:
  - Decision to "reassign" airwaves: most popular parts of the spectrum are already owned by TV broadcasters
  - FCC decided to design a new auction (**FCC Incentive Auction**) to buy these back so they can be sold to companies that will put it to better use, e.g. wireless broadband services
- **Double auction:** reverse auction to buy back licenses, forward auction to sell
- We will focus on the **forward auction** to sell licenses today

Setting Items Seperately

- Spectrum auctions are combinatorial in nature because corporations often want a subset of frequencies
  - Having a particular license may make others redundant/ more desirable
- Direct mechanisms are impractical to run in this setting
- What is a reasonable indirect/ asynchronous way to sell multiple items?
- A simple idea: instead of selling bundles, we can try to sell items separately
  - Already know truthful mechanisms for single-item auction
  - Can we just use it to sell multiple items?
- **Question.** Could selling items separately work?
- **Question.** How do we organize these single-item auctions?

Sequential Auctions

- **Question.** Is it a good idea to sell hold single-item auctions sequentially?
- Consider two nearly identical items, sold back to back using a second-price auction
- Suppose you are high-valuation bidder (likely to win any auction)
- What is your best strategy?
    - Suppose everyone is bidding truthfully
    - If you skip the first auction, second-highest bidder wins and is out of the auction
    - Now you can pay third-highest price in the second auction
- Not a dominant strategy to bid in a straightforward way

What Goes Wrong

- **Mistake 1.** Holding single-item auctions sequentially
- In March 2000, Switzerland auctioned off 3 blocks of spectrum via a sequence of Vickrey auctions
- Resulted in some unexpected variation in prices:
  - Two identical 28 MHz blocks sold for quite different prices: **121 million** and **134 million**
  - In a third auction, a larger 56 MHz block sold for **55 million**!
- It was clear the bids were far from equilibrium
- Reasonable to speculate that the revenue generated was far from optimal
- **Takeaway:** items should be auctioned simultaneously

What Goes Wrong

- **Question 2.** What we use a (private) sealed-bid format?
- Difficult for bidders to figure out how to bid in such auctions
- Suppose you want 1 out of 10 licenses
- What are some good bidding strategies?
    - Pick one at random and go for it
    - Bid less aggressively in a few different auctions in the hopes of getting lucky in one of them and getting a deal
- **Challenge:** how to trade off risk of winning too many licenses with the risk of winning too few

Sealed Bid Format

- **Mistake 2.** Use sealed-bid single-item auctions
- In 1990, New Zealand auctioned off almost identical licenses for TV broadcasting using simultaneous (sealed-bid) Vickrey auctions
- Revenue of auction was far below projected revenue
  - Actual revenue: **$36 million; projected: $250 million**
  - In contrast, most spectrum auctions exceed projected revenues
- In one auction, high bid was **$100,000 million** and 2nd highest was **$6**
- In another, highest bid was **$7 million**, and second highest **$5000**
- **Takeaway**: Bidding behavior should be more public (public drop outs)

Simultaneous Ascending Auctions

- So how are spectrum auctions run these days?
- Simultaneous ascending auctions (SAAs) form the basis of the state-of-the-art spectrum auction format
  - A bunch of English auctions (ascending clock) run in parallel
- Main component is that bids are visible to all
  - Even though this may lead to strategic behavior
  - Overall, leads to more-informed decisions

Simultaneous Ascending Auctions

- Why do SAAs work better?
  - Main reason is price discovery
  - Bidders have more information about likely selling prices and can change their strategy midway: abandon highly-competitive licenses, finding unexpected bargains, etc
  - What is another benefit of this format?
    - bidders only need to determine valuations on a need basis
- **General wisdom:** SAAs perform well and achieve good welfare and incentive properties