CS 357: Algorithmic Game Theory

Lecture 3: Intro to Auctions

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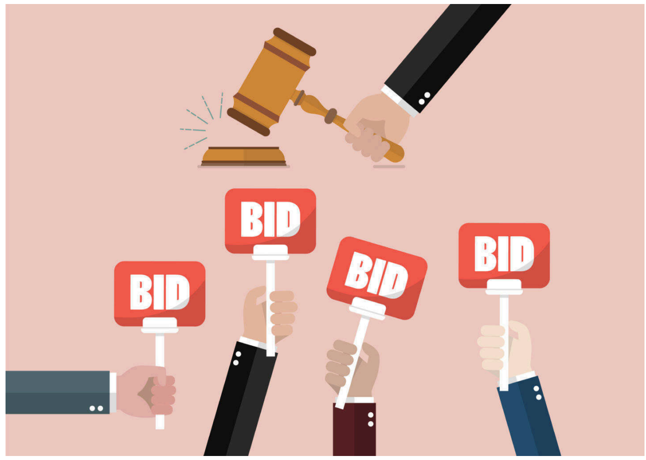
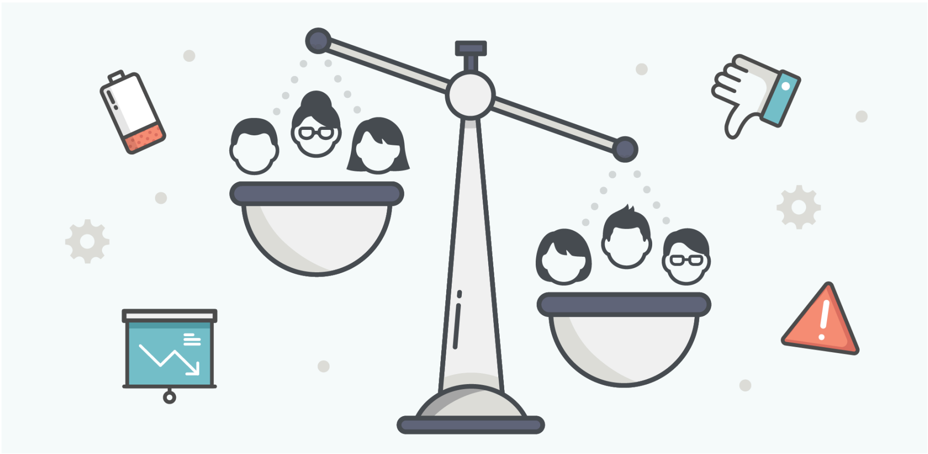
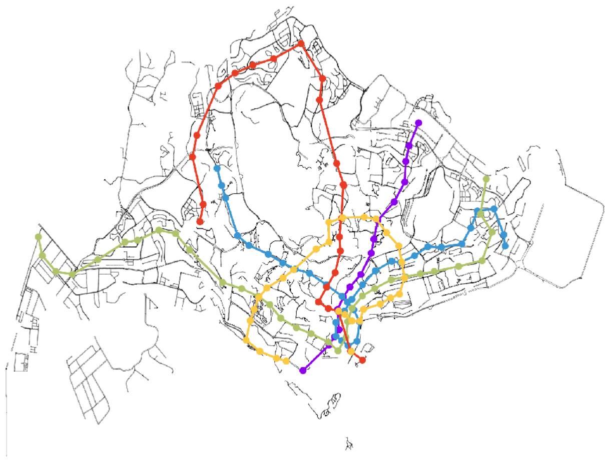


Announcements

- Turn in **Homework 1**
- Pick up **Homework 2**, due Friday in class
- **Assignment 1** (partner work) is due **Fri Feb 21 at noon**
  - Joint write up with partner to be submitted on Gradescope
  - Match pages to questions after uploading
- Help hours (TCL 304/ CS croom):
  - Monday 10-11.30, Wed 2.30-4 pm and Fri 9.30 -10.30 am
  - Come even if you don't have question, sometimes it helps to hear other people's questions

Recap from Last Time

- **"Drinking from a firehose"** intro to some game theory
- **Dominant action** a_i for player i: for all a_{-i}, an action that maximizes their utility u_i compared to all other a_i'
- **DSE**: each player plays their dominant action
  - Strong guarantee: may not exist always exist
- **Best response:** An action a_i^* is the best response to a_{-i} if playing a_i^* achieves the best-possible utility compared to any other action a_i, (keeping everyone else's action a_{-i} fixed)
- A pure-strategy Nash equilibrium is an action profile where each player plays their best response to others

Two Games

Prisoner's Dilemma

Guess 2/3rd average

Question. Which one is easier to participate in and reason about?

Problems with Nash

- Additional examples at the end of slides
  - Good idea to look them to answer HW 2 questions
- Based on what we know so far, what challenges do you foresee if we use Nash equilibrium as a "solution concept"
  - Most of the time we "guess and check"
  - But complexity of computing is extremely high
  - Multiple Nash equilibrium: problem of equilibrium selection
  - No pure Nash equilibrium may exist
- "Mixed Nash" (randomized strategies) later, always guaranteed to exist

Before Moving on to AGT

https://www.youtube.com/watch?v=LJS7Igvk6ZM

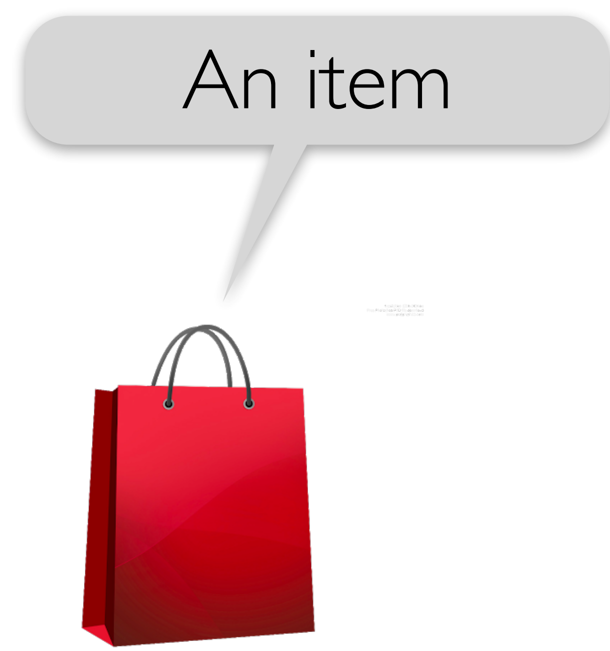
How to align individual ambition with that of the group?

Today: Introduction to Market Design

- How to approach algorithmic problems about markets
- Many types of markets:
  - Centralized or decentralized
  - With or without money
  - One-sided or two-sided
- Start with the most natural: centralized, single-item market
- **Goal:** how to design algorithms that induce "desirable behavior" as a dominant strategy of the participants
  - De-incentivize strategizing: align local and global objectives

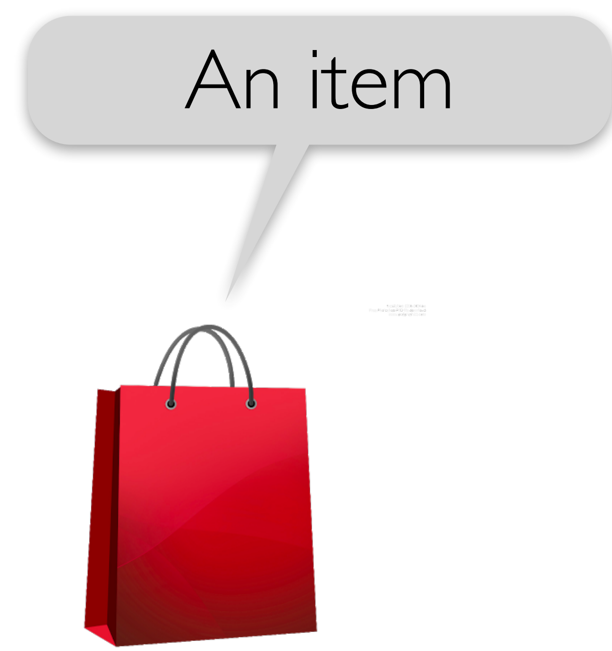
Single-Item Centralized Market

- Design questions:
  - How to capture buyer's preference? (Individual preference)
  - What is a good outcome for the market? (Global preference)



Single-Item Centralized Market

- Each player i has some personal (private) **valuation** v_i for the item
  - Quantifies how much they value it *(maximum they are willing to pay for it)*
- Suppose the goal is to give the item to the buyer with the maximum value *(generate the most social welfare)*



Getting Started

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Prefer receiving the item to not receiving the item.

n buyers, each have a **private** value v_i for the item

An item

First Attempt

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Prefer receiving the item to not receiving the item.

Algorithm:

1. Ask each player to report their value (their **"bid"** b_i)
2. Give the item to the player with largest bid for free

What would players do?

Problem With Earlier Approach

- Player's best response: try to bid as high as possible
  - For any fixed bid profile $\mathbf{b}_{-i}$, always incentive to bid $>$ max $\mathbf{b_{-i}}$
- No connection between true value and bid

Refined Goal. Need to incentivize truthful bidding, whenever possible.

- **Question.** Any ideas on ways to incentivize truthful bidding?

Lottery Auction

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Prefer receiving the item to not receiving the item.

Algorithm:

1. Select a player uniformly at random
2. Allocate the item to the selected player.

Any incentive to misreport value?

How good is this in terms of "social welfare"?

Social Welfare

- **Definition.** The social welfare of a mechanism is the total value it "generates"
    - If a player with value v_i gets the item, then value generated is v_i
- What is the "optimal" algorithm (one that maximizes social welfare)?
    - One that allocates item to $i^* = \text{argmax}_i \; v_i$ *(defined as the player with the maximum value)*
- How good is the lottery mechanism?
    - Expected social welfare $= \sum_{i=1}^{n} (1/n) \cdot v_i = (1/n) \sum_{i=1}^{n} v_i$
    - In the worst case, this is an n-approximation of the optimal

Takeaway from Lottery

- Achieving truthful reporting by itself is not useful if the optimization criterion (social welfare) achieved is also low

Question. Is it possible to achieve both truthfulness & optimal welfare?

Best Possible Without Payment

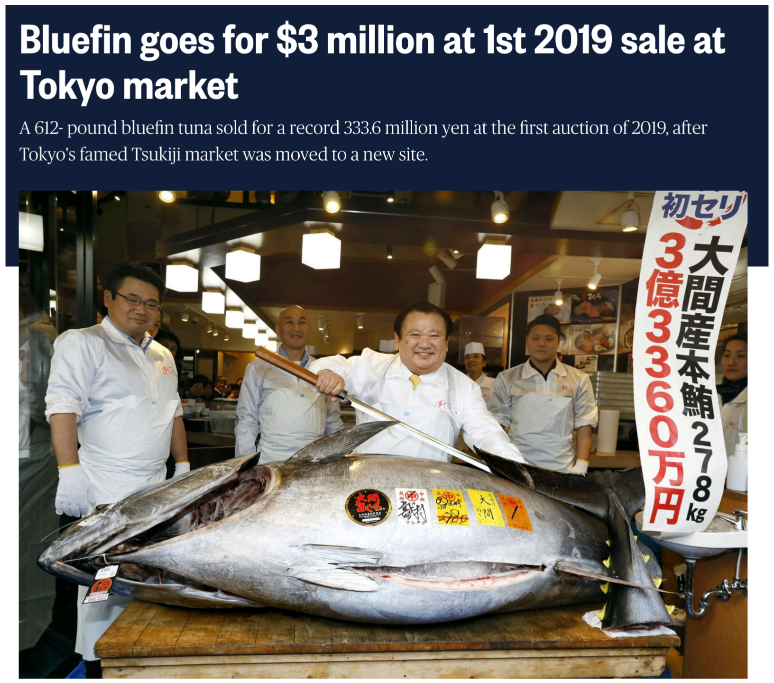
- Turns out the lottery mechanism is the best you can do without charging payments

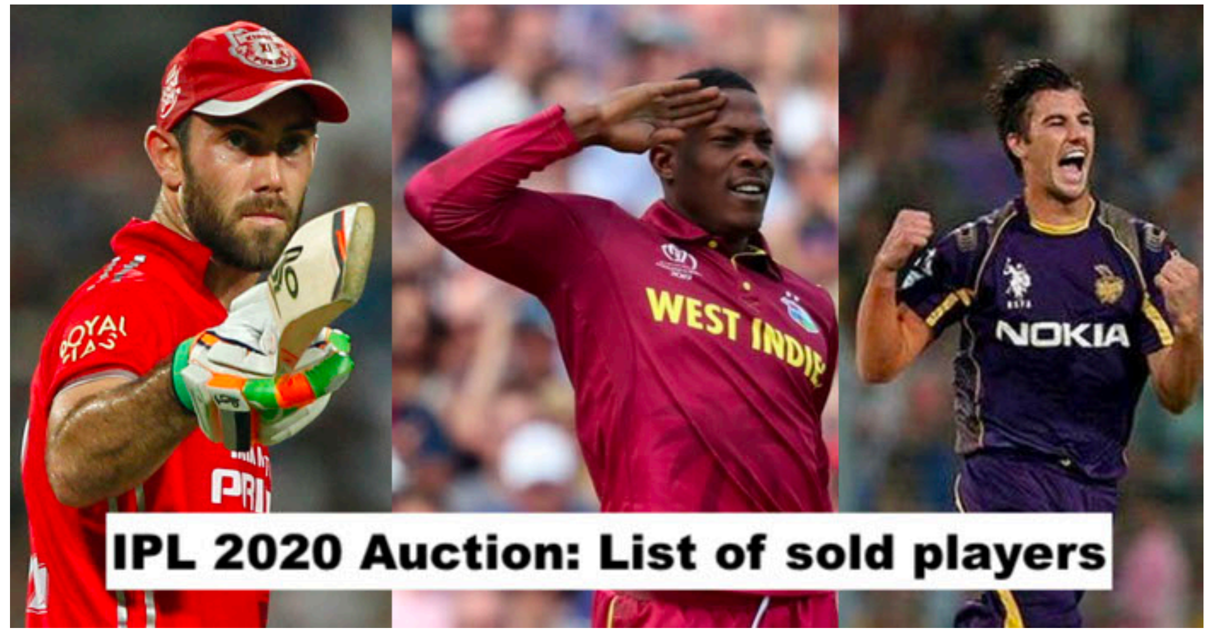
Need Prices. Even if goal is not to generate revenue, just social welfare.

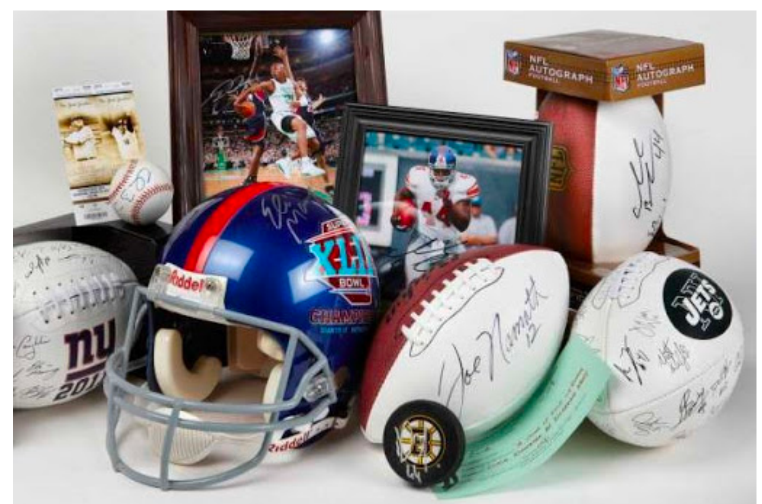
Auction: Allocation and Prices

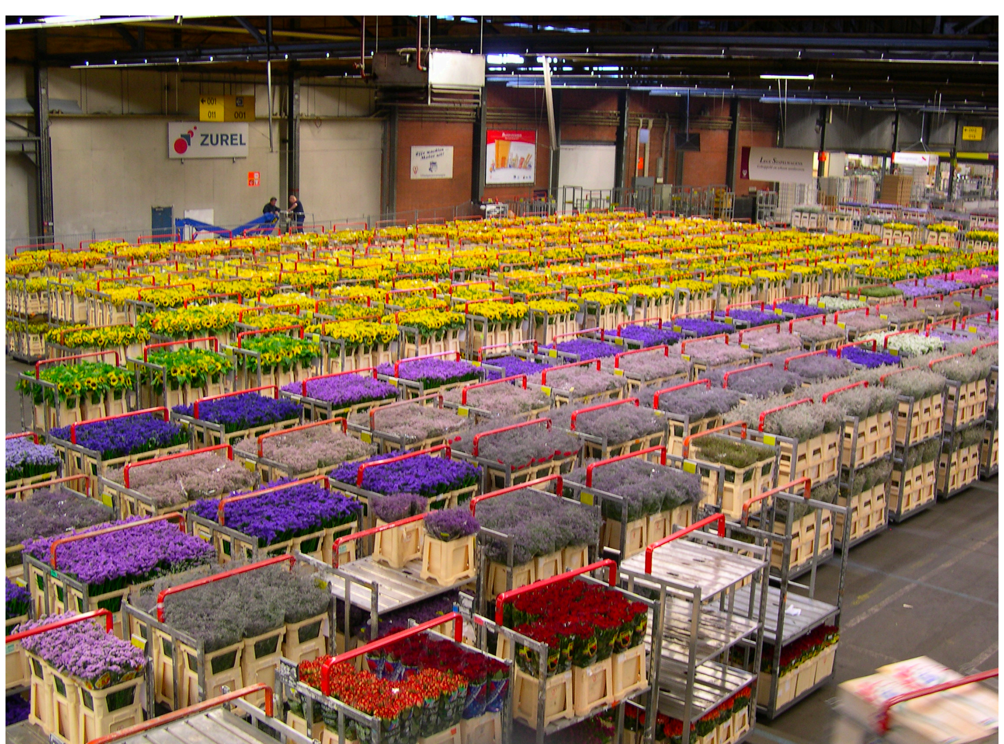
- Players submit bids
- Allocation rule: who gets the item?
- Payment rule: what do they pay?

Auction: Real-Life Examples?









Sealed-Bid Auctions

- Players submit sealed (private) bids
- Allocation rule: who gets the item?
- Payment rule: what do they pay?

A natural payment rule?

First Price Auction

- Each player submits a private (sealed) bid b_i
- Allocation rule: allocate to bidder with highest bid
- Payment rule: charge the winner a **payment equal to your bid**
- To reason about this auction, first model player utility:
  - Utility of player is value minus price if they win, otherwise $u_i = 0$
- Very natural and commonly used auction:
  - Real estate auctions, government auctions etc

First-Price Auction: More Formal

- Each player submits a private (sealed) bid b_i
- Allocation rule: allocate to bidder with highest bid
  - $\mathbf{x} = (x_1, \ldots, x_n)$ where $x_i = 1$ if i wins and $x_i = 0$ otherwise
- Payment rule: charge the winner a payment equal to your bid
  - $\mathbf{p} = (p_1, \ldots, p_n)$ where $p_i = b_i$ if i wins and $p_i = 0$ otherwise
- To reason about this auction, first model player utility:
  - Utility of player i : $u_i = v_i \cdot x_i - p_i$
- **Question.** How should players bid? How to reason about utility?

Class First-Price Auction

- Your value: **sum of the last four digits of your SID times 0.4 cents**
  - E.g. 3124578 leads to value 4+5+7+8 * 0.40 cents = $9.60
- There will be two auctions:
  - **Two-person:** you will be paired with a random person
  - **Three-person:** you will be paired with a random pair
- Need from you: two bids (one for each auction)
- If you win (highest bid), you get utility **(value - your bid)**
- Send your bids at https://tinyurl.com/357auction by **4 pm today**

Towards a Simpler Auction

- Is truthfully bidding a dominant strategy in first-price auction?
  - No, truthful bidding gives zero utility
  - Bidders ***shade*** their shade
- A first-price auction is an incomplete-information game
  - We need a "variant" of Nash to analyze it
  - Requires some comfort with probability and expectation
  - Lets defer this discussion for now
- Today: want an auction where no one has incentive to "strategize"
- What can we charge as payment to make players bid truthfully?

Second-Price (Vickrey) Auction

- Each player submits a private (sealed) bid b_i
- Allocation rule: allocate to bidder with highest bid
- Payment rule: charge the winner a payment equal to the second-largest bid

n bidders, each has a **private** value v_i for the item

Vickrey Auction. How good is this auction? How should bidders bid?

Strategy in Auctions

- Each bidder has a private value $v_i \in \mathbb{R}$ and submits a bid b_i
- Strategy $s_i : \mathbb{R} \to \mathbb{R}$ of bidder i
    - Defines a bid b_i for every possible value v_i
    - In general, strategy maps information available to the action

DSE in Auctions

- A **dominant strategy equilibrium (DSE)** in an auction is a bid profile $\mathbf{s} = (s_1, \ldots, s_n)$ such that for all bidders $\boldsymbol{i}$, valuation v_i and bids $\mathbf{s}_{-i}$, bidder $\boldsymbol{i}$'s utility is maximized by following the strategy $s_i(v_i)$

- **"Truthfull bidding"** is referred to the strategy profile where each bidder $\boldsymbol{i}$, has strategy $s_i(v_i) = v_i$

Truthtelling as DSE

- **Theorem.** A sealed-bid second-price (SBSP) auction is
  - **dominant strategyproof**, i.e., truthful bidding is a weakly dominant strategy for all bidders
  - truthful bidding gives you non-negative utility
- Proof. (On board)
- In fact, truth telling is the **unique dominant strategy** in a SBSP auction
  - **Exercise.** Think about how you would prove this!

Social Welfare

- Second-price (Vickrey) auction maximizes surplus:
  - At the unique DSE, highest valuation bidder gets item
- *Solves the surplus-maximization optimization problem as well as if the valuations where known in advance!*
- **Linear time.** All the auction needs to do is compute maximum and second maximum from a list of bids, and thus is linear-time

General Auction Design Goals

- *Strong incentive guarantees:*
  - Dominant strategyproof (truthful reporting is dominant strategy)
  - Truth-telling guarantees non-negative utility (individually rationality)
- *Strong optimization guarantees:*
  - Maximizes social surplus $\sum_{i=1}^{n} v_i x_i$, where $x_i = 1$ if i wins and 0 otherwise; and $\sum_{i=1}^{n} x_i = 1$ (single item case)
- *Computational efficiency:*
  - Can be implemented efficiently (polynomial time in the worst case)

Other Possibilities

- Analyzed sealed bid second-price auctions
- Will analyze first-price auction next time
- Both give away the item to the highest bidder:
  - Does it ever make sense to give the item to not the highest bidder?
- What about the other design choices?
  - How good/bad are other payment rules?
  - Are multi round auctions inherently "richer" than sealed bid ones?

Auctions Practice

- **Assignment 1 (Problem 2):** Show that charging the highest bidder the third-highest bid is not DSIC.
- **Assignment 1 (Problem 3).** Show that sealed-bid second price auctions are susceptible to collusion: give necessary and sufficient conditions
    - Even though for a single player truth telling is dominant
    - For a group, they can cheat and get better total utility

Multi-Round Auctions

Auctions

Sealed-bid (Simultaneous-move auctions)

Multi-round open-outcry style auctions where bidders respond to other bids

- Ascending
- Descending, etc