STAT131 Week 1 Notes

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1 Introduction

This document covers essential information and example problems with answers in Chapter 1 in the "Introduction to Probability, 2nd Edition" written by Joseph K. Blitzstein and Jessica Hwang.

2 Important Definitions

Contains important definitions for understanding Chapter 1 in textbook.

- *Sample Space* S is the set of all possible outcomes from the experiment.
- *Event* A is the subset of *sample space* S.
  - **Note:** We say an event *occurred* if the actual outcome is in A.
- *Complement* of A are the elements not in A, denoted as

$$A^c$$

- *Union* of A and B occurs if *at least one* of A or B occurs

$$A \cup B$$

- *Intersect* of A and B occurs if only and if *both* of A and B occur

$$A \cap B$$

- *De Morgan's laws* are a pair of transformation rules vital to set theory

$$(A \cup B)^c = A^c \cap B^c \text{ and } (A \cap B)^c = A^c \cup B^c$$

- *Naive definition of probability* is to count the number of ways an event can happen and divide by the total number of possible outcomes

$$P_{naive}(A) = \frac{|A|}{|S|} = \frac{\text{number of outcomes favorable to A}}{\text{total number of outcomes in S}}$$

  1. There is *symmetry* in the problem if outcomes are equally likely.
  2. Naive can be used when outcomes are equally likely *by design*.
  3. Naive is useful as a *null model*, which is when we apply the naive definition to see what predictions it would yield.

- The *multiplication rule* is used for *sampling with replacement* and *sampling without replacement*.

  - **Usage:** Compound experiment with experiment A, B, with a, b outcomes respectively. Then the compound experiment has *ab* possible outcomes.

- To adjust for overcounting, note the amount of times you count each probability (c) and adjust by dividing by c.

- *Story proofs* are proofs by interpretation. This usually means counting the same thing in two different ways, rather than doing an algebraic proof.

- A *probability space* consists of a sample space S and a *probability function* P that takes an event A is a subset of S as input and returns *P(A)*, a real number between 0 and 1, inclusive, as output. It must satisfy the following axioms.

  1. $P(\emptyset) = 0$, $P(S) = 1$.
  2. If A_1, A_2, ... are disjoint events, then

$$P(\bigcup_{j=1}^{\infty}(A_j)) = \sum_{j=1}^{\infty} P(A_j).$$

- For any events A and B, they hold the properties:

  1. $P(A^c) = 1 - P(A)$
  2. If $A \subset B$, then $P(A) \leq P(B)$
  3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- *Permutations* occur when <u>selecting without replacement</u> and <u>order is important</u>

- **Usage:** $\frac{n!}{(n-k)!}$, n = total number of items, k = number of chosen items, denoted as ${}_nP_k, P(k,n), P_{n,k}$

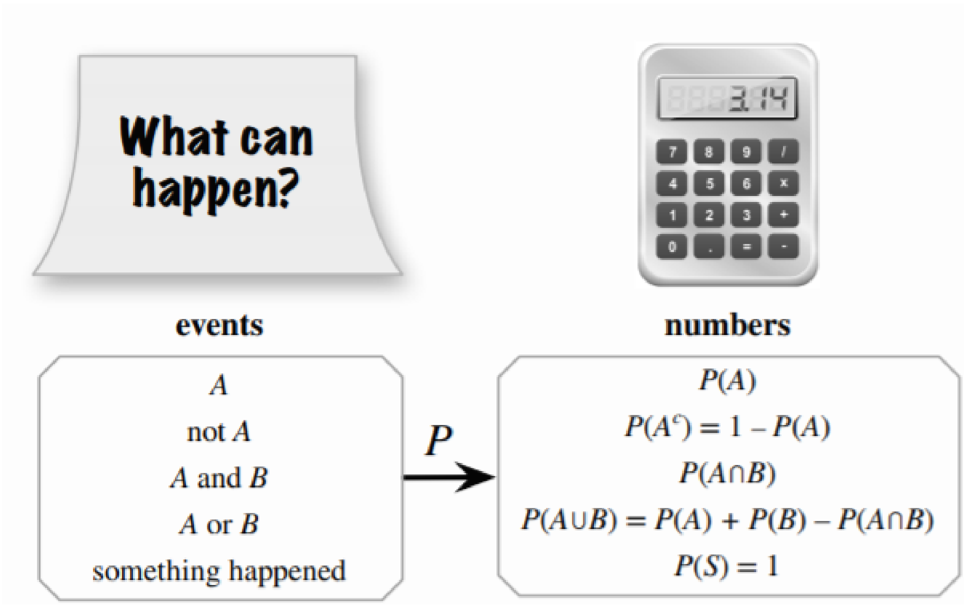
- *Combinations* occur when selecting without replacement and order isn't important
  - **Usage:** $\frac{n!}{(n-k)!k!}$, n = total number of items, k = number of chosen items, denoted as ${}_nC_k, C(k,n), C_{n,k}$

- Choosing an item *with replacement* means you can choose the same item multiple times
  - **Usage:** n^k, n = total number of items, k = number of chosen items

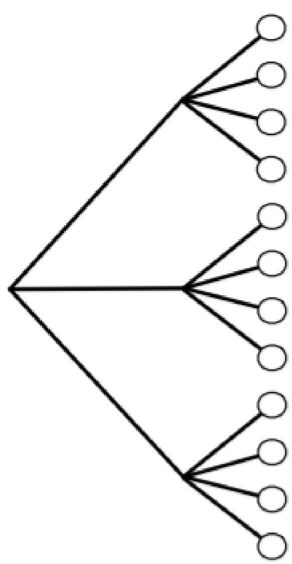
3 Screenshots

1.

English	**Sets**
Events and occurrences	
sample space	S
s is a possible outcome	$s \in S$
A is an event	$A \subseteq S$
A occurred	$s_{\text{actual}} \in A$
something must happen	$s_{\text{actual}} \in S$
New events from old events	
A or B (inclusive)	$A \cup B$
A and B	$A \cap B$
not A	A^c
A or B, but not both	$(A \cap B^c) \cup (A^c \cap B)$
at least one of $A_1, \ldots, A_n$	$A_1 \cup \cdots \cup A_n$
all of $A_1, \ldots, A_n$	$A_1 \cap \cdots \cap A_n$
Relationships between events	
A implies B	$A \subseteq B$
A and B are mutually exclusive	$A \cap B = \emptyset$
$A_1, \ldots, A_n$ are a partition of S	$A_1 \cup \cdots \cup A_n = S, A_i \cap A_j = \emptyset$ for $i \neq j$


What can happen?
events
numbers
A
not A
A and B
A or B
something happened
P
P(A)
P(A^c) = 1 – P(A)
P(A∩B)
P(A∪B) = P(A) + P(B) – P(A∩B)
P(S) = 1

2.



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