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Journal of Development Economics 84 (2007) 76 – 103

JOURNAL OF Development ECONOMICS

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Urbanization, informal sector, and development ☆

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Received 19 July 2004; received in revised form 30 June 2005; accepted 17 September 2006

Abstract

Urbanization is a universal phenomenon in the postwar world, but its qualitative nature is varied across countries. Some nations have experienced urbanization that accompanied skill upgrading, industrialization, and the expansion of the urban formal (modern) sector, but others urbanized without such modernization and underwent the expansion of the urban informal (traditional) sector simultaneously. The question that naturally arises is what are the underlying causes for the differences. The purpose of the paper is to tackle this question analytically based on a dynamic model of urbanization and development.

JEL classification: J62; O11; O15; O17
Keywords: Urbanization; Informal sector; Development; Income distribution

1. Introduction

Urbanization proceeded universally in the post-war world. The degree of urban concentration increases with a nation's income level, while the speed of urbanization shows an inverted U pattern: the speed of urbanization of middle-income nations is highest and that of developed nations is lowest (Mazumdar, 1987). Since middle-income nations are, on average, those that are undergoing highest economic growth and the most drastic changes in their economic structures, the evidence suggests that urbanization is related to the income growth, industrialization, and modernization of an economy.

However, the speed of urbanization seems not to be explained solely by economic growth, because many of nations that urbanized most rapidly, i.e. African nations among low-income

☆ Insightful comments and suggestions from two anonymous referees and an editor are gratefully appreciated. All remaining errors are the author's own.
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doi:10.1016/j.jdeveco.2006.09.004

nations and Latin American and Caribbean nations among middle-income nations, grew relatively slowly (Mazumdar, 1987; Fay and Opal, 2000).

Behind the changes in the aggregate variable are migration decisions of rural residents, which are mainly driven by economic factors, that is, better job opportunities in urban areas. However, immigrants do not necessarily enjoy higher welfare in urban areas, due to the dualistic nature of the urban economy in a typical developing country.

The urban economy of a typical underdeveloped nation can be classified into the formal or modern sector and the informal or traditional sector.[1] The informal (traditional) sector is characterized by small firm sizes, unskilled jobs, low wages, and the loose enforcement of laws and regulations, and it incorporates sectors such as petty trading, domestic services, repair services, and basic manufacturing. The formal (modern) sector, by contrast, possesses contrasting features and includes modern manufacturing and skill-intensive services such as finance and health care. According to Ranis and Stewart (1999), in most developing countries, the size of the urban informal sector is substantial, in many cases accounting for over half the urban workforce. Further, earnings of workers in the informal sector are close to or sometimes lower than those in rural areas after adjustment for the cost of living.[2] The large share of the informal sector in the urban economy implies that many immigrants do not attain noticeable improvements of their economic conditions.

It seems that urbanization is associated with sizable job creation in the formal sector and thus large welfare gains by immigrants occur, only if an economy achieves industrialization. For example, the case study of Thailand and the Philippines by Ranis and Stewart (1999) shows that, during the 1960s and the 1970s, both nations experienced large-scale urbanization accompanied by rapid industrial growth, resulting in the expansion of the formal sector and the contraction of the informal sector. After the 1980s, Thailand kept following this trend, while the Philippines encountered urbanization with stagnant (almost zero) industrial growth and had to deal with the rapid growth of the informal sector.

The question is why some economies attain urbanization accompanying the expansion of the formal sector and why others urbanize without such modernization but with the expansion of the informal sector. This paper presents a dynamic model of urbanization and development in order to answer the question.

The model economy is inhabited by a continuum of individuals and is composed of the rural agriculture sector, the urban informal (traditional) sector, and the urban formal (modern) sector. The rural sector and the urban informal sector employ unskilled workers, whereas the urban formal sector employs skilled workers and physical capital. The rural sector produces an agricultural good, and the urban formal sector produces a manufacturing good, both of which are tradables. The urban informal sector produces a nontradable good that would correspond to services such as petty trading and domestic services in real economy.[3] The rural agriculture sector

[1] Strictly speaking, the modern/traditional classification is based on production technologies, while the formal/informal classification is based on establishment size or social security coverage, so they are distinct. In this paper, however, the two taxonomies are used interchangeably, because the focus is on differences in skill requirements between the sectors, as detailed below.

[2] According to Jamal and Weeks (1988), for example, rural–urban income differentials are substantial in many sub-Saharan African nations but the income differentials for unskilled workers are very small and, in some nations, their urban incomes are actually lower than rural incomes.

[3] Ranis and Stewart (1999) find that about 70% of the labor force in the informal sector in Manila in 1986–87 are engaged in trading and only about 13% are in manufacturing. In Bangkok, the proportion of workers in the informal sector engaged in manufacturing is larger but still only about 17%. Similar numbers are found for many Latin American economies as well (Marcouiller et al., 1997).

exhibits diminishing returns to scale, reflecting the scarcity of land, and output is shared equally among workers, while the urban sectors have constant returns to scale technologies and wages are determined competitively. As different skill requirements of the formal and informal sectors indicate, this paper focuses on the role of human capital accumulation in development. The accumulation of physical capital is thus given only a minor role by assuming free international capital mobility.[4]

An individual in the model lives for two periods, the first period as a child and the second period as an adult. In childhood, he or she receives a transfer from the parent to invest in assets or education. Education is needed to become a skilled worker, is provided by teachers (skilled workers), and must be financed by the received transfer. That is, there are no credit markets for such investment in this economy. In adulthood, depending on human capital accumulation in the previous period, the individual becomes a skilled or unskilled worker, and makes occupational and associated locational choices. There are no barriers to interregional migration. Then, he or she receives labor and capital incomes, which are spent on the consumption of the three goods and a transfer to his or her sole child. The utility function is assumed to be Cobb–Douglas, hence the share of spending on each item is constant over time. Generations go by in this fashion. Individuals of the same generation are heterogeneous in terms of received transfers, and this heterogeneity translates into diverse choices of investments, occupations, and locations, and brings interesting dynamics.

In the model economy there exist two types of steady states. The first type called the equal opportunity steady state has features of a typical developed economy: many people are educated and skilled; most people live in urban areas and are engaged in non-agricultural activities; and inequalities between skilled and unskilled workers and between urban and rural residents are small. (In the model, distributions of net wage and wealth are equalized.) The second type called the unequal opportunity steady states, by contrast, characterizes a typical stagnant developing economy: many people live in rural areas and are engaged in agriculture; many of the urban residents are not skilled and remain in the informal sector; and the inequalities are substantial.

Given an initial distribution of transfers over the population, an economy approaches one of the steady states in the long run. If an economy starting with limited asset accumulation is on a path to the equal opportunity steady state, it experiences urbanization that accompanies skill upgrading, the expansion of the formal sector, the shrinkage of the informal sector, and falling inequality. Instead, if it is on a track to an unequal opportunity steady state, it urbanizes without skill upgrading or modernization but with the expansion of the informal sector. Thus the model exhibits two distinctive patterns of urbanization observed in real economy. The critical determinant of the long-run success of an economy is the fraction of the population who can afford education in the initial period, which reflects the degree of equality of initial asset distribution unless aggregate asset accumulation is very low. This is consistent with empirical findings, for example, by Deininger and Squire (1998) and Deininger and Olinto (2000), which show that an economy's growth rate is affected negatively by initial land inequality (a proxy for initial asset inequality).

These results are obtained with fixed productivities. The paper also examines effects of one-time productivity changes in the rural agriculture sector and the urban formal sector on the urbanization and development of an economy. Both a decrease in the agricultural productivity and an increase in the formal sector's productivity promote urbanization but make it more likely for an

[4] The model becomes analytically intractable once the closed capital market is assumed. In considering the urbanization of contemporary developing nations, this simplifying assumption would be defended, since the link between domestic savings and investment is becoming weaker with growing international capital flows.

economy starting with an unfavorable wealth distribution to end up in an unequal opportunity steady state. The result is suggestive since many sub-Saharan African nations actually experienced falling agricultural productivity,[5] rapid urbanization associated with the expansion of the informal sector, and stagnant growth for many years. The effects of the productivity increase in the formal sector suggest that skill-biased productivity growth is not necessarily beneficial for the long-run development of a poor economy.

This paper is related to the literature that explores the mechanisms of urbanization and the evolution of the informal sector. Many papers in this literature, including Fields (1975), Gupta (1997), and Ranis and Stewart (1999), are some variants of the Harris and Todaro (1970) model in the sense that the formal sector is characterized as the sector that pays higher wages to *identical* workers than the informal sector does, due to exogenous institutional factors, such as entry restrictions to the sector, compliance with minimum wage regulation, and the presence of trade unions. By contrast, the present paper distinguishes the two sectors in terms of skill requirements to workers. Empirical evidence supports this characterization: for example, Marcouiller et al. (1997) found that workers in the formal sector are significantly more educated than workers in the informal sector. Further, the conventional classification is at odds with some empirical findings: according to Funkhouser (1997), while there exist unexplained sectoral wage differentials after adjustment for workers' characteristics, the differentials seem not to be explained largely by the presence of labor regulations or unions.[6] Of course, each characterization would capture a part of reality and the present paper's skill-based classification reflects its emphasis on human capital as the determinants of development and the regional and sectoral distributions of workers.

In this literature, Bencivenga and Smith (1997) and Banerjee and Newman (1998) are particularly worth mentioning. Bencivenga and Smith (1997) constructs a two-period OLG model with the rural, urban formal, and urban informal sectors and analyzes how migration, unemployment (employment in the informal sector), physical capital accumulation, and economic development are related. In their model, individuals are *innately* skilled or unskilled, but their types are private information and an adverse selection problem arises. Unlike a conventional Harris–Todaro model, this adverse selection, not exogenous institutional factors, gives rise to the dual structure of the urban economy. In equilibria, only skilled workers migrate to the urban region, while unskilled workers stay in the rural sector. Depending on parameter values, there exist multiple steady state equilibria, where the high-income steady state is more urbanized and industrialized but has a *greater* share of its urban workforce in the informal sector.

Banerjee and Newman (1998) examine implications of differences in technological and institutional conditions between urban modern and rural traditional sectors in development. In the model economy, individuals can move freely between the sectors and earn higher wages in the more productive modern sector. This does not lead to full urbanization, however, since the rural sector has one advantage: loans for fulfilling opportunities to consume indivisible goods are available more easily, because the degree of information asymmetry between lenders and borrowers is lower in the close-knit community. Hence, there exists a tradeoff between the better access to credit in the traditional sector and the higher productivity in the modern sector, and thus some of them stay in the rural sector. Under this setting, the degree of urbanization could be too

[5] Fulginiti and Perri (1997) examine changes in agricultural productivity in 18 developing countries over the period 1961–1985 and find that at least half of these countries have experienced productivity declines in agriculture including all the sub-Saharan African nations in the sample.

[6] Also, Magnac (1991) constructs a Roy model to test the hypothesis of competitive labor market against that of segmented market for married women in the urban areas of Columbia and does not reject the competitive hypothesis.

low: if some of those staying in the rural sector and receiving loans are forced to migrate, loans available to the urban sector increase and the interest rate falls, which in turn promotes migration and raises total output.

The present paper is also related to the literature that investigates the interplay between income distribution and growth through human capital accumulation, which includes Galor and Zeira (1993), Ljungqvist (1993), Benabou (1996a,b), Durlauf (1996), Galor and Tsiddon (1997), and Yuki (2003). Closely related are Galor and Zeira (1993), Ljungqvist (1993), and Yuki (2003).

Galor and Zeira (1993) considers a small-open OLG model, where, as in the present paper, intergenerational transfers motivated by impure altruism (utility from making transfers) constrain educational decisions, while capital is freely mobile internationally. There are two sectors, one using skilled labor and physical capital and the other employing unskilled labor, to produce a single final good. They examine how initial wealth distribution affects the dynamics of the model economy. Ljungqvist (1993) examines a dynamic model with a single production sector where individuals are infinitely lived (or have perfect altruism), the final good is produced by skilled labor, unskilled labor, and physical capital, and education is required to remain skilled for a fixed units of time. He shows that there exists a continuum of steady states that are different in terms of the proportion of skilled workers and wage inequality. In these papers, wages are determined by supply factors, whereas, in the present paper, unskilled wage is affected by demand factors as well.

Banerjee and Newman (1993) examine the interaction between occupational choices and the distribution of wealth in the process of development. There exist three kinds of occupations – self-employed workers, entrepreneurs, and wage earners – in the economy. Due to credit market imperfections, certain amounts of wealth are required to become an entrepreneur or a self-employed worker, and thus occupational patterns are affected by the distribution of wealth. Occupational patterns in turn affect future wealth distributions and their interaction determines the fate of the economy.

Yuki (2003) constructs a similar model with the tradable modern sector and the nontradable traditional sector. However, its focus is on the mechanism by which initial distribution affects the long-run outcome of an economy and the effect of the sectoral shift of consumption associated with income growth (Engel's law) on development.

This paper is organized as follows. Section 2 presents the model economy and derives equilibrium conditions. Section 3 analyzes the model's dynamics and Section 4 presents and interprets the results. Section 5 concludes the paper.

2. Model

The model economy is of a discrete-time OLG variety. The economy is composed of a continuum of individuals who live for two periods, the first period as children and the second period as adults. There are two regions, urban and rural, and, as detailed below, each region has different production sectors and available jobs. This is the only difference between the regions in the model: there are free mobility of goods and factors of production; hence prices are equalized between the regions. There is no uncertainty in the model.

2.1. Lifetime of an individual

2.1.1. Childhood

In childhood, an individual receives a transfer from his or her parent. Then he or she allocates the transfer for two investment options, assets and education, in order to maximize future income.[7] The educational investment is required to become a skilled worker and enjoy higher earnings in adulthood. The investment is a discrete choice, making education optional and incurring a fixed cost. Consider an individual who was born into lineage i in period $t-1$. His or her generation is called *generation t*. Then, his or her education costs e_t, and its gross return is $w_{h,t}-w_{l,t}$ in the next period, where $w_{h,t}$ and $w_{l,t}$ are skilled and unskilled wages in period t, respectively. Assume that the education cost is the cost of hiring current skilled workers as teachers and it is proportional to $w_{h,t-1}$:$e_t=s_e w_{h,t-1}$.[8] The investment must be self-financed because loan markets for such investment are not available: the child's future income is not considered a valid collateral in the financially underdeveloped economy. The other option, the investment in assets, is a continuous choice, and brings a gross rate of return of $1+r_t$. It is easily shown that, in an equilibrium, the return from the investment in education becomes at least as high as the return from the investment in assets, i.e. $w_{h,t}-w_{l,t}\geq(1+r_t)e_t$.

Suppose that the individual has received b_t^i units of income as a transfer from the parent. He or she allocates the transfer between the investments in assets a_t^i and in education e_t^i in order to maximize future income. If the return from the educational investment is strictly higher than the one from the asset investment, optimal investment choices are given by the following equations:[9]

$$a_t^i = b_t^i, \quad e_t^i = 0, \quad \text{if} \quad b_t^i < e_t, \tag{1}$$

$$a_t^i = b_t^i - e_t, \quad e_t^i = e_t, \quad \text{if} \quad b_t^i \geq e_t. \tag{2}$$

Since innate abilities of individuals are identical, transfers solely determine the investment and resulting occupational choices.

2.1.2. Adulthood

At the beginning of adulthood, an individual makes occupational and associated locational choices. Available jobs are constrained by the human capital investment in the previous period. Then, the individual obtains income from assets and labor supply and spends the income on consumption and a transfer to his or her child. Each adult is assumed to have a single child. There are three different consumption goods, goods *uh*, *ul*, and *rl* in the economy. Characteristics of these goods are described later in this section. The utility function of agent i in generation

[7] Alternatively, one can suppose that the investment decisions are carried out by the parent in order to maximize the child's future income. Note that the transfer in the model corresponds to *total* intergenerational transfers including bequests, education, and other inter-vivos transfers in real life. The decision that the child has to make is the allocation of the whole transfers between education and assets.

[8] World Bank (1983) reports that about 95% of current expenses in primary school systems of low income countries are teacher salaries.

[9] Actually the relative return from education is determined as a result of people's investment decisions, since it depends on the numbers of skilled and unskilled workers in the economy. More formal analysis of the investment decision is described in Section 3.1.

t depends on the consumption of the three goods $c^i_{j,t}$, $j=uh$, ul, and rl, and the transfer to his or her child b^i_{t+1}:

$$u^i_t = \prod_j (c^i_{j,t})^{\gamma_j}(b^i_{t+1})^{\gamma_k}, \quad \sum_j \gamma_j + \gamma_k = 1. \tag{3}$$

The individual maximizes the utility function subject to the budget constraint,

$$\sum_j p_{j,t}c^i_{j,t} + b^i_{t+1} = w^i_t + (1+r_t)a^i_t, \tag{4}$$

where $p_{j,t}$ is the price of good j, w^i_t is the wage of the individual, r_t is the interest rate, and a^i_t is his or her asset holding in period t. Solving the maximization problem gives the following consumption and transfer rules:

$$p_{j,t}c^i_{j,t} = \gamma_j\{w^i_t + (1+r_t)a^i_t\}, \tag{5}$$

$$b^i_{t+1} = \gamma_k\{w^i_t + (1+r_t)a^i_t\}. \tag{6}$$

2.1.3. Generational change

At the beginning of period $t+1$, current adults pass away, current children become adults, and new children are born into the economy. Since each adult has one child, the population is constant over time. The population of each generation is normalized to be one.

2.2. Production

The urban region has two production sectors, sector *uh* and sector *ul*. Sector *uh* employs skilled workers and physical capital to produce good *uh*, and sector *ul* employs unskilled workers to produce good *ul*. The former sector may be considered as the urban formal sector such as modern manufacturing and the latter sector as the urban informal sector such as petty trading, personal services, and repair services. Good *uh* serves as a capital good as well. In contrast, the rural region has sector *rl* that employs unskilled workers to produce good *rl*. This sector corresponds to agriculture in real economy.

The production functions for sectors *uh*, *ul*, and *rl* in period t are given by

$$Y_{uh,t} = (A_{uh}H_{u,t})^{\alpha_{uh}}(K_t)^{1-\alpha_{uh}}, \tag{7}$$

$$Y_{ul,t} = A_{ul}L_{u,t}, \tag{8}$$

$$Y_{rl,t} = A_{rl}(L_{r,t})^{\alpha_{rl}}, \tag{9}$$

$$0<\alpha_{uh}<1, \ \ 0\leq\alpha_{rl}<1, \tag{10}$$

where A_j($j=uh$, ul, and rl) is the productivity in sector j; $H_{u,t}$ is the number of skilled workers employed in sector *uh*;[10] $L_{u,t}$ is the number of urban unskilled workers (employed in sector *ul*); $L_{r,t}$ is the number of rural unskilled workers (employed in sector *rl*); and K_t is the amount of physical capital employed in sector *uh*. Sector *rl* exhibits decreasing returns to its only input $L_{u,t}$, which intends to capture the fact that agricultural productivity is constrained by limited arable land.

[10] The rest of skilled workers is employed in the education sector. It is assumed that the sector is located in the urban area.

2.3. Equilibrium

Assume that goods *uh* and *rl* are tradables and their prices are determined in international markets. Remember that goods *uh* and *rl* correspond roughly to manufacturing goods and agricultural goods, respectively in real economy. Normalize the price of good *uh* to be 1, and denote the price of good *rl* by p_{rl}. By contrast, good *ul*, which may be considered as service goods produced with technologies intensive in unskilled labor, is assumed to be non-tradable, thus its price is determined in the domestic market.

As for factors of production, capital is assumed to be freely mobile internationally. The assumption would be more realistic than the other extreme of the closed market, and enables the paper to focus on human capital accumulation rather than physical capital accumulation as the primary source of development. Denote the exogenously determined interest rate by r. Wage rates are determined competitively in urban sectors *uh* and *ul*, and in the education sector. By contrast, in the rural sector *rl*, where the production technology exhibits decreasing returns, labor income is determined so that the product is equally shared among workers, as is assumed in Lewis (1954).[11] Thus, the average productivity of labor is higher than the marginal productivity in this sector, i.e. there exists disguised unemployment. Since the wage rate of the sector, $p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1}$, approaches infinity as $L_{r,t}$ goes to 0, good *rl* is always produced in the economy.

The wage of skilled workers is obtained by solving the profit maximization problem of a firm in sector *uh*:[12]

$$w_h = \alpha_{uh}A_{uh}\left[\frac{1-\alpha_{uh}}{r}\right]^{\frac{1-\alpha_{uh}}{\alpha_{uh}}}. \tag{11}$$

The wage rate w_h is exogenously determined. Since the cost of education e_t is the cost of hiring skilled workers as teachers, it is exogenous as well, $e_t=e(=s_e w_h)$.[13] The wage rate of unskilled workers must satisfy the following equations:

$$w_{l,t} = p_{ul,t}A_{ul} \tag{12}$$

$$= p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1}. \tag{13}$$

[11] This assumption reflects the fact that traditional agriculture in developing economies is typically engaged by collectives or family farms. After a minor modification, the assumption is also compatible with share-cropping systems widely observed in Asian countries.

[12] The profit maximization problem gives the following first order conditions:

$$r = (1-\alpha_{uh})(A_{uh}H_{u,t})^{\alpha_{uh}}(K_t)^{-\alpha_{uh}},$$

$$w_{h,t} = \frac{\alpha_{uh}}{H_{u,t}}(A_{uh}H_{u,t})^{\alpha_{uh}}(K_t)^{1-\alpha_{uh}},$$

Solving the first equation for K_t and substituting it into the second equation gives Eq. (11).

[13] The market clearing condition of the education sector is given as follows:

$$\begin{aligned} w_h H_{e,t} &= eH_{t+1}, \\ \text{or } H_{e,t} &= s_e H_{t+1}, \end{aligned}$$

where $H_{e,t}$ is the number of 'teachers' in the education sector and H_{t+1} is the number of 'students' in period t. Assume that the education technology is reasonably efficient, that is, s_e is not very high. Then the equation is satisfied without rationing; that is, all children who want to receive and can afford education are able to find their teachers.

The wage rates in the two regions are equalized, because there is free labor mobility between the regions, and there are no mobility costs of goods or factors of production. The unskilled wage is, in general, not constant over time.

The price of non-tradable *ul*, $p_{ul,t}$, is determined domestically. The market clearing condition is[14]

$$p_{ul,t}A_{ul}L_{u,t} = \gamma_{ul}\left\{ w_h H_t + w_{l,t}(1-H_t) + (1+r)\sum_i a_t^i \right\}. \tag{14}$$

In this equation, H_t is the total number of skilled workers and $\sum_i a_t^i$ is the aggregate asset.[15] The right-hand side of the equation is obtained by aggregating the consumption rule for good *ul* (Eq. (5)) over the population. Let B_t be aggregate intergenerational transfers. Then $\sum_i a_t^i = B_t - eH_t$ is satisfied, since currently skilled workers have paid the cost of education e out of transfers received from their parents. Substituting $\sum_i a_t^i = B_t - eH_t$ and Eq. (12) into the above equation and solving for $p_{ul,t}A_{ul}$, one can obtain

$$p_{ul,t}A_{ul} = \frac{\gamma_{ul}}{L_{u,t}-\gamma_{ul}(1-H_t)}\{w_h H_t + (1+r)(B_t - eH_t)\}. \tag{15}$$

The price level $p_{ul,t}$ is increasing in H_t and B_t, and decreasing in $L_{u,t}$. Further, from the substitution of Eq. (13) and $L_{u,t} = 1 - H_t - L_{r,t}$ into the above equation, the number of unskilled workers in the rural region, $L_{r,t}$, is determined:

$$p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1} = \frac{\gamma_{ul}\{[w_h-(1+r)e]H_t + (1+r)B_t\}}{(1-\gamma_{ul})(1-H_t)-L_{r,t}}. \tag{16}$$

The left-hand side of the above equation is decreasing in $L_{r,t}$, and the right-hand side is increasing in $L_{r,t}$. Further, LHS > RHS, as $L_{r,t} \to 0$, and LHS < RHS, as $L_{r,t} \to (1-\gamma_{ul})(1-H_t)$. Hence, the existence and uniqueness of $L_{r,t}$ as the solution for the above equation is assured.[16] In order to indicate the

[14] It is assumed that the urban sector *ul* supplies good *ul* to the both regions, which might seem unnatural considering the fact that good *ul* includes services such as petty trading and personal services in real economy. The assumption is made just for simplicity: all the results in later sections except those on sectoral allocations of workers remain intact with the alternative assumption that good *ul* consumed by individuals in each region is supplied by the nontradable sector located in the region, although the analysis becomes significantly more complicated. As for sectoral allocations of workers, as detailed in footnotes 19, 26, and 29 in Section 4, most of the qualitative results go through under the alternative assumption.

[15] Due to free international capital mobility, a large portion of the aggregate asset may have been invested abroad, if there do not exist enough investment opportunities within the economy.

[16] If unskilled wage in the rural region is determined competitively, i.e. $w_{l,t} = \alpha_{rl}p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1}$, $L_{r,t}$ is the solution to the following equation:

$$p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1} = \frac{1}{\alpha_{rl}}\frac{\gamma_{ul}\{[w_h-(1+r)e]H_t + (1+r)B_t\}}{(1-\gamma_{ul})(1-H_t)-\left(1+\frac{1-\alpha_{rl}}{\alpha_{rl}}\gamma_{ul}\right)L_{r,t}}. \tag{17}$$

By comparing the above equation with Eq. (16), it is clear that $L_{r,t}$ is smaller under the competitive rural labor market. This implies that, in terms of static output maximization, there are too many farmers (too few urban workers) under the income sharing. However, the competitive labor market does *not* necessarily lead to higher aggregate output in the longer term, since, as is detailed in later sections, the long-run development of the economy crucially depends on income levels of unskilled workers. If the profit share in the rural agriculture sector, $1-\alpha_{rl}$, is high or skilled workers take most of profits from the sector, unskilled workers might have lower incomes in the competitive market and thus long-run aggregate output could be lower.

dependence of $L_{r,t}$ on H_t and B_t, let $L_{r,t}=L_r\,(H_t,B_t)$, which is a decreasing function of H_t and B_t. Then, from Eq. (13), $w_{l,t}=w_l(H_t,B_t)$ is an increasing function of H_t and B_t; and from Eq. (12), $p_{ul,t}=p_{ul}(H_t,B_t)$ is also an increasing function of H_t and B_t. These results are summarized in the following lemma.

Lemma 1.

$$\text{(i)}\quad \frac{\partial L_r(H_t,B_t)}{\partial H_t}<0,\quad \frac{\partial L_r(H_t,B_t)}{\partial B_t}<0. \tag{18}$$

$$\text{(ii)}\quad \frac{\partial p_{ul}(H_t,B_t)}{\partial H_t}>0,\quad \frac{\partial p_{ul}(H_t,B_t)}{\partial B_t}>0. \tag{19}$$

$$\text{(iii)}\quad \frac{\partial w_l(H_t,B_t)}{\partial H_t}>0,\frac{\partial w_l(H_t,B_t)}{\partial B_t}>0. \tag{20}$$

3. Dynamics

The previous section has presented the model and derived the equilibrium conditions. In the model economy, individuals live only for two periods and take part in each market for one period alone, thus each market is populated by individuals of a single generation every period. Hence, the model economy can be considered as a sequence of static economies.

What connects these static economies across periods are intergenerational transfers. Because of the credit constraint, transfers directly affect individuals' investment and occupational choices, and consequently consumption and transfer decisions. Further, the distribution of transfers over the population determines a proportion of individuals who can afford education, and thus it affects the relative return from education and investment decisions. The distribution also affects supplies and demands of goods and their prices and allocations. Hence, in general, the time evolution of the distribution of transfers must be examined in order to understand how the structure of the economy, such as regional and sectoral distributions of workers, production and employment shares of each sector, and distributions of earnings and assets, changes over time.

This section first derives the dynamic equation relating the current transfer to the next period's transfer within a lineage (*individual dynamics*). The dynamics depend on the time evolution of two aggregate variables that in turn are determined by the dynamics of the distribution of transfers. However, it turns out that directions of motion of the two aggregate variables, which can be obtained *without* knowledge on the distributional dynamics, is enough to derive the model's implications. Hence the dynamics of the aggregate variables are characterized next. Although the two dynamics interact, for exposition, initially the dynamics of each variable are analyzed fixing the other, then the both dynamics are analyzed together by introducing a phase diagram.

3.1. Individual dynamics

Consider an individual born into lineage i in period $t-1$, who belongs to generation t. He or she allocates the transfer b_t^i between investments in assets a_t^i and in education e_t^i so as to maximize future income. If the transfer is less than the cost of education, i.e. $b_t^i<e$, the transfer is spent only on assets and he or she becomes an unskilled worker, as described above. On the other hand, if the transfer is at least as large as the cost of education, i.e. $b_t^i\geq e$, the investment decision

is more complicated. Since investment decisions of others affect unskilled wage $w_l(H_t,B_t)$ and the relative return from education, the individual has to take into account their actions in making the decision. The key variable affecting the decision is the fraction of individuals in generation t who have received transfers b_t^i larger than e, Fr_t. In short, when only a small number of individuals can afford education, all of them take education and become skilled, and when many individuals have access to education, not all of them become skilled workers.

Unequal opportunity case: More precisely, when the proportion of individuals who can afford education is small, the return from education is higher than the return from assets, even if all of them actually take education, i.e. $w_h-(1+r)e>w_l(Fr_t,B_t)$. In this case, the cut-off level of transfer $b_t^i=e$ determines whether an individual becomes a skilled worker or an unskilled worker, and $H_t=Fr_t$ is satisfied. This case is named the unequal opportunity case since access to the better investment opportunity, education, is constrained by received transfers. For a currently skilled worker, i.e. those who have received $b_t^i\geq e$, the dynamic equation linking the received transfer b_t^i to the transfer given to the next generation b_{t+1}^i is

$$b_{t+1}^i = b_s(b_t^i)\equiv\gamma_k\{w_h+(1+r)(b_t^i-e)\}. \tag{21}$$

The equation is derived by substituting $w_t^i=w_h$, $a_t^i=b_t^i-e$, and $r_t=r$ into the transfer rule (6). The assumption $\gamma_k(1+r)<1$ is made so that the fixed point for the equation, $(b_s)^*\equiv\frac{\gamma_k}{1-\gamma_k(1+r)}[w_h-(1+r)e]$, exists.

Similarly, for a currently unskilled worker, i.e. those who have received $b_t^i<e$, the corresponding dynamic equation is

$$b_{t+1}^i = b_u(b_t^i;Fr_t,B_t)\equiv\gamma_k\{w_l(Fr_t,B_t)+(1+r)b_t^i\}. \tag{22}$$

The equation is obtained from the substitution of $w_t^i=w_l(Fr_t,B_t)$, $a_t^i=b_t^i$, and $r_t=r$ into Eq. (6). The fixed point of the equation for *given* Fr_t and B_t is denoted by $b_u^*(Fr_t,B_t)\equiv\frac{\gamma_k}{1-\gamma_k(1+r)}w_l(Fr_t,B_t)$.[17]

Equal opportunity case: In contrast, when the return from education *fails* to be higher than the return from assets, *if* all of them invest in education, i.e. $w_h-(1+r)e\leq w_l(Fr_t,B_t)$, the number of skilled workers H_t is determined at the point where the two returns are equated, that is, H_t is the solution of $w_h-(1+r)e=w_l(H_t,B_t)$. Now *not* all of financially eligible individuals take education and become skilled workers, i.e. $H_t\leq Fr_t$. The dynamics of transfers of both skilled and unskilled workers are described by the single equation $b_{t+1}^i=b_s(b_t^i)$ (Eq. (21)).

The economy belongs to one of the two cases depending on levels of two aggregate variables, Fr_t and B_t. The dividing line separating the two cases on the (Fr_t,B_t) space is derived in the following lemma.

Lemma 2. *The dividing line between the unequal opportunity case and the equal opportunity case on the (Fr_t, B_t) space is*

$$B_t = D(Fr_t)\equiv\frac{1}{\gamma_{ul}(1+r)}\left\{1-\gamma_{ul}-Fr_t-\left[\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right]^{\frac{1}{1-\alpha_{rl}}}\right\}[w_h-(1+r)e]. \tag{23}$$

[17] In general, this is *not* the long-run transfer level of a lineage of a currently unskilled worker, since his or her descendants may become skilled workers *and* Fr_t and B_t may change over time. Although this fixed point may appear to have no economic importance, it turns out that the level of $b_u^*(Fr_t,B_t)$ is crucial for aggregate dynamics (detailed later).

Proof. In the Appendix. □

It is assumed $1-\gamma_{ul}-\left[\frac{P_{rl}A_{rl}}{w_h-(1+r)e}\right]^{\frac{1}{1-\alpha_{rl}}}>0$, which ensures that, for some combinations of positive Fr_t and $B_t, w_h-(1+r)e>w_l(Fr_t,B_t)$ is satisfied and thus individuals with $b_t^i \geq e$ become skilled workers and all the goods are produced. The economy belongs to the equal opportunity case when Fr_t or B_t is large enough that $B_t \geq D(Fr_t)$ is satisfied. Based on the above lemma, the number of urban unskilled workers and the number of skilled workers in the equal opportunity case are obtained.

Lemma 3. *In the equal opportunity case, i.e.* $B_t \geq D(Fr_t)$,

$$L_{r,t}=\left(\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right)^{\frac{1}{1-\alpha_{rl}}}, \tag{24}$$

$$H_t=D^{-1}(B_t)=1-\gamma_{ul}-\left[\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right]^{\frac{1}{1-\alpha_{rl}}}-\frac{\gamma_{ul}(1+r)B_t}{w_h-(1+r)e}. \tag{25}$$

Proof. In the Appendix. □

3.2. Aggregate dynamics

The previous subsection has shown that the individual dynamics depend on the dynamics of two aggregate variables, aggregate transfers B_t and the fraction of individuals who have received transfers $b_t^i \geq e$, Fr_t. These dynamics in turn depend on the time evolution of the distribution of transfers, which cannot be characterized without imposing strong assumptions on the distribution. However, in order to derive the model's implications, the information on directions of change of the two aggregate variables is sufficient. Thus this subsection analyzes the dynamics of the two aggregate variables qualitatively. For exposition, initially each of them is examined separately fixing the other variable, then their interaction is taken into account at the end.

3.2.1. Dynamics of B_t

The dynamic equation linking current aggregate transfers B_t to transfers in the next period B_{t+1} is derived by aggregating the individual dynamics of skilled workers (Eq. (21)) and of unskilled workers (Eq. (22)) over the population:

$$B_{t+1}=\gamma_k[w_hH_t+w_l(H_t,B_t)(1-H_t)+(1+r)(B_t-eH_t)]. \tag{26}$$

Remember that in the unequal opportunity case, i.e. $w_h-(1+r)e>w_l(Fr_t,B_t)$, H_t can be replaced by Fr_t in the above equation. By contrast, in the equal opportunity case, $w_h-(1+r)e=w_l(H_t,B_t)$ is satisfied. Hence the dynamic equations are expressed as,

$$B_{t+1}=\gamma_k\{[w_h-(1+r)e]Fr_t+w_l(Fr_t,B_t)(1-Fr_t)+(1+r)B_t\},\ \text{for } B_t<D(Fr_t), \tag{27}$$

$$B_{t+1}=\gamma_k[w_h-(1+r)e+(1+r)B_t],\ \text{for } B_t \geq D(Fr_t). \tag{28}$$

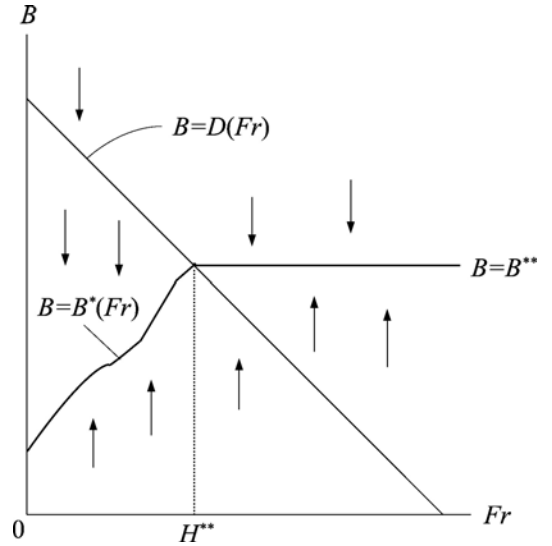

Fig. 1. Dynamics of aggregate transfers for fixed *Fr*.

Aggregate transfers, when the other aggregate Fr_t is *fixed*, are shown to converge to particular levels of aggregate transfers, as is illustrated graphically in Fig. 1:

Lemma 4. *Assume* $\frac{\gamma_k(1+r)}{1-\gamma_{ul}}<1$ *and let* Fr_t *be fixed.*

(i) If $Fr_t<H^{**}\equiv 1-\frac{\gamma_{ul}}{1-\gamma_k(1+r)}-\left(\frac{P_{rl}A_{rl}}{w_h-(1+r)e}\right)^{\frac{1}{1-\alpha_{rl}}}$, *aggregate transfers converge to* $B^*(Fr_t)$, *which is a solution to the following equation:*

$$B=\gamma_k\{[w_h-(1+r)e]Fr_t+w_l(Fr_t,B)(1-Fr_t)+(1+r)B\}. \tag{29}$$

Further, $\frac{\partial B^*(Fr_t)}{\partial Fr_t}>0$ *is satisfied.*

(ii) If $Fr_t\geq H^{**}$, *aggregate transfers converge to*

$$B^{**}\equiv\frac{\gamma_k}{1-\gamma_k(1+r)}[w_h-(1+r)e]. \tag{30}$$

Proof. In the Appendix. □

3.2.2. Dynamics of Fr_t

The dynamics of Fr_t, the fraction of the adult population who had received enough wealth to take education, depend on the dynamics of intergenerational transfers of skilled and unskilled workers. For skilled workers, the following result holds.

Lemma 5. *Assume* $\gamma_k>s_e$. *Then, descendants of currently skilled workers can afford education.*

Proof. It is enough to show that the child of a currently skilled worker can afford education. From Eq. (21), $b^i_{t+1}\geq\gamma_k w_h>s_e w_h=e$. □

The assumption implies that a parent has reasonably strong altruism towards his or her child and the education technology is not very inefficient. From the lemma, Fr_t is non-decreasing over

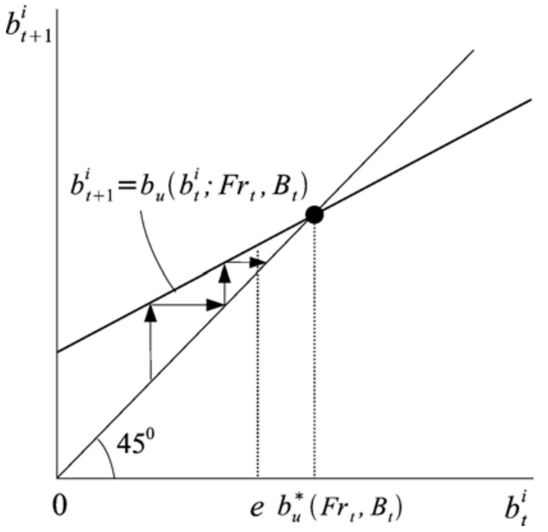

Fig. 2. Necessary condition for $Fr_{t+1}>Fr_t$.

time. The question is whether Fr_t remains constant or increases over time, and the answer depends on the level of transfers at the fixed point of the dynamic equation of unskilled workers, $b^*_u(Fr_t, B_t) \equiv \frac{\gamma_k}{1-\gamma_k(1+r)} w_l(Fr_t, B_t)$, relative to the cost of education e:

Lemma 6.

$$\text{If } b^*_u(Fr_t, B_t) \equiv \frac{\gamma_k}{1-\gamma_k(1+r)} w_l(Fr_t, B_t) \leq e,$$

$$\Longleftrightarrow \quad B_t \leq \frac{[1-\gamma_k(1+r)]e}{\gamma_k \gamma_{ul}(1+r)} \times \left\{ (1-\gamma_{ul})(1-Fr_t) - \left(\frac{p_{rl}A_{rl}}{e} \frac{\gamma_k}{1-\gamma_k(1+r)} \right)^{\frac{1}{1-\alpha_{rl}}} \right\} \tag{31}$$

$$-\frac{1}{1+r}[w_h-(1+r)e]Fr_t, \tag{32}$$

then $Fr_{t+1}=Fr_t$.

Proof. In the Appendix. □

From the lemma, $Fr_{t+1}>Fr_t$ happens only when $b^*_u(Fr_t,B_t)>e$, although $Fr_{t+1}=Fr_t$ is possible if the distribution of transfers $\{b^i_t\}$ is such that there do not exist unskilled workers satisfying $b_{t+1}^i \geq e$. However, if $b^*_u(Fr_t,B_t)>e$ continues to hold for many periods, lineages of current unskilled workers accumulate wealth gradually and Fr_t starts to increase eventually (see Fig. 2).[18]

3.2.3. Joint dynamics of B_t and Fr_t

Now the dynamics of the two aggregate variables are considered together by adding directions of motion of Fr_t to Fig. 1 that illustrates the dynamics of B_t for given Fr_t. To do so, one more result regarding the relative location of the two critical loci, $B_t=D(Fr_t)$ and $b^*_u(Fr_t,B_t)=e$, is needed.

[18] To be more accurate, the position of $b^*_u(Fr_t,B_t)$ on the figure moves as Fr_t and B_t change.

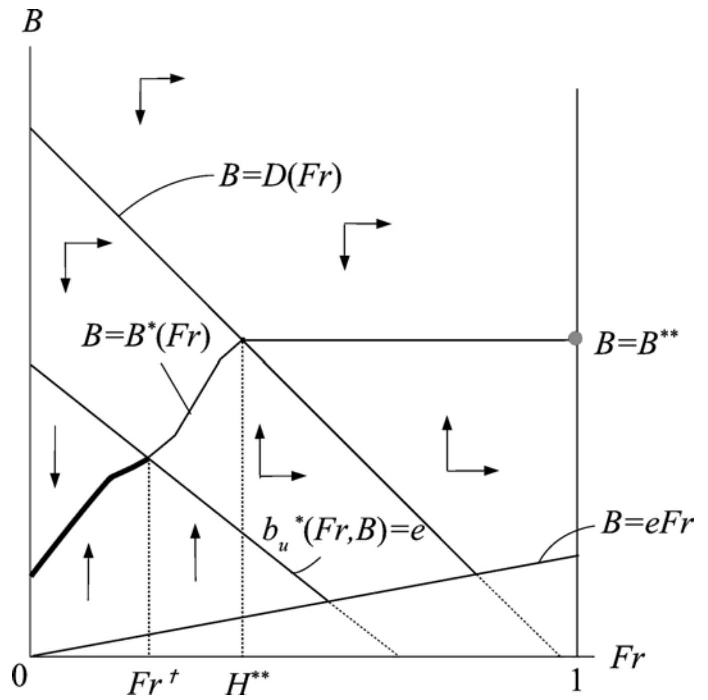

Fig. 3. Phase diagram.

Lemma 7. *$b_u^*(Fr_t, B_t)=e$ is located below $B_t=D(Fr_t)$.*

Proof. In the Appendix. □

Fig. 3 is the resulting phase diagram describing the joint dynamics of B_t and Fr_t. In the figure, feasible combinations of Fr and B must be in the area bound by $Fr=0$, $Fr=1$, and $B=eFr$. The economy must satisfy $B \geq eFr$ because Fr is defined as the fraction of adult individuals who had received transfers b^i larger than the cost of education e in their childhoods. Remember that the region below the locus $B=D(Fr)$ is the unequal opportunity case and $H=Fr$ is satisfied, and the region on or above the locus is the equal opportunity case and $H=D^{-1}(B)\leq Fr$ is true. Directions of change of B and Fr are expressed with vertical arrows and horizontal arrows, respectively. In the unequal opportunity case, the direction of motion of B is determined by the position of current (Fr,B) relative to $B=B^*(Fr)$, and in the equal opportunity case, it is determined by the current position relative to $B=B^{**}$. As for the direction of change of Fr, it is determined by the current location of (Fr,B) relative to $b_u^*(Fr,B)=e$. Denote Fr at the intersection of $b_u^*(Fr,B)=e$ and $B=B^*(Fr)$ by $Fr^\dagger$. In the next section, main results of the model are presented with the help of this diagram.

4. Analysis

4.1. Steady states and transitional dynamics

The inspection of Fig. 3 reveals that there exist two types of steady states in this economy, $(Fr_{ss},B_{ss})=(1,B^{**})$ and $(Fr_{ss},B_{ss})=(Fr,B^*(Fr))$, where $Fr\leq Fr^\dagger$ is satisfied. In the former steady state, the number of skilled workers is $H^{**}=D^{-1}(B^{**})$, skilled wage (net of the cost of education) is equal to unskilled wage, and all the individuals hold the same level of wealth. By contrast, in the latter type of steady states, the number of skilled workers is Fr ($\leq Fr^{\dagger}<H^{**}$), the net wage and wealth of skilled workers are higher than the corresponding ones of unskilled workers. Further, as for the distribution of the population over the rural and urban areas in the steady states, the following lemma is obtained.

Lemma 8. *(i) In the steady state $(Fr_{ss}, B_{ss})=(1,B^{**})$, the number of skilled workers and the numbers of rural and urban unskilled workers are*

$$H^{**}=1-\frac{\gamma_{ul}}{1-\gamma_k(1+r)}-\left(\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right)^{\frac{1}{1-\alpha_{rl}}}, \tag{33}$$

$$L_r(H^{**},B^{**})=\left(\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right)^{\frac{1}{1-\alpha_{rl}}}, \tag{34}$$

$$L_u(H^{**},B^{**})=\frac{\gamma_{ul}}{1-\gamma_k(1+r)}. \tag{35}$$

(ii) In the steady state $(Fr_{ss},B_{ss})=(Fr,B^{*}(Fr))$, $Fr\leq Fr^{\dagger}$, the number of unskilled workers in the urban area is

$$L_u(Fr,B^{*}(Fr))=\frac{\gamma_{ul}}{1-\gamma_k(1+r)}\left\{1+\frac{[w_h-(1+r)e]-w_l(Fr,B^{*}(Fr))}{w_l(Fr,B^{*}(Fr))}Fr\right\}. \tag{36}$$

Proof. In the Appendix. □

Based on this lemma, the following proposition compares the different steady states in terms of the distributions of the population over the two areas and the sectoral compositions of production.[19]

Proposition 1. *(i) The equal-opportunity steady state, $(Fr_{ss},B_{ss})=(1,B^{**})$, is more urbanized and has a smaller informal sector in the urban area than the unequal opportunity steady states, $(Fr_{ss}, B_{ss})=(Fr,B^{*}(Fr))$, $Fr\leq Fr^{\dagger}$:*

$$L_r(H^{**},B^{**})<L_r(Fr,B^{*}(Fr)), \tag{37}$$

$$\text{and } L_u(H^{**},B^{**})<L_u(Fr,B^{*}(Fr)),\ \text{ for } Fr\leq Fr^{\dagger}. \tag{38}$$

(ii) The outputs of goods rl and ul are lower and the output of good uh is higher in the equal-opportunity steady state in comparison to the unequal-opportunity steady states.

(iii) Among the unequal opportunity steady states, one with higher Fr is more urbanized (has smaller $L_r(Fr,B^{*}(Fr))$) and has a smaller informal sector relative to the formal sector in the urban region ($\frac{L_u}{L_u+Fr}$ is lower).

[19] The results remain unchanged in the modified model with two regional nontradable sectors supplying good *ul* (footnote 14) by defining L_u to be the total number of unskilled workers in the two nontradable sectors and L_r to be the number of unskilled workers in the rural tradable sector. (This is also true for Propositions 3 and 4 in the next subsection.) Further, it can be shown that the number of unskilled workers in *each* nontradable sector is smallest at the equal opportunity steady state and, among unequal opportunity steady states, the number of *total* rural workers and the proportion of informal workers in the urban region decrease with *Fr*. That is, the qualitative results of the original model are obtained in the modified model as well.

Proof. In the Appendix. □

The equal-opportunity steady state has features of a typical developed economy: many people are educated and skilled; most of the people live in urban areas and are engaged in non-agricultural activities; and inequality between skilled and unskilled workers is small (in the model, the distributions of net wage and wealth are equalized). In contrast, the unequal-opportunity steady states characterize a typical stagnant developing economy: many people live in rural areas and are engaged in agriculture; many of the urban residents are not skilled and in the informal sector; and inequality between skilled and unskilled workers is large. Among such stagnant economies, one with a greater portion of skilled workers is more urbanized, less dependent on agriculture, has a smaller informal sector relative to the formal sector in the urban region, accumulates larger wealth, and attains lower inequality between skilled and unskilled workers because of higher unskilled wage.[20]

Starting from the initial distribution of transfers over the population, the economy approaches one of the steady states in the long run. From Fig. 3, it can be seen that the critical determinant of success and failure of the economy is Fr_0, the initial proportion of individuals who can afford education:

Proposition 2. *Assume* $B_0 < B^*(Fr_0)$.

(i) *If* $Fr_0 \leq Fr^{\dagger}$, *the economy converges to the steady state* $(Fr_{ss}, B_{ss}) = (Fr_0, B^*(Fr_0))$ *in the long run. On the transitional path,* $Fr = H$ *remains constant and* B *increases, thus* $L_r(H,B)$ *decreases and* $L_u(H,B)$ *and* $w_l(H,B)$ *increase over time.*

(ii) *If* $Fr_0 > Fr^{\dagger}$, *the economy converges to the steady state* $(Fr_{ss}, B_{ss}) = (1, B^{**})$ *in the long run. On the transitional path, (a) in the region* $b_u^*(Fr,B) \leq e$, *the results in* (i) *hold; (b) in the region where* $b_u^*(Fr,B) > e$ *and* $B < D(Fr)$ *are satisfied, both* Fr *and* B *increase, thus* $L_r(H,B)$ *decreases and* H *and* $w_l(H,B)$ *increase over time; and (c)in the region* $B \geq D(Fr)$, *both* Fr *and* B *increase, but this time* $L_r(H,B)$ *and* $w_l(H,B)$ *remain the same, while* H *decreases and* $L_u(H,B)$ *increases over time.*

Intuitively, the reason why Fr_0 is critical for the long-run performance of an economy can be explained as follows.[21] In this model, the number of skilled workers increases, if a portion of unskilled workers accumulates wealth sufficient for their children to take education and get skilled jobs. Unskilled wage increases as the degree of urbanization rises through the migration of unskilled workers from ‘over-populated’ agriculture (sector *rl*) to the urban informal sector, which is possible if the price of the non-tradable good produced in the urban informal sector (good *ul*) goes up. The price of the good, which is determined jointly with the number of rural unskilled workers, depends positively on the number of skilled workers and aggregate assets: a greater number of skilled workers implies the higher demand and lower supply of the good, and larger asset accumulation leads to the higher demand. Thus, for the economy to initiate ‘modernization’, the proportion of people who have enough wealth to take education and aggregate assets must be above certain levels, i.e. $b_u^*(Fr,B) > e$.

If the initial distribution of wealth is such that a relatively large portion of the population can afford education and becomes skilled workers, the price of good *ul* is higher due to the higher

[20] Formally, from Lemmas 1(iii) and 4(i),

$$\frac{\mathrm{d}w_l(Fr_t, B^*(Fr_t))}{\mathrm{d}Fr_t} = \frac{\partial w_l(Fr_t, B^*(Fr_t))}{\partial Fr_t} + \frac{\partial w_l(Fr_t, B^*(Fr_t))}{\partial B_t}\frac{\mathrm{d}B^*(Fr_t)}{\mathrm{d}Fr_t} > 0.$$

[21] A more detailed explanation of the mechanics of a related model is found in Yuki (2003).

demand and the lower supply of the good, and the number of rural unskilled workers is smaller, resulting in higher unskilled wage. If the price level is above the critical level, i.e. $b_u^*(Fr_0,B_0) > e$, a richer portion of unskilled workers can send their children to school and the expansion of the modern sector starts immediately. Even when the initial price level is not high enough for the 'take-off' due to limited asset accumulation, the large pool of skilled workers makes rapid asset accumulation possible, hence the price level rises to the critical level eventually and the modernization process begins. Once the structural change is initiated, it continues *autonomously*. An increase in the number of skilled workers and asset accumulation lead to a further increase in the price of good *ul*, stimulate the migration of unskilled workers to the urban region, and boost unskilled wage. This allows children of less affluent unskilled workers to access education and thus increases the number of skilled workers further, which, in turn, inflates the price and unskilled wage even more. As long as skilled wage (net of the cost of education) is higher than unskilled wage, this process continues. In the long run, the economy reaches the state in which the return from education is equated with the one from assets, thereby attaining a state of equal opportunity.

In contrast, if the economy starts with relatively small Fr_0, either because wealth is concentrated in the few rich or because limited wealth is dispersed among the many poor, skilled labor is scarce, the price of nontradable *ul* is low, and many of unskilled workers remain in agriculture. Children of unskilled workers are not financially able to obtain education and the number of skilled workers does not increase. Still, if initial asset accumulation is low, i.e. $B_0 < B^*(Fr_0)$, further asset accumulation increases the relative price, promotes the migration of unskilled workers, and raises unskilled wages over time. However, asset accumulation alone is not enough to raise the relative price to the critical level necessary for structural change. Since skilled labor remains limited, inequality between skilled and unskilled workers does not disappear and the investment choices are affected by family income even in the long run.[22,23]

As for the issue of internal migration, the proposition shows that, as long as an economy starts with small asset accumulation, i.e. $B_0 < B^*(Fr_0)$, it *always* experiences urbanization along its transition to a steady state, which is consistent with the fact that urbanization occurred universally for most of the post-war period. However, the economy experiences different kinds of urbanization depending on its development path.

When an economy fails to initiate industrialization and ends up in the unequal-opportunity steady state, i.e. $(Fr_{ss},B_{ss}) = (Fr,B^*(\mathrm{Fr}))$, $Fr \leq Fr^\dagger$, urbanization takes place without an increase in well-paid jobs for skilled workers, hence the proportion of 'informal' unskilled jobs increases

[22] Using panel data of wider coverage and of higher quality than those of earlier studies, Deininger and Squire (1998) and Deininger and Olinto (2000) discover that an economy's growth rate is affected negatively by initial land inequality (a proxy for initial asset inequality) and positively by its mean years of schooling per working person (a proxy for human capital). Further, they find that the average educational attainment is negatively affected by initial land inequality; the effect of human capital is larger for a lower-income economy; and initial land and income inequalities affect negatively the income growth of the poor, but not of the rich. Also, Birdsall and Londono (1998) find that initial inequalities of land and education have strong negative effects on subsequent income growth, especially that of the poor. Their findings are consistent with predictions of the model, as long as low Fr_0 reflects high initial asset inequality, that is, unless asset accumulation is very low.

[23] The model may be used to understand growth experiences of two contrasting economies, Brazil and Korea. At the beginning of the 1960s, the income and wealth distributions of Brazil were far less equal than Korea (Deininger and Olinto, 2000). Both Brazil and Korea experienced rapid growth in the 1970s, but Brazil attained only a modest increase in the proportion of workers with secondary education from a much smaller starting value, resulting in much smaller decreases in returns to education (Birdsall et al., 1995). According to Williamson (1993), a large portion of differences in enrollment rates of secondary education between the two economies in the early 1970s can be explained by the greater income inequality in Brazil, not GNP per adult, the relative price of teachers, or school age population ratio. And Brazil stagnated after the 1980s, while Korea continued to grow fast. In terms of the model, Korea may be on a path to the equal opportunity steady state and Brazil on a path to an unequal opportunity steady state.

over time in the urban region. This result accords with growing urban slum dwellers observed in a typical stagnant developing economy. Wealth accumulation raises the demand for nontradable *ul*, which corresponds to services such as trading and housework in real economy, increases urban unskilled jobs supplying the good, and attracts rural unskilled workers into the urban region.

By contrast, when an economy is on a track to the equal-opportunity steady state, i.e. $(Fr_{ss}, B_{ss}) = (1, B^{**})$, it goes through three stages of urbanization. At the initial stage when an economy grows without an increase in skilled workers, i.e. $b_u^*(Fr,B) \leq e$, it urbanizes without an increase in skilled jobs, thus the share of informal jobs in the urban region increases. Once the economy 'takes off', i.e. $b_u^*(Fr,B) > e$, urbanization accompanies an increase in skilled jobs. At this stage, urbanization is driven not only by the increasing demand for non-tradable *ul*, but also by an increase in the number of educated people searching for skilled jobs. An increase in the number of skilled workers and asset accumulation lower wage differentials between skilled and unskilled workers over time, and urbanization stops once the economy attains equal opportunity. At this point the wage differential between the regions disappears as well.[24]

4.2. Roles of productivities

So far, the analysis has been performed for given levels of productivities. This subsection examines how changes in the productivities of sector *rl* (agricultural productivity) and of sector *uh* (manufacturing productivity) affect the distribution of the population between the regions, sectoral compositions of production, and the dependency of the initial distribution on long-run steady states. First, effects of a decrease in the productivity of sector *rl*, A_{rl}, are investigated.[25]

Proposition 3.[26] *Suppose that an economy experiences a one-time and permanent decrease in A_{rl} at the beginning of a period that is expected in the previous period. Then,*

- **(i)** *$Fr^{\dagger}$ increases. That is, the economy is more likely to end up in an unequal opportunity steady state, $(Fr_{ss}, B_{ss}) = (Fr, B^*(Fr))$, $Fr \leq Fr^{\dagger}$.*
- **(ii)** *Instantaneous effects on sectoral allocations of workers and wages are:*

[24] Larson and Mundlak (1997) find that the ratio of average labor productivity of agricultural workers to that of non-agricultural workers converges to one as an economy develops. Some evidence indicates that the size of the informal sector and the wage differential between the informal and formal sectors decrease with development through human capital accumulation (Marcouiller et al., 1997; Ranis and Stewart, 1999). Marcouiller et al. (1997) find sizable wage differences between formal and informal sectors after controlling for individual characteristics and industry-specific factors in El Salvador and Peru but not in Mexico in the late 1980s. Consistent with the model's implications, the Mexico's income level is higher than the others and the share of the informal sector is smaller. Higher economic growth of Mexico may be partly due to its more equal initial asset distribution: at the beginning of the 1960s, the distribution of land holdings in Mexico was much more equal than the other nations (Deininger and Olinto, 2000).

[25] Results are presented for the productivity decline rather than the more frequent productivity increase, since, as summarized in the proposition, the former promotes urbanization. As for manufacturing, for the same reason, the productivity increase is examined. In this way, one can see clearly how the productivity changes in the two sectors, both promoting urbanization, have different implications for development and inequality.

[26] In the modified model with two regional nontradable sectors, the following results are additionally obtained. As for the instantaneous effects in (ii), in the unequal opportunity case, the fraction of workers in the rural tradable (agriculture) sector declines and the number of workers in the urban informal sector rises; when the economy belongs to the equal opportunity case both before and after the productivity change, the fraction of workers in the informal sector falls in the urban region, while the number of workers in the rural nontradable sector decreases and that of the urban informal sector increases by the same amount. Regarding the long-run effects in (iii), in all the steady states, the number of workers in the rural nontradable sector falls and that of the urban informal sector rises.

(a) *(Unequal opportunity case)* $L_{r,t}$ *and* $w_{l,t}$ *decrease,* $L_{u,t}$ *increases, and* $H_t = Fr_t$ *does not change.*

(b) *(Equal opportunity case) If the economy belongs to the unequal opportunity case after the productivity decline,* H_t *and* $L_{u,t}$ *increase, and* $L_{r,t}$ *and* $w_{l,t}$ *decrease. If it remains in the equal opportunity case,* H_t *increases,* $L_{r,t}$ *decreases, and* $L_{u,t}$ *does not change.*

(iii) *Long-run effects on sectoral allocations of workers and wages are:*

(a) *(Unequal opportunity steady states) At the steady state with given Fr,* $B^*(Fr)$*,* L_r*, and* w_l *decline, and* L_u *rises.*

(b) *(Equal opportunity steady state)* H^{**} *increases,* L_r *decreases, while* B^{**} *and* L_u *do not change.*

Proof. In the Appendix. □

A fall in agricultural productivity A_{rl} decreases unskilled wage directly and makes it more difficult for unskilled workers to afford their children education ($b_u^*(Fr,B) = e$ shifts outward on the (Fr,B) space). The lower unskilled wage translates into the slowdown of asset accumulation and the reduction of long-run aggregate transfers ($B = B^*(Fr)$ shifts downward), which reduces the demand for non-tradable *ul* and unskilled wage. As a result, the minimum size of Fr_0 necessary for successful development increases (Proposition 3(i)). That is, as agricultural productivity is lower, a disadvantageous asset distribution in the initial period becomes a more serious hindrance to development. Further, the productivity decline has negative effects on the unequal opportunity steady states: unskilled wage falls and wage and wealth inequalities worsen (Proposition 3(iii)(a)). In contrast, it does not affect the wage and inequalities in the equal opportunity steady state (Proposition 3(iii)(b)).[27]

Proposition 3(ii) and (iii) show that urbanization is not necessarily associated with income growth. A fall of agricultural productivity depresses farmers' incomes and forces them to leave their lands and migrate to the urban area immediately. They move to the urban informal sector in the unequal opportunity case and to the modern sector in the equal opportunity case. Moreover, the decline of unskilled wage and the migration of rural workers are persistent. In particular, when Fr_0 is low and the economy remains in the unequal opportunity case in the long run, the long-run effect on unskilled wage is stronger and the one on migration is weaker than the short-run effects: the lower income due to the productivity decline slows down asset accumulation, and has a negative effect on the demand for good *ul* and thus unskilled wage. As can be shown easily, a fall of the price of the agricultural good *rl* has the same qualitative effects. Empirical findings suggest that agricultural productivity and prices of agricultural goods are important determinants of internal migration in many stagnant developing countries, especially in sub-Saharan African nations.[28]

[27] The divergent development and urbanization experiences of Thailand and the Philippines after the 1980s, which is mentioned in the introductory section, could be due to differences in initial conditions of the two economies. That is, Thailand started with the more equal distribution of land and higher agricultural productivity (more abundant fertile land) (Inral and Power, 1991; Siamwalla et al., 1993), both of which are conductive to urbanization with development according to the model. Although pro-industry policies, such as export taxes on agricultural products, overvaluation of the exchange rate, and various subsidies to industry, lead to industrialization in the Philippines in the 1970s, they were not sustainable.

[28] Becker and Morrison (1988) find that per capita caloric intake in rural areas is the most important determinant of urbanization in sub-Sahara African economies for the years 1970–80. Jaeger (1992), based on survey data from Ghana, shows that policy reforms that shift rural–urban terms of trade in favor of agriculture resulted in a significant reverse migration from urban to rural regions in the late 1980s.

The next proposition summarizes effects of a permanent increase in the productivity of sector *uh*.

Proposition 4.[29] *Suppose that an economy experiences a one-time and permanent increase in A_{uh} at the beginning of a period that is anticipated in the previous period. Then,*

(i) *$Fr^{\dagger}$ increases. That is, the economy is more likely to end up in an unequal opportunity steady state, $(Fr_{ss},B_{ss})=(Fr,B^*(Fr))$, $Fr \le Fr^{\dagger}$.*

(ii) *Instantaneous effects on sectoral allocations of workers and wages are:*

- ***(a)*** *(Unequal opportunity case) $L_{u,t}$, $w_{h,t}$, and $w_{l,t}$ increase, $L_{r,t}$ decreases, and $H_t=Fr_t$ does not change.*
- ***(b)*** *(Equal opportunity case) If the economy belongs to the unequal opportunity case after the productivity increase, H_t, w_h, and $w_{l,t}$ increase, and $L_{r,t}$ decreases. The effect on $L_{u,t}$ is ambiguous. If it remains in the equal opportunity case, H_t, $L_{u,t}$, w_h, and $w_{l,t}$ increase, while $L_{r,t}$ decreases.*

(iii) *Long-run effects on sectoral allocations of workers and wages are:*

- ***(a)*** *(Unequal opportunity steady states) At the steady state with given Fr, $B^*(Fr)$, L_u, w_h, and w_l increase, and L_r decreases.*
- ***(b)*** *(Equal opportunity steady state) H^{**}, B^{**}, w_h, and w_l increase, L_r decreases, and L_u does not change.*

Proof. In the Appendix. □

A rise in manufacturing productivity raises the cost of education, which is proportional to skilled wage, more than unskilled wage and makes it harder for children of unskilled workers to accomplish upward mobility ($b_u^*(Fr,B)=e$ shifts outward on the (Fr,B) space). It also raises total income and long-run aggregate transfers ($B=B^*(Fr)$ shifts upward), which raises the demand for good *ul* and unskilled wage. It turns out that the former effect dominates, thus the critical level of *Fr* for modernization, $Fr^{\dagger}$, goes up and the economy is more likely to end up in the unequal opportunity steady state (Proposition 4(i), see Fig. 3). In particular, if the initial condition satisfies $Fr_0 \in (Fr^{\dagger}_{\text{old}}, Fr^{\dagger}_{\text{new}}]$, the productivity increase drastically changes the economy's steady state from $(Fr_{\text{ss}},B_{\text{ss}})=(1,B^{**})$ to $(Fr_{\text{ss}},B_{\text{ss}})=(Fr_0,B^*(\text{Fr}_0))$. The result suggests that an uneven productivity growth of the more advanced manufacturing sector, that is, a skill-biased technological progress, is *not* always beneficial to the long-run development of a low-income nation.

For an economy starting with relatively large Fr_0 ($Fr_0 > \text{Fr}^{\dagger}_{\text{new}}$), however, the productivity increase is certainly advantageous. In such an economy, the rise of the cost of education does not become an impediment to convergence to the equal-opportunity steady state, although it slows down the convergence process somewhat. Further, the productivity growth raises the number of skilled jobs, income, and wealth in the equal-opportunity steady state (Proposition 4(iii)(b)).[30] The increase in manufacturing productivity also promotes urbanization. In the short run, the productivity increase

[29] In the modified model with two regional nontradable sectors, the following results are additionally obtained. As for the instantaneous effects in (ii), when the economy remains to be in the same (unequal or equal opportunity) case after the productivity change, the number of workers in the rural nontradable sector declines and that of the urban informal sector rises; otherwise, the number of workers in the rural nontradable sector decreases and the fraction of workers in the informal sector falls in the urban region. Regarding the long-run effects in (iii), in all the steady states, the number of workers in the rural nontradable sector falls and that of the urban informal sector rises.

[30] On the other hand, when the initial condition satisfies $Fr_0 \le Fr_{\text{old}}\dagger$, the positive productivity shock yields mixed results: while it raises wages of both skilled and unskilled workers (Proposition 4(iii)(a)), it worsens wage inequality between them, as can been seen from Eq. (36) in Lemma 8.

raises skilled wage, total income, and the demand for good *ul* and encourages the migration of rural workers to the urban informal sector in the unequal opportunity case (Proposition 4(ii)(a)), while, in the equal opportunity case, it promotes migration to the formal sector as well (Proposition 4(ii)(b)). Since the increase in total income stimulates asset accumulation and raises the demand for good *ul* further in the longer term, the effects are persistent (Proposition 4(iii)).

4.3. Discussion

Some of the model's assumptions are made for the purpose of simplifying analyses rather than for capturing reality. As for the assumption of the single nontradable sector *ul*, it has been shown in footnotes 14, 19, 26, and 29 that main results are not affected even if regionally separated nontradable sectors are allowed. In this subsection, the robustness of main results with respect to modifications of several other assumptions and the model's implications for international migration are briefly discussed.

4.3.1. Urban formal sector's production function

When the model is modified so that sector *uh* (the urban formal sector) employs unskilled workers as well as skilled workers, skilled wage changes with the ratio of skilled to unskilled workers in the sector. With this modification, the model becomes intractable with the present setting that skilled workers are employed as teachers in the education sector, hence the cost of education is assumed to be the cost of purchasing a fixed amount of good *uh*. Then, although the analysis becomes more complex, the results of the original model remain intact qualitatively for the most part. The only major difference is that now an increase in $Fr{=}H$ raises unskilled wage through the higher proportion of skilled workers in sector *uh*, as well as the higher demand for good *ul* and the lower number of unskilled workers, in the unequal opportunity case.

4.3.2. Utility function

In the model, the utility function is Cobb–Douglas and thus the proportion of spending on each item is constant and independent of income and price levels. In real economy, the proportion of the consumption of agricultural goods decreases with development (the Engel's law). This feature can be captured by a slightly modified utility function, $u_t^i=(c_{rl,t}^i-\overline{c_{rl}})^{\gamma_{rl}}\Pi_{j\neq rl}(c_{j,t}^i)^{\gamma_j}(b_{t+1}^i)^{\gamma_k}$. That is, an individual must consume at least $\overline{c_{rl}}$ units of good *rl* irrespective of his or her income. Qualitatively, the results remain intact with positive $\overline{c_{rl}}$, but quantitatively they are affected. In the unequal opportunity case, positive $\overline{c_{rl}}$ implies smaller demand for good *ul*, the lower degree of urbanization, and lower unskilled wage for given *Fr* and *B*, resulting in smaller asset accumulation and higher $Fr^{\dagger}$, i.e. low *Fr* becomes a more serious obstacle to development. In contrast, in the equal opportunity case, H^{**} increases and L_u decreases due to the lower demand for good *ul*.

4.3.3. Rural economy

When the production function of sector *rl* (the agriculture sector) exhibits constant returns to scale, the wage rate equals the marginal product of labor, i.e. there is no disguised unemployment. In this setting, when *Fr* and *B* are low, both sectors *rl* and *ul* exist, while when *Fr* and *B* are high, only sector *ul* operates, i.e. the economy is completely urbanized. The dynamics are also affected. If the agricultural productivity is high enough that $\frac{\gamma_k}{1-\gamma_k(1+r)}p_{rl}A_{rl}{>}e$ is satisfied, irrespective of initial conditions, the economy reaches the equal opportunity steady state in the long run. By contrast, when the productivity is low, the long-run outcome is determined by initial conditions as in the original model. In a transition to the equal opportunity steady state, as long as sector *rl* is in operation, urbanization proceeds without changes in unskilled wage, and once urbanization is completed, unskilled wage starts to increase.

If the production function of the rural sector is identical to the original model but the wage is determined competitively, given asset distribution, the number of workers in the sector is smaller and aggregate output is higher compared to the original setting, as is explained in footnote 16. This does not imply that unskilled wage and income are higher, since a portion of the output is distributed as profits. Main results would be qualitatively unchanged, but whether the economy goes through urbanization accompanying modernization or not now depends on the distribution of profits over the population as well.

4.3.4. International migration

The model has some implications for effects of international migration on the urbanization and development of low income nations. Migration is motivated economically by returns from migration and migration costs. Returns from migration would be greater for unskilled workers, since the return to education and wage inequality are lower in developed nations, while migration costs would inhibit migration of unskilled workers with limited wealth. When Fr and B are very low in the absence of international migration, unskilled wage is low, many of unskilled workers cannot cover migration costs, and thus skilled workers would be overrepresented in immigrants. If this is the case, the presence of international migration tends to decrease Fr, exacerbate wage inequality, lower the degree of urbanization, and raise the proportion of the informal sector in the urban region. On the other hand, when Fr and B are not very low, immigrants are disproportionately unskilled workers, hence international immigration would raise Fr and have the opposite effects to the previous situation. In particular, if Fr is only slightly less than $Fr^{\dagger}$ without international migration, increased migration allows the economy to attain equal opportunity in the long run. Further, remittances from immigrants may contribute to wealth accumulation by unskilled workers and lower $Fr^{\dagger}$, further easing the burden of low Fr in development.

5. Conclusion

Urbanization is a universal phenomenon in the postwar world, but its qualitative nature is varied across countries. Some nations have experienced urbanization that accompanied skill upgrading, industrialization, and the expansion of the urban formal (modern) sector, but others urbanized without such modernization and underwent the expansion of the urban informal (traditional) sector simultaneously. The question that naturally arises is what are underlying causes for the differences. The purpose of the present paper has been to tackle this question analytically based on a dynamic model of urbanization and development.

The main findings of the paper can be summarized as follows. (1) In the model economy there exist two types of steady states. The equal opportunity steady state has features of a typical developed economy and the unequal opportunity steady states have features of a typical stagnant developing economy in terms of the accumulation of human capital, the sectoral and regional distributions of the population, and sectoral and regional income inequalities. (2) Given an initial distribution of transfers (wealth) over the population, an economy approaches one of the steady states in the long run. If an economy starting with limited asset accumulation is on a path to the equal opportunity steady state, it experiences urbanization that accompanies skill upgrading, the expansion of the formal sector, the shrinkage of the informal sector, and falling inequality. Instead, if it is on a track to an unequal opportunity steady state, it urbanizes without such modernization but with the expansion of the informal sector. Thus the model exhibits two distinctive patterns of urbanization observed in real economy. The critical determinant of the long-run outcome of an economy is the initial fraction of the population who is sufficiently wealthy to obtain education. (3) Both a decrease in agricultural productivity and an increase in the formal sector's productivity promote urbanization but make it more difficult for an economy starting with an unfavorable wealth distribution to reach the equal opportunity steady state in the long run.

Appendix A. Proofs of lemmas and propositions

Proof of Lemma 2. From the definition of the two cases, the dividing line is equivalent to $w_h-(1+r)e=w_l(Fr_t,B_t)$. By substituting $p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1}=w_h-(1+r)e$ into Eq. (16) and solving for $L_{r,t}$,

$$L_{r,t}=1-\gamma_{ul}-H_t-\frac{\gamma_{ul}(1+r)B_t}{w_h-(1+r)e}. \tag{40}$$

By plugging $H_t=Fr_t$ into the above equation and substituting the resulting equation into $w_h-(1+r)e=p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1}$,

$$w_h-(1+r)e=p_{rl}A_{rl}-\left[1-\gamma_{ul}-Fr_t\frac{\gamma_{ul}(1+r)B_t}{w_h-(1+r)e}\right]^{\alpha_{rl}-1}. \tag{41}$$

Eq. (23) is obtained by solving the above equation for B_t. □

Proof of Lemma 3. $L_{r,t}$ is obtained from the condition $w_{l,t}=p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-1}=w_h-(1+r)e$. As for H_t, $w_h-(1+r)e=w_l(H_t,B_t)$ must hold by the definition of the equal opportunity case. From the proof of the previous lemma, it can be seen that $w_h-(1+r)e=w_l(H_t,B_t)$ is equivalent to $B_t=D(H_t)$. □

Proof of Lemma 4. First, the existence and uniqueness of the solution to Eq. (29) for given Fr_t is proved.

It is enough to prove that (a) $\frac{\partial \mathrm{RHS}}{\partial B_t}<1$ and (b) RHS > 0 when $B_t=0$ are satisfied. From Eq. (13),

$$\frac{\partial w_l(Fr_t,B_t)}{\partial B_t}=-(1-\alpha_{rl})p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-2}\frac{\partial L_{r,t}}{\partial B_t}. \tag{42}$$

From Eq. (16),

$$\frac{\partial L_{r,t}}{\partial B_t}=\frac{-\gamma_{ul}(1+r)}{p_{rl}A_{rl}(L_{r,t})^{\alpha_{rl}-2}[(1-\gamma_{ul})(1-\alpha_{rl})(1-Fr_t)+\alpha_{rl}L_{r,t}]}. \tag{43}$$

Substituting this equation into Eq. (42),

$$\frac{\partial w_l(Fr_t,B_t)}{\partial B_t}=\frac{(1-\alpha_{rl})\gamma_{ul}(1+r)}{(1-\gamma_{ul})(1-\alpha_{rl})(1-Fr_t)+\alpha_{rl}L_{r,t}}. \tag{44}$$

Hence,

$$\begin{aligned}\frac{\partial \mathrm{RHS}}{\partial B_t}&=\gamma_k(1+r)+\gamma_k(1-Fr_t)\frac{\partial w_l(Fr_t,B_t)}{\partial B_t},\\&=\gamma_k(1+r)\frac{(1-\alpha_{rl})(1-Fr_t)+\alpha_{rl}L_{r,t}}{(1-\gamma_{ul})(1-\alpha_{rl})(1-Fr_t)+\alpha_{rl}L_{r,t}}\leq\frac{\gamma_k(1+r)}{1-\gamma_{ul}}<1.\end{aligned}$$

(b) is obviously satisfied. Thus, the solution to Eq. (29), $B^*(Fr_t)$, exists and is unique for given Fr_t. If the dynamics of aggregate transfers are always described by Eq. (27), global convergence to $B^*(Fr_t)$ is assured by this result as well. Similarly, if the dynamics are always determined by Eq. (28), global convergence to B^{**} is obvious. The actual dynamics are given by Eq. (27) in the unequal opportunity case, i.e. $B_t<D(Fr_t)$, and by Eq. (28) in the equal opportunity case, i.e. $B_t\geq D(Fr_t)$. If $B^*(Fr_t)<D(Fr_t)$ is satisfied, the fixed point of Eq. (27) is below the critical level of B_t dividing the two cases on the (Fr_t,B_t) space, hence B_t converges to $B^*(Fr_t)$ globally (see Fig. 1). Otherwise, B_t converges to $B^{*\backslash *}$ globally (see Fig. 1). The critical level of Fr_t satisfying $B^*(Fr_t)= D(Fr_t)$ is H^{**}, which is obtained by substituting $B_{t+1}=B_t=D(Fr_t)$, Eq. (23), into Eq. (28) and solving for Fr_t. Finally, $\frac{\partial B^*(Fr_t)}{\partial Fr_t}>0$ is true, since the RHS of Eq. (29) is increasing in Fr_t in the unequal opportunity case. □

Proof of Lemma 6. $Fr_{t+1}=Fr_t$ is satisfied iff children of currently unskilled workers cannot afford education, i.e. for any lineage i satisfying $b_t^i<e$, $b_{t+1}^i=\gamma_k\{w_l(Fr_t,B_t)+(1+r)b_t^i\}<e$ (from Eq. (22)) must be true. The inequality (31) is derived from this condition. Since $w_l(Fr_t,B_t)$ is equal to the right-hand side of Eq. (16) when $H_t=Fr_t$, the inequality becomes

$$\frac{\gamma_k}{1-\gamma_k(1+r)}\frac{\gamma_{ul}\{[w_h-(1+r)e]Fr_t+(1+r)B_t\}}{(1-\gamma_{ul})(1-Fr_t)-L_{r,t}}\leq e, \tag{45}$$

$$\Leftrightarrow\ L_{r,t}\leq(1-\gamma_{ul})(1-Fr_t)-\frac{\gamma_k}{1-\gamma_k(1+r)}\frac{\gamma_{ul}}{e}\{[w_h-(1+r)e]Fr_t+(1+r)B_t\}. \tag{46}$$

Substituting Eq. (13) into the same inequality (31) and solving it for $L_{r,t}$, one can also obtain

$$L_{r,t}\geq\left(\frac{p_{rl}A_{rl}}{e}\frac{\gamma_k}{1-\gamma_k(1+r)}\right)^{\frac{1}{1-\alpha_{rl}}}. \tag{47}$$

Then, Eq. (32) is obtained from these two inequalities (46) and (47). □

Proof of Lemma 7. From Eqs. (23) and (32) the statement is true iff

$$\begin{aligned}
&\gamma_k\left[1-\gamma_{ul}-Fr_t-\left(\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right)^{\frac{1}{1-\alpha_{rl}}}\right][w_h-(1+r)e]>[1-\gamma_k(1+r)]\\
&\qquad\times e\left\{(1-\gamma_{ul})(1-Fr_t)-\left(\frac{p_{rl}A_{rl}}{e}\frac{\gamma_k}{1-\gamma_k(1+r)}\right)^{\frac{1}{1-\alpha_{rl}}}\right\}-\gamma_k\gamma_{ul}[w_h-(1+r)e]Fr_t,\\
\Leftrightarrow\ &\{\gamma_k[w_h-(1+r)e]-[1-\gamma_k(1+r)]e\}(1-\gamma_{ul})(1-Fr_t)>(p_{rl}A_{rl})^{\frac{1}{1-\alpha_{rl}}}\\
&\qquad\times\left(\gamma_k[w_h-(1+r)e]^{\frac{-\alpha_{rl}}{1-\alpha_{rl}}}-\gamma_k^{\frac{1}{1-\alpha_{rl}}}\{[1-\gamma_k(1+r)]e\}^{\frac{-\alpha_{rl}}{1-\alpha_{rl}}}\right).
\end{aligned}$$

Since $\gamma_k[w_h-(1+r)e]>[1-\gamma_k(1+r)]e$ is satisfied by the assumption $\gamma_k w_h>s_e w_h=e$, the left-hand side of the above equation is positive, while the right-hand side is negative, proving the inequality. □

Proof of Lemma 8. (i)H^{**} is obtained from the substitution of $B_t=B^{**}$, Eq. (30), into $B_t=D(H_t)$, Eq. (23). $L_r(H^{**},B^{**})$ is from Lemma 3. Finally $L_u(H^{**},B^{**})=1-H^{**}-L_r\,(H^{**},B^{**})$. (ii) By substituting $H_t=Fr$, $B_t=B^*(Fr)$, and $p_{ul,t}\,A_{ul}=w_l(Fr,B)$ into Eq. (36),

$$w_l(Fr, B^*(Fr)) = \frac{\gamma_{ul}}{L_u-\gamma_{ul}(1-Fr)}\{w_h Fr + (1+r)(B^*(Fr)-eFr)\}. \tag{48}$$

From Eq. (29),

$$B^*(Fr) = \frac{\gamma_k}{1-\gamma_k(1+r)}\{[w_h-(1+r)e]Fr + w_l(Fr, B^*(Fr))(1-Fr)\}. \tag{49}$$

By plugging the above equation into Eq. (48) and solving for L_u, Eq. (36) is obtained. □

Proof of Proposition 1. (i) The first relation is derived from the facts that $L_r(H,B)$ is a decreasing function of its arguments (Lemma 1(i)), and $H^{**}>Fr$, $B^{**}>B^*(Fr)$ are satisfied for $Fr\leq Fr^\dagger$. As for the second relation

$$L_u(H^{**}, B^{**}) < L_u(Fr, B^*(Fr)), \tag{50}$$

$$\Leftrightarrow\ 1<1+\frac{[w_h-(1+r)e]-w_l(Fr, B^*(Fr))}{w_l(Fr, B^*(Fr))}Fr,\ \ \text{(from Lemma 8)}, \tag{51}$$

$$\Leftrightarrow\ [w_h-(1+r)e]-w_l(Fr, B^*(Fr))>0. \tag{52}$$

(ii) is directly obtained from (i). The first result of (iii) is straightforward from Lemma 1(i) and the result $\frac{\partial B^*(Fr_t)}{\partial Fr_t}>0$ of Lemma 4(i), and the second result is from Eq. (36) of Lemma 8, Lemma 1 (iii), and $\frac{\partial B^*(Fr_t)}{\partial Fr_t}>0$. □

Proof of Proposition 3. (i) Recall that $Fr^\dagger$ is the intersection of $B=B^*(Fr)$ and $b_u^*(Fr,B)=e$. From Eqs. (29) and (32), the exact form of $Fr^\dagger$ is

$$Fr^\dagger = \frac{[1-\gamma_k(1+r)]e\left\{[1-\gamma_k(1+r)]\left[1-\gamma_{ul}-\left(\frac{p_{rl}A_{rl}}{w_h-(1+r)e}\right)^{\frac{1}{1-\alpha_{rl}}}\right]-\gamma_k\gamma_{ul}(1+r)\right\}}{\gamma_k\gamma_{ul}[w_h-(1+r)e]+[1-\gamma_k(1+r)][1-\gamma_{ul}-\gamma_k(1+r)]e}. \tag{53}$$

(ii)(a) $H_t=Fr_t$ is not affected, since the educational investment of currently skilled workers has been made in the previous period. From Eqs. (16) and (13), $L_r(H_t,B_t)$ and $w_l(H_t,B_t)$ decline with the fall of A_{rl}, hence the results follow. (b) Since the productivity decline shifts $B_t=D(Fr_t)$, Eq. (23), outward, there is a region that used to be in the equal opportunity case but now belongs to the unequal opportunity case. For such region, $w_{l,t}$ obviously drops. H_t increases since $H_t=Fr_t$ rather than $H_t\leq Fr_t$ is satisfied now. Then, from Lemma 1, $L_{r,t}$ decreases. Regarding $L_{u,t}$, the substitution of $p_{ul,t}A_{ul}=w_{l,t}$ into Eq. (15) gives

$$w_{l,t}(L_{u,t}-\gamma_{ul}) = \gamma_{ul}\{[w_h-(1+r)e-w_{l,t}]H_t + (1+r)B_t\}. \tag{54}$$

Since H_t rises and $w_{l,t}$ falls, the right-hand side of the equation shifts upward, while the left-hand side shifts downward. Hence, $L_{u,t}$ increases. In contrast, if the economy remains in the equal opportunity case, the results follow from Lemma 3.

(iii)(a) Since the decrease of $w_l(H,B)$ lowers the right-hand side of Eq. (29), $B^*(Fr)$ and thus w_l decline. Then, the increase of L_u is straightforward from Eq. (36). (b) Straightforward from Lemmas 4(ii) and 3. □

Proof of Proposition 4. (i) Remember that w_h and $e=s_e w_h$ are proportional to A_{uh}. Then, the result is straightforward from Eq. (53).

(ii) (a) From Eq. (16), $L_r(H_t,B_t)$ declines with the increase of $w_h-(1+r)e$, hence from Eq. (13), $w_l(H_t,B_t)$ rises. (b) If the economy belongs to the unequal opportunity case after the productivity increase shifts $B_t=D(Fr_t)$, Eq. (23), outward, H_t increases since $H_t=Fr_t$ rather than $H_t \leq Fr_t$ is satisfied now. Then, from Eq. (16), $L_{r,t}$ declines, and from Eq. (13), $w_{l,t}$ rises. From Eq. (15), the effect on $L_{u,t}$ is ambiguous. On the other hand, if the economy remains in the equal opportunity state, the results follow from Lemma 3 and Eq. (13).

(iii) (a) Since the increases of $w_h-(1+r)e$ and $w_l(H,B)$ raise the right-hand side of Eq. (29), $B^*(Fr)$ and $w_l(Fr,B^*(Fr))$ increase. Then, from Eq. (36), L_u rises. (b) Straightforward from Lemma 4(ii), Lemma 8(i), and Eq. (13). □

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