1.1 Each row is:

$$u_i' u_i f_{11} + u_i' v_i f_{12} + u_i' f_{13} + v_i' u_i f_{21} + v_i' v_i f_{22} + v_i' f_{23} + u_i f_{31} + v_i f_{32} + f_{33}$$

if we set $f_{33} = 1$ and move it to the right side, then we have: $Af = -1_8$

- We have some linear combination:

$$\lambda_1 \begin{bmatrix} 1 \\ a_1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 \\ a_2 \\ 1 \end{bmatrix} + \dots + \lambda_8 \begin{bmatrix} 1 \\ a_8 \\ 1 \end{bmatrix} = 0$$

$$\hookrightarrow \begin{bmatrix} & A & \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_8 \end{bmatrix} = 0$$

However, we just showed $Af = -1_8$ where $f_{33} = 1$

Thus, we can define a new Q which satisfies $p_i^T Q p_i = 0$ where $q_{33} = 0$

- $p_i'^T Q p_i = 0$

$$(M' P_i)^T Q (M P_i) = 0$$

$$P_i^T M'^T Q M P_i = 0$$

Thus, the points P lie on the quadric $M'^T Q M$

- $C^T M'^T Q M C$

  $MC = 0 \rightarrow$ C lies on the quadric

$C'^T M'^T Q M C'$

$C'^T M'^T = 0 \rightarrow C'$ lies on the quadric

1.2

- $p_2^T F p_1 = 0$

$e'^T F p_1 = 0$

$(Hp_1)^T F p_1 = p_1^T H^T F p_1 = p_1^T H^T [e']_\times H p_1 = p_1^T [e]_\times p_1 = 0$

Thus, p_2, Hp_1, and e' are all aligned

- Since these 3 points are aligned, we can write p_2 as a linear combination:

$p_2 = a Hp_1 + b e'$

$p_2 = Hp_1 + \frac{b}{a} e'$ define $\rho = \frac{b}{a}$

$p_2 = Hp_1 + \rho e'$

- $p_2^T F p_1 = 0$ $F = [e']_\times H$

$p_2^T [e']_\times H p_1 = 0$

$p_2^T [e']_\times p_3 = 0$

Since $E = F$ and $E = [t]_\times R$, we see there's only a translation

- $t = e'$

1.3

- $H_0 = P_0' P_0^{-1}$

$= P_0' P^{-1} Q^{-1}$

$= Q P' P^{-1} Q^{-1}$

$= Q H P P^{-1} Q^{-1}$

$H_0 = Q H Q^{-1}$

$H_0 v = \lambda v$

$Q H Q^{-1} v = \lambda v$

$H Q^{-1} v = \lambda Q^{-1} v$

- $\pi_\infty = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$

$$T^{-T} \pi_\infty = \begin{bmatrix} R^{-T} & 0 \\ -t^T R^{-T} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Thus, π_∞ is an eigenvector

- If Q is the correction, we know $Q^{-T} \pi_\infty = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Now, $H^{-T} \pi_\infty = Q^T H_0^{-T} Q^{-T} \pi_\infty$

$= Q^T H_0^{-T} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = Q^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$H^{-T} \pi_\infty = \pi_\infty$, since $Q^T \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \pi_\infty$

1.4

- $p \times (p \times w) = (p^T w) p - (p^T p) w$

$[p]_x^2 w = (p p^T) w - (p^T p) w$

$[p]_x^2 w = [p p^T - (p^T p) I_3] w$

$[p]_x^2 = p p^T - (p^T p) I_3$

- $F = [e']_x S = [e']_x [e']_x F$

$F = \underbrace{e' e'^T F}_{0} - \|e'\|^2 F$

$= -\|e'\|^2 F \cong F$

- S has rank less or equal to 2 since $\text{rank}(F) = 2$

  S cannot be rank 1 since there is no other point p besides e in the nullspace of F.

- Since we know S is rank 2, we know that all points in image 1 map to a line in image 2. Thus, the plane induced by S must pass through C' or else S would map to a plane in image 2.

- It follows from the previous statement that the plane must intersect the image plane along a line.

  $e'^T S p = 0$, thus all points lie on the line defined by e'

- $Sp = [e']_x Fp$

  $= [e']_x l_{e'}$

  So we see that it's the intersection of $\langle e' \rangle$ and $l_{e'}$

Approach

Plane Segmentation

I roughly followed the method described in (*Detecting Planar Homographies* in an *Image Pair*. Etienne Vincent and Robert Laganibre).

1. It is kind of like a 2 loop RANSAC. The idea is that we randomly pick 4 correspondences and then fit a homography H to those points. Then, from the set of all correspondences, we select *compatible* pairs that agree with the homography. The candidate set is selected by computing dist(Hx1, x2) for all correspondences, where dist is Euclidean distance. We only proceed to the next step if we have more than some amount of compatible pairs(I chose 20-30) that is within a distance threshold (I chose 3).

2. The idea of the first step is that we will have computed an H which should be roughly close to the true homography associated with the scene plane. So in this second step, we start tightening the distance threshold (from 3 to 0.03), while continuously using the inliers to re-estimate H. After we have reached the final threshold (0.03), we consider to have found a plane. Now, we take the convex hull of the inliers which we have found, and then remove from our total set of matches, all the matches which are within that convex hull.

Reconstruction

1. Before computing the plane equation, we first estimate the fundamental matrix using all our correspondences. Before computing the projection matrices, we first need to compute K, the intrinsics. For this, I am using a routine I found online for vanishing point detection (Bo Li, Kun Peng, Xianghua Ying, Hongbin Zha. Simultaneous Vanishing Point Detection and Camera Calibration from Single Images). This approach directly makes the assumption that there are 3 orthogonal vanishing points to be found in the image and constrains their positions based on the geometry of the orthocentre formed by their triangle. With K, we recover M1 and M2, the projection matrices.

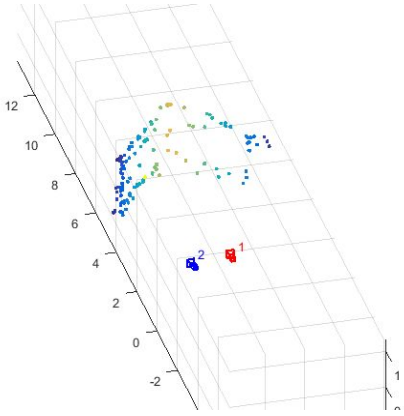
2. In order to compute the plane equation, we take the corresponding points that were classified to a particular plane. Using these correspondences, we triangulate the points and then fit a plane to their 3D position. We simply take a least squares solution.

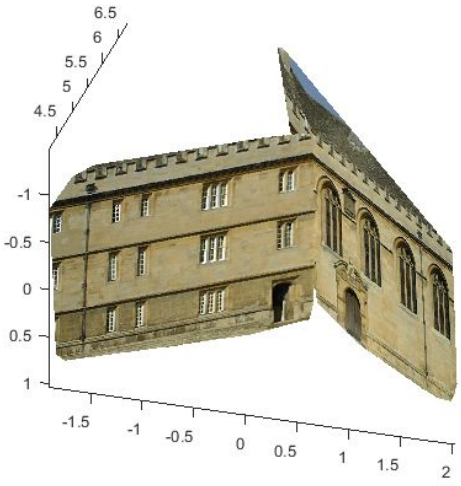
3. The last step is to backproject the planes we detected using the plane equations we found. Again, we use the convex hull of the correspondences which we associated with a particular plane. We take all the pixel points in the convex hull and then backproject them to the 3D plane. Finally, we assign the points with their respective colors in the image.

Results

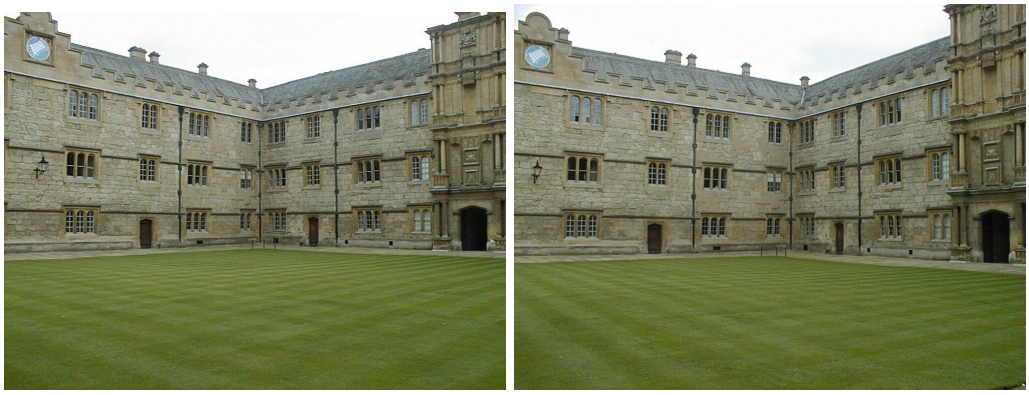
Input

Output

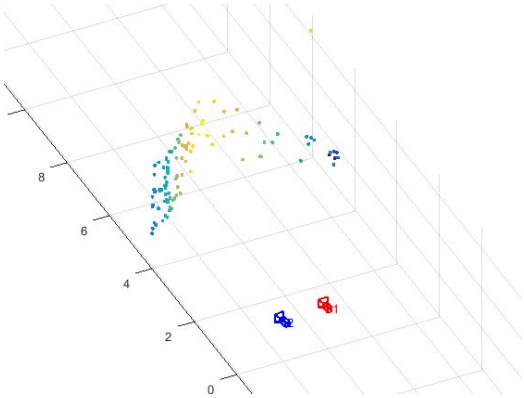


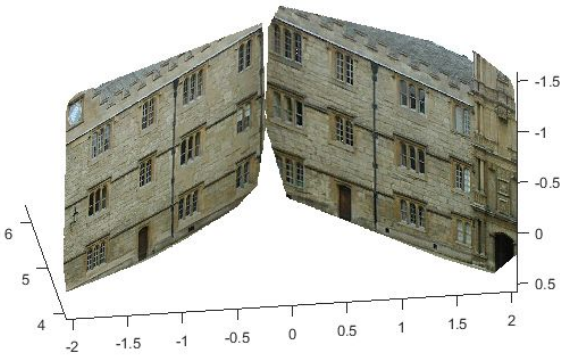


Input



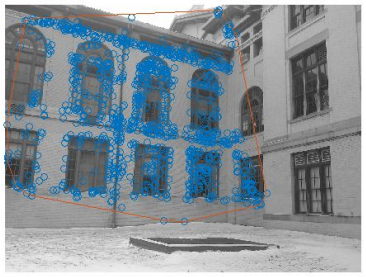
Output

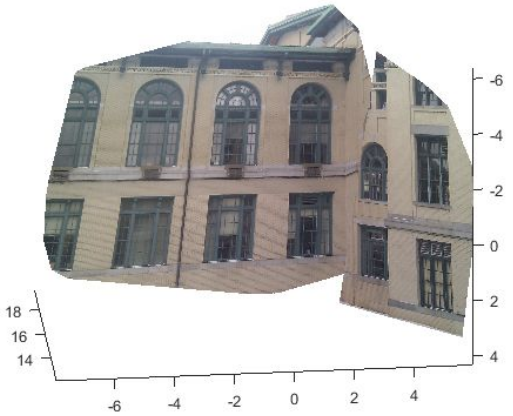


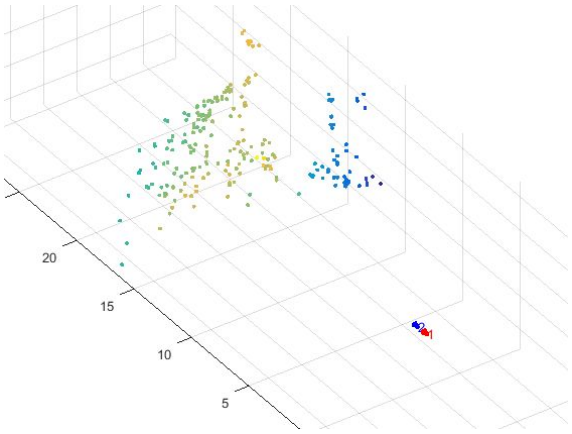


Input


Output

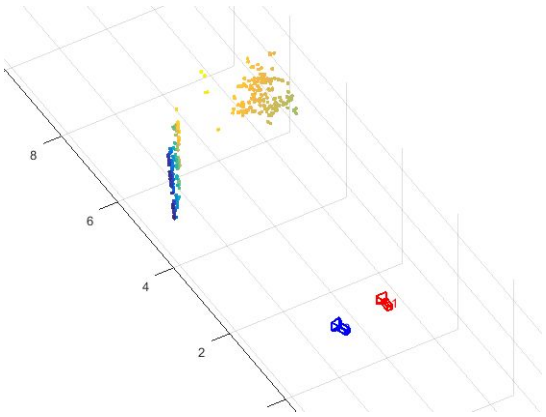


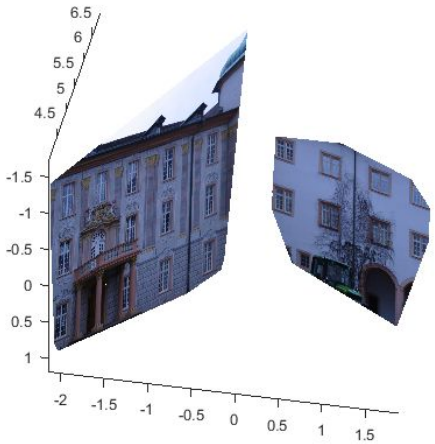





Input


Output



Input

Output