Writing a book with Barry Mazur

(on Prime Numbers and the Riemann Hypothesis)

William Stein

SageMath, Inc. and University of Washington

June 4, 2018 at Harvard University

Abstract

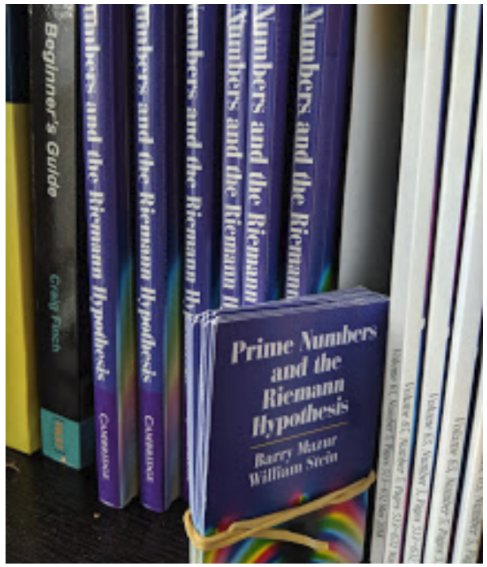
Abstract

In 2004, Barry Mazur and I started a decade project to write the book "Prime Numbers and the Riemann Hypothesis", which we recently published with Cambridge University Press. This talk is about what's in the book and why, some of the technical aspects of production of the book, and tradeoffs between self publishing online versus with a traditional publisher.

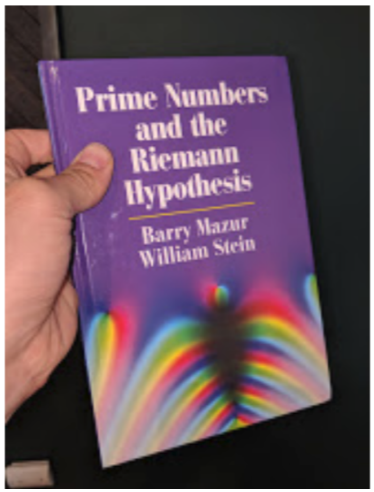
Overview

The book...


Sage
ON BULLSHIT
Harry G. Frankfurt


Beginner's Guide
Numbers and the Riemann Hypothesis
Prime Numbers and the Riemann Hypothesis
Barry Mazur
William Stein


15 Cesàro Smoothing


Prime Numbers and the Riemann Hypothesis
Barry Mazur
William Stein

§1. A Public Lecture

Clay Math Institute public lecture (MIT, May 3, 2005)

Are there still unsolved problems about the numbers 1, 2, 3, 4, ... ?

"Sell" number theory to the general public

- Immediately accessible
- Immediately interesting
- Primes and how eratic they are
- Cicada's every 13, 17 years...
- Lots of examples that are "open, interesting questions/issues."
- People can immediately make computations of their own
- Barry Mazur got his father, who had done NO math, hooked on the Goldbach Conjecture, so thought primes would work.

SageMath

I started SageMath

I launched Sage a few months before this 2005 CMI public lecture.

- Sage is a free Python based open source alternative to Mathematica, Maple, Magma, and Matlab.
- Early Sage development was motivated by this talk
  - Linking Sage to Mathematica to compute Γ.
  - Early plotting functionality.

More about what was in the public lecture...

The Public Lecture was about

- Primes as atoms (factorization)
- Record large primes (kept changing...)
- Enumerating primes (Sieve of Eratosthenes)
- Twin primes
- Counting primes
- Gauss's Conjecture – Prime Number Theorem
- Riemann: Fourier Adjustments to get smooth approx to $\pi(x)$
- Riemann Harmonics (zeros of $\zeta(s)$)

§2. Write a Book

Let's write a book...

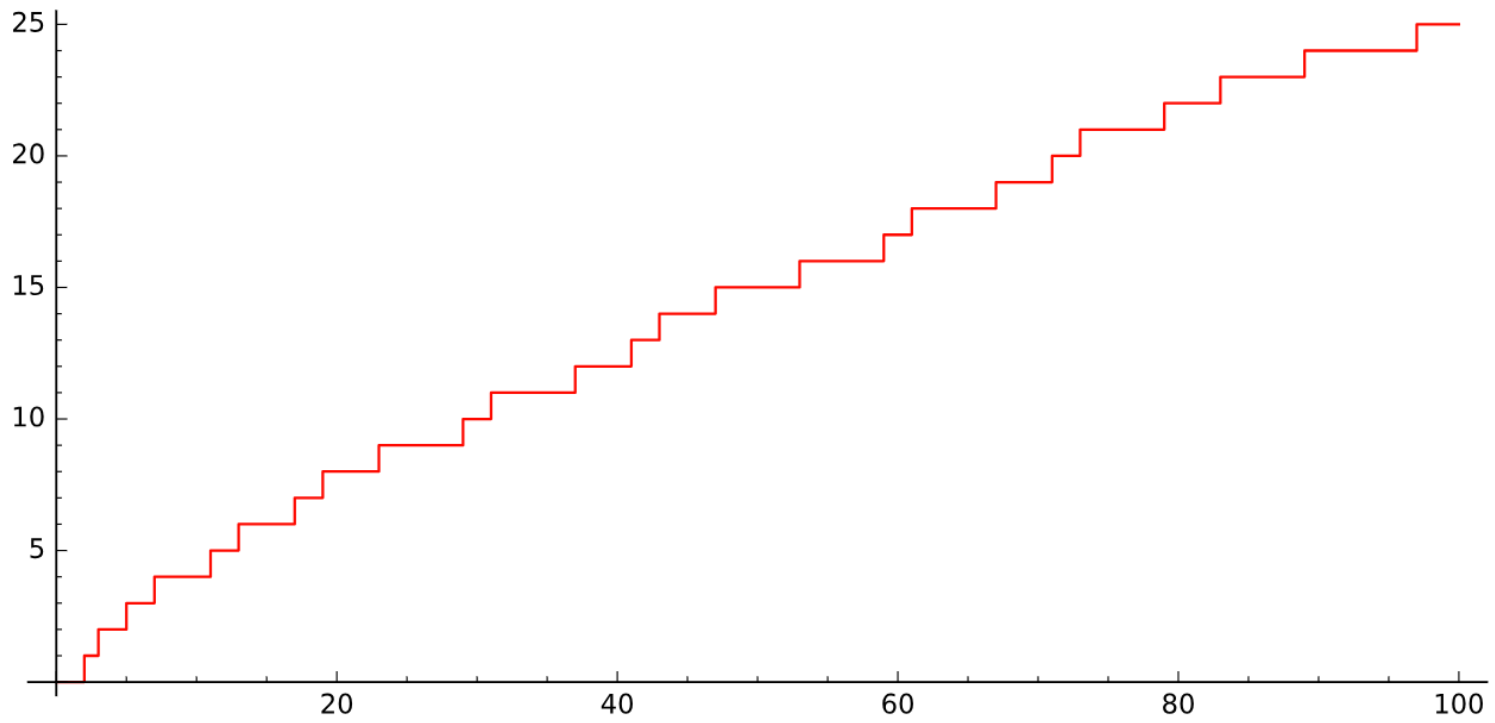
Turn this public lecture into a book?

- Write something for a popular audience.
- Small and readable.
- Full of *mathematics*, not stories of people.
- Profusely illustrated.
- Meet for a few weeks and focus on this.

The Prime Counting Problem

Let $\pi(x)$ be the number of primes $\leq x$.
Problem: give a "good approximation" for $\pi(x)$.

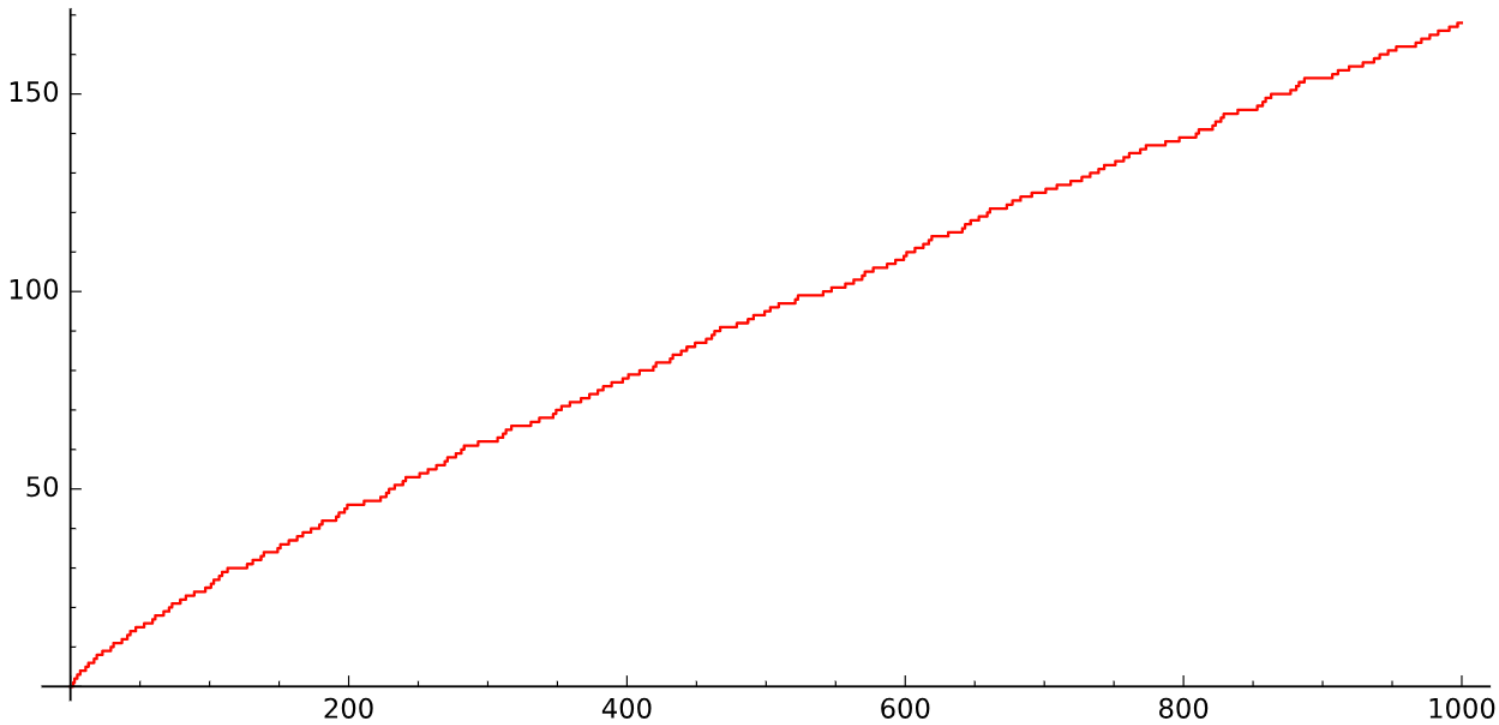
```
plot(prime_pi, 0, 100, color='red', figsize=[8,4])
```



The Prime Counting Problem

Let $\pi(x)$ be the number of primes $\leq x$.
Problem: give a "good approximation" for $\pi(x)$.

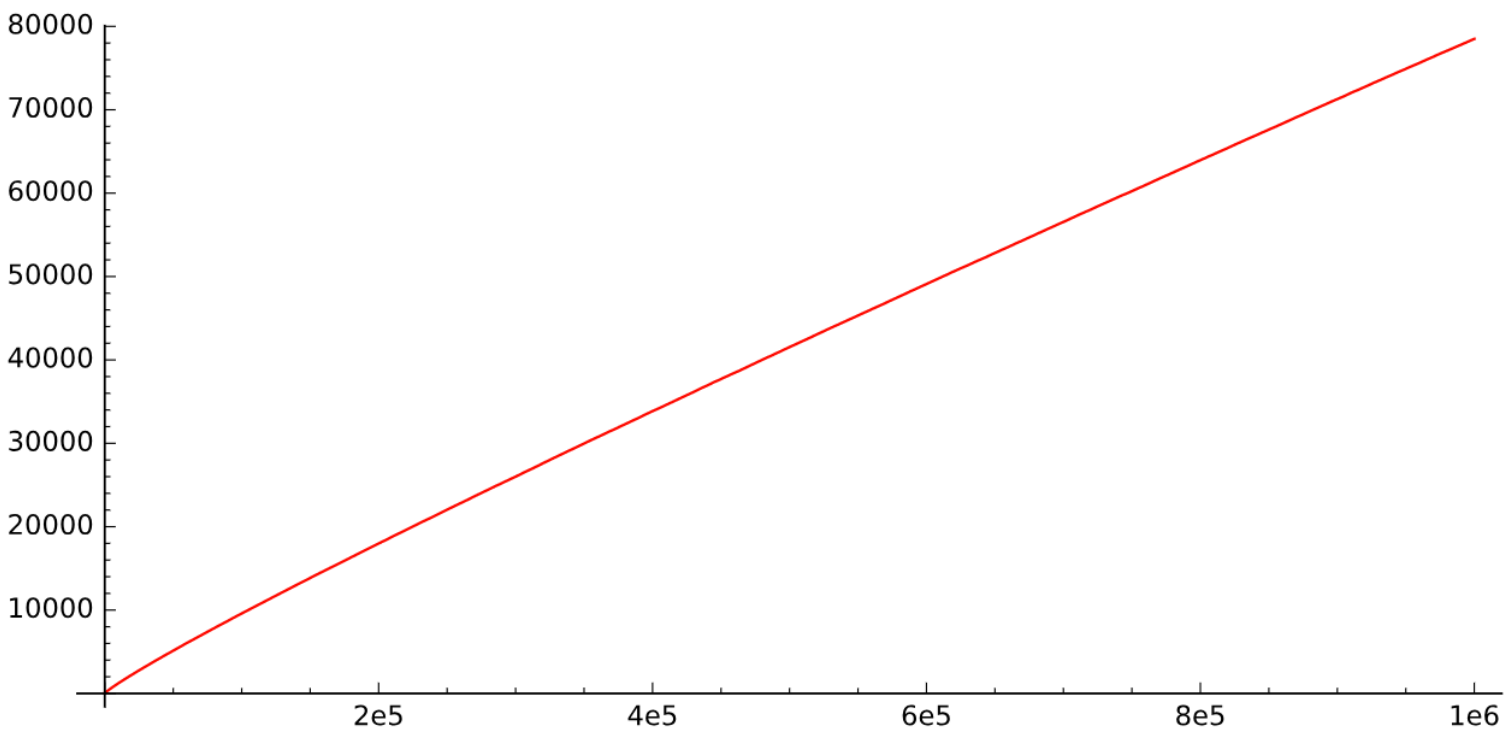
```
plot(prime_pi, 0, 1000, color='red', figsize=[8,4])
```



The Prime Counting Problem

Let $\pi(x)$ be the number of primes $\leq x$.
Problem: give a "good approximation" for $\pi(x)$.

```
plot(prime_pi, 0, 10^6, color='red', figsize=[8,4])
```



Answer: The Riemmann Hypothesis (first formulation)

The number of prime numbers less than X is approximately $\mathrm{Li}(X)$ and this approximation is essentially square root accurate.

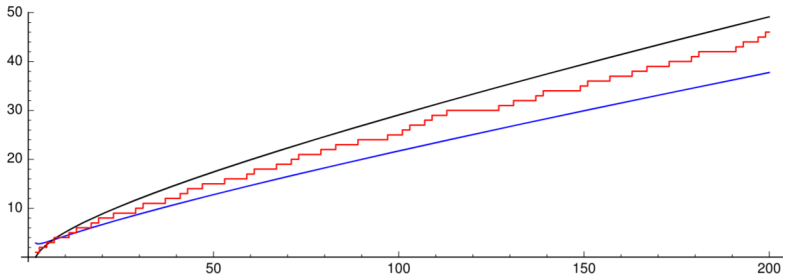


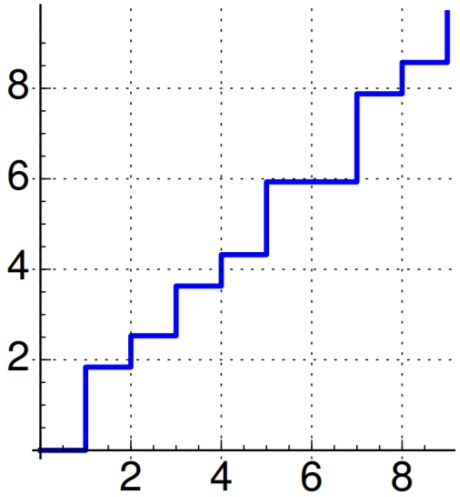
Figure 13.1: Plots of $\mathrm{Li}(X)$ (top), $\pi(X)$ (in the middle), and $X/\log(X)$

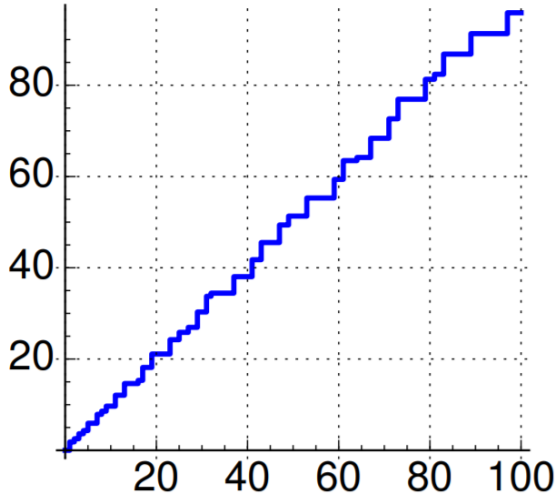
$$\begin{aligned}
\pi(X) &= 18{,}435{,}599{,}767{,}349{,}200{,}867{,}866 \\
\mathrm{Li}(X) &= 18{,}435{,}599{,}767{,}366{,}347{,}775{,}143.10580\ldots \\
X/(\log(X)-1) &= 18{,}429{,}088{,}896{,}563{,}917{,}716{,}962.93869\ldots \\
\mathrm{Li}(X)-\pi(X) &= 17{,}146{,}907{,}277.105803\ldots \\
\sqrt{X}\cdot\log(X) &= 55{,}262{,}042{,}231{,}857.096416\ldots
\end{aligned}$$

Answer: The Riemmann Hypothesis (second formulation)

The prime power staircase $\psi(X)$ is essentially square root close to the 45 degree straight line.

$\psi(x)$: "Our staircase starts on the ground at $x = 0$ and the height of the riser of the step at $x = 1$ will be $\log(2\pi)$. The height of the riser of the step at $x = p^n$ will not be 1 but rather: the step at $x = p^n$ will have the height of its riser equal to $\log p$."

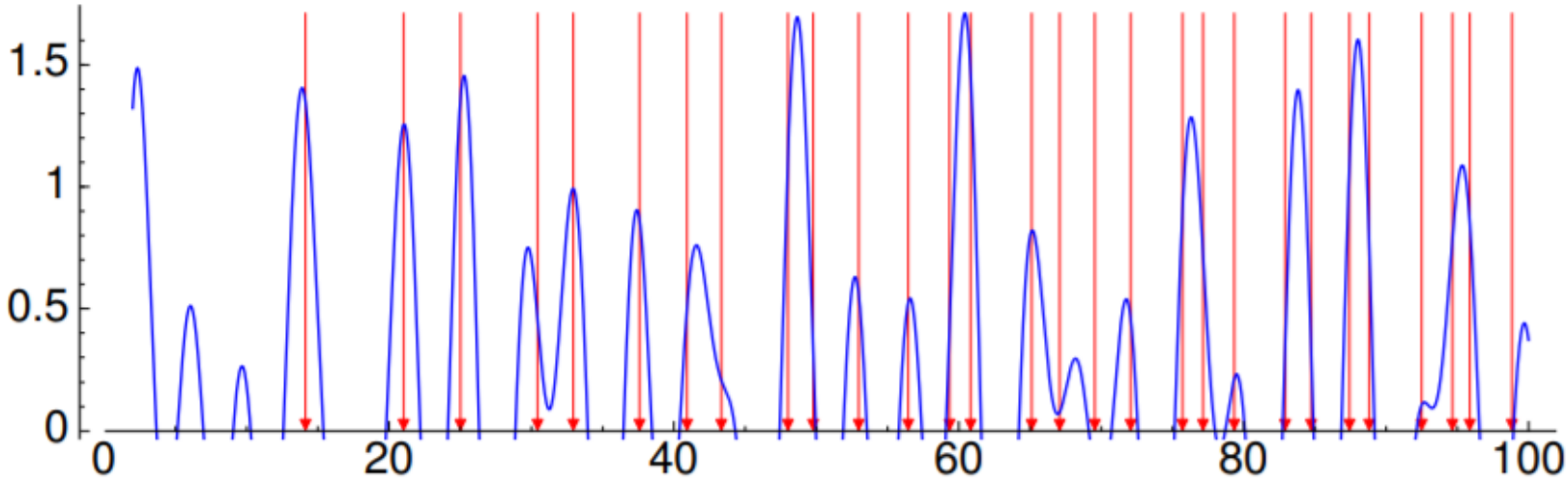




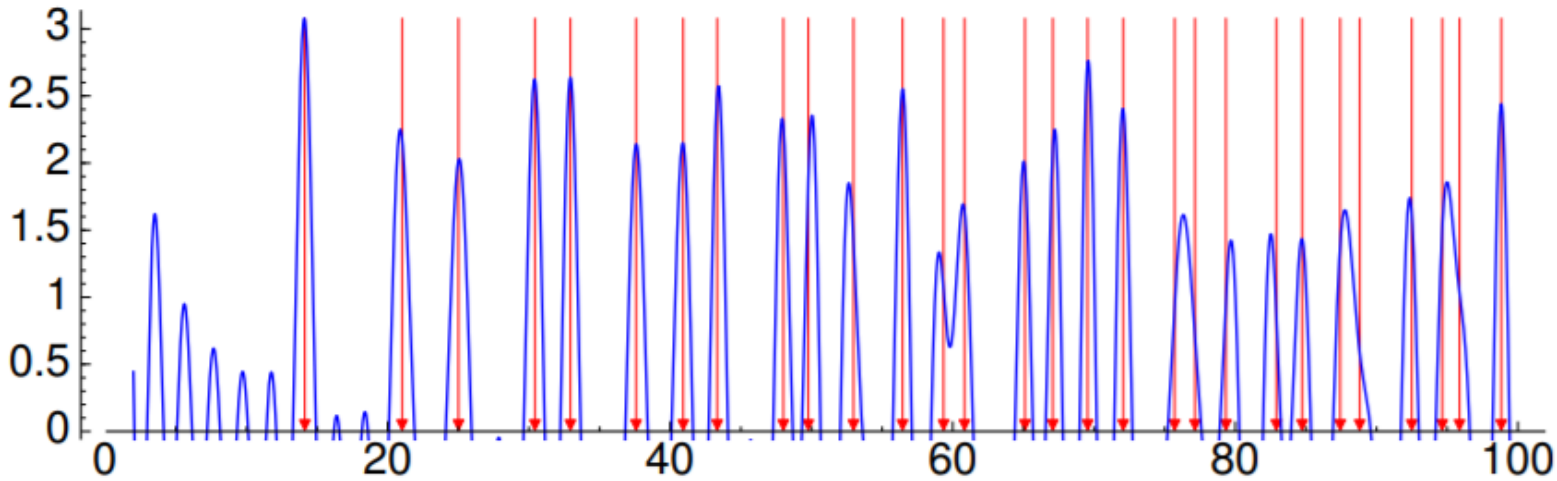
Answer: The Riemmann Hypothesis (third formulation)

Basically, *the explicit formula*: the Fourier transform of the derivative of $\psi(X)$ "is" a discrete distribution (supported at the real parts of the zeros of the Riemann Zeta function). Deleted from book, since it was too technical (but didn't renumber!). Instead, we illustrate the heck out of this, as follows...

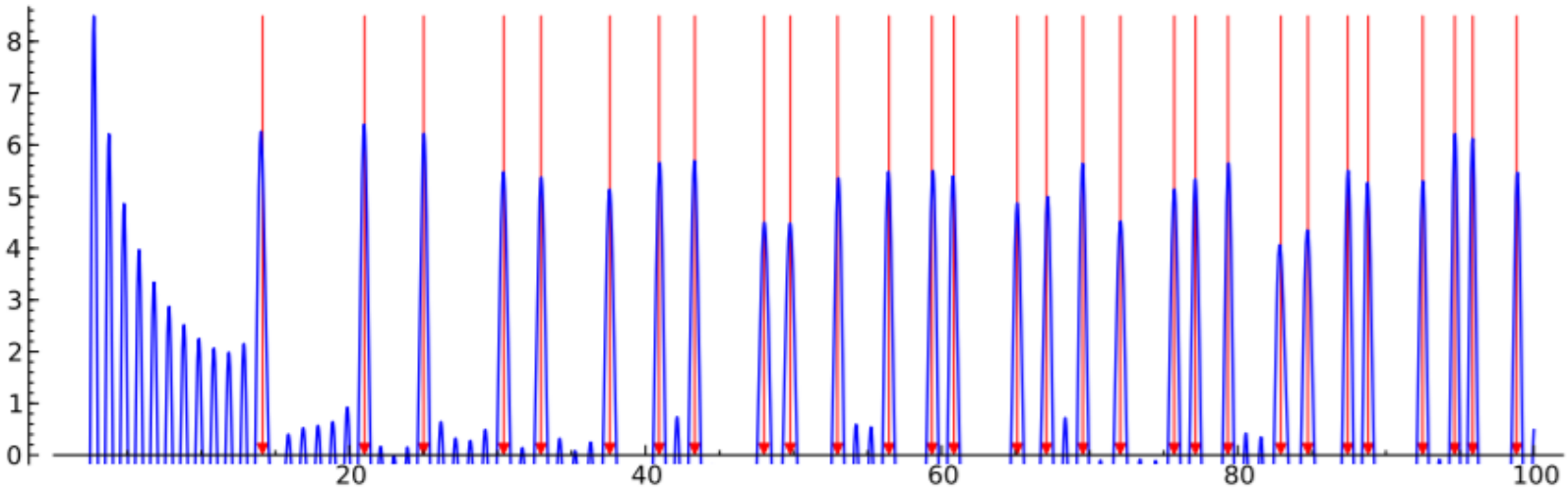
(add formula)



(add formula)



(add formula)



(add formula)

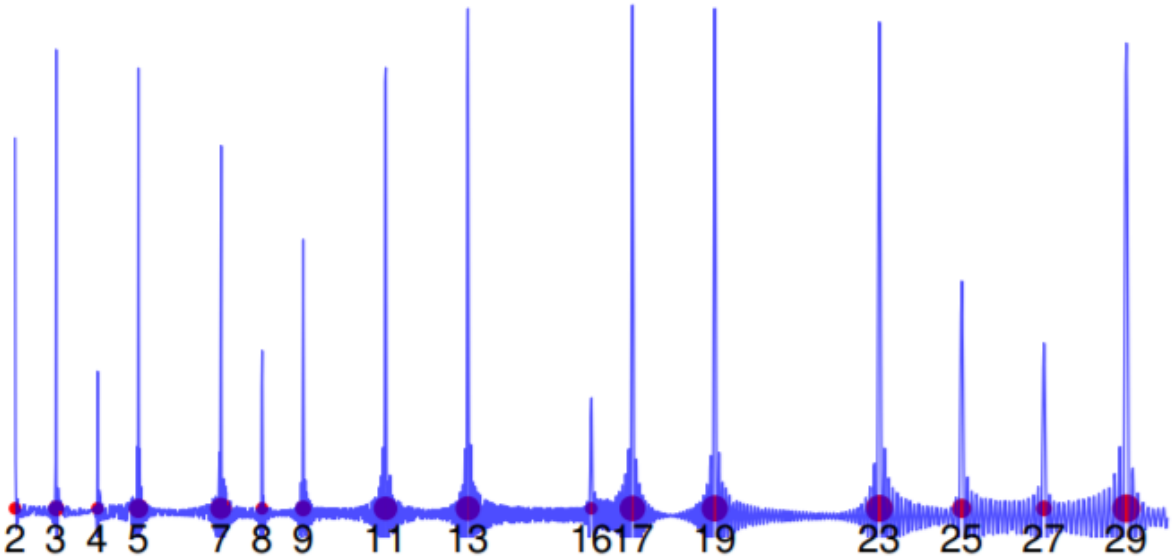


Figure 30.6: Illustration of $-\sum_{i=1}^{1000} \cos(\log(s)\theta_i)$, where $\theta_1 \sim 14.13, \ldots$ are the first 1000 contributions to the spectrum. The red dots are at the prime powers p^n, whose size is proportional to $\log(p)$.

(add formula)

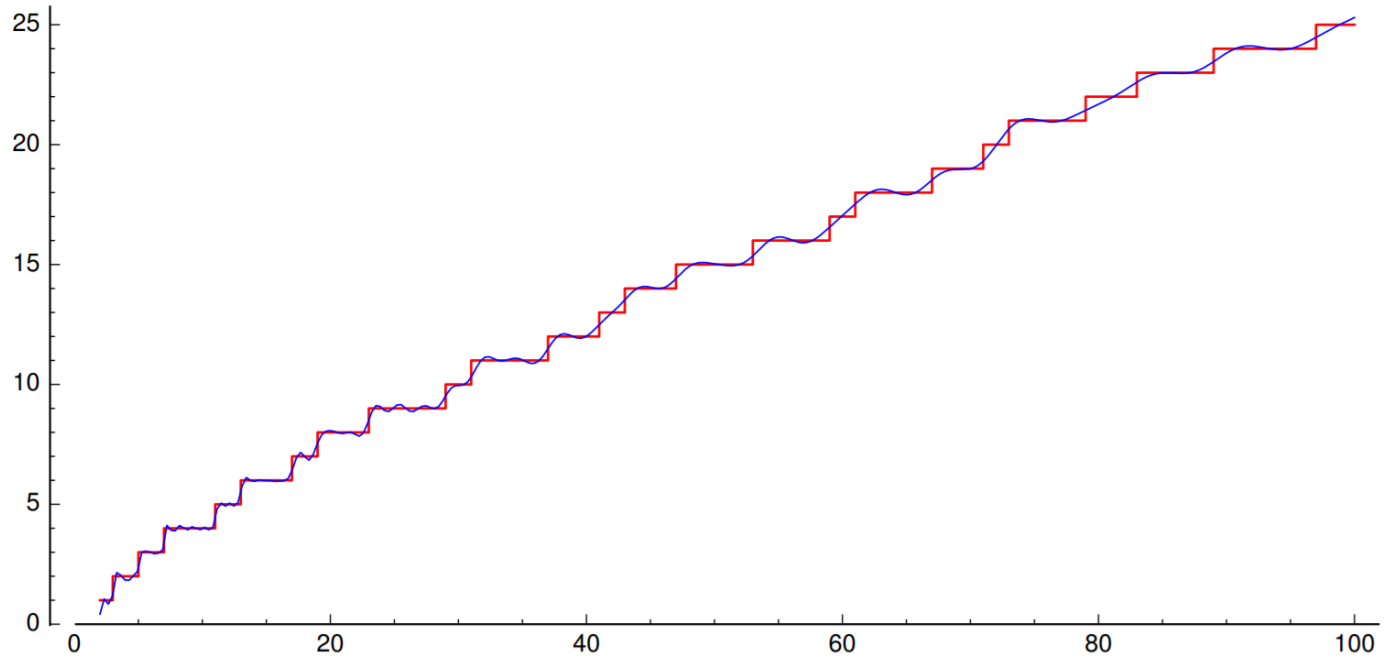


Figure 36.8: The function R_{25} approximating the staircase of primes up to 100

(add formula)

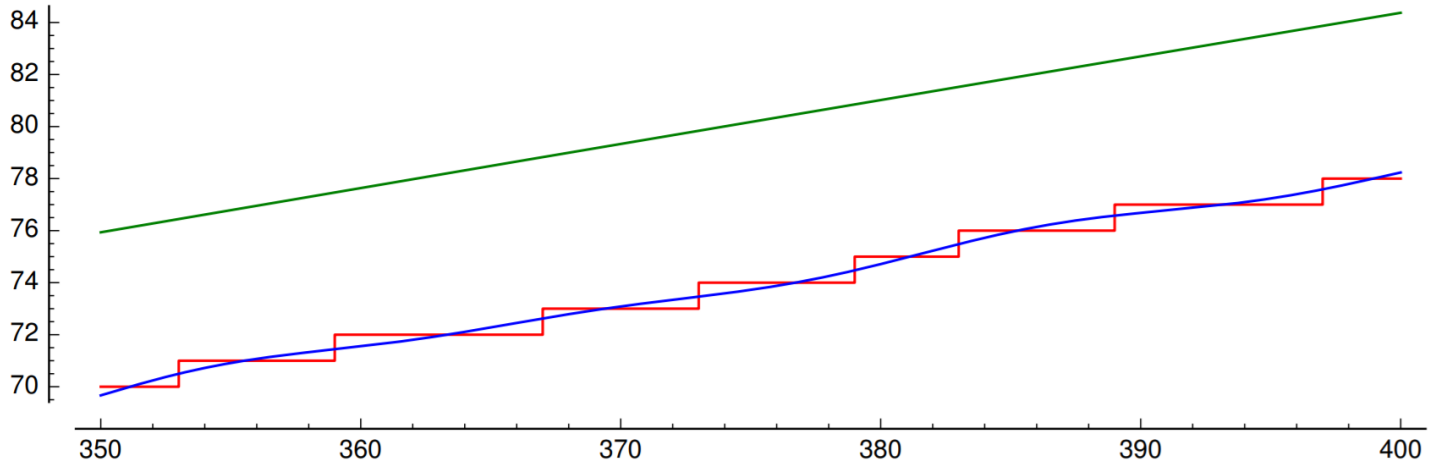


Figure 36.11: The function $\mathrm{Li}(X)$ (top, green), the function $R_{50}(X)$ (in blue), and the staircase of primes on the interval from 350 to 400.

Answer: The Riemmann Hypothesis (fourth formulation)

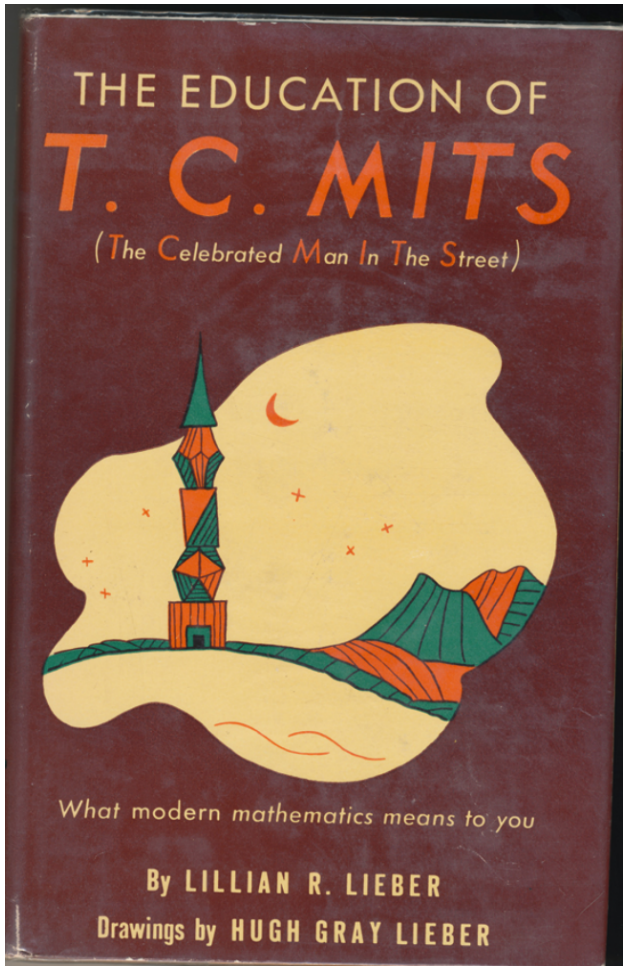
All the nontrivial zeroes of $\zeta(s)$ lie on the vertical line in the complex plane consisting of the complex numbers with real part equal to $1/2$.

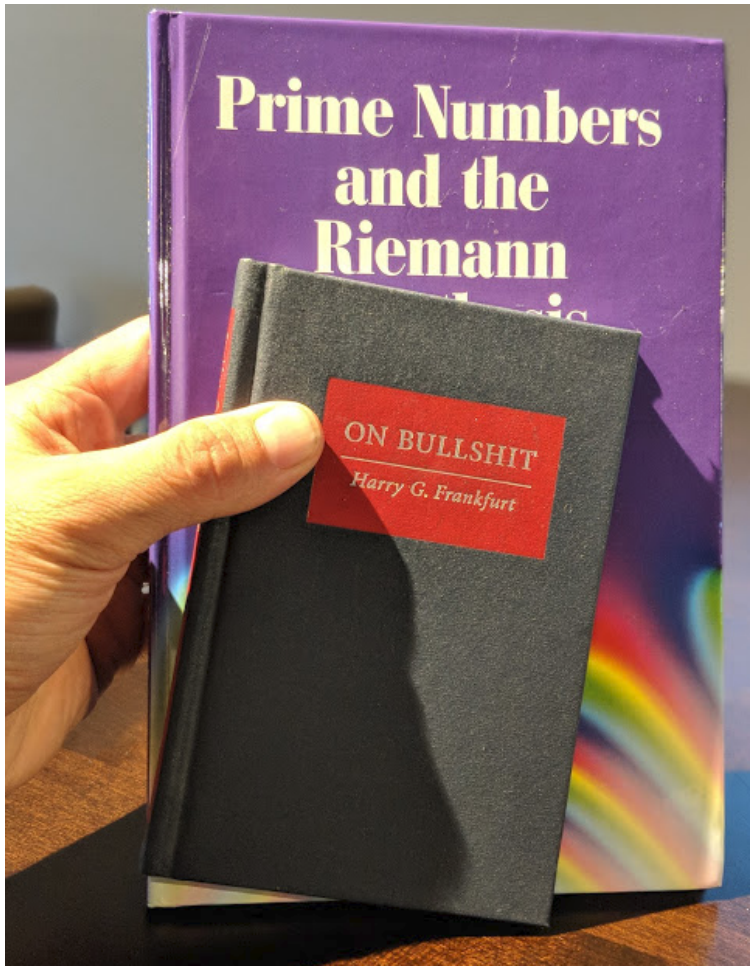
What kind of book?

- Book motivated by the prime counting problem (connect with other project – explicit formula projects...).
- Mostly math and no "stories of people", since many other books on RH do that already.
- Similar problem to guessing rank of an elliptic curve from a_p, which we had been thinking about.

What Sort of Book? Big? Small?

Like T. C. MITS or like ON BULLSHIT?





Our Approach

- Do not emphasize people/history/stories, since that's done already in many other books on RH.
- Go back 150+ years and explain what RH is more from the point of view of real classical Fourier analysis.
  - We embraced this mid-19th century real perspective.
  - We left complex numbers to the very, very end.

Target Audience?

Who are we writing this book for? Those who want to read about mathematics (not people and history).

- **High school students?** Tested at SIMUW 2007.
- **Retired electrical engineers?** Tested with original MIT talk, and online materials we shared.

SageMath again

Used Sage to compute with prime numbers, zeros, etc., and generally to plot everything in the book.

- We drew tons of plots to illustrate everything.
- The plots are absolutely essential to the exposition, and in fact really drove it!
- It is surprising that you see the spikes with such little computation.
- Central observations:
  - Fourier transform of discrete distribution at primes, gives discrete distribution of zeros of ζ.
  - The reverse gets the discrete distribution of prime powers.
- This what got us thinking about "how explicit is the explicit formula?" (another research project...)

Collaborative LaTeX via CoCalc

How we wrote the book

- Using CoCalc's LaTeX editor.
  - Web browser...
  - Gives a sense of the collaborative spirit
  - Plug: a brand new version of this was just released.
- We put a rough PDF of book on the web at every stage.
- GitHub: `https://github.com/williamstein/rh`

§3. Publish a Book

How to Publish the book: Self publish!?

- Self publishing: just put it on my website and see what happens.
  - Enormous number of people that proof read it.
  - Will Hearst pushed us for proper publication.

Finding a commercial publisher

- Finding a publisher
- What expected to give is LaTeX...

Typos and Mistakes

- Dozens of people carefully read drafts of the book and provided incredibly useful feedback. **THANK YOU!**
- The publisher also had a copy editor read the book, and provided complementary feedback.
- Don't expect your publisher to catch the sort of mistakes a mathematician catches:

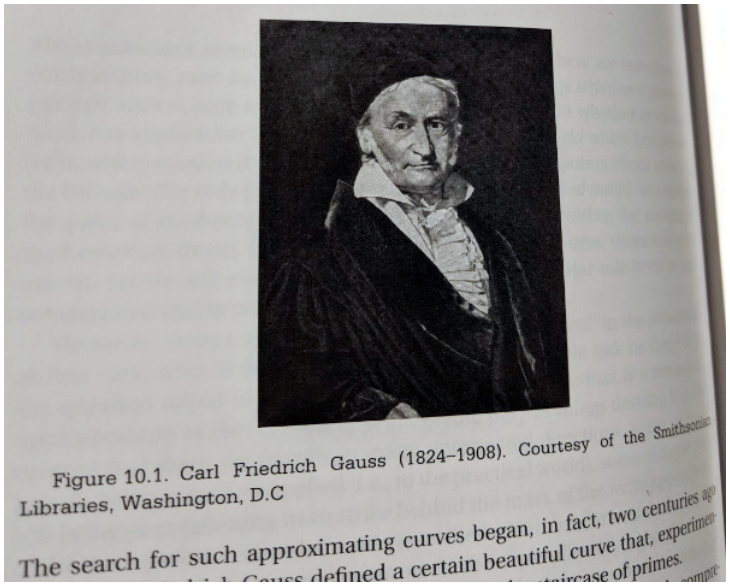


Figure 10.1. Carl Friedrich Gauss (1824–1908). Courtesy of the Smithsonian Libraries, Washington, D.C

The search for such approximating curves began, in fact, two centuries ago

Creating a Cover

- ...

Endorsements for the back cover

- ...

Production

- Cambridge Univ Press might have made some positive steps toward LaTeX (mention Ogus).
- Working with CUP has been a very positive experience overall.

Published!


amazon prime
Kindle Store
Deliver to William
Seattle 98122
Departments
Browsing History
William's Amazon.com
Today's Deals
Gift Cards
Registry
Sell
Buy a Kindle
Kindle eBooks
Kindle Unlimited
Prime Reading
Best Sellers & More
Kindle Book Deals
Free Reading Apps
Kindle Singles
Kindle Monthly Deals Up to 80% off top titles
Kindle Store › Kindle eBooks › Science & Math
Prime Numbers and the Riemann Hypothesis 1st Edition, Kindle Edition
by Barry Mazur (Author), William Stein (Author)
37 customer reviews
Prime Numbers and the Riemann Hypothesis
Barry Mazur
William Stein
eTextbook
$16.38
Hardcover
from $44.03
Paperback
from $14.68
Other Sellers
See all 6 versions
Buy
eTextbook features:
Highlight, take notes, and search in the book
Page numbers are just like the physical edition
Create digital flashcards instantly

Reviews by Readers

Top customer reviews

Oliver

★★★★★ **short, gorgeous, inspring and affordable.**

June 27, 2016

Format: Paperback | Verified Purchase

A couple of books on the Riemann hypothesis have appeared for the general public: Derbeshire 2003, Du Sautoiy 2003, Sabbagh 2003, Rockmore 2005, Watkins 2015, van der Veen and van der Craats 2015 and now Mazur-Stein 2016. More for mathematicians are Koblitz 1977, Edwards 2001, and Stopple 2003. From general expositions, one should also mention the paper of Conrey of 2003 which won the Conant prize for expository writing as well as a nice paper of Bombieri of 1992. Is this too much for the subject? No. A problem like the Riemann hypothesis can never be written too much about, especially if texts are written by experts. It is the open problems which drive mathematics. The Riemann hypothesis is the most urgent of all the open problems in math and like a good wine, the problem has become more valuable over time. What helped also is that since the time of Riemann, more and more connections with other fields of mathematics have emerged. The book of Veen-Craats and Mazur-Stein have emerged about at the same time. They are both small and well structured. Veen-Craats has been field tested with high school students and has focus mostly on the gorgeous Mangoldt explicit formula for the Chebychev prime distribution function, sometimes called the "music of the primes". Mazur-Stein do it similarly, however stress more on the Riemann spectrum and go didactically rather gently into the mathematics of Fourier theory as well as the theory of distributions. The book is carefully typeset, has color prints and some computer code for Sage. While Veen-Craats has many nice exercises, an exercise of Mazur-Stein led me to abandon other things for a couple of weeks, since it was so captivating. So be careful! A student who has taken basic calculus courses, should be able to read it. By the way, except Sabagh's book "Dr Riemann's zeros", which was written by a writer and journalist, the other books were created by professional mathematicians. The Mazur-Stein book has probably the best "street cred" among the RH books for the general audience: both have done important work in number theory, also related to zeta functions: Mazur's name is on one of the grand generalizations of the Riemann zeta functions, the Artin-Mazur zeta function which has exploded into a major tool under the lead of Ruelle who made it

discussion of the Reimann spec
Some good graphics and a good Reimann spectrum. I found Derb Obsession" overall more interest for the non-expert.
Published 1 year ago

Ricardo J. Menéndez

★★★★★ **Five Stars**
Excellent presentation ! Ricardo
Published 1 year ago

Saul

★★★★★ **Very Good for Ge**
Some Math Background
Very informative. Not for the prc about it already, of course.
Published 1 year ago

david bailey

★★★★☆ **Four Stars**
satisfactory
Published 1 year ago

Negative reviews due to *production issues*, both with the physical book and the Kindle edition....

Reception by Readers

- Feedback
- Sarnak's review in Bulletins
- Other reviews
- Prizes

Royalties

We sold some copies, so Cambridge University Press sent us some money. I bought a puppy named Bella!

Translation: what to expect?

Dear Professor Stein,

Prime Numbers and the Riemann Hypothesis

I am delighted to inform you that we are currently concluding an agreement with Nippon Hyoron Sha for a Japanese language edition of your book. They plan to print an edition of 2,500 copies initially, which will be sold at approximately 2,200 JPY per copy.

What to expect?

- Also Korean?
- Will they bother with French, etc.?

Future Plans

- Online fully interactive version.
- Related research on L-series of elliptic curves, etc.