Life only avails, not the having lived. Power ceases in the instant of repose;
it resides in the moment of transition from a past to a new state,
in the shooting of the gulf, in the darting to an aim.

— Ralph Waldo Emerson, "Self Reliance", *Essays, First Series* (1841)

O Marvelous! what new configuration will come next?
I am bewildered with multiplicity.

— William Carlos Williams, "At Dawn" (1914)

Announcement

No Tutorials next Friday

Lemma

L is a language.

For every distinguishable pair x and y and every DFA M that recognizes L

$$q(x) \neq q(y)$$

For every DFA M that recognizes L if x, y distinguishable w.r.t. L then $q(x) \neq q(y)$.

Consequently, a language L w/ ≥ 1 distinguishable pair canot be recognised by a 1-state DFA.

<u>Def</u> (Fooling Set)

A set of strings F is a fooling set for L iff every pair $x, y \in F$ is distinguishable.

$x \neq y$

<u>Fooling Set Lemma</u>

L is a language, F is a fooling set for L.

If F has size k (i.e. $|F| = k$)

then every DFA that recognizes L has at least k states.

Moreover, if there exists fooling set F_k w/ $|F_k| \geq k$ for every k, then L is not regular.

Fooling Set Lemma

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Moreover, if there exists fooling set F_k w/ $|F_k| \geq k$
for every k, then L is not regular.

Proof Suppose M is a DFA w/ $k-1$ states.
By PHP, $\exists\, x, y \in F$ s.t. $q(x) = q(y)$
$\Rightarrow \forall z$, M accepts both xz, yz or rejects both.

Examples

$L =$ language of strings ending in 10

$F = \{ \varepsilon, 1, 10 \}$.

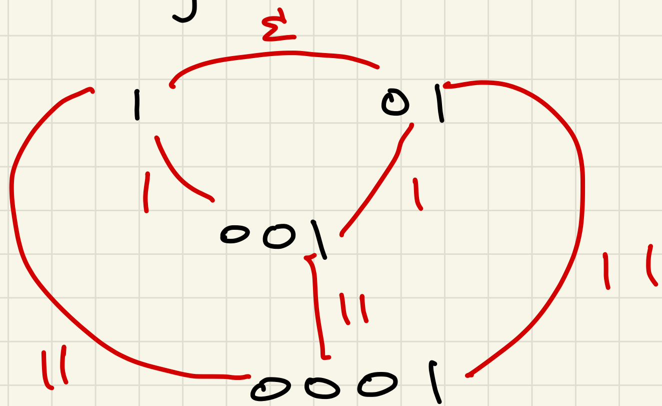
- $(\varepsilon, 1)$ distinguishing suffix 0
- $(\varepsilon, 10)$ ε
- $(1, 10)$ ε.

$\Rightarrow L$ needs ≥ 3 states.

$L = \{ 0^n 1^n \mid n \geq 0 \}$.

$\varepsilon, \; 01, \; 0011, \; 000111$

Fooling set of size k for every k?



$$F_k = \{ 1, 01, 001, 0001, \ldots, 0^{k-1} 1 \}.$$

$0 \leq i < j \leq k-1 \qquad 0^i 1 \qquad 0^j 1 \cdot 1^{j-1}$

$z = 1^{j-1}$

Strategies for fooling sets

1. Consider prefixes of strings in L
2. Counter of $\geq k$.
3. Find common distinguishing suffixes.

Nonregularity via Closure Properties

If A, B regular, then $A \cup B$ regular.

If $A \cap B$ not regular and A regular then B not regular.

If A regular, then A^c regular

If A^c not regular, then A not regular.

Examples

B language of strings w/ equal # of 0 and 1

ε, 01, 10, 0011, 1010, ...

$\underset{\text{regular}}{A} = 0^* 1^*$ $\quad A \cap B = \underset{\text{not regular}}{\{0^n 1^n \mid n \geq 0\}}$ $\Longrightarrow$ $\underset{\text{not regular}}{B}$

Example

B = strings w/ $\#0 \neq \#1$

B^c = strings w/ $\#0 = \#1$

B^c not regular $\Rightarrow$ B not regular

x, y dist. z $\Rightarrow$ $xz \in L$ $\quad yz \notin L$

$\Rightarrow q(xz) \neq q(yz)$

$\Rightarrow q(x) \neq q(y)$

w, x dist $\Rightarrow q(w) \neq q(x)$

w, y dist $\Rightarrow q(w) \neq q(y)$.

$$\begin{array}{ccc} & q(w) & \\ \neq & & \neq \\ q(y) & \neq & q(x) \end{array}$$