

# Part1: Simulation Exercise

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## Overview

This is a simulation practice which we would like to compare the simulation means and variances with the theoretical means and variances of a certain distribution. The distribution is an exponential distribution with  $\lambda = 0.2$ , 40 simulations will make up a sample and totally there are 1000 samples. This simulation will produce mean and variance plots and verify the assumption that the mean and the variance distribution is approximately normal.

## Set Up the Simulation

We use the code as follows to set up the simulation results:

```
setwd("C:/Users/Haonan/Desktop/Study/R/statistical inferring project")

library(ggplot2)

random <- data.frame()
for (i in 1:1000){
  a <- rexp(40,.2)
  meanr <- mean(a)
  stdr <- sd(a)
  random[i,1] <- meanr
  random[i,2] <- stdr
}
colnames(random) <- c("Mean", "Std_Dev")

head(random)
```

```
##      Mean Std_Dev
## 1 4.445450 4.079561
## 2 4.407662 3.950253
## 3 5.236816 6.142477
## 4 5.113871 5.427024
## 5 5.481151 6.559435
## 6 5.129172 4.418907
```

## Comparison

## Sample Means

The theoretical mean of exponential distributions is  $1/\lambda$ , which, in this case, was  $1/0.2 = 5$ . Firstly, it is reasonable to conduct a t test to see whether there's a significant difference between the sample mean and the theoretical mean

```
t.test(random$Mean, mu = 5)
```

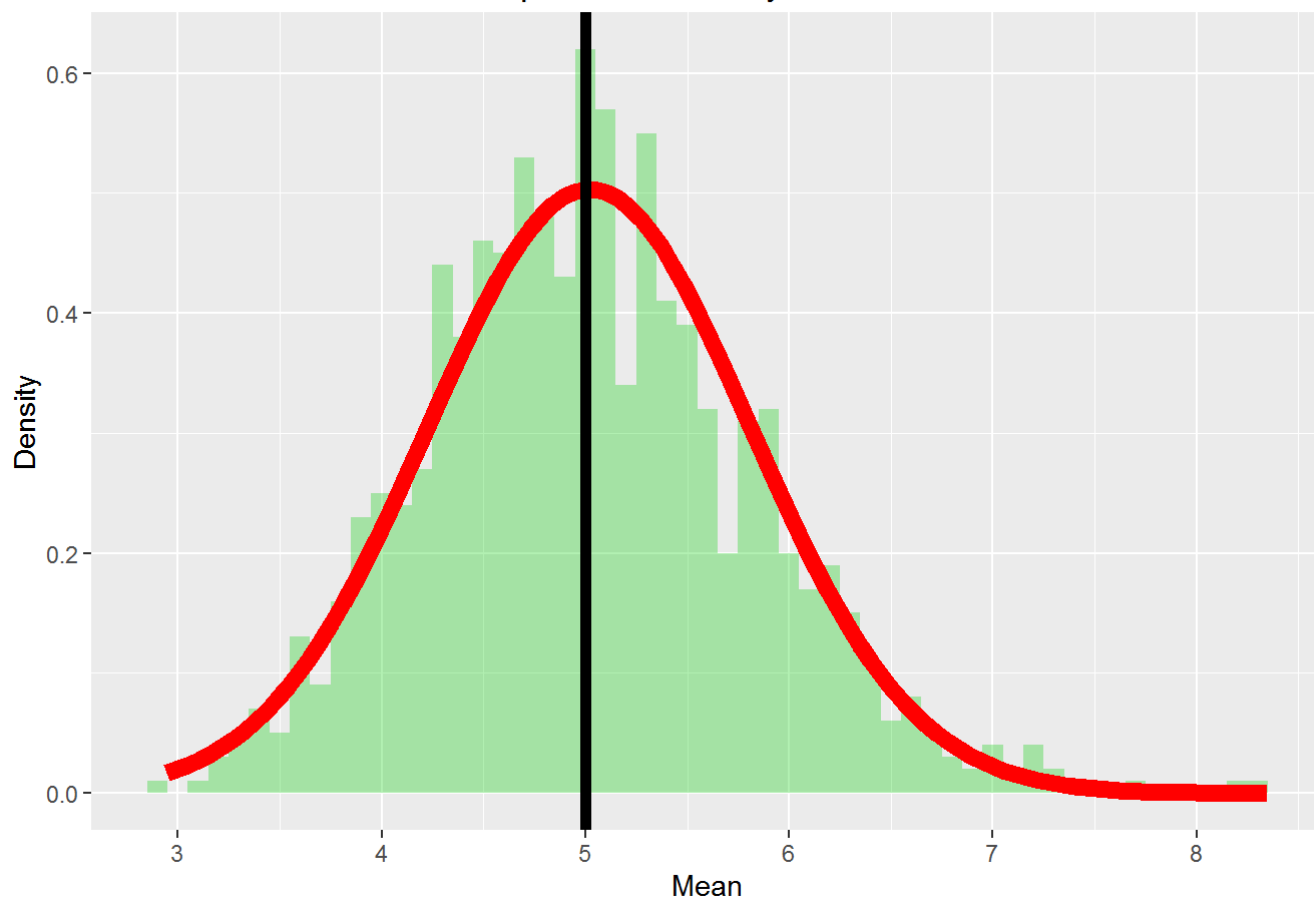
```
##
## One Sample t-test
##
## data: random$Mean
## t = 0.74452, df = 999, p-value = 0.4567
## alternative hypothesis: true mean is not equal to 5
## 95 percent confidence interval:
##  4.969461 5.067879
## sample estimates:
## mean of x
##  5.01867
```

From the result we could clearly identify that the p-value is very large. This means there's no significance between the sample mean and the theoretical mean, they're equal.

Then we graph it, add a vertical line which indicates the theoretical mean = 5, and finally add a normal line as follows:

```
gg1 <- ggplot(random, aes(Mean)) + geom_histogram(alpha = .3, fill = "red", binwidth = .1, aes(y =
  ..density..)) + stat_function(fun=dnorm, size = 3, color = "red", args=list(mean=mean(random$Mean),
  sd=sd(random$Mean))) + geom_vline(xintercept = 5, size = 2) + labs(x = "Mean", y = "Density", tit
  le = "Sample Mean Density Distribution")
gg1
```

### Sample Mean Density Distribution



From the plot, the sample mean is almost normally distributed with mean = 5. Compared with the red line, the sample mean is approximately normal

## Sample Variance

The theoretical variance of exponential distributions is  $1/\lambda^2$ , which, in this case, was  $1/0.2^2 = 25$ . Firstly, it is reasonable to conduct a t test to see whether there's a significant difference between the sample mean and the theoretical mean

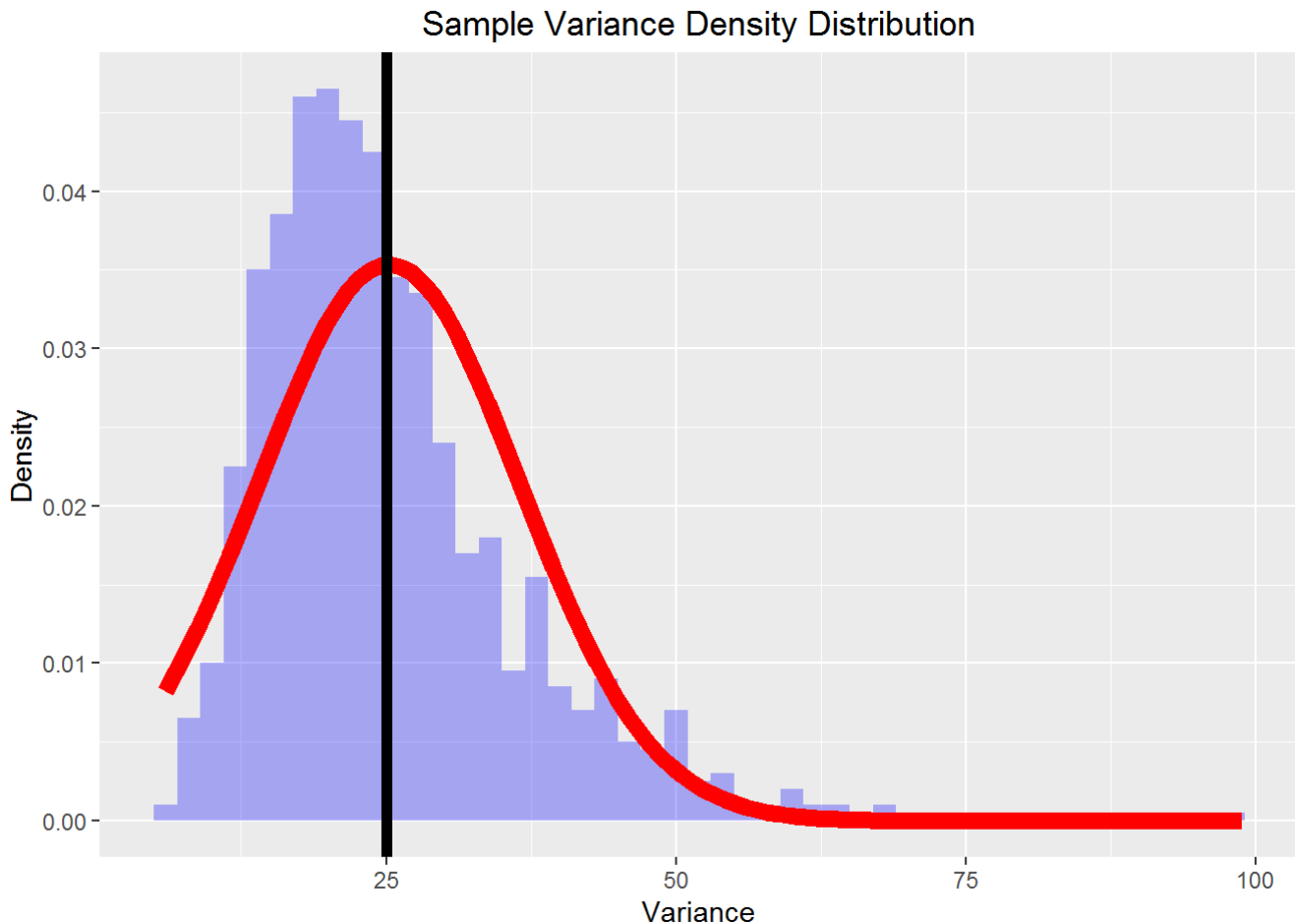
```
with(random, t.test(Std_Dev^2, mu = 1/.04))
```

```
##
## One Sample t-test
##
## data: Std_Dev^2
## t = 0.6666, df = 999, p-value = 0.5052
## alternative hypothesis: true mean is not equal to 25
## 95 percent confidence interval:
##  24.53738 25.93860
## sample estimates:
## mean of x
## 25.23799
```

From the result we could clearly identify that the p-value is quite large. This means there's no significance between the sample variance and the theoretical variance, they're equal

Then we graph it, add a vertical line which indicates the theoretical variance = 25, and finally add a normal line as follows:

```
gg2 <- ggplot(random, aes(Std_Dev^2)) + geom_histogram(alpha = .3, fill = 4, binwidth = 2, aes(y = ..density..)) + stat_function(fun=dnorm, size = 3, color = "red", args=list(mean=mean(random$Std_Dev^2), sd=sd(random$Std_Dev^2))) + geom_vline(xintercept = 25, size = 2) + labs(x = "Variance", y = "Density", title = "Sample Variance Density Distribution")
gg2
```



From the plot, the sample variance seems like normally distributed with mean = 25, but still, a little skewed to the right. Compared with the red line, the sample variance distribution is approximately normal

## Conclustsion

The simulation exercise has produced the desired results that the sample mean and the sample variance is nearly normal